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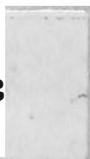
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BEING, NEGATION AND LOGIC

BEING, NEGATION AND LOGIC

By

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CONTENTS

	<i>page</i>
ACKNOWLEDGMENTS	ix
PART I. ORTHODOX LOGIC AND BEING ...	1
Ch. I. TYPES AND ABSTRACT BEING	1
1. The need for discussion of types	1
2. Linguistic theories of types	2
3. The nature of a linguistic system	8
4. The logical theory of types	10
5. The ontological theory of types	17
(a) Full being and abstract being	17
(b) Aristotle and the meaning of 'ghostly being'	18
(c) Aristotle's proof of ghostly being	20
6. The identity of the logical and ontological theories	22
7. Ideal being	23
(a) Confusion regarding the subject of existential predication	23
(b) The concept as a result of conceiving	24
(c) The concept as a natural sign	26
8. Two meanings of 'exists'	28
Ch. II. THE ONTOLOGICAL PRESUPPOSITIONS OF LOGIC ...	30
1. The relevance of logic to questions of existence ...	30
2. The older empiricist criticism	31
3. Vacuous occurrence	32
4. The 'rule' theory of logic	34
5. Formal and intuitive thinking	35
(a) The significance of formalisation in modern logic	35
(b) The invalidity of the proof that a contradiction implies every proposition ...	37
(c) Kleene's formalistic proof also invalid ...	38
(d) A defence of these proofs, and reply ...	39
(e) Intuitionism not intuitive thinking	41
(f) The paradoxes of implication	41

	<i>page</i>
(g) The Rule of Inference presupposes intuitive thinking	43
6. Lewis Carroll's paradox of inference	44
(a) Statement of the paradox	44
(b) Analysis of the paradox	46
(c) Solution in terms of a rule	48
(d) Solution by termination of the regress	52
(e) The regress of relations	54
7. The general argument for logical empiricism	55
(a) A statement of the logical empiricist position	55
(b) Failure of the argument that a logical principle is trivial	58
(c) The positive, intuitive aspect of the 'analytical'	60
(d) The negative aspect, and its destruction by the positive aspect	61
8. Logical conceptualism	62
9. The matter of logic	65
(a) Vacuous occurrence	65
(b) Hypotheticals	67
(c) Falsehood and negation	68
Ch. III. NON-EXISTENCE AND UNIVERSALS	71
1. Historical background	71
2. Existence of a subject-matter	72
3. The factual form of the paradox... ..	74
4. Negative facts	75
5. The argument for universals	77
6. The more general issues of universals and negation	79
 PART II. NEGATION AND UNORTHODOX LOGIC	 81
Ch. IV. THE PARADOX OF NEGATION	81
1. Conceptions and theories of negation	81
2. Propositions and possibilities	82
3. Opposition	84

page

4.	Difference	87
5.	The indispensability of non-existence	88
6.	The irreducibility of non-existence	89
7.	The meaning of non-existence	93
	(a) The kind of object to which negation applies	93
	(b) Inveterate assumption of a way out	95
	(c) Modern philosophy presupposes that there is only one root meaning of non-existence ...	96
	(d) Plato also appears to presuppose this... ..	96
	(e) This presupposition, hence the paradox, unavoidable	97
8.	The denial of negative fact	98
	(a) The first three methods of denial	98
	(b) Criticism of subjectivist methods of denial ...	99
	(c) The sceptic's objection	100
	(d) The Type objection	102
	(e) The double negation method of denial ...	103
	(f) The paradox itself as a denial	104
Ch. V.	THE PARADOX OF THE PAST	106
1.	The implication of ceasing to be... ..	106
2.	Solutions in terms of decriptions	107
3.	The conception of history	108
4.	A Whiteheadian solution	111
5.	Rationalisation by the idea of memory	112
6.	Rationalisation by the idea of mind	114
7.	The status of events in time	116
CONCLUSION (A)	118
(B)	120
INDEX OF MAIN SUBJECTS DISCUSSED	125

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I am grateful to the Editor of the *Philosophical Quarterly* for permission to re-publish (chapter III) the bulk of an article of mine entitled *Non-Existence and Universals*, which appeared in Vol. 6, No. 23 (April 1956) of this journal. The only substantial changes are that the final section (§7) has been removed, and a conclusion added to the previous section. I wish to thank the Editors of the *Philosophical Review* for permission to re-publish (parts of chapter I, §4) the substance of arguments which first appeared in Vol. 65, No. 4 (Oct. 1956), and Vol. 67, No. 1 (Jan. 1958), of this journal.

Finally my thanks are due to the University of Glasgow and to the Carnegie Trust for generously making a financial contribution needed for publication.

PART I

ORTHODOX LOGIC AND BEING

CHAPTER I

TYPES AND ABSTRACT BEING

I. THE NEED FOR DISCUSSION OF TYPES.

The relevance of the theory of logical types to questions about being has been brought to the fore in recent times by Prof. Ryle's treatment of an Aristotelian category as a logical type.¹ The categories, for Aristotle, are the widest classes of the things that are; but the sense in which a member of one category 'is' is different from the sense in which a member of another category 'is'. If, therefore, a category is to be identified with a logical type (and it will be shown later that it is), a decision as to whether there are or are not type distinctions will carry with it a decision as to whether there are or are not the corresponding ontological distinctions, i.e. distinctions between different meanings of 'being'. It will be my contention that there can be no such distinctions within an orthodox logic, i.e. if the standard laws of thought (Identity, Non-Contradiction, and Excluded Middle)² are upheld.

There are two reasons why I have chosen to discuss the question of types now rather than later. The first is that some of my important arguments have a structure which might be condemned as violating type distinctions, so that by demolishing type distinctions I shall anticipate these criticisms. The second concerns my use of expressions such as 'exists', 'is', 'is real', 'is a fact'. Some writers have thought fit to use these different expressions in an attempt to signify ontological distinctions, i.e. type distinctions. But since I deny such distinctions within an orthodox logic, it follows that for me there is no fundamental difference in meaning

¹ G. Ryle, 'Categories', *Proceedings of the Aristotelian Society*, Vol. 38 (1937-8), pp. 189-206. See also Manley Thompson, 'On Category Differences,' *Philosophical Review*, Vol 66 no. 4 (Oct. 1957).

² Subsequently 'NC' and 'EM' for the laws of Non-Contradiction and Excluded Middle respectively.

between them, and if my denial of these distinctions is justified it will further follow that I will not be in danger of serious equivocation in such usage. Still, one has to use these different expressions in order to be understood, e.g. I could not single out 'is' and discard the others, for certain forms of proposition are traditionally described as 'existential'. The word 'fact' perhaps needs special consideration, but it is not necessary to give this additional explanation until the occasion arises.

In Part I, I shall in general assume the standpoint of orthodox logic. The fact that this standpoint is upset by the argument of Part II means that criticisms, such as those I give in the present chapter against types, are ultimately to be viewed not as directly destroying the theories criticised, but rather as transplanting such theories from their foundations in orthodox logic to radically different foundations in unorthodox logic. In the case of the theory of types, the criticism will also contribute in the end to the discredit of orthodox logic, for the contradiction subsequently shown to affect the theory of types, is of such a radical nature as to suggest forcibly that there is no satisfactory alternative to types for avoiding the contradictions (reflexive paradoxes) which it is the purpose of the theory of types to overcome.

2. LINGUISTIC THEORIES OF TYPES.

The primary division between theories of types is between the linguistic and the realistic theories. On the realistic side there is the Aristotelian theory of categories which, with more recent theories essentially similar to it, I shall classify under the heading 'the ontological theory of types'; also there is the early form of Russell's theory of logical types, which I shall classify as 'the logical theory of types'. I shall later argue that the ontological and the logical theories are virtually equivalent, the former occurring at the level of general philosophical discussion, the latter at the level of formal logic. On the linguistic side we have primarily the formalistic approach to the problem of reflexive paradoxes, which grew up as an alternative to the logical theory of types when the latter met with apparently fatal criticisms. There was, for instance, the mathematical criticism concerning real numbers, but Max Black's criticism¹ is of greater importance philosophically.

¹ *The Philosophy of Bertrand Russell*, Schilpp (Evanston, U.S.A., 1946), p. 235.

This points out that if L belongs to the same type as K, and M does not belong to the same type as K, it is (according to the logical theory of types) meaningful to assert both of L and of M that they belong to the same type as K, which in turn implies (according to the logical theory) that L and M belong to the *same* type. As a result of such criticisms as this, the previously assumed realistic interpretation of the logical symbols was held to be mistaken, and the attempt was made (e.g. by Black) to maintain the theory of types, or something like it but weaker, only in the linguistic sense as concerning the right to substitute certain symbols in certain contexts. Thus owing to the apparent failure of realistic theory to solve the problem of the paradoxes, a linguistic outlook supervened, which gave the logician a free hand to introduce any set of postulates which would allow for proof of consistency without putting him under the necessity of feeling that the postulates must be true.

It will now be shown, however, that a realistic outlook in logic is necessitated by the facts that logic, whether linguistic or otherwise, treats a contradiction as a paradox, and that the treatment of a contradiction as a paradox implies its acceptance not merely as a verbal form but as having essential reference to reality.

From the commonsense standpoint this result is plain enough. That an object may be *said* or even believed both to have and not to have a certain property, everyone knows to be possible, alas! Thus there is no problem about the actual occurrence of contradictions in language. Contradictions can and do occur in language, and no one thinks it particularly strange that they do so occur. It may be a matter for regret, but it is not a matter for puzzlement. The problem arises only when the contradiction seems to be true, i.e. to *represent a real object* both as possessing and as not possessing a certain property. Thus in so far as a contradiction presents a problem at all, this would seem to demand a realistic interpretation of a common subject-term, without which there would be no basis for a conflict between affirmative and negative predications.

Except for the supposed need to treat logic linguistically, I do not think anyone would dispute this simple account of the problem set by a contradiction. But associated with linguistic theory there seem to be two ways in which this account has been combatted; a contradiction is said to be meaningless, or alterna-

tively it is said to be logically false. In either case it is denied that a contradiction is false in the simple sense implying a reference to reality. The reason why we find no fact corresponding to a contradiction is said to be not that some cosmic law of the real world has ordained it so, but that the verbal form of contradiction violates the rules of language, either the rules for the meaningful use of an expression (rules of formation) or the rules of formal proof (rules of transformation).

As to the first of these two alternatives, although an expression of the form $p \cdot \text{not-}p$ is never treated as a postulate, it is nevertheless always treated by logicians as a proposition (formula), which implies that it is accepted as meaningful. To be meaningless it would have to be neither a theorem nor a formula. If it were not even a formula the whole programme of proving consistency would be absurd, for such a programme presupposes that, in the system under consideration, we are not as yet sure that none of the theorems are contradictions. But if all contradictions were debarred by the rules of formation from being formulae, we would know from the outset that no contradiction could be a theorem.

Again, the method of *Reductio ad Absurdum*, as used in Mathematics, in Logic and elsewhere, is a special case of *modus tollens*:

$$\begin{array}{l} \text{If } p \text{ then } q \\ \quad \underline{\text{not-}q} \\ \quad \text{not-}p \end{array}$$

For we at once obtain the *Reductio* argument by putting a contradiction in place of q . This is assumed, of course, to make $\text{not-}q$ true, i.e. the contradiction given as a special case of q is assumed to be a special case of a *simply false* proposition, leading to the parallel rejection of the hypothesis as simply false.

It might perhaps be thought that a reflexive paradox exhibits a special sort of contradictoriness which *is* properly describable as 'meaningless'. The standard analysis of a reflexive paradox is that a contradiction would arise if a certain expression were counted as meaningful, e.g. from the assumed premise that ' $\phi(\phi)$ ' is meaningful it would be claimed that a contradiction of the form ' $P(P) \cdot \text{not-}P(P)$ ' follows. This, by an ordinary *Reductio* argument, would reject the hypothesis that ' $\phi(\phi)$ ' is meaningful. Allowing, then, that ' $\phi(\phi)$ ' is meaningless whatever ϕ may be, it

follows that 'P(P)' and 'not-P(P)' are meaningless, hence that the contradiction is meaningless.

This however has no tendency to prove that any sentences are meaningless in virtue of the contradictory form as such, or in virtue of any special sort of contradictoriness. It only means that, given any meaningless expressions, the result of combining them in any way (whether or not the contradictory form is so involved) will also be a meaningless expression. Moreover, the expression 'P(P). not-P(P)' functions as a simply false proposition in the *Reductio* argument, so there is no sign anywhere that the contradiction, *qua* contradiction, is associated with meaninglessness.

On the second alternative account of the problem of contradiction, it is contended that the falsehood of a contradiction is strictly logical falsehood, which like meaninglessness results from the breaking of certain rules of language, in this case the rules of transformation.

Now if we could eliminate contradiction merely by adhering to certain rules of transformation, consistency would be capable of final proof. But the formalist programme of obtaining such a proof and finally eliminating contradiction has in fact broken down. For Gödel's epoch-making theorem has shown that, for the more important, wider systems, completeness implies contradiction. In such a system at least one true proposition is to be found such that, if it is provable within that system, a contradiction is incurred.

The question therefore is not, When shall we succeed in finally eliminating contradiction? but What conditions must be imposed in order provisionally to avert contradiction? It is therefore apparent from developments within linguistic theory itself that the problem of contradiction lies beyond language, viz. in the nature of the real world to which language has reference. So in the end we come back to the conclusion, which is surely obvious in any case, that the problem of contradiction arises solely because it seems to be proved that a certain object both has yet lacks a certain property. In other words, the mere fact that a contradiction such as a reflexive paradox is regarded as setting a problem at all presupposes that a basic realistic interpretation must underlie whatever linguistic methods are pursued in the attempt to solve that problem.

In consequence the linguistic development can have relevance

to the problem of contradiction only if a system can be produced which is not only proved consistent but is capable of a realistic interpretation wide enough to include the fields in which the paradoxes occur. Now Gödel's theorem suggests (if I interpret it aright) that these two demands cannot *both* be met. In any case, logicians seem to be content with the mechanical development of linguistic systems with their proofs of consistency, without a thought for the demand for realism, and thus, by implication, without a thought for the original problem which alone makes it sensible to look for consistency. At the same time, the re-introduction of an explicitly realistic logic would be looked down upon as a retrograde step. Yet this would appear simpler and no less hopeless than traversing the long circle through linguistic systems and back to a realistic interpretation. Logicians abandoned realism because it seemed that the paradoxes were not solvable at this level; they overlooked the fact that at the level of language the paradoxes are not a problem at all, and that if the problem is not eventually solvable at the level of reality it is not solvable at all.

In the next section I shall cite another reason for taking the realistic theory of types seriously, viz. that a linguistic logic, when carefully examined is found not to be an *alternative* to realistic logic, but an *instance* of realistic logic. Meanwhile let us look at certain linguistic tendencies which might (mistakenly) be supposed to constitute independent linguistic theories of types. In the first place, distinctions between languages are commonly enough made in popular philosophical discussion, (i.e. in discussion outside the field of symbolic logic) e.g. the distinction between the languages of sense-data and material objects, the distinction between electron language and physical object language, etc. Principally such distinctions have arisen, I think, out of techniques such as that of logical constructions and that of reduction. These distinctions are used either to explain certain apparent existencies away as non-existent, or to maintain the view that philosophies are never about facts other than the facts admitted by empiricists, but only deal with the same set of facts in various ways. (This view either begs the issue by surreptitiously applying some kind of verification test for deciding what are facts, or else it is simply dogmatic). It is only by accident or by way of application that a type distinction might be involved

here, e.g. it might be involved in the distinction between language about nations and language about persons, since a nation is a class of persons. But the conception of a class is a realistic conception, and a theory of types which creeps in on this account is therefore realistic, not linguistic.

In a similar way Ryle's theory of types is, in my opinion, realistic, not linguistic. If his question is, e.g., How is this word used? his answer comes only after delving deeply into every-day experience, without restriction to linguistic behaviour, which appears only as a special case of behaviour as such. Again, where he develops his theory of types, he appeals undisguisedly to what we mean by a *class of objects*. Of a foreigner visiting Oxford or Cambridge, who after seeing the colleges asks to be shown the university, he says: 'His mistake lay in his innocent assumption that it was correct to speak of Christ Church, the Bodleian Library, the Ashmolean Museum *and* the University, to speak, that is, as if 'the University' stood for an extra member of the class of which these other units are members. He was mistakenly allocating the University to the same category as that to which the other institutions belong.'¹ Categories to which actual colleges or a university belong are realistic, not linguistic categories. Nor *could* Ryle's distinction between types depend finally upon language. For his method is empirical ('ordinary language'), not the formalistic one of setting up a list of syntactical rules *a priori*; consequently if his principle of type distinction were linguistic, it would be derived from an empirical study of actual linguistic usage; but a generalisation *from* actual linguistic usage obviously cannot be used as a principle for *correcting* actual linguistic usage,² a task which a theory of types certainly has to perform. His ultimate appeal must therefore be to something beyond linguistic rules, e.g. to the realistic conception of a class. So far as types are concerned it therefore seems plain that Ryle's theory is not a further linguistic theory which we have to take account of, since in essence it is not a linguistic theory at all.

¹ *The Concept of Mind* (London, 1949), ch. I, section (2), p. 16, cp. the example involving a *pair*, p. 22.

² This reflects upon those 'ordinary language' methods which, on the contrary, do claim to be linguistically self-contained. See Ernest Gellner's vigorous discussion of the Argument from Paradigm Cases, in *Words and Things* (London, 1959), ch. II, section 4.

Ryle's theory may, of course, contain aspects other than that of types, e.g. reductionism (to behaviour). If so, it is not relevant to my purpose to decide whether or not these other aspects are essentially linguistic. For this reason I have not contradicted anyone who claims, without further specification, that Ryle's theory is *obviously* linguistic in essence.

3. THE NATURE OF A LINGUISTIC SYSTEM.

Typically, a linguistic system begins with a set of, say, twelve *formal symbols* corresponding in the main to the logical words 'if', 'not', 'all', etc., and including symbols called variables. A *formal expression* is any series (ordered class) of formal symbols, and since repetitions are allowed both within and beyond a given expression, the signs are types (universals or classes) not tokens (particulars). *Formal objects*, comprising the formal symbols and the formal expressions, in a sense constitute the elements of the system. (This does not mean that the formal symbols are combined merely for the purpose of producing a large number of elements, for the subsequent rules for developing the system show that the internal structure of a formal object is of first importance). *Formulas* (corresponding to meaningful sentences) constitute a subclass of formal objects, and are picked out from these by the *rules of formation* (corresponding to rules of significance or meaningfulness). *Provable formulas* (corresponding to theorems) are a subclass of formulas, picked out from them by a further set of rules, viz. the *rules of transformation* (corresponding to rules of inference).¹

Now a system as so constructed is not peculiarly linguistic, since the formal symbols are not treated *as* symbols but as objects. A logically identical system could be constructed on the basis of any twelve numerically different objects (universals) whose instances are such that any set of them can be ordered in any required way. For instance, we could construct the system from twelve kinds of seeds like peas, beans, maize, etc., from which linear arrangements could easily be made.

It might be thought that the system is not realistic because abstract, and therefore consists only of a set of operations, not the

¹ I follow S. C. Kleene, *Introduction to Metamathematics* (Amsterdam, 1952), ch. IV.

objects operated upon. The mistake of this is that abstraction is not from existence but from particularity; it does not matter *what* (e.g. twelve) universals we have, but it does matter that we do have some particular set (e.g. of twelve). For when we work with an abstract system we do work with a particular set, the abstraction consisting not in a certain quality of the system, but in the way we treat it, viz. by disregarding the particular natures of the members of the set. In this respect abstraction is a special, complex case of vacuous occurrence (to be discussed in Chapter II).¹ Thus the expression 'an abstract system' is misleading. Abstraction is not a feature of any one actual system, but an attitude which we adopt towards an actual system. And *one* abstract system is any one of a certain class of actual systems treated abstractly.

Now a concrete system is one in which we cannot abstract from the particular natures of the basic elements, because the operations and theorems of the system result from those natures. Thus if the proposition $(a+b)(a+c)=a+bc$ is considered as a theorem in the concrete class calculus, it arises because a , b and c are classes, and addition a certain operation possible only with classes. In the corresponding abstract system the same 'proposition' is a theorem because it is a sequence of marks admitted by the rules of formation and transformation. But it would be a mistake to suppose that abstraction is complete in the latter case. For a sequence of marks is an ordered class of entities, so we have failed to remove existence, order and classes from the basis of the system. The result is that all our operations with the system are governed by a concrete logic of ordered classes. This is why a metalanguage is necessary. The metalanguage expresses our intuitive operations with sequences of marks, i.e. with certain ordered classes. Therefore, in so far as abstraction is the antithesis of intuition, no purely abstract logic is possible, and a so-called abstract system is a relatively complex concrete system transformed into a relatively simple system, which is still, however, concrete and so realistic. In particular, it is impossible to obtain a complete abstraction from the concrete calculus of classes, because the transformation obtained by abstraction is a calculus whose operations depend upon an intuitive handling of ordered

¹ Chapter II, 3, 9.

classes of symbol tokens. In the case of other concrete calculi, such as ordinary algebra, complete abstraction from the concrete *class* calculus is similarly impossible. In all such cases the notion of a class has still to be understood intuitively, and the consequent logic arises from our understanding of the nature of a class (and order, etc.), so cannot be expressed in *arbitrary* rules, even if rules can be formulated.

It follows that Russell's paradox concerning classes is not exhaustively discussed on the linguistic level. The whole linguistic discussion presupposes actual classes and a concrete logic of classes, and therefore implies that Russell's paradox must occur as a fact about actual classes unless there are some other facts about actual classes which exclude it. Therefore after all investigations of linguistic systems and their consistency or otherwise, we are still left with Russell's paradox (and perhaps others) on our hands, and the need to discover those facts (if there are any) which show how the paradox is averted. Realism is still needed after linguisticism has done its work. It is the realistic theory of types which is ultimately relevant to the problem of the paradoxes.

4. THE LOGICAL THEORY OF TYPES.

The two theories falling under the heading of the realistic theory of types are the logical and the ontological theories. The first of these will now be considered, for in spite of the fact that Black's criticism destroys it as it stands, there are weaker forms which are not immediately affected.

The theory claims that not all values of a variable x are such as to make a given propositional function ϕx a meaningful sentence, i.e. a proposition, i.e. true or false. In particular, if x has the value ϕ or anything involving ϕ , ϕx is not a proposition. The range of values of x making ϕx a proposition constitutes the *range of significance* or *scope* of ϕx (or of ϕ). The scope of ϕ constitutes a *type*, viz. the type lower than the type in which ϕ itself occurs (normally). All functions having the same scope as ϕ belong to the same type as ϕ . If it be supposed that there is a level of elementary objects or 'individuals', these will belong to the lowest type, type 0; functions of them to type 1, functions of these functions to type 2, and so on. If such a supposition be rejected, a relative distinction between types can still be maintained.

The original theory applied to every function ϕ , but weaker forms were subsequently held in which the same thing was asserted either of one specified function or of the functions of a specified class. Black's criticism does not immediately apply to these weaker forms of the theory. But I shall now contend that the principle underlying his criticism can be extended to them. In that case it will be shown that every function belongs to its own scope.

Suppose, then, that there were even one function ϕ not belonging to its own scope, i.e. such that the sentence ' $\phi(\phi)$ ' is meaningless. This means that ϕ is not any x making ' ϕx ' meaningful, i.e. ϕ is not any x making ϕx true, nor any x making ϕx false. The first alternative can be expressed by:

$$\phi \sim \epsilon \hat{x} \phi x$$

(ϕ is not a member of the class of x 's making ϕx true). From this it follows that ' $\phi \sim \epsilon \hat{x} \phi x$ ' is true, and hence that both this and ' $\phi \epsilon \hat{x} \phi x$ ' are meaningful. Now to admit that ' $\phi \epsilon \hat{x} \phi x$ ' is meaningful is to admit that we know under what conditions ϕ would be something satisfying ϕ . But the conditions under which ϕ would be something satisfying ϕ are obviously the conditions in which ϕ would satisfy ϕ , so that to know the former conditions would be to know the latter. Thus, since we know the conditions under which ϕ would be something satisfying ϕ , we know the conditions under which ϕ would satisfy ϕ , and ' $\phi(\phi)$ ', like ' $\phi \epsilon \hat{x} \phi x$ ', is meaningful. But this contradicts the hypothesis, which is therefore disproved. Thus there is no function ϕ for which ' $\phi(\phi)$ ' is meaningless, and every function is a member of its own scope.

The corresponding criticism for classes is very much simpler. Here the logical theory of types claims that, for any class k , ' $k \epsilon k$ ' is meaningless. Suppose, however, that for even one class k it were the case that ' $k \epsilon k$ ' is meaningless, k is then not any of the x 's for which ' $x \epsilon k$ ' is meaningful, i.e. neither any of those for which ' $x \epsilon k$ ' is true nor any of those for which ' $x \epsilon k$ ' is false. But the first alternative means that k is not any of the members of k . Thus ' $k \sim \epsilon k$ ' is true, therefore meaningful; therefore ' $k \epsilon k$ ' is likewise meaningful and the hypothesis is self-contradictory.

Prof. Ushenko has given what I consider to be the analogue of

these criticisms for the Liar Paradox.¹ Below is a simplified version of it.

No true sentence is written
within the rectangle of fig. 1.

Fig. 1.

The Liar Paradox arises because a sentence such as that in the rectangle of fig. 1 leads to a contradiction both if we assume it true and if we assume it false. The solution, according to the theories of types and language levels, is that the sentence is meaningless. Let us suppose, however, that it *is* meaningless. This means that the sentence expresses no proposition at all, and is neither a true sentence nor a false sentence. But since it is not a true sentence, and since it is the only sentence written within the rectangle of fig. 1, it follows that *no* true sentence is written within the rectangle. Thus the sentence 'No true sentence is written within the rectangle of fig. 1' is true, therefore meaningful; and as the sentence just stated is the sentence written within the rectangle, the sentence within the rectangle is meaningful, which contradicts the hypothesis.

For our purposes it will be necessary to give a formalised version of the whole argument, including the arguments of the Liar Paradox itself, in order to discuss adequately the standard criticism of Ushenko from the standpoint of Type theory (i.e. the theories of types and language levels).

Let 'f' mean 'written in the rectangle of figure 1'.

Let 'T' mean 'belongs to the class of true sentences.'

Let 'M' mean 'belongs to the class of meaningful sentences.'

Let 's' be a variable name for which the name of a sentence may be substituted.

Let 'a' designate the sentence '(s) (fs \supset \sim Ts)'

(s) (fs \supset \sim Ts)

Figure 1

¹ *The Problems of Logic* (London, 1941), pp. 78-80; *Mind*, LXIV (1955), p. 543; and my discussion in *The Philosophical Review* LXXV (1956), pp. 542-7.

We have

(1) fa (i.e. a is written in the rectangle of figure 1).

(2) $(s) (fs \supset . s = a)$ (i.e. only a is written in the rectangle of figure 1).

(A) Suppose Ta . Then

(3) $(s) (fs \supset \sim Ts)$ (assertion of the sentence supposed true).

(4) $fa \supset \sim Ta$ (from (3), for the value a of s).

(5) $\sim Ta$ ((1), (4), modus ponens).

This alternative therefore leads to a contradiction.

(B) Suppose $\sim Ta$. Then

(6) $(s) (s=a. \supset \sim Ts)$.

(7) $(s) (fs \supset \sim Ts)$. ((2), (6), syllogism).

(8) Ta (since (7) asserts a as true).

This alternative also therefore leads to a contradiction.

(C) Suppose $\sim Ma$. Then

(9) $\sim Ta$ (since T is a narrower classification of sentences than M).

(10) Ta (by the argument of (B)).

(11) Ma .

This alternative likewise therefore leads to a contradiction, and the Liar Paradox does not appear to be solved by claiming that the paradoxical sentence is meaningless.

The standard criticism of this argument from the standpoint of Type theory is that it fails to distinguish sentence tokens from sentence types. Type theory, it is said, holds only that the sentence token *within the rectangle* is meaningless, not that the sentence type a itself is meaningless, thus leaving open the possibility that a token such as the one occurring at step (7) may be true.¹

That this way out of the contradictions is not possible may be shown by concentrating upon two points. In the first place the property f (of being written within the rectangle) appears to belong only to one token of type a , and this would of course invite the Type theory criticism. But in fact f is interpretable as a unique property of the sentence *type* a . For the fact that the type a has a token which *is* in the rectangle is itself a property

¹ See e.g. Mr. Keith S. Donnellan's Note in *Philosophical Review* LXVI (1957), pp. 394-7.

which belongs to the type *a* and only to the type *a*. This property is commonly referred to as the property of being *written* or *expressed* in the rectangle, but this is not the same as the property of *being* in the rectangle; it signifies *having a token* which has this simpler property of *being* in the rectangle. This ensures that (1) and (2) are true of the type *a*, and that throughout the argument we are entitled to treat 'a' as indicating the type.

Secondly, however, there are two places in the argument, viz. at propositions (3) and (7), where a *token* of type *a* is used to assert a proposition, and the question arises whether the inferences to or from these propositions involve invalid transitions from type to token or vice versa. Thus (7) asserts the truth of a token while (8) infers the truth of the type. Such an inference is plausible only on the supposition, normally true I suppose, that if one token of a certain type is true then all tokens of that type are true. But since in the present circumstances the issue is precisely whether a token of type *a* outside the rectangle can be true though the one inside be meaningless, the assumption inevitably begs the question.

This raises the whole issue as to what exactly is meant by the truth of a sentence type,¹ since only a sentence token is immediately associated with an asserted proposition. So far we seem to have assumed that a sentence type is true or not according as all of its tokens are true, or all not true, which leaves out of account the possibility that some might be true and some not true. On this assumed definition it would not be paradoxical if a sentence type turned out to be neither true nor not true (i.e. neither true nor false nor meaningless), for the case of some tokens true and some not true would not fall decisively under either alternative.

To avoid this failure to exhaust the alternatives, the idea of the truth of a sentence type can be tightened up in either of two ways, (a) by taking the truth of one (relevant) token to be sufficient and necessary to ensure the truth of the type, or (b) by taking the non-truth of one (relevant) token to be sufficient and necessary to ensure the non-truth of the type. If a paradox occurs on either definition it will be valid in itself; it could not be avoided by

¹ In *Philosophical Review* LXVII (1958), pp. 101-5, I replied to Mr. Donnellan's criticism, but did not raise this second point concerning the definition of the truth of a sentence type. Consequently my argument there contains an invalid inference at the step corresponding to propositions (7)-(8). I am grateful to Prof. A. N. Prior for drawing my attention to this fallacy.

appealing to the other definition, since this would be to set up a different set of arguments related to a different paradox. I shall choose a definition of the kind (a), since it will be found that this does lead to paradox.

A token will be said to be true if and only if it states or expresses a fact, i.e. if and only if the person using it is asserting a true proposition, i.e. stating what is the case. But we could conceivably fix upon any fact, and conventionally determine the meaning of a particular token such as '(s) (fs \supset \sim Ts)' in such a way as to express that fact. In this way any token whatsoever could be made true. It is evident that we must restrict the field of *relevant* tokens in a certain way; they are those tokens alone whose meanings are determined by the meanings of the constituent signs and symbols, as those meanings are recognised within the given context. The only fact which can ensure the truth of the token is therefore the fact (if there is one) indicated by using a string of signs and symbols to assert a proposition on a particular occasion, on the understanding that the meanings accorded them in the act of assertion are their meanings as recognised in the context (either standard meanings or meanings assigned by special definition). So much is assumed, I think, by the Type theory criticism under discussion, for what it maintains surely is that, *given the meanings of the elementary signs and symbols within the context*, the normal tokens of a certain type are meaningful and even express a fact, whereas a certain special token of that same type cannot, *under these conditions*, be accorded any meaning at all. But of course it could at once arbitrarily be accorded meaning as soon as we remove the conditions regarding the meanings of the constituent signs and symbols.

A sentence *token*, then, is true within a certain context if and only if it can be used to state a fact, with the recognised meanings of its constituent signs and symbols, and a sentence *type* is true in the context if and only if at least one token of that type is true. Analogously a sentence type is meaningful if and only if, under like conditions, at least one token of that type is meaningful. If s is true, one of its tokens expresses a fact and so is meaningful, so s is meaningful. s is false if and only if none of its tokens is true though at least one is meaningful, and is meaningless if and only if none of its tokens is meaningful. NC and EM now ensure that

the alternatives *true*, *false* and *meaningless* for a given sentence (type) are mutually exclusive and exhaustive. Also, the alternative *not true* includes and is exhausted by the alternatives *false* and *meaningless*.

With the same definitions of symbols as before we now proceed to the arguments of the Paradox, with the revised meaning of the truth of a sentence.

$$\boxed{(s) (fs \supset \sim Ts)}$$

Figure 1

(1) fa

(2) $(s) (fs \supset . s = a)$

(A') Suppose Ta .

This means that there is at least one token of type a which expresses a fact. Let this be a_1 .

Any token of type a within this context contains symbol and sign tokens ('s', 'f', etc.) all of which have recognised meanings (either standard or specially defined) within this context.

The fact expressed by a_1 is therefore indicated by using a_1 to assert a proposition, on the proviso that the constituent tokens are concurrently accepted and used with their recognised meanings (according to type).

But since a_1 is of the type '(s) (fs \supset \sim Ts)', so to use a_1 to assert a proposition is to indicate, as the fact expressed by it, that no sentence (type) written within the rectangle of figure 1 is true. (This does not exclude the possibility that some other token a_2 of type a , *not* given as expressing a fact, may be meaningless).

Thus we can assert in the ordinary way as true

(3') $(s) (fs \supset \sim Ts)$,

and the argument follows as before, making this alternative self-contradictory.

(B') Suppose $\sim Ta$

Thence we have, as in (B).

(6') $(s) (s = a. \supset \sim Ts)$

(7') $(s) (fs \supset \sim Ts)$ ((2'), (6'), syllogism).

But (7') is the use of a token of type a to assert a proposition, which implies that this token expresses a fact and is true. This is therefore sufficient to prove the truth of the type a in the

required sense, and the supposition has led to a contradiction.

(C') Suppose $\sim Ma$.

This implies $\sim Ta$, thence Ta by the argument of (B'), thence Ma , and this alternative also leads to a contradiction.

The final conclusion therefore still holds, viz. that it is no escape from the Liar Paradox to suppose that the paradoxical sentence is meaningless.

5. THE ONTOLOGICAL THEORY OF TYPES.

(a) *Full being and abstract being.*

Many philosophers have maintained that there are certain beings, e.g. universals, which 'do not exist in the full sense'. Such beings I shall call 'abstract'. Sometimes the being of certain abstract entities has been regarded as mental, which does not mean, I think, that these abstract entities have been regarded as mental constructions, since the elements of a mental construction are not necessarily mental. I shall use the words 'ideal' and 'ideality' in reference to abstract beings of this sort. Examples are Aristotle's beings of reason and Bradley's ideal content of the judgment. Alternatively, the emphasis has been simply on the lower ontological status of abstract beings, without regard to whether this is or is not due to the mind. In reference to abstract beings of this sort I shall use the expression 'ghostly being'. Examples are Aristotle's accidents and Whitehead's eternal objects. I shall use the expression 'full being' in reference to beings which 'exist in the full sense'.

In the remainder of this section (5) I shall be concerned with full and ghostly being as it occurs in the Aristotelian categories, individual substances being full beings, and the members of the other categories ghostly beings. The present argument will indeed have a bearing upon ideal being at one point, but in the main this question will be reserved to a later section (7). I shall argue that, within an orthodox logic (which Aristotle obviously tried to uphold) the Aristotelian distinction of ghostly being from full being is untenable. Perhaps a distinction between types or categories is necessary in an unorthodox logic, but if so its significance will be entirely different from its significance in orthodox logic. But this question will not arise until, in Part II, it becomes fairly clear that an unorthodox logic is inescapable.

There are strong evidences of unorthodoxy in the very manner in which the distinction commonly arises, viz. as a contradiction between common sense or empirical philosophy which deny the existence of certain entities, and Platonic realism which affirms their existence. The situation is then *rationalised* by positing a ghostly or ideal being. But this in itself is nothing more than a flat denial, in the face of contradiction, that there *is* a contradiction. To make the compromise plausible not only would we have to furnish proof that there must be a ghostly being as distinct from full being, we would also have to give a clear meaning to 'ghostly being'. Aristotle attempted to meet both of these demands, but I shall argue that he failed in both.

(b) *Aristotle and the meaning of 'ghostly being'.*

Aristotle claims that all things 'are' only by relation to what 'is' in the fullest sense, viz. individual substances. Some things are said to 'be' because they are substances, others because they are modifications of substances, etc.¹ This is an attempt to give a meaning to 'ghostly being' by picturing a ghostly being as having no native being, but as deriving such being as it has only from that which has being in the full sense. It is not necessarily mistaken to use a picture, however crude, to imply or suggest support for a certain position. The question is whether the argument so implied is a valid one.

The picture in question works by representing being as becoming *less* as we proceed away from the source, full being, and *more* as we proceed towards it, into closer relationship with it. Thus a body becomes less hot as it is moved away from the fire, and the branches of a tree become less thick as we proceed away from the trunk. But whether we conceive the diminution intensively or extensively, it is certain that throughout the whole series of diminishing quantities we have *only one quality*, such as heat or thickness, which varies. Each object in the series represents something which is said to be (the degree of its being depending upon its place in the series). Thus the varying quality belongs to each object representing something which is said to be, and

¹ Warner Wick, *Metaphysics and the New Logic* (Chicago, Ill., U.S.A., 1942), pp. 98-9. Aristotle, *Metaphysics* 2 (p. 116), 7 (p. 16) (Page references here and subsequently to the Everyman edition, London, 1956). I am grateful to Miss Bernardine Cate for drawing my attention to important passages in these two books.

accordingly being itself is represented (by this varying quality) as behaving like a quality of that which is said to be. But since a given quality can apply only to members of one category, such a picture can give no indication of the kind of relation that holds between beings of *different* categories.

It is worth adding that it is in any case fallacious within orthodox logic to treat being as a property of that which is said to be. It is not however enough to say, as critics of the Ontological Argument for the existence of God commonly do, merely that existence cannot be a property. Even Kant made existence¹ (as occurring in a proposition) a property, viz. a relational property, for he said that to assert existence is to assert that an object stands in relation to the idea, e.g. the existence of God would mean that something corresponds to the idea of God. This makes existence a relational property of the idea of God. Thus the critics of the Ontological Argument themselves make existence a property (viz. a certain relational property)—a property however not of the very object said to exist but of the *idea* of such an object. Accordingly it is no logical criticism of the Ontological Argument to point out that it assumes existence to be a property, unless it can be shown further that it assumes existence to be a property *of that which is said to exist*. But the Argument clearly does not make this further assumption, e.g. Descartes assumed the existence of God to be a (relational) property not of God himself but of his 'immutable essence'. On the other hand, in the case of the Aristotelian picture of ghostly being it *is* presupposed that being is a property of the very things said to be, and therefore the logical criticism which was wrongly applied to the case of the Ontological Argument is here fully justified.² The criticism is that, on any account of being or existence which makes existence a property of that which is said to exist, every affirmation of existence becomes trivial and every denial of existence self-contradictory. The Aristotelian picture therefore cannot give a clear meaning even of 'being', let alone 'ghostly being'.

¹ The relevant sense of 'existence' is sense (2)—see 8. Since the basic sense is sense (1), Kant's statement is not subjectivistic. For Kant see *Critique of Pure Reason*, Dialectic, Bk. II, Ch. III, Section V (Everyman, London 1934), p. 350.

² I now suspect (Dec. 1961) that this criticism of Aristotle is after all invalid, since his meaning of 'being' would correspond to my sense (1) of 'existence'. It is too late to make the appropriate changes in the text.

But Aristotle gives an analogy. He says that things are said to 'be' by relation to that which 'is' in the fullest sense, just as e.g. a kind of exercise is said to be 'healthy' only by relation to what is properly healthy (persons).¹ More precisely, the analogy is that a statement such as 'This exercise is healthy' is to be interpreted in same such way as 'All people who exercise in this way tend to become healthy'. What this means in relation to being is that 'Whiteness is' must be interpreted as meaning 'There are white things'. But this is compatible with nominalism, and is indeed a natural way of expressing the nominalist view. We can of course imagine, consistently with the proposed reduction, that whiteness is immanent in white things, but if so we make an addition which the reduction as stated does not require. Consequently the reduction in itself brings us no nearer to any new sense of to 'be', while the addition that whiteness is immanent is merely a repetition of the claim *that* there is a new meaning distinct from the meaning of to 'be' in the full sense.

Aristotelians and others often suppose that a new meaning of to 'be' is given by the idea of an *accident*, as that which *depends for its existence* upon a substance. Now if there are accidents and substances in this sense, the implication is only that during any time in which the accident exists the substance must also exist, not that the meaning of the accident's 'existence' during that time is different from the meaning of the substance's 'existence'. On the contrary, any such difference in meaning would constitute an ambiguity modifying the stated dependence, and accordingly we must presume that the meanings are the same unless there is an explicit statement that they are not the same. Once again, therefore, we are left without any indication as to what the alleged new meaning of to 'be' is. Whether, in the cases commonly supposed, the existence of one thing does depend upon the existence of another, is a matter to be discussed later, but even if there are cases of such dependence this does not imply that any distinction of *ontological* status is involved.

(c) *Aristotle's proof of ghostly being.*

Once it is made apparent that Aristotle failed to show what any new sense of to 'be' could be, his argument *that* there must

¹ Ibid 2.

be different senses of to 'be' inevitably loses much of its force. However, let us examine the argument. What he aims at proving is that being is not a genus, therefore not *one* genus but several. For a genus such as *animals* there are certain respects or qualities which differentiate one species from another. These respects are extraneous to the genus, e.g. the quality of rationality which distinguishes men from other animals is not itself an animal. But rationality is a being, and any respect which served to differentiate one species of being from another would also have to be a being, since anything that *is* at all is a being. Therefore, because the differentiae of being are themselves beings, whereas the differentiae of a genus are not themselves members of the genus, being is not a genus.¹

Now a genus is usually assumed to be a genus of individual substances, and at this level there is an obvious reason why the differentiae should be extraneous to the genus. For while in itself an individual substance (such as a particular man or horse) is something more than what can be immediately present to sense perception, we recognise and classify such an individual only by means of qualities which can be immediately present to sense perception. This suggests that the practical reason why we have to resort to differentiae extraneous to the genus is that the members of the genus are in themselves inaccessible to sense. It is not obvious that the differentiae *must* be extraneous to the genus.

Let us see whether we run into absurdity by supposing that a differentia of a genus falls within the genus, i.e. is a member of it. This supposition means that a member of the genus is a property determining a species of the genus. *Qua* member of the genus, this member *could* also be a member of any particular given species of the genus, in particular it could be a member of the species *determined by itself*; in other words it would make sense to say of such a member that it is, or is not, an instance of itself. Now the denial of this is the logical theory of types, and Aristotle's premise is accordingly a tacit appeal to the logical theory of types. But since, as we have seen, the logical theory of types involves a contradiction, Aristotle's argument that being cannot be a genus falls to the ground.

The outcome of the argument is that the usual conception of

¹ Aristotle, *Metaphysics*, B.3. (Problem 7), (I 2 also perhaps relevant).

'modes of being' or 'different meanings of to 'be'' cannot be sustained within an orthodox logic. It is natural to assume therefore that the contradictory of this is the only possible conclusion, viz. that being is one uniform nature or property belonging to everything whatsoever that is at all. This common nature is the sense (1) of 'existence' distinguished subsequently (8). Within orthodox logic this is, I think, the best we can do. There is, however, serious evidence that there is no such uniform common nature, for instance even with certain limited genera such as the genus *colour* there seems to be no common nature. How can such evidence (if it is admitted) be accommodated, seeing that the usual resort to modes of being has turned out to be impossible? The answer is, I think, that in the end orthodox logic has to be replaced by an unorthodox logic. This need not be argued yet. I mention the point here only in case anyone should think that evidences of the kind just indicated discredit the argument of this chapter.

Underlying the tendency and the desire to distinguish different meanings of to 'be' there is, I think, a significant truth, viz. that there is a more fundamental kind of differentiation in the Universe than the simple, commonsense classification of things into different sorts by means of properties. The thing-property apparatus seems quite incapable of explaining how unity is possible in the face of differences, for instance in explaining the unity of a class by means of an abstract common property, this property has to be separated from each of its instances, and before the original explanation can be made effective we have first to explain how each instance can be one with its property, in the face of its separation from it. I quite admit therefore that there must be a more fundamental kind of differentiation than that of classification by properties, but I would add as before that it is to be found in an unorthodox logic, not in types or different meanings of to 'be'.

6. THE IDENTITY OF THE LOGICAL AND ONTOLOGICAL THEORIES.

This raises the whole question of the connection between the logical and the ontological theories of types, for the Aristotelian argument tacitly claims that the latter is deducible from the former. And this claim seems to be valid, even if the theory of logical types is untrue. Now in so using the theory of logical types, it is plain

that Aristotle was determining the ontological distinction between categories in accordance with the logical distinction between types, since the proof could not guarantee the validity of any way of distinguishing different senses of to 'be' except the way implied by the logical theory of types. Thus a category, the widest possible genus, can be no other than a logical type, at least in extension. In other places Aristotle has also distinguished categories in a way similar to the characteristic method of distinguishing logical types, viz. in relation to the question whether a predicate does or does not become a meaningful sentence when a certain subject term is inserted. Thus two considerations go to show that the two theories are the same in logical structure, firstly that the outstanding feature of the ontological theory (viz. the distinction between different senses of to 'be') is deducible from the logical theory, secondly that the essential features of the logical theory are accepted as determinants of the ontological theory. Thus the arguments discrediting the one theory automatically discredit the other, and in particular the ontological theory cannot escape the logical criticisms which we have seen to be valid against the logical theory.

There is one rather obvious objection to this correspondence between the ontological theory and the logical theory, viz. that there are about ten ontological types and an infinite number of logical types. The reply is almost as obvious. Fundamentally there is only one distinction in each case, and therefore only two *relative* types. But Aristotle elaborated the side of ghostly being thus producing several sub-types, and Russell on the other hand produced an infinite series of types by indicating that the *relation* can be repeated indefinitely.

7. IDEAL BEING.

(a) *Confusion regarding the subject of existential predication.*

There are special reasons prompting the admission of ideal beings, which do not apply in the case of ghostly beings. The non-existence of something is commonly expressed by saying that it is imaginary or exists only in the mind. Now if Father Christmas does not exist, this does not deprive the mental image or idea of Father Christmas of existence, in fact the existence of the idea is usually supposed necessary in order to think the non-existence

of the man himself. Nevertheless, because the idea is associated with possible or actual non-existence, the quality of non-existence is, by a confusion, partially transferred to the idea, and we tend as a result to regard an idea, and mental entities generally, as enjoying only a partial existence.

The error is obvious if the subject of a predication of existence or non-existence is clearly recognised to be the *idea* of that which is said to exist or not to exist, or at least to be something other than the thing itself which stands grammatically as subject. If however this *thing itself* is taken to be the subject of the existential predication, the error becomes insurmountable. In Meinong's theory, for instance, existence and non-existence are treated as properties of the thing itself, with the result that the 'objects' of which existence and non-existence are predicated enjoy only a dubious existence which falls short of existence 'in the full sense'. This is inevitable on such a view, since otherwise an explicit contradiction would occur in a predication of non-existence: an existing thing would be said not to exist. Consequently the being of an 'object' has somehow to lie between existence and non-existence; it is a neutral ground capable of either determination.

But is the contradiction really removed by introducing a twilight existence, or is it only driven underground? If the 'object' is precisely that of which non-existence is predicated, then it is just this twilight existence which is denied existence, so in the case of a true denial of existence how can even a twilight existence *remain* as subject of the denial? A true negation of existence cannot be about any subject at all if, as this view holds, the subject is precisely that which is said not to exist. Consequently no justification of ideal being is possible in this direction.

(b) *The concept as a result of conceiving.*

For the remaining reasons which predispose philosophers to admit ideal beings, let us consider the example of a *concept*, which is a typical Aristotelian 'being of reason', i.e. ideal being. A concept has a relation to the mind and a relation to reality. On the one hand it is the result of a mental act of conceiving, and on the other it is a *natural sign* for certain real objects. On both of these counts a concept has been accorded the status of abstract being.

The argument that it is the result of an act of conceiving may

in the first place depend upon treating a concept in terms of the substance-attribute relation: the being of the concept depends upon the being of the mind conceiving it. But we have seen that this fails to demonstrate, or show the meaning of, a difference in ontological status, but instead presumes an identity of ontological status. However, the argument may take its stand upon the special nature of the substance in this case, viz. as involving an *act* of thinking (whether fully conscious or not). The general premise of the argument would thus be that an originating cause must have a higher ontological status than its effects. We are concerned here with existences in time, i.e. occurrences. Why should it be supposed that the sense in which an originating cause 'occurs' is different from the sense in which one of its effects 'occurs'? The one occurrence is perhaps conscious, the other not conscious; the one is perhaps living, the other not living, etc. But can any such method of distinguishing them signify a difference in the *meanings* of the word 'occurrence' on the two occasions of its use? Obviously not. Any such supposed difference in meaning would be an addition to the *stated* differences (between being conscious and not being conscious etc.). Consider the statement 'The occurrence A is conscious, but the occurrence B is not conscious'. Can this imply that the meaning of the second instance of 'occurrence' is different from the meaning of the first instance? But if so, the difference between *occurrence A* and *occurrence B* is not only the difference due to the one's having the quality of being conscious and the other's not having it, but also the difference due to the difference in the meanings of 'occurrence' in 'occurrence A' and 'occurrence B'. The effective meaning of the statement as a whole is therefore that of a statement which duplicates the difference explicitly asserted, and this in turn duplicates the difference between the meanings of the two instances of 'occurrence'. The difference implicitly asserted by the statement is now triple the difference explicitly asserted, and so on. Of course, we could arbitrarily assert, on extraneous grounds, that the two instances of 'occurrence' have different meanings, but as soon as this difference is made a consequence of the statement itself, its meaning shifts indefinitely. To possess a clear and unequivocal meaning, the meanings of the terms of a statement must be given independently of what is asserted by the statement. Moreover the

presumption is that a given term has the same meaning in each instance of it.

(c) *The Concept as a natural sign.*

The argument that a concept is a natural sign depends upon relating it to the objects signified instead of to the act of conceiving. I suppose it is assumed that a natural sign is less real than the objects signified, in the same way as an image in a mirror is less real than the object of the image. But although an analogy of this kind may show that the image is in a certain way 'less' than the object, the 'less' always turns out to refer to the properties of the image, not to the meaning of asserting its existence. Thus the image in the mirror does have a certain appearance (like the object), but it lacks position in physical space (unlike the object). If by mistake we ascribe too many properties to the image, the result is that the image *so* described does *not* exist, whereas if we keep within the proper limits the result is that the image *does* exist. There is no way of describing it which suspends it *between* existence and non-existence. The impression that it has a kind of half-being arises because the description of the natural sign as it actually is is confusedly overlaid by a stronger description (on the model of the signified objects) which has no application at all.

In the so-called 'intentional logic', based on Aristotle's metaphysics, a distinction is made between *instrumental* signs and *formal* signs (the latter being the same as natural signs for our purposes).¹ In using an instrumental sign we are aware of the sign itself before we proceed to an awareness of the object signified, the latter is, as it were, calculated from the former, like knowing where to go by reading a sign-post. But in the case of a formal sign, for instance a concept, we proceed straight to a knowledge of the thing itself which is signified, normally without awareness of the sign. In the latter case, but not in the former, it is in the nature of the sign to become identified with its object, thus making knowledge of things in themselves possible—not merely representational knowledge. Apparently it is because the nature of the formal sign is to become identified with something else, and thus has *no nature peculiarly its own*, that it is assigned the status of merely ideal being.

¹ Veatch, *Intentional Logic* (New Haven, U.S.A., 1953), pp. 12-14.

The picture seems to be that of a thin fluid adapting itself to the shape of any object present. Surely the expression 'become identified with' cannot be taken seriously. The concept does not cease to be the concept and become the object. If the fluid takes on different shapes, it is still the fluid, it is still *adaptable*, and so has a nature of its own. I confess I see nothing beyond a physical picture in all this, and very little indeed in the way of argument.

Another of Veatch's arguments for ideal being arises from the characterisation of logical entities (concepts, propositions, and arguments) as constituted by 'relations of identity'.¹ Thus a proposition in respect of its subject-predicate form is taken to be an identification, identifying the 'what' of a thing with its 'it', which would presuppose first dividing the 'what' from the 'it'. But, it is added, such a division of a thing from itself obviously does not exist in *rerum natura*, but is only for thought, and similarly the identification is not in the real but is only in our means of knowing the real. Here it seems that the famous paradox of judgment (viz. that if 'S is P' is meaningful then S and P are different, and 'S is P' is false) is invoked to distinguish the logical, as ideal, from the real.

It is hardly enough to point out that the theory of descriptions has overcome the immediate contradiction of the paradox of judgment (viz. by showing that in the crucial case of a judgment of identity, the paradox disappears so long as one of the terms is understood to be not a name but a description). For if the above argument is correct, ideal beings occur simply because we *could* (if we are ignorant or perverse enough) use self-contradictory forms of proposition. But why should a proposition or a concept containing a self-contradictory form enjoy only ideal being, not full being? If there is a fact or thing 'corresponding' to it, we may be sure (on an orthodox logic) that at least the correspondence does not hold in respect of the contradiction. In this respect therefore we may be said to have non-being. But this applies only to the fact or thing. It is surely only by a confusion that this can be taken to reflect upon the ontological status of the proposition or concept. The situation appears to be parallel to that in which an idea is counted as in itself only semi-existent because there is not or need not be an object corresponding to it.

¹ Ibid, p. 24.

8. TWO MEANINGS OF 'EXISTS'.

In this chapter I have not argued that words like 'is' and 'exists' are never used in different senses. I have argued against such differences only in certain well specified cases, viz. those which are associated with a difference of type, and the similar distinction between full being and ideal being. I do not wish to suggest that *all* distinctions between different meanings of 'exists' are mistaken and misleading, for I believe that there is at least one such distinction which has been widely overlooked by philosophers, with serious consequences.

This concerns a basic, trivial and non-propositional sense of 'exists' (sense (1)), and a derivative, propositional sense (sense (2)). In sense (1) we can, indeed, truly assert of a given individual subject that it 'exists', but this assertion is trivial since the word 'exists' only repeats what is already presupposed by the fact that it is predicated of *something*, i.e. of an *existing* thing. Sense (1), then, is rightly describable as 'non-propositional' in nature, since its propositional expression is trivial. But sense (2) occurs because certain *significant* propositions are customarily also called 'existential', and are often expressed in terms such as 'exists' and 'there is'. This is simply an historical fact which we have to put up with, and because of this historical fact it is to that extent not 'mistaken' to suppose that there is this sense (2) of 'exists'. But sense (2) has nevertheless resulted in serious trouble because, so I believe, these so-called 'existential' propositions, although of course distinguishable from other forms of proposition, are not distinguishable from them by any *existential* character.¹ What they say, in effect, is that a certain idea, universal, or property (e.g. the property of being a unicorn) has instances or has no instances. This of course *presupposes* existence (in sense (1)). (This is why I say that sense (2) is derivative from sense (1)). But singular propositions presuppose existence (sense (1)) still more obviously, and on the usual accounts of symbolic logic even universal and hypothetical propositions presuppose existence in so far as they employ a variable presupposed to have values. The peculiarity of 'existential' propositions (sense (2)) lies not in their existential character but in the special kind of *generality* which they possess, expressed by the word 'some'.²

¹ Ch. IV, 7 (c).

² See II, 9, for arguments in support of these points.

If, now, a certain property or idea is said not to 'exist', or is said to be 'unreal', this ought to be meant in sense (2), i.e. it ought to mean simply that everything lacks that property. A contradiction would result from meaning it in sense (1). But if sense (1) is confused at all with sense (2), then on the one hand sense (2) straightforwardly presupposes the property to exist but on the other sense (1) infects its existence with contradiction; the impression therefore inevitably arises that the property or idea 'does not exist in the full sense'. This is perhaps the principal source of the belief in abstract entities.

CHAPTER II

THE ONTOLOGICAL PRESUPPOSITIONS OF LOGIC

1. THE RELEVANCE OF LOGIC TO QUESTIONS OF EXISTENCE.

In the previous chapter I have been concerned with questions concerning the meaning or meanings of 'existence' rather than with the question, What classes of things exist?

In the present chapter I shall be discussing the nature of logic in relation to existence, and this bears upon the latter question rather than upon questions of the meaning of 'existence'. At the same time the particular way in which a consideration of the nature of logic bears upon this question does reveal an important link with the previous chapter. There I argued that abstract being was inadmissible. But this conclusion is capable of two interpretations; either the beings usually supposed to be abstract do not exist at all or else they must exist 'in the full sense' (i.e. the only sense, so far as we are concerned). I may perhaps have hinted in many places that the latter, realistic, interpretation is in general the correct one. In any case, the present consideration of the nature of logic goes to substantiate the realistic interpretation. The same point connects the present chapter with the subsequent discussions of non-existence and negation, for the outcome of these later discussions is to reveal an inherent logical difficulty in the very notions of non-existence and negation, with the result that the question, What classes of things exist? has to be inverted. For the question as it stands suggests that we must be very careful not to admit the existence of any particular class of things unless and until their existence has been fully justified. An examination of non-existence strongly suggests that what we should be careful not to do (until we are fully justified) is to *deny* the existence of the particular class of things considered. For the problem set by my eventual conclusion is, How, if at all, is negation possible? The realistic consequences of the study of non-existence and negation begin to emerge in Chapter III where, long before the ultimate difficulty of the notion of negation is broached, it is found that a consistent account of non-existence implies the existence of universals.

2. THE OLDER EMPIRICIST CRITICISM.

The view that logical principles reflect general features of reality is strongly contested by nearly all forms of empiricism, more particularly by the idealistic forms of empiricism, which in these times are linguistic. The view is also contested by some forms of idealism which do not boast of empiricist leanings. I shall first consider the older empiricist criticism and afterwards that of modern or logical empiricism.

The older empiricist criticism, which is associated with the Kantian criticism of the ontological argument for the existence of God, is based upon a consideration of the hypothetical form of a logical principle. It claims that no existential conclusion can be deduced from purely logical premises.¹ For logical principles are all fundamentally hypothetical in form, and hypothetical propositions do not imply existence.

Now when it is said that a hypothetical proposition 'does not imply existence', this has obvious reference to a hypothetical of the form 'If anything is A it is B', which does not imply the existence of anything which is A or which is B. This results from the fact that a hypothetical does not assert the truth of its antecedent or of its consequent. The fact that, e.g. in regard to its antecedent, it fails to imply existence, rests simply upon the fact that it fails to assert the antecedent. In general, if a compound or a complex proposition fails to imply existence in respect of one of its component propositions, this is due simply to its failure to assert that component proposition. Consequently the claim that a proposition does not imply existence *at all* would be justifiable only on the basis that it fails to assert *anything at all*, and so fails to be a proposition in the proper sense. Now if something is properly a hypothetical proposition, it is a proposition and does assert something. It asserts something distinct from what is asserted by its antecedent and by its consequent considered separately. Thus to deny *any* implication of existence by pointing to the antecedent and to the consequent is to put the blind eye to the telescope.

The only way out is to deny that a principle of logic is properly a proposition at all, and this is to shift to a completely different

¹ See Prof. Ryle's discussion of Collingwood and the Ontological Argument *Mind*, 1935.

ground for criticism. It is in fact to shift from the older form of empiricist criticism to the more recent form. But before discussing this, let us see just how the appearance of an ontological implication arises, by considering more exactly the form of the hypothetical 'proposition' which expresses a logical principle.

3. VACUOUS OCCURRENCE.

Let us assume for the moment that a logical principle is properly a proposition, hence a true proposition, since where we might question the empirical truth of the premises of a valid argument, however well they are confirmed, we would not dream of questioning the proposition that the conclusion follows from the premises. Consider a logically true statement such as 'If all men are mortal and Socrates is a man, then Socrates is mortal'. Here all the empirical terms occur vacuously, i.e. the statement is true (in other examples we might have a false statement) independently of the fact that these terms have the particular values which they do have.¹ Let us call this quality of the statement its 'truth-independence'. It is on account of its truth-independence that the statement remains true if its empirical terms are replaced by any other empirical terms. In consequence it is possible to generalise and arrive at the corresponding logical principle, in which variables occur in place of the empirical terms.

But it would be a mistake to suppose that the quality of universality present in the resulting logical principle expressed the special character of a logical truth, and that an account of the nature of logic in terms of universality would be adequate. For some sets of values of its variables truth might be due to one reason, for other sets of values to another reason. Thus considered simply as a *universal* proposition, the logical principle does not necessarily possess the quality of truth-independence. In a similar way we could regard the above example as not properly an example of logical truth, by imagining that its truth does depend upon the particular meanings of 'Socrates', 'man' and 'mortal'. Truth-independence is something that we have to think into a proposition, and we still have to do this even after generalisation. Generalisation fails to express it. We *could* invent

¹ Prof. Quine explains vacuous occurrence at length in his introduction to *Mathematical Logic* (New York, 1940).

a symbolism for truth-independence, e.g. by adding a suffix to the 'x' of universal quantification. Unfortunately the existing symbolism leads to confusion in the theory of logic because it implies that simple universal quantification is a sufficient expression of the essential character of logical truth. The 'tautology' theory of logical truth arises from a parallel confusion in the propositional calculus.

Bearing in mind this distinction between truth-independence and simple universality, we find that the *logical terms* occurring in a logical principle have a very special significance. These are the remaining non-empirical terms, upon which the truth of the logical principle must depend, since it does not depend upon empirical terms, which are the values of the variables. Now in the case of a true empirical proposition, in what way does its truth depend upon the meanings of its empirical terms? In the first place, the word 'meanings' is perhaps used loosely, at least if we accept the more modern accounts of meaning. Clearly, the truth of the proposition depends upon the extra-linguistic references of the empirical terms. It depends upon the individual things and the particular properties to which the empirical terms *refer*. What makes the proposition true is, e.g. the *fact* that a certain individual thing has a certain particular property. In the case of a logical principle, in what way does its truth *depend upon* the meanings of its logical terms? Unless we are to introduce some unknown meaning of 'truth', the answer must be that the logical terms refer to some extra-linguistic objects (called 'logical constants') and that the principle is made true by some fact concerning these objects. This is the ontological view of logic. But of course the argument no longer holds if it can be maintained with justification that a logical principle is not in the proper sense a 'proposition', and therefore not in the proper sense 'true'.

Nevertheless, the argument at least points to a *correspondence* between the acceptability of a logical principle, and the meanings of the logical words occurring significantly in it. If we replace one logical word by another, e.g. 'some' by 'all', we are liable to destroy the acceptability of the principle. Again, we cannot affect the acceptability of an instance of the principle by replacing its empirical terms by any others. It seems that this correspondence is explicable only in two ways: either, as the ontological view

claims, the acceptability of the principle is determined by independently given logical constants referred to by the logical words, or else the meanings of the logical words are determined by principles laid down (hence 'acceptable') independently of those meanings.

4. THE 'RULE' THEORY OF LOGIC.

The theory of logic associated with the second of these two alternatives may be called the 'rule' theory, since a principle of logic is here treated as a *rule* for determining the uses (meanings) of the logical words occurring essentially in it, *not* as a *proposition* contained in, or implied by, the meanings of these logical words. The theory is associated with the tendency, in philosophical analysis, to treat the meanings of the key words in a given philosophical discussion not as entities given in themselves and open to intuition, but as mental constructs out of the ways in which the words are used. On the more technical side the theory occurs most obviously in the practice of formalist logicians, but it is also essential to the logistic school. For perhaps the most vital step in the development of the *Principia Mathematica*¹ theory of deduction is the separation of inference as governed by a mere rule incapable of truth, from implication which, by contrast, occurs as a constituent in postulates and theorems capable of truth. The basic argument used to support this step is Lewis Carroll's famous paradox of inference,² and accordingly this paradox will be discussed later. Again, the theory is a vital aspect of the general logical empiricist position, which will also be discussed later because of its bearing upon the general question whether logic has ontological presuppositions.

At this point I shall not attack the various *arguments* for the 'rule' theory which these tendencies and theories imply, I shall directly attack the *conclusion* embodied in the 'rule' theory itself. The point is that a rule of the relevant kind presupposes that at least two logical constants are given independently of any rule, viz. the meanings of 'all' and 'same'. As to the first, a rule is obviously not applied in *vacuo* but only in certain circumstances which are indicated by a stated condition. Traditionally logical

¹ *Principia Mathematica*, by A. N. Whitehead and B. Russell, Cambridge 1910-13, 1935. Subsequently 'PM'.

² *Mind*, 1895, p. 278.

rules have been stated as propositions, e.g. 'Anything implied by a true proposition is true' and 'Anything true in general is true in a special case'. But since on the 'rule' theory a rule is a direction for the use of a certain expression, we have presumably to interpret e.g. the first of the above rules as 'Any formula of the form Q may be substituted for any formula of the form P. (P implies Q).' Such a transformation of the rule is bound to retain the word 'any' or an equivalent such as 'all', for it is in the very nature of the rule to apply to *any* case (or *all* cases) of a certain kind. The conception of identity or sameness is also presupposed, since the application of a rule presupposes that we must be able to recognise any case of the kind to which the rule applies, i.e. all these cases are implied to be the *same* in a certain respect. Therefore we cannot suppose either that the basic meaning of 'any' or that the basic meaning of 'same' can be given *by* any rule; on the contrary, at least these meanings must be given *before* any rule is possible. As a result, the mere abiding by the rules of the game is not such a trivial matter as Ayer supposes.¹ In relation to any ordinary game, the essential constituent meanings of a rule as such are so general that they seem to afford us complete liberty to frame what rules we please in the construction of a particular game. But in the case of the game of logic, the constituent meanings of any possible rule are highly relevant to the special rules to be constructed, so this arbitrariness disappears. Whatever rules we 'freely' set up for a given game must conform at least to such principles as are rendered necessarily true as a result of the given meanings of 'any' and 'same', and in particular any logical game is subject to these principles. But since these principles are themselves logical in character, the principles of logic are in the end beyond any free decision; they must be necessary in the full sense of the word.

5. FORMAL AND INTUITIVE THINKING.

(a) *The significance of formalisation in modern logic.*

In the present section I give another criticism of the *conclusion* embodied in the 'rule' theory of logic. In sections 6 and 7 I

¹ *A Modern Introduction to Philosophy* (Glencoe, Ill., U.S.A., 1957), ed. Pap and Edwards, pp. 601-2. See also C. A. Campbell's criticism 'Contradiction', 'Law' or 'Convention'? In *Analysis* 18, March 1958, and K. W. Rankin's Rule and Reality in the *Philosophical Quarterly*, April 1961.

shall analyse and criticise *arguments* for this conclusion and for the general philosophy which it represents.

My argument in this section is that the proofs of certain theorems of the propositional calculus associated with the 'paradoxes of implication' are logically fallacious, and that these fallacies arise because logicians are unwarrantably confident of the validity of purely 'formal' thinking, i.e. of thinking based upon the 'rule' theory. Let us first make sure of the meaning of the word 'formal' in the context of modern logic.

Inherent in the development of symbolic logic there is the tendency to treat reasoning as a mechanical process. This in fact is one of the principal ways in which modern logic is distinguishable from the traditional logic. The word 'formal' has suffered a corresponding change in meaning. From signifying that reasoning is independent of the particular subject-matter of propositions, it has come to mean that reasoning is the mechanical arrangement of objects (normally symbols or words) in certain patterns. Thus to 'formalise' a concept is to obtain the rules governing the use of the word or symbol standing for it, e.g. to 'formalise' negation would be to treat NC and EM as rules governing the use of the word 'not' or the symbol ' \sim '. The previous 'intuitive' thinking associated with the concept, a kind of thinking which by implication or explicitly is condemned as essentially confused, is then dispensed with (so far as possible) in favour of the 'formal' thinking consisting in the manipulation of the corresponding word or symbol according to the rules.

This tendency reaches its extreme development in Formalism, but it deeply affects even the logicism of PM. The primary difference between the traditional logic of propositions and the *Principia* propositional calculus seems to be that the latter distinguishes *rules of inference* from the axioms and theorems which, by contrast, are mere *logical truths*. Since rigour and hence validity are now made to depend upon applying the rules mechanically, the axioms, together with the theorems mechanically generated from them, are, from the point of view of validity, mere arrangements of marks. Formalism is inherent in logicism.

Now if validity is *defined* in terms of the correct application of the rules, we could not raise the question whether a particular theorem is validly proved, so long as the mechanical operations

involved have been satisfactorily checked. But in so far as any system claims to be *logic* as well as an efficient machine, it claims that its theorems 'follow from' its axioms not only in the mechanical or technical sense, but also in the non-technical sense that, supposing the axioms were true, this theorem would also be true. In fact formalisation is supposed at the very least to make this step certain as a result of making the rules explicit, by contrast with intuitive reasoning which, by failing to make the rules explicit, may fail through confusion to guarantee consequent truth. It is in so far as the formal proofs are tacitly claimed to be interpretable as proofs in the non-technical sense, that I contend that certain theorems of the propositional calculus do not follow from the axioms.

(b) *The invalidity of the proof that a contradiction implies every proposition.*

The foremost of these is the theorem that a contradiction implies every proposition. Interpreted, this theorem involves that if propositions of the forms, p , $\sim p$ were both true (or theorems) then any other proposition q we liked would also be true (or a theorem). The proof in PM depends upon the rule of Inference, which asserts that anything implied by a true proposition is true. Now if $\sim p$ is given true, $\sim p \vee q$ follows, since this only means that one of $\sim p, q$ is true. By the definition of 'implies', ' $\sim p \vee q$ ' means ' p implies q '. Thus if the other component p of the contradiction is also given, q follows by the Rule of Inference, since now we have not only ' p implies q ' but p . But q can be any proposition whatsoever. Thus on the usual argument, if we are given that p and $\sim p$ are both true, any proposition q whatsoever can also be proved true.

Logicians seem usually to take it for granted that the Rule of Inference is the same as *modus ponens*, but the definition of 'implies' shows clearly enough that it is strictly a case of *modus tollendo ponens*, being of the form:

$$\frac{\sim p \vee q}{p},$$

$$q$$

in which the minor premise p *negates* the alternative $\sim p$ in the major, leaving the other alternative q as the only one which can be

true. Now this can work only for so long as p does negate $\sim p$. If however we are given that $\sim p$ is still true even though p is true, then p evidently fails to negate $\sim p$. But this is exactly what we are given if we are given the truth of the contradiction $p \cdot \sim p$. In the present case, therefore, *modus tollendo ponens* breaks down, and the conclusion q is not proved. Since by hypothesis the alternative $\sim p$ in the disjunction $\sim p \vee q$ is not negated by p , the other alternative q is not the only alternative that can be asserted, so q is not proved.

(c) *Kleene's formalistic proof also invalid.*

Kleene¹ gives a different proof of the theorem, depending upon his particular formulation of the principle of *reductio ad absurdum*, viz. If $\Gamma, A \vdash B$ and $\Gamma, A \vdash \neg B$, then $\Gamma \vdash \neg A$. (If the theorems Γ with an additional proposition A yield B and also not- B , then not- A ; i.e. A is 'blamed' for the contradiction B and not- B). As a special case we take Γ to be the theorems $B, \neg B$,² i.e. a contradiction is supposed to be given true. Thus Γ, A becomes $B, \neg B, A$. Hence we have

(1) $B, \neg B, A \vdash B$ (since B is one of the three propositions
 $B, \neg B, A$)

(2) $B, \neg B, A \vdash \neg B$ (similarly)

Hence $B, \neg B, A$ yield a contradiction, and by the *reductio* principle A is blamed for the contradiction $B, \neg B$, whence we have $\neg A$. Thus if A is taken to be of the form $\neg C$, any proposition C we choose can be proved true, so long as we are given some contradiction $B, \neg B$.

To analyse this argument, let us first consider the *ordinary reductio* principle, as against Kleene's. This is a special case of *modus tollens*:

$$\begin{array}{l} \text{If } p, \text{ then } q \text{ and not-}q \\ \text{not-}(q \text{ and not-}q) \\ \hline \text{not-}p \end{array}$$

The conclusion not- p is *not* proved if the minor premise is not true, and the minor premise cannot be assumed true if the con-

¹ *Introduction to Metamathematics*, esp. p. 101.

² In Kleene's example p. 101 there is a confusing change of lettering which is avoided here (though perhaps it is needed in Kleene's wider context).

tradiction q and not- q is assumed true instead of, as normally, false. These considerations apply to Kleene's formulation. It is justifiable to 'blame' A for the contradiction $B, \neg B$ only in so far as the contradiction itself is 'blamed' i.e. recognised to be false. But here on the contrary the hypothesis consists in asserting this contradiction to be true.

The position is not rectified by appealing to the way in which Kleene's *reductio* principle is proved. The proof, which Kleene does not actually give, obviously comes from postulate 7 (p. 82):

$$(A \supset B) \supset ((A \supset \neg B) \supset \neg A).$$

The proof is as follows. Given $\Gamma, A \vdash B$ and $\Gamma, A \vdash \neg B$, it follows by the Deduction Theorem that $\Gamma \vdash A \supset B$ and $\Gamma \vdash A \supset \neg B$. Thus both $A \supset B$ and $A \supset \neg B$ are theorems, from which it follows, by two applications of the rule of inference (postulate 2, p. 82) to postulate 7 above, that $\neg A$. Thus if $\Gamma, A \vdash B$ and $\Gamma, A \vdash \neg B$, then $\Gamma \vdash \neg A$; which is Kleene's *reductio* principle. However, if the contradiction $B, \neg B$ is given true, postulate 7 itself ceases to be acceptable as true if interpreted in the normal way as an 'if . . . then . . .' statement. For its justification involves *modus tollens* in the same way as does the ordinary *reductio* principle. Given $A \supset B$ and $A \supset \neg B$ we should, if postulate 7 is to be justified, be able to derive $\neg A$, i.e. from $A \supset (B, \neg B)$ we should be able to derive $\neg A$, which is possible only by applying *modus tollens* on the understanding that $B, \neg B$ is *false*. But this is just the contradiction which is given true.

(d) *A defence of these proofs, and reply.*

In defence of the standard arguments that a contradiction implies every proposition, it might be said that in spite of the hypothesis p and $\sim p$ we are still bound to maintain $\sim(p, \sim p)$. For, it will be argued, we must still use the sign ' \sim ' in the proper sense, and this proper sense of negation is maintained only by adhering to NC and thus taking $\sim(p, \sim p)$ as true. Otherwise, it would be said, the hypothesis $\sim p$ would not be the genuine *negation* which it is supposed to be, and the assumption of p and $\sim p$ together would not properly be the assumption of a *contradiction*.

Now in one way this objection *supports* the view that validity in logic is ultimately to be settled by resort to intuition rather

than to rules and their mechanical application, for the objection clearly depends upon an *intuition* (so far as it goes) of the nature of negation. However let us consider the content of the objection rather than its form—by sharpening the same intuition of the nature of negation.

So far as its content is concerned, the objection is an attempt to rehabilitate the machinery of the formal system as a reliable guide to validity. It is a mistake, however, to suppose that the function of the proposition $\sim(p. \sim p)$ is, under the circumstances, to uphold the meaning of ' \sim '. So far as the meaning of negation is concerned, the damage is already done once we have made the joint assumption of p and $\sim p$, for at every point at which this assumption is used in the ensuing argument we are allowing the truth of $\sim p$ without supposing that this destroys the truth of p , i.e. we are changing the meaning of ' \sim ' in precisely the sense objected to. The function of the proposition $\sim(p. \sim p)$ is not to prevent this change in the meaning of ' \sim ', but to produce additional limbs of the argument independent of the joint assumption of p and $\sim p$. If we call the joint assumption of p and $\sim p$ ' P ', the effect is thus to introduce the opposite assumption $\sim P$ operating at points in the argument different from the points at which P operates. Thus P operates at the point where we construct the premises $\sim p \vee q$, and p , as true; but $\sim P$ operates at the point where we combine these premises to produce the conclusion q . We now have a *second* contradiction, viz. P . and $\sim P$, which brings about a *second* deterioration in the meaning of ' \sim '. For whereas without the assumption $\sim P$, it was only in its application to p that the meaning of ' \sim ' was destroyed, the meaning of ' \sim ' is now in addition destroyed in its application to P , since the truth of $\sim P$ is admitted without its destroying the foregoing assumption P .

The second deterioration in the meaning of ' \sim ' is recorded in ordinary terms by saying that the argument as a whole is inconsistent. There is a formal fallacy of equivocation on the meaning of ' \sim '. The assumption $\sim P$ is gratuitous, i.e. it is not included among the explicit premises p and $\sim p$, which on the contrary amount to P . The only excuse for supposing $\sim P$ to be contained in the explicit premises is that $\sim P$ is part of the definition of ' \sim ' as occurring in the explicit premise $\sim p$. But it has

been shown that the very admission of the premise p alongside $\sim p$ is enough to ensure that $\sim P$ is *not* now part of the definition of ' \sim ' as occurring in $\sim p$, and that the introduction of $\sim P$ as a premise, as if it *were* part of this definition of ' \sim ', far from upholding the definition produces a second change in it. The argument as a whole can therefore be regarded as valid only if $\sim P$ is disallowed as a premise, and accordingly the standard arguments for the theorem that a contradiction implies every proposition is invalid because it uses $\sim P$ as if it were a premise.

(e) *Intuitionism not intuitive thinking.*

What I have called 'intuitive thinking' must not be confused with the theory of logic known as 'Intuitionism'. It would seem that the first method of proving that a contradiction implies every proposition, would hold within an intuitionistic system, since Intuitionism admits the theorem $p \supset \sim \sim p$, though it does not admit the theorem $p \equiv \sim \sim p$. Thus it would admit the validity of the above-mentioned special case of *modus tollendo ponens*, viz:

$$\frac{\sim p \vee q}{\frac{p}{q}},$$

since all that is required for this is that the minor premise p should imply $\sim p$, i.e. the negation of the alternative $\sim p$ in the major premise. In the intuitionistic system as in the classical system it is assumed justifiable to apply the rule of inference even when the hypothesis to which it is applied is a contradiction. Again, postulate 7 of Kleene's system is true in the intuitionistic system as well as in the classical system, and we have seen that, interpreted as an implication, it can no longer be maintained as true if the contradiction involved subordinately in it is assumed true. Thus I do not see how any modifications in the formal systems, such as those suggested by Intuitionism, can remedy the defects which I have indicated.

(f) *The paradoxes of implication.*

The fallacy can take two forms: either the alleged theorem is not a theorem, or the application to it of the rule of inference is invalid. Thus on the assumption that p and $\sim p$ are both true, the

alleged theorem $p. \sim p: \supset q$ would cease to be a theorem. For the truth table for $p. \sim p: \supset q$ would yield T (for $p. \sim p$), F (for q), for a value F for q , and this would give the value F for the implication itself, i.e. the implication could not conform to the condition needed for it to be a tautology. Alternatively, let us start with the theorem $\sim p \supset . p \supset q$ (PM *2.21), supposing again that both p and $\sim p$ are given true. Here we apply the rule of inference once, giving $p \supset q$, but when we come to apply it a second time we have the syllogism

$$\frac{\sim p \vee q}{p},$$

$$q$$

which as shown previously is invalid if $\sim p$ as well as p is given true. In this case we can, if we like, regard the theorem as genuine, but if so we cannot apply the rule of inference to it. Either the truth of the theorem, or the application of the rule of inference to obtain $p \supset q$ in a given case of $\sim p$, requires that not both p and $\sim p$ are true.

The remaining paradox of implication, viz. $q \supset . p \supset q$, is not I think subject to such criticisms. If a proposition q is given true, then given the truth of any proposition p , the hypothesis q remains, i.e. if q is true, then if p then q . Even if it so happened that p implies $\sim q$, this would only mean that we have a contrafactual hypothetical, which presumably is possible.

A fallacy arises, however, as soon as we attempt to prove the other two paradoxes from this one. It might be thought that since $p \supset q. \supset . \sim q \supset \sim p$ we can argue from $q \supset . p \supset q$ to $q \supset . \sim q \supset \sim p$. But transposition depends upon *modus tollens*:

$$\frac{p \supset q}{\sim q},$$

$$\sim p$$

and this we have seen to be invalid if the contradiction $q. \sim q$ is assumed true.¹ But before we can reach the conclusion $\sim p$ by applying the rule of inference (twice) to $q \supset . \sim q \supset \sim p$, we do have to assume that q and $\sim q$ are simultaneously true. Therefore if $q \supset : \sim q \supset \sim p$ be admitted, the rule of inference cannot

¹ See (b) in this section.

be validly applied to it, and the implications are sterile, in practice meaningless.

(g) *The Rule of Inference presupposes intuitive thinking.*

As remarked earlier, the Rule of Inference is of the form

$$\frac{\sim p \vee q}{\frac{p}{q}},$$

and works only because p implies $\sim(\sim p)$, and thereby eliminates the alternative $\sim p$ from the disjunction $\sim p \vee q$, leaving only the alternative q as true. Since the significant component of NC is ' p implies $\sim(\sim p)$ ' (the other component being ' $\sim p$ ' implies $\sim(p)$ ', which is correspondingly presupposed by the *normal* form of *modus tollendo ponens*), we can conclude that the Rule of Inference does, in effect, presuppose NC, and it therefore gives quite a wrong impression to ' prove ' NC at a late stage in the propositional calculus. Moreover, ' p implies $\sim \sim p$ ', as so presupposed, is not simply a material implication. From p we have to *infer* $\sim \sim p$. If the implication were only a material implication, then to perform the inference of $\sim \sim p$ from p validly we would require the validity of the Rule of Inference. But this is obviously impossible since the valid inference of $\sim \sim p$ from p is something required in order to make the Rule of Inference valid.

There seems to be only two ways of accounting for this relatively fundamental inference from p to $\sim \sim p$: either it is a case of *modus ponens* proper, or else it is simply intuitive. I do not see any real difference between these two alternatives. In both cases the characteristic separation between inference and implication disappears. In *modus ponens* proper, there is nothing to justify the inference of q from the premises p , p implies q , except the meaning of ' implies ', i.e. the inference is known to be valid by an intuitive perception of the meaning of ' implies '. Conversely, given that p implies q , the intuitive perception of the meaning of ' implies ', tells us that, if it so happens that p is true, it is also the case that q is true, i.e. we can infer q from p .

A mechanical mode of thinking cannot, in the nature of the case, produce any ultimate guard against fallacy. A rule may seem correct, but so long as thinking is confined to the application

of rules there is no understanding, i.e. no intuitive perception, of the way in which the rule works; and thus no way of knowing whether the rule has exceptions, and if so what they are. By studying the rule we find this out, but this studying is intuitive thinking. By studying the Rule of Inference we discovered an exception and qualifications, and this through perceiving intuitively just how it works. Correctness or validity are wholly unknowable without intuition. Thinking consists fundamentally not in proceeding according to a rule, but in perceiving whether or how far the rule is valid, and because it is not the mere operation of the rule but its *valid* operation that counts, thinking is fundamentally intuitive.

6. LEWIS CARROLL'S PARADOX OF INFERENCE.¹

(a) *Statement of the paradox.*

Suppose that p is a true proposition or conjunction of propositions. It is usually supposed that, for some other proposition q to be true, it is necessary and sufficient that if p then q . However it may be said that this merely augments the true premises: instead of ' p ' we have ' p and (if p then q) '. Consequently, for q to be true, it now seems (by the same principle) that we need the truth of ' if p and (if p then q), then q '. But as before, this seems only to add to the premises, so that still another hypothetical proposition is needed in order to guarantee the truth of q , and so on *ad infinitum*. Thus what we take to be valid reasoning seems to be incapable of performing its task.

The paradox can be given a slightly different twist by emphasising that the hypotheticals successively obtained in this infinite regress assure us of nothing more than a *relation* between the premises and the conclusion, instead of, as is needed, the truth of the conclusion q by itself without hypothesis. This aspect has special relevance on the one side to Bradley's well known infinite regress of relations, and on the other to the particular way in which Russell has argued for the need of a rule, as distinct from a hypothetical proposition, for inference.

Russell's (and Whitehead's) theory of the Rule of Inference highlights the importance of Lewis Carroll's paradox for logic. The Rule as stated in PM is:

¹ *Mind*, 1895, p. 278.

Anything implied by a true elementary proposition is true. (The qualification 'elementary' does not concern us.) The authors comment: "It is not the same as 'if p is true, then if p implies q, q is true.' This is a true proposition, but it holds equally when p is not true and when p does not imply q. It does not, like the principle we are concerned with, enable us to assert q simply, without any hypothesis. We cannot express the principle symbolically, partly because any symbolism in which p is variable only gives the *hypothesis* that p is true, not the *fact* that it is true."¹ A footnote refers the reader, for further remarks on the principle, to *Principles of Mathematics* (London 1903, 1937) §38, where Russell argues that, to avoid Lewis Carroll's paradox, we need the notion of *therefore*, which is quite different from the notion of *implies*, and holds between different entities. He says that true propositions have the distinctive property of being *asserted*, by contrast with propositions which may be either true or false, and that *therefore* relates propositions having this peculiar property, whereas *implies* only relates propositions taken as either true or false. When we can say *therefore* we can thus *assert* the conclusion, and it becomes no longer necessary to maintain its relation to the premises. The dropping of the premises is clearly needed for the practice of Symbolic Logic, and the implied dropping of the *relation* appears to solve Lewis Carroll's paradox. So far as *implies* and 'if . . . then' are concerned, their use solely to express a proposition (material implication) as against a rule of inference is strongly defended both in the above quoted passage from PM and elsewhere.²

Ryle insists on the other hand that 'if . . . then . . .' ought to be used primarily to express not an informative proposition but a rule (licence) for inference. But he maintains the distinction between a proposition and a rule of inference just as strongly as Russell does, viz. by saying that in using 'if . . . then . .' to express a rule of inference, we preclude its use to express a *premise* for an inference, a premise being necessarily a proposition. He argues that if we do make this mistake of treating a principle of inference as a premise, we run into Lewis Carroll's paradox

¹ PM 1.1 (2nd edition, p. 94).

² PM * 1.01 and *Introduction to Mathematical Philosophy* (London, 1919), pp. 153-4.

(Achilles and the Tortoise). He concludes "The principle of an inference cannot be one of its premisses or part of its premiss. Conclusions are drawn from premisses in accordance with principles, not from premisses that embody those principles".¹

Russell and Ryle therefore disagree only on a verbal issue as to the proper use of 'if . . . then . . .'. Both believe that a principle of inference is something entirely different in nature from a hypothetical *proposition*, viz. it is a *rule*; and both accept Lewis Carroll's paradox as proof, even *the* proof, of this result.

(b) *Analysis of the paradox.*

The analysis of a paradox or of any other problem is to be carefully distinguished from a solution. Indeed, an analysis need not even have a solution in view. Its aim is to understand the paradox, i.e. to see how it works, by separating its component parts and perceiving their relationship. Success in such a task is likely if one is interested in structure for its own sake. It is unlikely if one's foremost aim is to produce a solution. It is still less likely if one approaches every philosophical paradox *via* the prejudice that such paradoxes are really non-existent and are to be explained away as purely verbal puzzles.

What are the factors necessary and sufficient to make the paradox operate?

(1) Given a true premise p and a conclusion q validly drawn from it, there must be some relation between them in virtue of which the conclusion is validly inferred. We must therefore appeal to a logical principle P_1 , in virtue of which the conclusion is validly inferred, and without which it cannot be validly inferred. Though P_1 is usually supposed to be hypothetical in form, I do not think it is essentially so, and it is not necessary to insist on the hypothetical form in order to ensure the production of the paradox.

(2) But there is one feature of P_1 which is normally insisted upon, and is vital for the production of the paradox, viz. that P_1 must be true. In fact, in *appealing* to P_1 we appeal to its truth. If P_1 were false, then however certain we were of the truth of p , P_1 would no longer guarantee the truth of q . It is the *objectivity* of the relationship between p and q which guarantees the validity,

¹ 'If,' 'So' and 'Because', *Philosophical Analysis*, edited Max Black. (Ithaca, N.Y., 1950), pp. 327-8.

and this objectivity is the fact making P_1 *true*. No amount of subjective belief or acceptance of P_1 is enough for validity. Clearly there is a natural and well-founded basis in commonsense for this factor in the paradox. Thus, the argument goes, it is not the truth of p alone which guarantees the truth of q , but the truth of p together with the truth of P_1 .

(3) But if $(p$ and $P_1)$ is *necessary* to guarantee the truth of q , q , is it *sufficient*? No, for by the same argument as before a principle P_2 is necessary to connect these true premises with the conclusion q , and P_2 must be true. Again, P_2 is different from P_1 since the premises in this case are different, viz. p and P_1 instead of p .

(4) In a similar way the demand for further principles for guaranteeing the truth of q goes on indefinitely. At no point therefore do we find any principle P_n which, with the premises $(p$ and P_1 — $P_{n-1})$ is sufficient to guarantee the truth of q . Hence there is no discoverable principle for the validity of the argument. This contradicts the fact that logicians have discovered many principles which seem indisputably to guarantee validity.

Corresponding to these four stages in the analysis, there are four assumptions constituting the conditions for the paradox to work:

- A. In every valid inference, i.e. whenever the truth of certain propositions logically guarantees the truth of another proposition, a principle of inference in addition to these propositions is needed.
- B. A principle of inference is a (true) proposition.
- C. If two inferences have the same conclusion, but the premise or premises of the one is contained conjunctively within the premises of the other, the principles required by the two inferences are different.
- D. There is no way, alternative to the way indicated in the paradox, of exhibiting the succession of principles of inference.

The meaning of assumption D will be made clear soon, when an alternative way of exhibiting the succession of principles is indicated. This assumption would be sufficient, with the others, for the working of the paradox, but it might not be necessary (as the others are), for even if alternative ways are found they might

meet with the same difficulties as the original way of exhibiting the principles.

(c) *Solution in terms of a rule.*

This type of solution obviously attacks assumption (B), seeking to deny the implied possibility of a principle being a premise, and seeking to establish firmly a difference in nature between a principle and a premise. A principle of inference is said to be something more than, even different in type from, a proposition. It is a *rule* or *licence* to pass from one proposition (the premises) to another (the conclusion). If a hypothetical 'proposition' (if p then q) is still used to express a principle, it is said to be not strictly a proposition (in this case a true proposition or fact) but a licence to perform certain inferences. Prof. Ryle still wants to use the hypothetical form in this way, but denying its propositional nature. Russell, on the other hand, refrains from using 'if' and ' \supset ' to express a principle of inference; on his view only axioms and theorems are expressible by such means.

But this method of stopping the regress by establishing a difference of type between principle and premise has very serious, even disastrous, consequences. The trouble is that, to avoid retrogression to the form of a hypothetical proposition merely *connecting* the premises to the conclusion, the hypothesis must not be incorporated within the principle, but instead becomes an extraneous conditioning of the principle itself, in so far as it serves simply to determine the class of conclusions, each of which the principle in effect directly asserts to be true. Consider the Rule of Inference in PM: Any proposition implied by a true proposition is true. The word 'any' applies not to 'proposition', but to 'proposition implied by a true proposition'. For if it applied to 'proposition' the meaning would be 'Any proposition, if it is implied by a true proposition, is true'. But the hypothetical form is specifically rejected in PM. The only other interpretation is that the word 'any' here presupposes that there are 'propositions implied by true propositions', and that the Rule claims that *any* of these is true. Thus 'any' is equivalent to 'each' and to 'every'.

But this has an extra-ordinary consequence regarding the

method by which the Rule *justifies* the conclusion. In effect the Rule says that every conclusion satisfying the required conditions is true. Satisfying the conditions is not the *reason* for the assertion of truth, since this would make the Rule an 'if' proposition. The Rule justifies a particular conclusion only because this falls among the class of conclusions which it justifies, and these are 'justified' merely by being asserted. In other words the conclusion is to be accepted as true because we are told on authority that it is true. For the justification of the conclusion we are referred to the assertion of the conclusion itself, and the premises (with the appropriate principle) are, *qua reason* for the conclusion, redundant. But of course the problem set by Lewis Carroll's paradox is not —How is the conclusion of a valid inference justifiable? but, How is it justifiable *by reference to the premises?* To justify it in terms other than the premises (and the appropriate principle), as if these were irrelevant, is to admit a still greater paradox than Lewis Carroll's.

Of course, symbolic logic does require that we should be able to assert the conclusion 'by itself, without any hypothesis'. But the only sense in which this is required is that the premises, with the principle, must warrant the truth of the conclusion. It is a confusion to suppose that the conclusion has somehow to be actually asserted. Our ability to write the conclusion or assent to it are not in question, but even if they were, neither the actual writing of it nor the assent to it is sufficient guarantee of its truth. If the premises with the principle do not actually contain an assertion of the conclusion, it is not to be inferred that they fail to guarantee the truth of the conclusion, and that we must therefore introduce something which does directly assert the conclusion.

In reply to this it might be claimed that the purpose of the rule theory is not to state a justification at all; that it presupposes that we can *see* the justification, but that the justification cannot or need not be stated. In that case, however, the rule theory cannot replace the ontological theory, even if the latter incurs Lewis Carroll's paradox. For the paradox occurs only if one assumes the need for a justification, and is therefore solved only by an alternative justification avoiding the paradox. Moreover, if the justification is in fact *seen*, why are there varying theories of its nature? And if a *stated* justification (the ontological theory)

is claimed not to be free of paradox, how can we be sure that an *unstated* and therefore unexamined justification will be free of paradox? The failure to state it looks like a device for sheltering it.

More plausibly one might associate the rule theory with the notion of *therefore*, as Russell does in *Principles of Mathematics* §38. But neither can this bear a close examination. In any case of validly inferring q from p , the validity (expressed by using the word 'therefore') depends upon two factors, (1) the truth of p , (2) certain formal relationships between p and q . It is a matter of elementary logic that these two factors are mutually independent. The required formal relationships between p and q may be present although p is not true or is not known to be true. But their independence of the truth of the component p means precisely that the indicated relationship between p and q is *hypothetical*, appropriately expressed by 'if . . . then . . .'. Thus in so far as 'therefore' expresses the relation involved in inferring, this relation is indistinguishable from the one expressed by 'if . . . then . . .'. This is perhaps consistent with Ryle's treatment of 'if . . . then . . .' as the proper means of expressing inference, but it destroys the supposition that in the notion of *therefore* we have a relation of necessitation distinct from the relation which, according to the 'rule' theory, is responsible for Lewis Carroll's paradox. A hypothetical *relation* is what the 'rule' theory condemns as leading to the paradox; Ryle escapes only if he takes 'if . . . then . . .' not to express a relation at all, but quite literally a rule. In that case he could not consistently fall back on Russell's notion of *therefore*. And since we have now seen that this attempt of Russell's to produce an alternative justification for inference fails, the rule theory is still without any means of justifying inference.

It might be supposed that the difficulties of the 'rule' solution could be avoided by a modification on the following lines. In the first place it seems necessary to admit (assumption B) that a principle of inference is a true proposition, and moreover that its being a true proposition is all that is required to ensure validity, i.e. to ensure that, in any case of true premises, the conclusion is also true. However, it is added that there are two ways in which a true proposition can function in an inference, (a) as a premise, (b) as a principle. Consequently the inference from p to q *via* the principle P_1 cannot be correctly represented as an inference

from p and P_1 to q , for P_1 operates as a principle in the first case and as a premise in the second. It is now argued that the need for P_2 , and hence the regress, arises because the functioning of P_1 as a principle is mistakenly identified with the functioning of P_1 as a premise.

This method of solution in effect attacks assumption A instead of assumption B. It claims that the truth of certain given propositions can logically guarantee the truth of another proposition, yet no principle of inference *in addition* to them be needed, the reason being that the required principle of inference is already contained in the set of given propositions. And it does seem at first sight that assumption A is plausible only so long as this possibility is overlooked. I think that for ordinary purposes this solution could count as sufficient.

On the other hand it may be felt that Lewis Carroll's paradox depends upon a more fundamental trouble regarding the nature of any significant inference, viz. that it is a passage from one proposition (in general conjunctive in form) to a *different* proposition. It is just the difference which requires that a principle of inference be introduced to *relate* them. Now if p and P_1 be given, this given is different from q , even if it be stipulated that P_1 is to operate as a principle, not as a premise. From one thing we have to get to a different thing, even if the 'getting' to it is denied the name 'inference'. A further relation is therefore needed in any case, and after that a further one, and so on. Consequently there seems little point in denying that the relation is really a principle of inference. On the one hand there is no discernible difference between this kind of 'getting' from one proposition to another, and inference from one to the other; and on the other there would still be an infinite regress indicating that the conclusion q had never been reached, even if it were not stated as a regress of principles of *inference*.

It will soon be seen that, from this more fundamental standpoint, Lewis Carroll's paradox is a special case of Bradley's regress of relations. But before discussing the matter in this connection, it will be necessary to see if any other solution to the paradox is forthcoming.

(d) *Solution by termination of the regress.*

The criticism of the 'rule' solution of the paradox would perhaps fail to carry conviction if I left the matter at this point, as if the paradox were insoluble anyway. If no alternative solution were discovered, the impression would arise that something is amiss with the criticisms of at least one of the existing solutions, since *some* solution presumably has to be correct. Fortunately, however, an alternative solution arises from the fact that assumptions C and D are both false.

Assumption D is false because there is an alternative way of exhibiting the series of principles P_1, P_2 etc. P_1 indeed can only be, as before, the principle of the inference from p to q , but P_2 can be taken to be the principle of the inference from P_1 to (if p then q), instead of the principle of the inference from p and P_1 to q ; and so on for the subsequent principles.

Now let us follow up the regress by considering its application to any example, e.g. the inference from p_0 to p_0Vq_0 , where p_0 and q_0 are particular given propositions. The corresponding logical principle directly underlying this inference can provisionally be expressed in the form 'if p then pVq ', where p and q are now variables. Of course, Russell objects to this way of expressing a logical principle, on the ground that 'if — then —' expresses the very different concept, material implication; and Ryle accepts it only on condition that it expresses a licence, not a statement of fact. But here we are taking it to express a fact, in accordance with assumption B, and in view of the fact that the 'rule' theory has now been discredited. In the light of what has been said on vacuous occurrence, we can hardly take this fact to be one of mere universality, though this I think would work for the following argument. We must take it to be a fact of logical necessity, *qua* defined and determined by the feature of truth-independence. It will be convenient to use the expression 'nec' to signify logical necessity so defined, since 'if — then —' has to be used elsewhere in its ordinary meaning. It will also be convenient to abbreviate 'The inference from A to B' to 'inf (A to B)'. Thus the first term of the regress is inf (p_0 to p_0Vq_0), P_1 is (p nec pVq), and the second term of the regress is inf ((p nec pVq) to (if p_0 then p_0Vq_0)).

Now on what principle does this last inference depend, i.e. what is P_2 ? If P_1 had been a merely universal proposition, P_2

would have been the familiar principle that if anything is true in general it is true in a particular case. As it is, we have to express P_2 in some such form as 'If anything has truth-independence relative to a certain class of cases, it is true in any one of these particular cases', or more strictly, and in symbols: $\phi(x, y \text{---}) \text{ nec } \overset{\text{nec}}{\phi}$ ($x_0, y_0 \text{---}$), where $\left(\overset{\text{nec}}{\underset{\text{if-then}}{\phi}} \right)$ signifies the function obtained by substituting 'if — then —' for 'nec' in ϕ .

The third term of the regress therefore is: $\text{inf } (\overset{\text{nec}}{\phi}(x, y \text{---}) \text{ nec } \overset{\text{nec}}{\phi}(x_0, y_0 \text{---}) \text{ to if } (p \text{ nec } p \vee q) \text{ then } (\text{if } p_0 \text{ then } p_0 \vee q_0))$. The important point to notice is that *the principle underlying this inference is still P_2* , not some other principle P_3 . For what P_2 essentially demands is that the right hand side of the necessitation should be a special case of the left hand side of the necessitation, modified by the replacement of ϕ by $\overset{\text{nec}}{\underset{\text{if-then}}{\phi}}$; and a careful examination of the above inference will show that its right hand side is in fact related to its left hand side in this way. To see this, it must be realised that the function operating for ϕ is now not the original unspecified ϕ but the identity function, ϕ being now treated as one of the variables within the identity function. The special case on the right hand side is obtained as follows:

- (1) For the variable ϕ put the particular function $x \text{ nec } x \vee y$,
- (2) Replace the variables x and y by p and q respectively.
(This change is purely verbal.)
- (3) Omit the remaining variables. (The general form ' $\phi(x, y \text{---})$ ' means, of course, not that the number of variables is infinite, but that the number is not yet stated, and is subject to later specification.)

The specification (1) clearly falls under P_2 , since we may take particular values for a variable function as for any other variable. Again, in so far as the number of variables in ' $\phi(x, y \text{---})$ ' is itself a variable, and in so far as specification (3) gives this variable number the particular value *two*, this case also falls under P_2 . It is moreover clear that any other logical principle, treated in a similar way, would be found to fall under P_2 . In all cases the

regress ends as soon as P_2 is reached, because P_2 is found to be the principle underlying the *next* inference in the regress (viz. the inference beginning with P_2 itself as premise). Thus assumption C as well as assumption D turns out to be false, and there is no longer a paradox.

(e) *The regress of relations.*

There is a traditional dilemma to the effect that if an inference is valid the conclusion cannot be a new fact and the inference is trivial, whereas if the conclusion is a new fact different from the premises the inference is invalid. Lewis Carroll's paradox develops the second horn of this dilemma, but its solution in terms of a rule is impaled on the first horn of the dilemma and consequently, as we have seen, is involved in an even worse paradox. The second horn has surely to be faced, since in normal inferences the conclusion is significantly different from the premises considered as isolated facts. This difference between conclusion and premises sets the problem of justification, and we try to solve it by finding a certain sort of *relation* between the two numerically different terms.

In fact Lewis Carroll's paradox is a special case of Bradley's problem of the infinite regress of relations.¹ If A and B are two terms numerically different yet connected, then a relation C, numerically different both from A and from B, seems to be required in order to account for the given connection. But if C were unconnected with A and B it would obviously fail to relate them, so that further relations D and E are now required to relate C to A and to B respectively. But these relations in turn create the need for still more relations, and so on ad infinitum. The modification of this situation in Lewis Carroll's paradox is that the question of relationship arises only on the side of the second term B of the relation. If A is the premise of a valid inference and B the conclusion, validity requires a relation C between A and B. But C and A together now form a term A_1 numerically different from B, so another relation C_1 is required between A_1 and B. C_1 and A_1 now form a term A_2 numerically different from B, and another relation C_2 is required, and so on. The form of Lewis Carroll's paradox considered in the last section (d) is still more obviously

¹ F. H. Bradley, *Appearance and Reality* (London, 1893, 1897), Ch. II.

a case of Bradley's paradox. If C is the relation (of inference) between A and B, the problem then was that this relation can be maintained only if there is another relation (of inference) between C and (A and B).

In the case of Lewis Carroll's paradox, the regress was brought to a close by a principle which was in a certain sense 'reflexive'. Similarly Bradley's regress can be brought to a close by a relation which is in a similar sense 'reflexive'. (This sense of 'reflexive' is not quite the same as the usual sense in which a *relation*, such as e.g. numerical equality, is said to be 'reflexive', but derives from the use of the word in the phrases 'reflexive proposition', 'reflexive paradox'). Thus it would seem that, in order for two things to be actually related by a so-called two-term relation, this apparently two-term relation would have to be an aspect of a more complex relationship involving at least three terms. If the regress stopped there, the third term of the relationship would be not only a term but a *reflexive relation*; not only would it relate the original two terms, but it would also relate *itself* to them considered as a pair, thus being a term as well as a relation. At this point we part somewhat from Bradley's conclusions. As a result of the regress, Bradley inferred that external relations are *unreal*, they are a mark of *mere appearance* and cannot occur in reality. The account which I am suggesting does not imply this, it only implies that external relations are a mark of *incompleteness*, always requiring a reflexive relation to *complete* the reality of which they characterise at most a part. The regress can be brought to a close or *completed* only by a reflexive relation. There can be *one* universe, a *complete* universe, only in virtue of some reflexive feature. In the end the *one* of the universe cannot fall outside the *many* items comprising the universe (as it does in Whitehead's Category of the Ultimate¹), but must be one of these items.

7. THE GENERAL ARGUMENT FOR LOGICAL EMPIRICISM

(a) *A statement of the logical empiricist position.*

In this section (7) I shall discuss and reject a second argument for the 'rule' theory, viz. what seems to me to be the general argument in support of logical empiricism.

According to logical empiricism there is a field beyond the

¹ *Process and Reality* (Cambridge, 1929), Part I, Ch. II.

facts in which there is room for the free determination (normally by convention) of general principles having the form of statements of fact. Although such principles as are so determined are laid down as 'true', they are in the strict sense neither verifiable nor falsifiable, and it is just because they are not falsifiable that they *can* be laid down as 'true'. As 'true' they seem to be propositions, but properly they are rules, commands, prescriptions, or definitions. Our freedom to assert such principles has been called by Carnap 'the principle of tolerance'.¹

It is obvious that the 'freedom' underlying the principle of tolerance does not imply 'freedom' in the sense claimed by libertarians. It means primarily that the 'truth' of a logical principle is not determined by any general fact, such as a logical principle would seem to express as a result of being propositional in form. Any principle which *is* determined by such a general fact I shall call 'objectively necessary'. Thus ontologists would claim that logical principles are objectively necessary, whereas logical empiricists would deny this. Nevertheless logical empiricists would normally allow that there are non-objective causes which determine the 'truth' of a logical principle, indeed if they failed to provide some such explanation their view would clash with the obvious fact that as individuals we cannot reason validly on whatever principles we please.

This leads to the consideration of an important variant of the principle of tolerance (or even a special case of it), which may be called the 'Kantian variant'; though it must be understood that Kant himself used the conception only in the field of transcendental logic, not in the field of formal logic. Whereas the ordinary logical empiricist view is that logical principles are determined by convention (i.e. by past tacit agreement to define the key logical words in a certain way), the Kantian variant goes further by attempting to explain the universal conviction of necessity associated with logical principles. It points out that if we failed to uphold them all communication would collapse, and possibly even experience itself would become chaotic. For this reason we are psychologically unable to believe that the negation of a logical principle is true, and thus the explanation tacitly assigned for logical necessity is that it is really a species of *psychological* neces-

¹ R. Carnap, *Logical Syntax of Language* (London, 1937), section 17.

sity. The fact that the appeal here is to psychological necessity shows that objective necessity has been abandoned, for if the latter had been admitted there would have been no need for the former; and because objective necessity is abandoned we have the principle of tolerance in fact if not in appearance. It should be added that, as stated in 4, the only alternative to the theory that logical necessity is objective would seem to be the 'rule' theory. Thus the argument against the 'rule' theory in 4 applies not only against the principle of tolerance in its usual form but also against the Kantian variant.¹

The general position of logical empiricism rests upon a criticism of 'mere speculation', corresponding to the Kantian derogatory conception of 'dialectic'. A mere speculation is a theory or hypothesis so constructed as to avoid the relevance of any sensible evidence either in its favour or against it; yet it is asserted as a genuinely true proposition, i.e. as expressing a fact, and so, it is said, it comes into an absurd collision with the opposite speculation, since there is no way of deciding between them. A logical principle, it is said, is like a mere speculation in so far as no sensible evidence is relevant to its truth or falsehood, but is unlike it in so far as it is explicitly not to be regarded as properly a proposition or true; there is no fact determining whether it is true or false, but on the contrary it is freely asserted for extraneous reasons such as convenience or psychological stress. Nevertheless, it is said, there is a trivial necessity associated with a logical principle, in so far as, having decided to use a certain word in a certain way, this use must be upheld in subsequent discourse in so far as the principle in question remains a *logical* principle. Evidently, habit is not enough to ensure correct reasoning. This is here to be interpreted as meaning that our former decision to use words in a certain way brings about our present use of them in that way, only through our present awareness of the former decision. This awareness may be disguised, e.g. we may think we are contemplating ontological principles instead of merely remembering past decisions. Let us call the awareness involved here 'verbal awareness'. When a proposition is 'true' in virtue of 'our decision to

¹ Cp. C. I. Lewis, 'A Pragmatic Conception of the A Priori' in *Readings in Philosophical Analysis*, ed. Feigl and Sellars, New York, 1949. This 'pragmatic variant' is closely similar to the 'Kantian variant'.

use certain words in a certain way' it is called 'analytical', and it is clearly only by verbal awareness that we can ascertain analyticity in a given case. The whole point of this way of defining 'analytical' is that it makes a logical principle trivial, depriving it of ontological status.

(b) *Failure of the argument that a logical principle is trivial.*

However, something like the Law of Identity is implied by the very idea of the 'analytical', for an analytical proposition is one of the propositions contained in the conjunction of the definitions of the words occurring in the proposition, so that the step from the conjunction of definitions to the analytical proposition itself involves the logical principle '(p and q) implies p'. Thus the alleged triviality of an analytical proposition *assumes* that at least one logical principle is trivial. This illustrates a general method of suggesting the triviality of logical truth, viz. by reducing all logical truths to one which seems more obvious than the others. For instance the 'tautology' definition of logical truth similarly reduces all logical truths to one, viz. EM (or its analogue for a many-valued logic); for EM (or its analogue) is the obvious basis for the construction of truth-tables, the limitation of all the possibilities to a certain number being precisely a reflection of the limitation of alternatives to two in EM (or correspondingly for an analogue of EM). In any such reduction the status of logical truth *as such* (whether analytical, synthetical, etc.) is unaffected, since one logical truth remains unreduced and its nature undecided. To infer triviality is to beg the issue in regard to this remaining logical truth. Moreover, if we pay attention not so much to a particular reduction, but to the fact that no reduction would seem to be capable of reducing all logical truths to *nothing* rather than to *one* logical truth, the inference surely is that logical truth is fundamentally irreducible, and that awareness of it must at least be an irreducible awareness of fact, making it synthetical rather than analytical. Thus the logical empiricist's use of words like 'analytical' and 'tautological' to imply triviality is wholly illegitimate because question-begging.

This conclusion is further verified by considering the way in which the undoubted psychological attractiveness of the idea of triviality (in explanation of the necessity of logical truths and

principles) arises. To the layman, the standard of truth is empirical truth, not only because empirical objects constitute the centre of his interest, but because empirical things are normally the only ones upon which he is required to form a judgment. It is by *contrast with empirical truths* that the truths of logic seem trivial. This means not that the layman does not *use* any truths beyond empirical truths, but that normally he has no awareness of them because his interests are elsewhere. But if someone draws his attention to them, he at once senses their obviousness, which he interprets as triviality because the work of judging upon evidence is not required. Nevertheless this obviousness also shows that a judgement has been made, and the felt triviality is therefore a matter of psychology and not logic, arising from a contrast between the ease with which a logical truth is admitted and the relative difficulty of making an empirical judgment, not from the observation of any intrinsic character of the logical truth. Consequently the characteristic obviousness of a logical truth to the plain man in no wise shows the presence of an intrinsic character of triviality, but only shows, what we know already, that logical truths are known with certainty and independently of experience.

According to logical empiricism, a logically true statement records a mere decision to use words in a certain way, and it is for this reason that both these and mathematical truths are claimed to be analytical, not synthetical. This alleged ground for analyticity is really distinguishable into two separate grounds: (a) the assumption, already noted, that the proposition is a component in the conjunction of definitions of the logical words in the proposition, (b) the assumption that the so-called definition is verbal, a 'mere decision'. The conception of triviality is by implication applied to both grounds whenever they are present. We have seen that, in the case of the ground (a), there is really no indication of triviality in any logical sense, but rather a strong indication that logical necessity is indefinable, hence ontological and synthetical. Now in the case of ground (b) there is not indeed any indication that the logical necessity so defined is ontological, but there is a direct and immediate inference that it is synthetical. For whether we say that the decision is effected for convenience, by psychological necessity, or by an arbitrary act of will, it is certainly determined by a principle *extraneous to*

any presupposed meanings of the logical words, since these meanings are supposed to be given only by the definitions, and thus only after the decision has been made. This corresponds exactly to the meaning of 'synthetical' as established by Kant, indeed the whole situation is logically identical with the situation as envisaged by Kant, when he claimed that transcendently necessary propositions are *determined as true by the mind* (as a condition for the possibility of any experience), and so are synthetical. Consequently the logical structure of the view under discussion directly implies that a necessary truth, whether a mathematical proposition, or more basically a logical principle, must be synthetical and so cannot be logically trivial, even if an analytical feature is present too. Although the more basic necessary truths may conceivably be divested of the apparently analytical feature, which is merely the step from the conjunction of definitions to a particular one, the synthetical feature is never eliminable, since this is ascribed to the definitions with which we begin. If we think of a definition as a 'mere decision, not a proposition' this makes the conception of triviality seem applicable even in respect of ground (b). But if it is, this in no way upsets the demonstration that the logical empiricist view is Kantian in its formal structure; and 'trivial' as so applied cannot have the logical sense of 'analytical' but still implies 'synthetical'. The logical empiricist argument is thus seen to be based upon a radical ambiguity of the word 'trivial', and once this ambiguity is resolved the strange result emerges that the premises of the argument imply precisely the opposite conclusion from the one intended.

This incidentally shows the worthlessness of appealing to the stock argument, against Kant, that mathematical propositions have now been proved analytical. Either this falsifies logical empiricism too, or else it leaves Kant unscathed.

(c) *The positive, intuitive aspect of the 'analytical'.*

The general thesis of logical empiricism can be stated in two stages: firstly it is claimed that no proposition is factually meaningful (i.e. can have that characteristic relevance to facts which, in taking it to be assertable, we suppose it to have) unless there are means relevant to the determination of its truth or falsehood. This point goes little further than EM: if anything *is* a proposition

it is either true or false, and it is idle to assert in the abstract that it is true or false if there are no means whatsoever of moving towards a decision. For the sake of argument I shall therefore concede that this is a *logical* point. It is important to see that the word 'logical' here implies a contrast between intuition and mere speculation, where 'intuition' indicates that immediate discernment or awareness of truth which Descartes expressed in the phrase 'clear and distinct ideas'. The means of deciding whether a proposition is true or not must ultimately be some form of immediate awareness. The decisiveness of intuition or immediate awareness stands over against the indecisiveness of mere speculation. Again, it is important to see that the word 'analytical' has a positive sense which is precisely the meaning here given to 'intuitive'. It is this positive aspect of the meaning of 'analytical' which expresses and justifies the logical empiricist's opposition to mere speculation and to the meaninglessness arising therefrom.

(d) *The negative aspect, and its destruction by the positive aspect.*

The second part of the logical empiricist's thesis can, however, by no stretch of the imagination be counted as a *logical* point. Here the attempt is made to set up a *criterion* for decisiveness and a corresponding *limitation* of intuition to certain forms, viz. to *sensible* awareness primarily, but including *verbal* awareness too, this being probably regarded as a special case of sensible awareness. But in the first place, a criterion for the limitation of intuition would have to be *a priori* in relation to all intuition, so could not itself rest upon a particular intuition; it would have to rest upon mere speculation in the end, and in fact criticism of logical empiricism has continually brought this out. That this is not a point to be settled by laying down a definition is shown by the fact that a question of existence is involved; the logical empiricist restricts the field of facts along with the restriction of intuition to the stated forms. Unfortunately the word 'analytical' also gives expression to this part of the thesis of logical empiricism since, owing to the restriction in the field of facts there is simply *no* fact such as a logical principle appears to express, with the result that a logical principle is trivial and 'analytical'. But this aspect of the meaning of 'analytical' is negative, and is clearly distinguishable from the other, positive, aspect of the meaning.

And curiously enough the positive meaning discredits the attribution of the negative meaning. For if the opposition of the analytical to the merely speculative rests upon the factor of immediate awareness or intuition in the analytical (as it surely does), then this element in the analytical must exclude the speculative elimination of certain existences, i.e. it must exclude the triviality of analytical propositions which results just from such elimination of existence.

This shows, then, that the first part of the logical empiricist thesis overthrows the second part, and that the argument which in general justifies the first part by characterising it as logical, *de facto* destroys the second part as well as denominating it 'non-logical'. The primary principle underlying logical empiricism does not militate against metaphysics as such but only against a merely speculative metaphysics. It cannot militate against a metaphysics which is intuitive or 'analytical' in the positive sense. Moreover it cannot be said that there is a secondary principle in logical empiricism which repairs the deficiency. For the primary principle positively supports a metaphysics which is intuitive, at the same time destroying the secondary principle of logical empiricism. And in case anyone should claim that in point of fact a form of metaphysics which is characteristically intuitive in method has not been found, we have only to point to the intuition of logical principles, which leads directly to a metaphysics or ontology whose subject-matter consists of facts beyond either the facts of sensible awareness or the facts of verbal awareness. This kind of metaphysics can be countered only a pitiful speculation which is demolished by the very means used to justify it, and which supports the kind of metaphysics in question by its implicit dependence upon EM (see (c)).

8. LOGICAL CONCEPTUALISM

'Logical subjectivism' can be defined as the view that logical constants are not realities, not real forms of structure affecting everything in the universe, but are nothing but concepts in the mind, or worse still nothing but words. In that case although, in accordance with the foregoing arguments, logical constants might be admitted to be logically prior to logical principles, and although logical principles would accordingly express facts about the

constants, these facts would be trivial ones concerning certain contents of the human mind or of language, not significant facts about everything whatsoever. 'Logical conceptualism' can be defined as that form of logical subjectivism which interprets logical constants as concepts, and 'logical nominalism' as that form of it which interprets logical constants as mere words.

If we can distinguish a logical nominalist view in practice from logical empiricism, presumably it must be based upon the theory of incomplete symbols. A logical word would not be a complete symbol, it would be a connecting word by means of which other complete or incomplete symbols are transformed into one complex complete symbol. Only a complete symbol, such as a proper name, a simple sentence, or a compound sentence, has significance by itself. 'If the sky is clear, there will be a frost' has meaning by itself and is a complex complete symbol, so have 'The sky is clear' and 'There will be a frost'; but 'if', 'is' and 'will be' are connecting expressions, incomplete symbols having no meaning by themselves, but only contributing meaning to the complex symbols of which they are constituents.

Words like 'if' and 'or' cannot *merely* connect, since the meanings of 'if p then q' and 'p or q' are distinct. One would think that they could *contribute* to the meaning of the corresponding compound proposition only by indicating certain features of the fact which would make such a proposition true, and then e.g. the feature indicated by the use of the word 'if' would be different from that indicated by the use of 'or', and so distinguish the meanings of 'if p then q', 'p or q'. This suggestion appears to be combatted by the theory of truth functions, according to which 'p or q' is a general symbol for any situation in which either p or q is a fact, and 'if p then q' is a general symbol for any one of the following situations: those in which both p and q are facts, those in which only q is a fact, and those in which neither p nor q is a fact. According to this, the function of the connective 'or' (and of any other connective such as 'if' in so far as 'or' occurs in its definition) is not to indicate any feature of the facts, but to indicate generality in our use of symbols. It seems however that there is no parallel argument for the use of 'and', since according to the theory of truth functions 'and' indicates conjunction of facts, and a conjunction of facts is obviously itself a fact distinct

e.g. from either of the constituent facts. Again, it has been shown that the use of 'if' in logic is not always reducible to the sense of material implication, i.e. not always to the truth-functional sense.

Logical conceptualism would normally go along with the treatment of logical constants as ideal beings (I, 7), but it does not imply that they *must* be treated as ideal beings, since a concept could be regarded as existing in the same sense as other things, even though it be a very special sort of thing. The point of the theory is that logical principles are not, as their explicit statement suggests, facts about *all* things whatsoever, but are facts essentially about concepts, and therefore trivial because signifying something not about things in themselves but only about the ways in which we think about things. This implied Kantian standpoint results in a duality between appearance and reality. Things as they are in themselves are not *really* self-identical, distinct from others, subjects of properties, etc. These forms which they seem to have are only appearances. At the same time, the forms are more real than anything else, since they cannot be thought away, although the rest can be thought away. This for instance is one of the ways in which Kant arrived at the *a priori* conception of space. Consequently the reduction of the forms to mere appearance reduces everything else in experience, imagination and thought to the same status, and although we may posit a 'matter of sense' pointing to things in themselves, we cannot disentangle it from the forms of sense. If logical constants are treated as such Kantian categories, logical principles would be reduced to mere appearance, and since everything conforms to these principles and since they are more certain than anything else, everything would be reduced with them to appearance.

The trouble with this theory is that on the one hand it reduces itself, along with everything else, to the status of appearance, especially on the grounds that a *theory* is of the nature of thought and openly appeals to logical principles for its support; but on the other hand, like all *bona fide* theories, it claims that things *are* as it states, i.e. that things in themselves accord with its descriptions. From the latter point of view, it is assumed for instance that mind *in itself* is related in a certain way to reality *in itself*. To deny this would be simply to set the theory aside. The two points of view, embodied in one and the same theory, are clearly

contradictory; no forms of thought apply to things in themselves, yet the form of this very thought is assumed to apply to things in themselves.

These arguments also apply, I think, against the view which I have called 'logical nominalism'. If the latter does not construct a subtle theory about the relation between appearance and things in themselves, at least it makes many positive claims about words and symbols, assuming thereby that all the logical constants used and the principles appealed to occur as stated, and so are not mere words. In particular there is that vital *negative* assertion, common to logical nominalism and logical conceptualism, to the effect that logical constants are *not* real. As later discussion will bring out more clearly,¹ this posits a negative fact of which the logical constant *negation* is taken to be a real form.

9. THE MATTER OF LOGIC

(a) *Vacuous occurrence.*

So far I have considered what is by far the more important issue concerning the ontological presuppositions of logic, viz. whether that *general* aspect of logic indicated by the expressions 'logical constant' and 'logical principle', is presupposed to be real. In this final section I consider whether logic presupposes the existence of any subject-matter.

It is really this that was denied by the older empiricist criticism on the basis of the nature of a hypothetical proposition, though the denial was mistakenly generalised to cover all ontological presuppositions whatsoever. But when we come to consider the nature of a hypothetical proposition more exactly, it will be found that even the denial of the presupposition of a *matter* of logic is a mistake.

The vital point depends, as before, upon the idea of vacuous occurrence. Consider again the hypothetical 'If all men are mortal and Socrates is a man, then Socrates is mortal'. In calling this a 'logical truth', we mean that it is true independently of the facts that the particular things and properties referred to are what they are. This of course means that they could be anything else, but it does not mean that they could be nothing at all. In place of Socrates we could have Churchill or Fido, and the

¹ III, 4, and IV, 8 (a), (b).

proposition would still be true, but if it is true it must be about Socrates or something else in his place. Consequently when we generalise, e.g. with respect to Socrates, it is implied that there must be some values of the variable x : '(x) (if all men are mortal and x is a man, then x is mortal)'. This is logically true because the function is true for any one of the presupposed values of x . It is true for any one of them because, in accordance with the theory of vacuous occurrence, it does not matter *which one* we take. But this presupposes that there *is* at least one to take, and the whole theory of vacuous occurrence would be reduced to nonsense if we replaced 'it does not matter which one we take' by 'it does not matter whether there are any to take'.

The situation here is the same as the situation in the case of an abstract system. The kind of generalisation involved is generalisation with respect to the particularity of given existences, not with respect to the feature of existence as such¹. The relation between an abstract system and a model or interpretation of it can be understood only in terms of vacuous occurrence. Any abstract system with which we actually operate *is* either a model or interpretation or some other kind of example of the 'abstract system'; the system is called 'abstract' not on account of any feature intrinsic to the system, but because of the way we treat it, viz. by disregarding all features additional to the purely abstract aspect. For this reason, if a theorem follows from the axioms in the 'abstract system', the truth that it does follow from them is independent of these additional features of the system, and so holds equally for *any* example, e.g. for a model or interpretation. The principle justifying generalisation here is the same as the principle justifying generalisation in the case of vacuous occurrence. Formalistic logic is occupied from first to last with actual systems of existing elements, and falls to the ground if the abstraction involved is supposed to be not only abstraction from particularity but abstraction from existence.

It seems to me that this argument from the nature of vacuous occurrence is enough to prove that logic presupposes the existence of a matter. But the mistake of confusing abstraction from particularity with abstraction from existence is so widespread that it is worth considering it in other contexts relevant to the issue of the presupposition of a matter of logic.

¹ II, 3.

(b) *Hypotheticals.*

The first concerns hypotheticals of the form 'if anything is an A it is a B', which are said 'not to imply existence'. This means first of all that such a proposition does not imply that there is something which is an A, or that there is something which is a B. But sometimes it seems to be supposed that the hypothetical does not pre-suppose the existence of any subject-matter at all. This seems to me quite mistaken. The natural meaning of 'anything' is 'any one of the given class *things*', which would presuppose at least the existence of a universe of discourse, or in symbolic logic the class of the values of the variable. If we try to avoid this presupposition, we have to interpret the hypothetical in some such way as 'If there is something and it is an A, then it is a B'. The issue here does not depend upon the hypothetical form of proposition, but upon the form of the so-called 'existential' proposition 'There is something which is an A', symbolically ' $(\exists x) x$ is an A'. If this means that, of the class of existing things or *x*'s, at least one of them is an A, then existence (of the class *things*) is implied, and the hypothetical as well as the existential presuppose existence. But if it means that the very existence of that which is classified as an A is explicitly *asserted*, then neither the hypothetical nor the existential *presuppose* this existence.

But the whole question is whether we *can* significantly assert the existence of anything without presupposing the existence of something.¹ Suppose firstly that such an assertion were directly the assertion of the existence of a member of the kind or class A, not a complex assertion involving first an indefinite assertion of existence and afterwards the classification of one of the resultant existing things as an A. Such an assertion would have to concern the class A as a subject. There would not yet *be* anything else for it to be about. But if we *have* to begin with a class A, then the existence of A is presupposed. Therefore if the presupposition of existence is to be avoided there must be a more basic, indefinite assertion of existence which is not *about* anything at all. It seems to me clear that there is no such assertion and that there is nothing either in ordinary thought or logic to show that there is. Moreover

¹ The whole argument of this section (9) is to be compared with that of I8. I am saying that the *basic* sense of existence (sense (1) is *non-propositional*).

the very idea of absolute nothingness from which an assertion could conceivably begin is self-contradictory. Nothingness and something are opposite determinations, so each is determinate and therefore a *something*. So what we imagine to be nothing is a determination, an existence. The symbolism ' $(\exists x)$ ' is to be interpreted as indicating generality, not existence. ' $(\exists x) \phi x$ ' means not that there is an x and it satisfies ϕ , but that ϕ is true of at least one of the given values of x . The latter is the proper meaning of 'For *some* x , ϕx '. The quantifiers ' $(\exists x)$ ' and 'some' are strictly particular, not existential, i.e. they select some member (part) of a given class, presupposing the existence of such members. 'Some' indicates generality with respect to particularity, not with respect to existence.

Although particular quantification is *prima facie* open to this double interpretation, the same is not true for universal quantification. Interpreted symbolically, 'If anything is an A, it is a B' is ' $(x) (x \text{ is an A implies } x \text{ is a B})$ ', i.e. 'For all values of x , x is an A implies x is a B'. Following the usual interpretation of universal propositions as hypotheticals, it might be thought that this could be rendered 'If anything is a value of x , then x is an A implies x is a B', thus avoiding the presupposition that there are values of x . But symbolically this is ' $(y) (y \text{ is a value of } x. \text{ implies: } x \text{ is an A. implies } .x \text{ is a B})$ '. Similarly, to avoid the presupposition that there are values of y we would have to introduce a third variable z , and so on *ad infinitum*. Thus it is one important function of the variable and its values, at least in the case of universal quantification, to mark the point at which we decide to question existence no longer in view of the fact that at *some* point or other the decision to presuppose existence has to be made. There remains the possibility that universal quantification might be defined in terms of particular quantification, but this is discussed below under (c).

(c) *Falsehood and negation.*

Doubt about whether a proposition presupposes existence is most serious in the cases of falsehood and negation. Thus 'The present king of France is bald' is false (or its negation is true), but this is due not to the fact that there is a present king of France who is not bald, but to the fact that there is no present king of

France. Since *non*-existence is one way of accounting for falsehood and for negation, it seems *prima facie* unplausible to hold that *existence* is presupposed by false and negative propositions. The question here is whether a negative existential proposition itself presupposes existence. But a negative existential proposition is equivalent to a universal negative proposition; $\sim(\exists x) \phi x$ is equivalent to $(x) \sim \phi x$.

If we take the latter form as basic, then negative existential propositions become definable in terms of negative *singular* propositions of the form $\sim \phi x$, and these directly presuppose existence. What this amounts to in practice is that although e.g. there is no king of France, there is at least a country France and people who *could* reign over France. And if in turn we deny that there is a country France, we could do so only by referring to more ultimate *x*'s which lack a certain description. We cannot expect this regress to proceed *ad infinitum*, since a negation is ultimately made effective only by strictly contradicting some affirmative proposition. But contradiction is possible only because an affirmation and a negation are applied to the *same* subject-matter. Thus in the regress we eventually reach negations which deny something concerning actual existences.

Sometimes, however, the form $\sim(\exists x) \phi x$ is taken to be more fundamental than $(x) \sim \phi x$, in so far as ' $(x) \phi x$ ' is sometimes defined as ' $\sim(\exists x) \sim \phi x$ ', suggesting that ' $(x) \phi x$ ' is merely a *shorthand* for the more basic ' $\sim(\exists x) \sim \phi x$ '. But in this case the shorthand is hardly more simple than the original expression, and the latter is not really so complex as to require a shorthand at all. The fact is that universal quantification is needed not as a symbolic device, but to give the cash value of a negation of existence. The meaning of ' $\sim(\exists x) \phi x$ ' is not made clear until we see that it requires the negation of *every* proposition of the form ϕx .

The conclusions of this chapter can be summed up as follows. Logicians commonly claim that in logic we are in no way concerned with the subject-matter of propositions and arguments. But this betrays a confusion between two different sorts of subject-matter and between two ways of presupposing a subject-matter. The only truth in the common view is that an *empirical* subject-matter is not presupposed *essentially*. But on the one hand an empirical

subject-matter is presupposed *vacuously*, and on the other hand a *non-empirical* subject-matter (logical constants) is presupposed essentially. Associated with the latter type of presupposition there is an intuition which is both ineradicable from logic and of fundamental importance to logic despite efforts to replace it by mechanical procedures.

CHAPTER III

NON-EXISTENCE AND UNIVERSALS

I. HISTORICAL BACKGROUND

It is well known that Russell's theory of descriptions on the one hand offers a plausible solution to certain traditional problems concerning the logical forms of existential propositions and identification propositions, and on the other hand avoids assuming the existence of classes and, with reservations, the existence of universals. Here I shall be concerned, on the one side, only with the problem concerning existential propositions, not with the one concerning identification propositions. This problem, which I shall call the 'paradox of non-existence', is that a denial of existence seems to presuppose, as its subject, the existence of the very things whose existence it denies. On the other side I shall be concerned with the denial of the existence of universals, not with the denial of the existence of classes. I have not selected these aspects of the theory of descriptions arbitrarily. The selection has already been made by writings which at least to some extent are inspired by the theory of descriptions. I refer particularly to Professor Ryle's celebrated article, *Systematically Misleading Expressions* (Sections I and II),¹ to Professor Quine's article, *On What There Is*,² and to the subsequent symposium of the same title.³ The reason for the association between universals and existence in this literature is not hard to find. On the one hand Ockhamism, in the shape of nominalism, finds in universals a very special enemy, because universals are traditionally one of the main bulwarks of metaphysics. On the other hand, Russell's solution of the paradox of non-existence consists partly in eliminating Meinong's 'objects', viz. those alleged entities constituting the logical subjects of which existence is affirmed or denied. At the same time Russell's solution is very markedly logical—it is a logical answer

¹ *Proceedings of the Aristotelian Society*, Vol. XXXII (1931-2).

² *Review of Metaphysics*, Vol. II (1948) and appendix to *Arist. Soc. Supplementary*, Vol. XXV (1951).

³ *Arist. Soc. Supplementary*, Vol. XXV (1951).

to the logical difficulty of self-contradiction. Therefore it is no wonder that the modern nominalist, who is frequently well versed in the more subtle aspects of modern logic, should see in the elimination of universals a natural culmination of the elimination of 'objects', should see in the logic of the latter a logical justification of the former, should even perhaps confuse universals with 'objects'.¹ There is no doubt, too, that the connexion follows to some extent the pattern already laid down in the theory of descriptions.

It is a pity, however, that the nominalist should have been so intent upon hitching his waggon to the purely logical requirements of the paradox of non-existence as to forget to investigate for its own sake the logic of the situation resulting from the elimination of 'objects'. If more attention had been paid to this situation, a very different conclusion would, I am persuaded, have been reached. In fact I am going to argue not only that the elimination of 'objects' does not point to the elimination of universals as its logical continuation, but that it implies the existence of universals, and so makes their elimination a logical absurdity. To this end I shall once more trace the path which has its logical beginning in the paradox of non-existence.

2. EXISTENCE OF A SUBJECT-MATTER

The paradox of non-existence is most simply stated by saying that, in so far as a negative existential proposition seems to be about the very object or objects denied existence, it presupposes their existence. A person who says that there is no Loch Ness monster seems to be talking about the Loch Ness monster, and a person who denies the existence of unicorns seems to be denying *their* existence. There have indeed been ways of parrying the admission of a serious paradox, the principle one being to deny that a proposition does necessarily presuppose the existence of some subject-matter which it is about. Although it has already been shown that all propositions presuppose the existence of a subject-matter,² it is worth considering here one important view which denies this result.

¹ The description of Meinong's 'objects' as 'Plato's beard' seems to me very near to such confusion. Plato is popularly associated with the admission of universals, but his theory of negation towards the end of the *Sophist* is a way of dispensing with 'objects'.

² II, 9.

The conclusion that a negative existential proposition presupposes the existence of a subject-matter is seriously threatened by Meinong's theory, according to which only 'objects' are required as subjects for propositions, and 'objects' might or might not exist—are beyond being and not-being.¹ So far as I know, Russell does not set out to refute this view, he merely opposes to it a 'robust sense of reality',² taking it for granted that if an account of non-existence is possible on this securer basis, it will naturally be more acceptable than Meinong's account. But Russell's attitude is hardly satisfactory, since Meinong's view is backed by a criticism of a theory like the theory of descriptions. This criticism I shall discuss later. Meanwhile Meinong's positive position can, I think, be refuted on the grounds that it is thoroughly ambiguous, confusing the existence of a thing with some property of it other than existence. Consider a device which the plain man might well use when faced with the paradox of non-existence. Admitting that it is not the existing gold mountain which does not exist, he adds that of course we are only talking about the idea of a gold mountain. If he is not careful, he will fall into the trap of saying that it is really the idea, not the real thing, which does not exist. But this is obviously wrong, so long as we keep to the same meaning of 'existence'. For he resorted to the idea precisely because, unlike the non-existing thing, the idea *could* be talked about because it existed. That the idea (cf. Meinong's 'object') is beyond existence and non-existence, therefore, turns out to be only a pretence, and if we persist in expressing our example in the form 'The idea of the gold mountain does not exist', then this asserted non-existence of the idea comes to mean not that there just isn't such an idea, but that it lacks some (relational) property such as that of being embodied in the world. This is exactly the fate of Meinong's 'object'. Because he insisted that it was beyond existence and non-existence the meanings of these words were destroyed, and instead he was talking misleadingly about very wide properties of things instead of their existence.

¹ See Findlay, *Meinong's Theory of Objects* (London, 1933), p. 49.

² *Introduction to Mathematical Philosophy*, p. 170. I have since noticed, however, that Russell does give a criticism of Meinong about half way through the final essay, 'Knowledge by Acquaintance', in *Mysticism and Logic*, but he still seems to be unaware of Meinong's implicit criticism of the theory of descriptions.

As a corollary, the alleged different orders of being (existence, subsistence, etc.) collapse, as being only mis-named properties. My contention, then, is that Meinong's theory is not the theory that it intends to be. It is a translation theory in so far as the very conception of existence has disappeared from it, but this feature of it is heavily disguised by the retention of the word 'existence' and other ontological words to apply to the new non-ontological properties. Viewed as the translation theory which it is, Meinong's view falls within a class of theories to be discussed later.

At least a true proposition, then, must be credited with an existing subject-matter to which it somehow refers, and what makes the proposition true is a certain fact concerning this subject-matter (or having this subject-matter as a constituent). The purpose of translating a negative existential proposition, in the way, e.g. that the theory of descriptions demands a translation, is not to *remove* the subject-matter, but to replace what appears to be the subject-matter by a different subject-matter which does not involve paradox.

3. THE FACTUAL FORM OF THE PARADOX

To talk in terms of existing subject-matter and fact is not to make a *naïve* inference from propositions to an alleged reality beyond them, it is merely to take account of the 'inside view' of a proposition and of what is meant by an admission of its truth. When someone asserts a proposition he is not talking about propositions, he is talking about things, and to say that the proposition is true is simply to say that the things are as he states, i.e. it is simply to repeat the proposition with emphasis. Again, to say that there is a fact making the proposition true is only to say that the things are as he states, i.e. it is only to repeat the proposition emphatically. So the admission of truth is quite inseparable from the admission of fact.

But if you insist upon taking up a thoroughly sceptical position, if you insist that the most we can be certain of is that language or thought have a certain form and never that facts have a certain form, my reply is that such a view is self-contradictory. For what the view maintains, if it maintains anything, is that there actually need not be facts, that the forms of language are so indifferent to facts (if there are any) that they need contain no clue at all to the

nature or existence of any facts—all this and probably much more is maintained *as fact*. It is obviously self-contradictory to regard as justifiable a view which maintains as fact that nothing can justifiably be maintained as fact.¹

Associated with the problem concerning negative existential propositions there is therefore a metaphysical problem concerning facts of non-existence, and the latter, I venture to state, is the more important. Facts of non-existence are specific; they are the non-existence of something, e.g. of satyrs. But, as a constituent of a fact, satyrs must be existing satyrs. It therefore seems that the fact of the non-existence of satyrs has existing satyrs as an indispensable constituent, that non-existence demands the existence of the very object or objects which do not exist. Just as a negative existential proposition must be translated in such a way as to avoid mentioning that very thing which it asserts not to exist, so the only possible structures for a fact of non-existence are such as do not possess the non-existent object as a constituent. The translations free from logical difficulties are nothing but representations of these possible structures of the fact.

4. NEGATIVE FACTS

Facts of non-existence, and the larger family of negative facts to which they belong, are among the least reputable of the entities of philosophical discourse. One characteristic criticism is that whereas a positive fact can be admitted as one which makes an affirmative proposition true, the truth of a negative proposition is simply the falsehood of the opposite affirmative, and that a proposition is false means simply that there is *no* fact making it true. So no fact, not even a negative fact, seems to be required in the case of a true negative proposition. The fault with this criticism is that it cannot say consistently what it intends to say. It intends to say that there is no fact making a negative proposition true, but the condition that there must also be no fact making the opposite affirmative proposition true is an indication of the state of affairs required to make the negative proposition true, and this state of affairs is, of course, the *fact* which on this theory makes the negative proposition true. More specifically, the theory avoids the admission of a fact making 'A is not B' true by pro-

¹ See II 8, and IV 8 (b).

posing an alternative condition for its truth, viz. the condition that 'A is B' must not correspond with any fact. It fails to notice that this is merely to substitute for the simple negative fact concerning A another and more complex negative fact concerning the relation between 'A is B' and the facts.

I suspect that the motive for any theory of negative fact in terms of propositions is to explain away negative facts. It is taken for granted that a proposition is merely language, or a mere thought, and therefore a mere nothing. Now if propositions are really "merely subjective", this means only that they are liable to be false and are not the same things as the facts which they claim exist. But if they are false and different from these facts, this proves not their non-existence but their existence. And even supposing it true that they were somehow non-existent, this would show that no theory could explain negative facts or anything else in terms of propositions, for there would be no such terms and consequently no explanation. Perhaps the critic who believes that he has so explained away negative facts would hedge; perhaps he would say that propositions do exist, but not in the full sense—they depend for their existence upon mind or language. *Prima facie* such a move does make the view respectable, and I shall subsequently consider theories of this kind. For the moment I want to draw attention to what is brought to light by this concession. A theory of negative facts in terms of propositions cannot be taken as immediately explaining away negative facts, it can be accepted only for what it is, viz. the claim that a negative fact has a certain internal structure, and that propositions are indispensable elements in its structure. From this it will follow that if a proposition suffers from certain limitations regarding its existence, e.g. dependence upon a mind or upon language, then a negative fact will suffer from at least those limitations. Conversely, if it is manifest that a negative fact cannot suffer from existential limitations of a certain kind, and if certain entities such as 'propositions' or 'concepts' have been shown to be indispensable constituents of negative facts, it will equally follow that neither can these constituent entities suffer from the said limitations.

5. THE ARGUMENT FOR UNIVERSALS

I have at last arrived at the point at which the argument for universals can be stated, with a reasonable assurance that the false ways of escape from it have been destroyed. I shall begin with the theory of descriptions, or rather its analogue for facts of non-existence. According to this, the fact, e.g. that unicorns do not exist is the fact that the proposition 'This is a unicorn' is false for everything to which the word 'this' could apply. The paradox of non-existence is avoided because the latter fact does not contain actual unicorns as a constituent, but only certain propositions (whose predicates indicate the property of being a unicorn) and 'everything', i.e. all *existing* things (not unicorns). An awkwardness might arise on account of the *falsehood* of these propositions, for the falsehood of a proposition ordinarily implies that the reason for falsehood lies in a fact distinct from the proposition itself, and in that case propositions would no longer figure in the final explanation; for the set of negative facts, concerning each thing in the universe that it is not unicornish, would then by itself be sufficiently equivalent to the fact of non-existence. The latter theory will have to be considered in its place. Meanwhile the proposition theory can plausibly be saved by a special theory of falsehood, viz. p is false if and only if there is some proposition q which is both true and incompatible with p .

Bearing in mind the previous remarks about negative facts, the theory under discussion is to be regarded seriously only in so far as it is taken to be assigning propositions as some of the elements in the internal structure of facts of non-existence. But what are propositions? One view is that they are essentially linguistic, either momentary assertions of sentences or the longer-term sentences themselves, but in either case dependent for their existence upon the existence of language. Now language is presumably something which would cease to exist in certain circumstances, e.g. if the human race ceased to exist. In that case, of course, propositions would also cease to exist, and with them facts of non-existence, since on the theory under discussion propositions are indispensable constituents of facts of non-existence. But this leads to results not only ludicrous but self-contradictory: if language ceased to exist, the non-existence of unicorns, ghosts, etc. would cease to be facts, and every conceivable sort of mythical

entity would come into existence; even language itself, which by hypothesis has ceased to exist, would come into existence.

A second view of propositions is that they are judgments in the sense of acts of thought dependent upon minds for their existence. Taking minds to be finite, and assuming (what is possible for finite minds) that they all ceased to exist, we reach a series of absurdities exactly similar to those resulting from a linguistic view of propositions. The linguistic and mentalistic views of propositions can be saved only by counting language and mind as eternal. But this in effect is to take a third and quite different view of propositions, viz. that they are eternal entities, in principle universals.

The fact that the only surviving form of the theory of descriptions is that in which propositions must be interpreted as eternal entities, strongly suggests that the whole programme of explaining away universal properties in terms of propositions is misguided. For the aim of this programme seems principally to be, not to dispense with universal properties *qua properties*, but to dispense with them *qua eternal*, i.e. the nominalist objects to universals only in so far as they are understood to be eternal. The significance of the theory of incomplete symbols seems similarly nominalistic in the end, viz. it dissolves away the constituents of a proposition into the whole proposition, which in turn is conveniently conceived as linguistic, an ordinary everyday object having a beginning and an end in time. But all these subtleties lose their point if propositions are inevitably eternal entities, though of another kind from universal properties. We might just as well have been satisfied with exhibiting the structure of a fact of non-existence directly in terms of the universal property corresponding to those things which there are none of. For example, the non-existence of unicorns would be the fact that each thing in the universe is not an instance of the property *unicorn*. Indeed this account is much simpler; the perhaps infinite series of propositions and the relation of incompatibility are replaced by one universal property and the relation of instantiation. But on either choice the end is the same: the indispensable constituents of a fact of non-existence must be eternal, since the reasons already given in the case of the proposition theory obviously apply with equal force in the case of the universal property theory. And because it is just the feature

of eternity which is primarily intended by the word 'universal', it is right to conclude that any theory of non-existence, of the general type exemplified by the theory of descriptions, is bound to entail the existence of universals.

6. THE MORE GENERAL ISSUES OF UNIVERSALS AND NEGATION

So far I have only contended that a certain type of theory normally accepted by logicians and philosophers, even by those of them who are explicitly nominalistic,¹ *implies* the existence of universals. I shall now ask the much wider question, *Do* universals exist? The only way now open for denying that any exist is to deny the truth of any theories of this type, and to offer in their place a fundamentally different account of facts of non-existence. I shall first mention one such possible alternative which I believe is easily disposed of, and then go on to discuss a fundamental objection to any theory of the type exemplified by the theory of descriptions. This objection, so I shall claim, certainly necessitates a radically different attack on the whole question of non-existence, so much so however that the very possibility of an orthodox logic is threatened. Thus the conclusion will remain that anyone adhering to an orthodox logic is committed to the existence of universals.

The first-mentioned alternative could be called 'the exclusion theory'. According to this, the fact e.g. of the non-existence of flying horses is said to be the mutual exclusion of two classes such as flying things and horses. But the theory as so stated would break down for a case in which either class were null, e.g. for the fact of the non-existence of flying unicorns. For an account of non-existence based upon the assumption of a null class obviously *assumes* a theory of non-existence, and the theory as a whole would be rendered either circular or self-contradictory. On the other hand, if the existence of a null class is not assumed, the exclusion theory has to be extended to accommodate cases in which there are more than two mutually excluding classes, e.g. in the example of flying unicorns there would be three classes, viz. horses, one-horned animals and flying things. But this extension of the theory points emphatically to the need, not apparent in the original statement, to formulate general criteria for deciding whether a

¹ e.g. Prof. Quine.

given set of classes does constitute the fact of non-existence which it is alleged to constitute. These criteria clearly are (a) that the classes must be mutually exclusive, (b) that the class properties of the classes of the set must, when conjoined, form the property specifying the fact of non-existence, e.g. the class properties of the classes *horses*, *one-horned animals* and *flying things*, when conjoined, form the property *flying unicorn*. But in consequence of condition (b), the property specifying a given fact of non-existence is indispensable to that fact, and so by previous arguments can be neither a linguistic nor a mental entity but must be eternal. It follows that the exclusion theory can in the end no more escape the implication of the existence of universals than can the theories belonging to the same category as the theory of descriptions.

It will be found that all these various accounts of non-existence possess one important feature in common, viz. they all explain non-existence in terms of *negation*. This would be of little significance if negation in turn were explicable in purely affirmative terms, but the next chapter will show that it is not; for instance incompatibility is definable in apparently affirmative terms by means of identity and implication, but even this method of defining negation has in the end to be qualified in such a way as to require the re-introduction of negation. Thus the thesis that non-existence is definable only in terms of negation is quite genuine. But its consequences are nothing short of devastating. For Meinong has pointed out that the defining characteristic of a negative fact is its *exclusion* of the opposite positive fact i.e. its implication that the opposite positive fact *does not exist*. And this naturally makes a theory explaining facts of non-existence in terms of negative facts circular; the paradox of non-existence is in no wise removed, and is not only not removed but spreads throughout the whole field of negation.¹ We cannot resort to Meinong's solution of the difficulty, for it has been shown that the constructive side of his theory rests upon a confusion of terms. But since so much depends upon the possibility of finding the solution to this wider *paradox of negation*, the following chapter is devoted to a discussion of it.

¹ Findlay, *Meinong's Theory of Objects*, pp. 54-5.

PART II
NEGATION AND UNORTHODOX LOGIC

CHAPTER IV
THE PARADOX OF NEGATION

I. CONCEPTIONS AND THEORIES OF NEGATION

There are, I believe, four conceptions of negation which are fundamental in the sense that any particular view of negation normally takes the form of attempting, explicitly or by implication, to reduce all conceptions of negation to a favoured one of these four. They are:

- (1) the conception of a negated proposition or possibility,
- (2) the conception of opposition,
- (3) the conception of difference,
- (4) the conception of non-existence or non-being.

It will be convenient to use the letters P, O, D, E in reference respectively to these conceptions and to the theories favouring one or other of the conceptions.

My argument is that each of the conceptions P, O, D proves to be worthless as a definition of negation or as an effective means of understanding the nature of negation; that only conception E is capable of this, and that accordingly an adequate theory of negation can be given only in terms of E; but that this need to resort to conception E as the basis of the conception of negation results in a scandalous paradox, called here the 'Paradox of Negation'.¹

A theory of negation is often stated in the form that negation *means* so-and-so. But since a given term notoriously varies in meaning from person to person, it is implied that there is some focussing point which makes a standardised meaning possible. The focussing point is the conception of negative fact, which I define as the actual state of affairs which makes a true negative proposition true. Why do we confidently apply the ordinary logical

¹ Subsequently 'PN'.

tests even to a theory of the *meaning* of negation? The reason surely is that we always regard the facts themselves as conforming to these tests, and that in consequence we admit only those forms of propositions which, by conforming to the tests, *could* be true. A meaning given by some other form of proposition would be only of psychological interest.

One of these tests is so vital, and has been so widely overlooked by those who have seriously discussed the nature of negation, that I cannot do better than state it with emphasis at once, with an eye to its frequent and deliberate application. I call it the 'test of the Law of Con-Contradiction'.¹ The NC test is the application of NC to a proposed conception of negation with the object of ascertaining whether the said conception is genuinely negative, e.g. if *otherness* were given as the meaning of negation, one would apply the NC test by insisting that A's being other than B must exclude A's being the same as B. This illustration might suggest that the NC test is too trivial to be worth stating. Such an impression however is very misleading, so much so that many widely accepted theories of negation have been able to persist only because this obvious requirement has remained unformulated.

2. PROPOSITIONS AND POSSIBILITIES

Perhaps the most popular of all views of negation is the one founded upon the P conception. According to this, what is negated is fundamentally a proposition. We can signify any affirmative proposition by 'A r B', including relational as well as subject-predicate propositions. The corresponding negation 'A not-r B' does not of course negate 'A r B' in the sense of negating its *existence*. It negates its *truth*. It means that 'A r B' is not true, i.e. that 'A r B' does not apply to the facts. Hence the proposed definition of 'A not-r B' is:

'A r B' does not apply to the facts.

First it should be noticed that this *definens* is itself a negation. Next it should be realised that the theory proposes a view not merely of first order negation (involving ordinary things and their properties and relations) but of negation as such. It is felt that an account, definition or analysis is required for negation as such,

¹ Subsequently the 'NC test'.

because the meaning of that simple form of negation expressed by inserting the word 'not' after the verb is not clear as it stands. This simple form of negation may be described as 'relational' in a wide sense which includes the relation between substance and attribute as a special case, i.e. in this simple form the negation negates a relation, of which A and B are the terms. It does not matter what sort of entities the terms are. The point is that the simple form of negation appears to negate a *relation*, whereas the proposed analysis aims at displacing this appearance by the claim that negation really negates a *proposition*, on the ground that what is meant by negating a relation is not clear. The antithesis is not exactly between a second and first order definition of negation but between the application of negation to a proposition and its application to a relation.

But when we come to examine the above proposed *definiens*, we find that although it is a second order definition (because about a proposition) to the exclusion of being a first order definition, it fails to apply negation to a proposition in such a way as to exclude the application of negation to a relation. It applies negation to the special relation of *applying to* in exactly the same way as the original proposition 'A not-r B' applies negation to the unspecified relation r. It is only as a result of mistaking the one antithesis for the other than the definition appears at all plausible.

For the proposed *definiens*, being still of the form 'A not-r B', is as much in need of analysis as the original negation, which is merely *any* proposition of that form, instead of a special case of it. To complete the definition we have therefore to exhibit the meaning of this *definiens*. The stipulated method of doing this is first to specify the proposition negated, and then to deny that it applies to the facts, and this gives:

" 'A is B' applies to the facts " does not apply to the facts.

But this as before is a negation applying to a relation. It is still of the form 'A not-r B', and stands as much as ever in need of analysis. Obviously the process of definition goes on *ad infinitum* without bringing us a step nearer the desired end of eliminating the implied obscurity in the simple form of negation and of showing what the meaning of negation really is. The terms in which the meaning is supposed to be clarified assume that the

meaning is already clear to start with, and therefore that no clarification is necessary.

The theory under review might be expressed not as a theory of the *meaning* of 'A not-r B' but as a theory about the form of *fact* which alone could make 'A not-r B' true, i.e. about the negative fact 'A not-r B'. It would claim that the negative fact 'A not-r B' is (exhibiting its form) the fact that the proposition 'A r B' does not apply to the facts. But the fact that 'A r B' does not apply to the facts must similarly really be the fact that "'A r B' applies to the facts" does not to the facts; and so on. Consequently, by this means, we never do move a step nearer to the exhibition of the form of the fact 'A not-r B', since the very form which is supposed to require exhibition re-appears in the exhibition of it. If we cannot conceive a fact to be simply of the form 'A not-r B', equally we cannot conceive a fact to be of the form "'A is B' does not apply to the facts", for the latter is a special case of the former. Whether we talk in terms of meaning or negative fact, propositions seem to be of no avail at all in helping us to understand negation.

To conceive the negation of 'A r B' as the non-actualisation of the possibility 'A r B' is no more enlightening than to conceive it as the negation of the proposition. In this case, the fact that A has not r to B is really the fact that the possibility 'A r B' is not actualised; but this fact in turn is really the fact that the possibility "'A r B' is actualised" is not actualised, and so on. The real form of the fact is never reached, but instead we are always required to look for it in the next stage of an infinite regress. (This regress, whether for propositions or possibilities, may conveniently be called the 'P regress').

3. OPPOSITION

The theory in terms of propositions or possibilities can be provisionally saved, I think, only by transferring the emphasis from propositions (or possibilities) themselves to the relation of *opposition* between them. If opposition is supposed to be better understood than negation, there is no immediate circularity in defining the latter in terms of the former. The negative fact 'A not-r B' is the fact that the proposition 'A r B' is *incompatible* with some true proposition, or *discords* with some fact. Here

incompatibility and discordance are two species of opposition, but it will for the most part be sufficient to treat opposition as if it were the same as incompatibility, since the argument in the case of discordance is parallel to the argument in the case of incompatibility.

If opposition is to yield an adequate account of negation, then, by applying the NC test, we see the need to impose the condition that two opposite propositions are not both true. Without this stipulation we would be free to maintain, in any given case, that both 'A r B' and 'A not-r B' are true, or are facts. Admitting, in a given case, the fact that not both are true, what is the form of this *negative* fact? According to the O theory, the fact that not both p and q are true is the fact that the proposition 'Both p and q' is opposite to some true proposition, say s. We have now, in turn, to make sure that not both s and 'Both p and q' are true, and so on. As in the cases of the P theory, the process of exhibiting the structure of the negative fact is never ending, and in effect never moves a step nearer its goal.¹

There is in fact a way of escaping this infinite regress, but only at the expense of a complete departure from the O theory. As a first step, 'not both p and q' means 'Only one of p, q is true'. The word 'only' need not be interpreted in explicitly negative terms, but in a way similar to the way in which the word 'the' is interpreted in the theory of descriptions. Let C be the class whose members are just the propositions p, q, and let x be a variable restricted in its range to C, i.e. whose only values are p, q. Let y be any proposition. Then the proposition that only one member of C is true is expressed by:

There is a y such that, for every x, x implies (x=y).

This seems to replace negation by implication and identity, in the definition of 'not both'. It should be noticed, however, that this proposition would not necessarily be capable of a negative meaning if the number of the members of C were not specified, since it does not have a negative meaning in the special case in which C has only one member. The possibility of a negative meaning arises only if C has more than one member, and it clearly

¹ Russell has given substantially this criticism of the O theory in *On Propositions, Logic and Knowledge* (London, 1956), pp. 288-9.

arises, in such cases, from the *numerical difference* which then exists between any one member of C and any other member. It is only when there are members which *are not* a certain given member y, that any *restriction* is expressed by saying that only y is true. Underlying the present proposal for a definition of 'not both' we therefore have an implicit definition of negation in terms of difference, and this case therefore falls under the heading of D theories, not O theories.

The apparently affirmative interpretation of 'only' might suggest a general method of reducing negation to affirmation, viz. by obversion. Thus "No X's are Y's" is not only equivalent to "Only non-X's are Y's" ('only' having reference here to a class, not to a number such as *one*) but also to "All Y's are non-X's". Now it is consistent with the affirmative quality of the last proposition to allow that the negative class X of the class non-X is null, in which case however the proposition would cease to have any negative meaning, since it would cease to imply a restriction upon the class of Y's. Conversely, it has a negative meaning only on condition that there are entities (members of X) *numerically different* from the members of non-X, i.e. this case also properly falls under the heading of D theories.

Before proceeding to the D theories however, it is worth drawing attention to a common type of theory which, in so far as it is at all plausible, reduces to a P theory or an O theory. The negation of 'A r B' is treated as one alternative to 'A r B', or as a disjunction of several alternatives to 'A r B'.¹ In the case of one alternative, the NC test shows that it must imply that 'A r B' is not actualised (or does not apply to the facts), which brings us back to the ordinary P theory. In the case of a disjunction of several alternatives, the question is whether each alternative excludes the actualisation (or truth) of 'A r B'. If not, the NC test fails and this is no genuine theory of *negation*, but if it does we revert again to a P or an O theory. It is of no use to introduce a clause to the effect that the one or many alternatives must be *exhaustive* of all the possibilities other than 'A r B'. It is quite possible to have exhaustiveness without mutual exclusiveness of alternatives, so the NC test still needs to be applied. Theories of

¹ Prof. Price criticises theories of this type in the symposium on 'Negation', in *Proceedings of the Aristotelian Society*, Supp. Vol. IX.

this class are prominent among those which, as mentioned earlier, have survived only by dodging the NC test.

4. DIFFERENCE

The criticism of P theories strongly suggests that the negation of 'A r B' must ultimately have reference to r, A and B themselves rather than to the whole proposition or possibility, since the P theory itself is apparently expressible only as the negation of a relation (*applying to*) between two terms. The D theory is an essentially first-order theory which does treat negation as applying to a certain relation, viz. identity, and which further treats the result of negation as also a relation, viz. numerical difference. The criticism of O theories also suggested the need to consider the D theory.

Traditionally, numerical difference or otherness has been used by Plato in the *Sophist* to furnish an account of negation. This kind of theory contrasts with second-order theories, and seems to be more appropriate if we have negative fact in mind rather than the thought or meaning of negation. If B is a thing, we would naturally mean by 'A is not B' that A and B are two different things. In the case of a subject-predicate or relational proposition a like view can be upheld by interpreting B as a class, and by treating 'A is not B' as the assertion that A does not belong to the class, i.e. is different from every member of the class. The basic conception remains that of numerical difference between one individual thing and another.

The application of the NC test shows that difference must exclude identity. Evidently 'A is not B' is not properly a negation if it fails to negate 'A is B'. Therefore if the D conception is used in an account of negation, it would also be necessary to use some other conception to supplement it, since by itself it pre-supposes negation so cannot define it.

The conclusions to be drawn in the cases of the P and the O conceptions are parallel to this. The P regress, for example, proves not that the P conception is *not* necessary to the conception of negation, but that some conception other than the P conception *is* necessary to it. One is thus free to try the device of using two of the three conceptions, P, O and D to supplement one another, especially as the application of NC to the D conception throws up

the need to relate 'A is not B' to 'A is B', and so might suggest a return to second-order theories. In my opinion the device is a desperate one. As well suppose that two half-wits could make an intelligent man. Our idea of negation does not seem to be complex in the way which would be required if two different conceptions were needed to give an account of it. The alternative to this view is that a fourth conception is indispensable to negation. That this is indeed correct will be shown independently in the next section—which relieves us of the need to disprove the view that negation is definable in terms of at least two conceptions each by itself insufficient.

5. THE INDISPENSABILITY OF NON-EXISTENCE

In the last section the application of the NC test has shown that the D conception is *not sufficient* to define negation. In the present section the NC test will be applied in such a way as to show that the E conception, i.e. the conception of non-existence, is *necessary* for any adequate definition of negation.

As yet I am not intending to imply any special analysis of the E conception. I do not assume, for instance, the analysis contained in the theory of descriptions, nor on the other hand do I assume the *naïve* paradoxical analysis which takes non-existence to be direct negation of an individual subject such as a given thing or fact. The proper analysis will appear only when, in the next section, the untenable alternatives are eliminated. For the moment, the negation 'A not-r B' is rendered, in terms of the E conception, as "There is no fact 'A r B'". This expression is deliberately selected because it avoids, I think, the presupposition of any special analysis. By contrast other expressions such as "No facts are of the form 'A r B'" and "This fact 'A r B' does not exist" do presuppose special analyses.

The NC test for any theory of the meaning of 'A not-r B' requires that not both 'A not-r B' and 'A r B' are true, and if the truth of 'A not-r B' is given it requires that 'A r B' is not true. Now it is empty talk to claim the untruth of 'A r B' if it is allowed at the same time that there is a fact 'A r B'. Thus given 'A not-r B', it must be admitted that: there is no fact 'A r B'.

Therefore, in so far as NC expresses something essential to the meaning of negation, there being no fact 'A r B' constitutes an

essential aspect of the negative fact 'A not-r B' in respect of its negative quality, i.e. *the E conception is essential to the meaning of negation.*

It might be said by way of objection that since NC can likewise be expressed in terms of the P, the O and the D conceptions, the above argument does not single out the E conception as more fundamental. However it is sufficient for the argument at this point that the E conception is indispensable; it does not matter as yet whether the P, the O and the D conceptions are or are not indispensable to negation. But it is worth anticipating the outcome of the argument as a whole, in order to see what the *relative* status of E, and the P, O and D conceptions will be shown to be. The next section 6 consolidates the present point that the E conception is indeed indispensable to negation. The next vital step in the argument occurs in section 7, where it is maintained that there is only one form of the E conception, viz. the paradoxical form which occurs in asserting the non-existence of a given individual subject. Now propositions and the D conception are necessary only in so far as they provide an escape from this paradox; for instance we need false propositions only because the idea of a non-existent fact is self-contradictory. But once the E conception is seen to be necessary, despite its paradoxicalness, the escapes from it in the P, the O and the D conceptions are seen to be no longer necessary, since not only is an escape not necessary, it is not even possible.

This, I repeat, is to anticipate. It is sufficient for the present stage of the argument to insist that the E conception is indispensable to negation; it is irrelevant to insist that the P, O and D conceptions are dispensable.

6. THE IRREDUCIBILITY OF NON-EXISTENCE

The thesis that non-existence is indispensable to negation would be emptied of importance if non-existence were reducible to one of the other basic conceptions.

Is the E conception reducible to the P or to the O conception? An attempt to make such a reduction forms a part of Russell's theory of descriptions. 'Unicorns do not exist' is reduced to " 'This is unicornish' is false for every object to which 'this' could refer ". In the case of the P conception consider the impli-

cations of asserting the falsehood of 'This is unicornish', where 'this' refers to some particular given object X. The NC test shows, as an implication, that there is no fact 'X is unicornish'. This is in accordance with the conclusion of 5, that non-existence is indispensable to negation. But it is a negation of existence, so according to the theory of descriptions we are to understand it as meaning that "This fact is of the form 'X is unicornish'" is false for every fact to which 'this fact' could refer. But the NC test shows once more that further negative existentials are implied, viz. it shows that for any particular fact Y (to which 'this fact' could refer), there is no fact "Y is of the form 'X is unicornish'". Evidently this regress is unending, and it is plainly a form of the P regress. It shows that it is quite unavailing to try to account for non-existence in terms of the P conception, since after the completion of any stage in the attempted explanation the factor of non-existence remains as securely as ever.

It is valuable, though not essential to the argument, to observe that the present form of the P regress is in all probability basic, since the E conception has been found indispensable to negation. The present form of the P regress shows the probable cause of its infinitude, viz. the stubbornness of the E conception, which necessitates that the attempt to explain it away has to be renewed unendingly. The previously encountered form of the P regress now appears to be nothing but a reflection of the present form, due to our ignorance of the cause of the regress. The cause could at that time be regarded only as *some* extraneous factor, since all that was proved was that *some* conception other than the P conception was indispensable. Since this extraneous conception was not then identifiable with the E conception, we were free to take the step of identifying it with e.g. the D conception. However if we anticipate the argument as a whole as at the end of 5, we see that the D conception will not work because dispensable, and that only the E conception will work here.

Returning to the main argument, we have to consider whether non-existence can be reduced to the O conception. Here again the analysis in the theory of descriptions is the relevant one, but it has to be backed by a special theory of falsehood, viz. that a false proposition is one which is either directly discordant with some fact or incompatible with some true proposition. The false-

hood of 'X is unicornish' is interpreted either as its discordance with e.g. the fact that X has no horn, or as incompatibility with the true proposition that X has no horn. In either case, the NC test demands that there is no fact 'X has no horn but is unicornish'. (This demand results, of course, not from assuming that having no horn is incompatible with being unicornish, but from the fact that otherwise the given fact or true proposition would not contribute to ensure the non-existence of unicorns.) The application of the NC test therefore brings us back to the E conception, and the attempt to explain *this* case of non-existence by the O conception will, by a similar path, bring us back to the E conception, and so on *ad infinitum*. The fact that no amount of explaining away on the basis of the theory of descriptions can remove the E conception clearly shows that it is not by this means reducible to the O conception.

The only way of escaping the regress is to follow the course previously adopted in the case of the O regress. What happens is not that an alternative means of reducing non-existence to the O conception is brought to light, but that a way is given of apparently reducing non-existence to the *D conception*.

This raises the general question whether non-existence can be reduced to the D conception. The NC test shows that an assertion (or fact) of difference between A and B must be accompanied by an implication (or fact) that there is no fact of the identity of A and B. However we attempt to use the D conception to explain non-existence away, there is thus an immediate dependence of the D conception upon the E conception, which shows that the latter cannot be effectively reduced to the former.

I once thought that a reduction of non-existence could be effected by means of a special form of the D conception which could be called 'limited non-existence'.¹ In limited non-existence the negation of existence is limited to a certain class. Thus if B is a class, 'A does not exist in B' limits the negation of existence to the class B, and in effect denies A's membership of B, or asserts that A is different from every member of B. The importance of this conception is (a) that by incorporating the basic conception of non-existence into the D conception, it indicates an intuitive

¹ The original article (*Philosophical Quarterly*) April 1956, upon which ch. III was based contains a section 7 expounding limited non-existence.

basis for NC and so seems to anticipate the NC test; consequently it promises to give an effective account of negation by contrast with the ordinary form of the D conception, (b) that by limiting non-existence to a class it avoids the paradox, to be discussed later, associated with the *naïve* conception of non-existence. However, in spite of (a), the application of the NC test, given that A does not exist in B, requires that there is no fact 'A exists in B', and this is *unlimited* non-existence. To avoid this we might perhaps treat unlimited non-existence in terms of properties of a peculiar kind, interpreting the non-existence of the fact 'A exists in B' as the non-existence of the property (form of fact) 'A exists in B' in the class of all the properties of all things and facts. But this leads to a regress exactly corresponding to the P regress. Thus non-existence *qua* unlimited is not reducible to limited non-existence. But limited non-existence seems clearly to be the form of the D conception containing the highest promise of an adequate account of non-existence, therefore since even this has failed nothing further can be expected from the D conception.

Consider finally an example of reduction of non-existence which does not fall so clearly under one of the standard headings. Non-existence is sometimes conceived as the 'emptiness' of a universal. *Having an instance* and *not having an instance* are here represented as two alternative conditions of the universal, which we may call respectively its 'reality' and its 'emptiness'. It does not matter whether these conditions are conceived as qualities of the universal, or as relations of the universal to the world; the point is that 'emptiness' is a positive condition, not simply the absence of 'reality'. However, the NC test does require of an 'empty' universal that there should be no fact of its 'reality'. But this brings us back to non-existence again, and as in the standard cases the reduction is shown to be circular, either directly or in the form of an infinite regress.

It would be tedious to enumerate the various subtle theories which might be devised to avoid this outcome. In all cases NC has to be applicable to ensure that the non-existence of something excludes its existence, and in the act of applying NC non-existence is reintroduced as an indispensable term of explanation. Thus it seems abundantly clear that there is no conception at all to which it would be possible to reduce non-existence.

Consequently we can supplement the claim, made in the last section, that non-existence (the E conception) is indispensable to negation. This claim cannot be discounted by alleging that non-existence is reducible to some more basic conception, and that it is therefore really this more basic conception which is indispensable to negation. Non-existence has been found *not* to be reducible to some more basic conception. Therefore non-existence is *significantly* indispensable to negation.

7. THE MEANING OF NON-EXISTENCE

(a) *The kind of object to which negation applies*

An important view of the nature of negation seems so far to have been overlooked, viz. that it is indefinable. I am not concerned here to urge that negation is completely definable. It is enough that, as sections 5 and 6 have shown, negation can be *partly* definable in terms of non-existence, in so far as the E conception has proved to be indispensable to negation. This result is in fact complementary to one meaning of the thesis that negation is indefinable. Simples are not definable significantly, and it is natural enough to treat negation in the abstract¹ as indefinable because simple and unanalysable. This is not inconsistent with the admission that negation as *instantiated* is definable, for in all instances of negation we have negation in the abstract plus something (whether a proposition, existence, etc.) to which the abstract conception of negation is 'applied', or upon which it 'operates'. The result cannot be simple and indefinable, because it is obviously analysable into abstract negation and that to which it is applied. If that to which it is applied is taken to be existence, then instantiated negation becomes non-existence. We should bear in mind that the thesis that non-existence is indispensable to negation was demonstrated by considering a typical *instance* of negation (e.g. a negative fact or a negative proposition), so that 'indispensable to negation' means 'indispensable to any instance of negation', and the practical consequence of the thesis that non-existence is indispensable to negation is the proposition that every instance of negation is an instance of non-existence. For instance the attempt to *dispense* with non-existence (for a given instance of

¹ I.e. as isolated from its instances, not implying any diminution of ontological status (as in I, 5 (a)).

negation) by appealing e.g. to the P conception turns out to be circular, and in general the attempt to explain non-existence *away* in terms of a species of negation assumed not to involve non-existence turns out to be circular.

This basic, irreducible non-existence arises, we saw, not for abstract negation itself, but only when abstract negation is applied to some object which is thereby 'negated'. That non-existence is indispensable to negation means that the complex resulting from the application of negation to an object always has or contains the form of non-existence. This suggests that the object negated is always an *existence*, while the prefix 'non' seems to express the application of abstract negation to such an existence.

As against this, the structure of the P conception suggests that the object to which negation is applied is not always an existence, but may be a *proposition*. Let us examine this to make sure whether *proposition* can be a significant alternative to *existence*, as regards the kind of object to which negation is applied. To negate a *proposition* does not indeed mean to negate its existence, it means to negate its truth. In other words what is negated is not exactly and solely the proposition (this could refer only to its existence) but its application to the facts. But in thus specifying what *exactly* is negated, we seem to return to the point that what is negated is after all an existence (a thing or a fact), for what seems now to be negated is the *fact* of the proposition's application to the facts. The attempt to deny that this negated 'fact' is properly a particular individual existence results in the demand for an account of it in terms e.g. of the theory of descriptions, and this leads to a P regress.

Much less clearly the D conception may also be said to offer an alternative to existence as the object to which negation is applied, viz. a *relation*. A relation *r* either *could* relate certain terms (say A and B), in which case it is propositional for those terms, or it actually does relate them, in which case it is both propositional and factual for those terms. If A does not have *r* to B, what is meant by saying that the *relation* *r* is here negated? We mean that, in the context of the terms A, B in this order, no instance of the relation *r* occurs.

Now this in effect is to fall back upon a more general account of the object to which negation is applied. For at this point the

critic would avoid saying that negation applies to particular instances of the relation, since this would be to return to *existence* as the object negated. He would have to say that negation applies in some way to the *universal* (in this case a universal relation). But how does it apply to the universal? Obviously it does not apply exactly and solely to the universal itself, for this could only mean that there is no such universal, which is not the negation intended. We can only say that what is negated is the *proposition* that the universal has some instance, and this brings us back to an alternative already dealt with.

(b) *Inveterate assumption of a way out*

By considering the different alternatives as to what that can possibly be to which negation is applied, the answer therefore always comes back the same—an *existence*, a particular given thing or fact. Now a devastating contradiction results from accepting non-existence in this *naïve* sense of the simple non-being of a given individual thing or fact. This is the paradox of non-existence already encountered in earlier chapters. This contradiction is surely the primary and basic reason why philosophers take it for granted that there simply must be some way out of the *naïve* conception of non-existence. A secondary reason, built firmly upon this one, applies particularly to modern philosophers. It is that some way out more or less resembling the theory of descriptions has become widely accepted not only as a true theory but as a deep-seated way of thinking constituting a background presupposition. To the modern philosopher, influenced as he is by the half-understood authority of mathematical logic, the P conception of non-existence contained in the theory of descriptions is the *natural* one, and the paradox of non-existence (PNE) is, by contrast, a superficial verbal puzzle easily explained away. As for the paradox of negation,¹ this, if he ever encounters it at all, appears as nothing but a sophisticated game. (By contrast, what our argument goes to show is that PNE is not only unshakable in its own field, but in the form of PN inevitably spreads throughout the wider field of negation in general.)

¹ Subsequently 'PN'.

(c) *Modern philosophy presupposes that there is only one root meaning of non-existence.*

But properly speaking the conception of non-existence contained in the theory of descriptions is not a conception of non-existence at all. We have here a way of escape *from* a certain conception of existence but not *to* an alternative conception of existence. For no properly existential feature can be detected in the proposition labelled 'negative existential proposition'. Instead we have a class of singular negative subject-predicate propositions (or relational propositions) of the form not- $f(x)$, every one of which is to be asserted. The conceptions occurring here are those of abstract negation, the subject-predicate (or relational) form of proposition, and joint assertion. Anything distinctively existential is conspicuous by its absence. Nor does the affirmative 'existential' proposition contain any distinctively existential feature, for the only difference from the negative is that there is no feature of abstract negation, and joint assertion is replaced by disjunctive assertion. Of course existence is *presupposed*, but only in the way in which any other proposition presupposes existence, viz. by presupposing values of the variable. The name 'existential' therefore arises not from any peculiarly existential character actually belonging to the proposition called 'existential',¹ but from the fact that this proposition *translates* a proposition which is indeed existential, but paradoxical. There seems therefore to be *no* non-paradoxical conception of non-existence which could form an alternative to the paradoxical conception, for if there were why, in translating the latter, should logicians resort to conceptions with no sign in them of a distinctively existential character? But in that case our self-confident modern philosopher is adding a lighted match to the gunpowder beneath his feet. For on the one hand non-existence is indispensable to negation, and on the other he himself takes it for granted that there is no conception of non-existence apart from the paradoxical one. Conclusion: every negative proposition and every negative fact is self-contradictory.

(d) *Plato also appears to presuppose this.*

In ancient as in modern philosophy, there are obvious attempts

¹ Concerning this lack of existential quality, see also I, 8.

to escape paradoxes concerning non-existence. Just as Russell believed that the theory of descriptions avoided the existential paradoxes associated with Meinong's 'objects', so Plato believed that his account of negation in the *Sophist* avoided the Parmenidean paradoxical conception of non-being, which eventuated for Parmenides in his treatment of non-being as impossible and even meaningless. Now Plato's theory is in terms of the D conception, which like the P conception is not properly a conception of non-being. It does not appear that Plato interpreted the D conception as limited non-existence, and even if he had done so, that of course would not have helped him. It seems therefore that Plato, like the modern philosophers, presupposed that non-being *as such* is self-contradictory.

(e) *This presupposition, hence the paradox, unavoidable.*

Now in accordance with section 6, it must be admitted that both the modern account of non-existence in terms of the P conception, and the ancient account in terms of the D conception, are circular. But the common presupposition that there is basically only one conception of non-existence, viz. the self-contradictory conception, I see no way of disputing. In the course of the argument, the only other conception of non-existence which has emerged has been that of limited non-existence, but this turned out to be dependent upon unlimited non-existence, and with it therefore self-contradictory. Again, the existential character revealed by the NC test as essential to negation, bears every mark of similarity to the non-existence which, through its paradoxicalness, urges philosophers to produce 'escape' accounts of non-existence. It is because the new occurrences of the existential character are never new in nature that the outcome is always an infinite regress instead of a satisfactory escape. There seems to be no conception of non-existence at all which does not depend upon the paradoxical non-existence. And when we find that the P, the O and the D theories must ultimately fall back upon the E conception as an indispensable term of explanation, this E conception upon which so much depends can be no other than the *self-contradictory* conception of non-existence.

Negatively, this discussion of the meaning of non-existence has shown that the usual escapes from, or, rationalisations of,

non-existence are inadmissible; positively, it has left us with only one basic meaning of non-existence, viz. the *naïve* paradoxical meaning; and this in conjunction with the proposition that non-existence is indispensable to negation makes PN inevitable.

8. THE DENIAL OF NEGATIVE FACT

(a) *The first three methods of denial.*

If the paradoxicalness of non-existence affects every negative fact, it would seem that there is now left one and only one way in which to avoid admitting the actuality of the contradiction involved in non-existence, viz. to deny that there are any negative facts. I think there are, broadly, four theories or attitudes of thought aimed at denying negative fact.

The first theory was mentioned in Chapter III, 4.¹ It claims that if 'A is not B' is true, it is so not on account of a negative fact which makes it true, but on account of the absence of any positive fact making it false. This is to give, as condition for the truth of 'A is not B', either that there exists no positive fact 'A is B' or that the proposition 'A is B' does not apply to the facts. But whether this condition be regarded as a negation of existence or as a negation of correspondence, it refers to that state of affairs which must actually obtain in order that the proposition 'A is not B' be true, i.e. we have a negative fact in the sense defined in section 1.

Secondly, a subjectivist account of negation may be stated or assumed. For example, a P theory may be given in which propositions are treated as thoughts or sentences, or an O theory in which incompatibility is conceived psychologically rather than logically. In place of a negative fact we then seem to have only an attitude of thought, or alternatively a functioning of the word 'not' according to certain rules. Now whatever form the positive account of negation takes, the negative or subjectivist aspect of the theory works only in so far as it guarantees that there are no negative facts. This negative guarantee is the logical part of a subjectivism; the rest is a pretty veneer. But if it really is guaranteed that there are no negative facts, then of course it is a fact that there are no negative facts, and since this is a fact verifying the *negative* proposition 'There are *no* negative facts', it is a *negative* fact. No

¹ See not only III, 4, but II, 8.

addition of psychological or linguistic factors can affect this implication. Indeed, whether we employ subjectivism or any other method to ensure that there are no negative facts, this which is ensured is implied to be a negative fact, so the denial of negative fact by whatever means is self-contradictory.

The third method of denying negative fact is to deny it not *qua* negative but *qua* fact. This theory, too, occurs I think only as covered up by a form of subjectivism, either linguistic or mentalistic. It is felt that *fact* is essentially a concept or the way we use the word 'fact', not anything to do with some reality beyond our thought or language. The expression 'the fact that . . .' seems to identify a fact with a proposition. Thus by calling a proposition a 'fact' it is held that we are merely ascribing the property of *truth* to the proposition in an emphatic manner. Again, when we speak of a certain fact as 'obtaining' we imply that it makes sense to suppose that it does not obtain, and thus that a fact is not necessarily something indisputably real such as we try to suppose it to be.

(b) *Criticism of subjectivist methods of denial*

Ultimately the anomalies concerning 'fact' are due, I think, to PN, but like other paradoxes of this origin they can be provisionally stemmed by a careful use of the theory of descriptions. According to this theory it is not strictly correct to say that such-and-such a fact obtains, what we should say is that such-and-such a description (or *form* of fact) applies to some fact or other. Here the reference of the word 'fact' is to some member of the universal class of given facts. Thus in accordance with the theory of descriptions it is an easy enough matter to introduce a (provisionally) strict use of the word 'fact' on the lines of the use of the words 'existence' and 'exist'. The expression 'the fact that . . .' does not present such a serious difficulty. All it implies is that a fact can have the same form as a proposition; it does not *identify* a fact with a proposition unless the critic question-beggingly assumes a Kantian position.

The self-contradictoriness of a subjectivism of this kind has been briefly pointed out in Chapter III, 3. A subjectivism, like any other theory, can proceed only by making assertions. If it is hedged about with qualifications, at least some assertion is still

adhered to, unless the subjectivism is given up altogether (in which case no one is called upon to reject it!) But no *bona fide* assertion is made without an equal committal to belief in a certain *fact*. Even if the only assertion made is that reality is for ever beyond the mind, this is to commit oneself to a certain *fact* of relationship between the mind and reality. The whole trouble here arises from a confusion between external reality and reality as such.¹ Reality as such includes all that is, and thus includes mind (at least for the subjectivist, who obviously assumes that there is such a thing as mind). Anything falling within reality as such will be an existence, e.g. a thing or a fact. Since subjectivism posits facts (included within reality as such), its denial of facts can be consistent with this only if 'reality' now refers to a limited or external reality. Thus an absolute denial of facts is self-contradictory. The self-contradiction is sharpened if we deny only *negative* facts, since it is plain that at least one fact posited by a *denial* of anything must be a negative fact. Thus the fundamental criticism of the second type of theory also applies to the third.

(c) *The sceptic's objection*

In connection with subjectivism the following general objection to my argument for negative facts might be made. It is first pointed out that the sceptic does not directly deny the thesis (say T) under consideration, but rather he refrains from asserting T and shows reasons why we ought generally to refrain from asserting T. My argument, it is now said, proceeds to 'refute' the sceptic by reducing to absurdity the proposition "There are no negative facts," which is the direct denial of T, and so not at all what the sceptic is claiming. So my argument does not refute the sceptic.

Prima facie this objection is surely an *ignoratio elenchi*. My argument is a formal one of the type *reductio ad absurdum*, and if valid does prove the conclusion "There are negative facts". The argument does not claim that any of its formal components (in particular the supposition "There are no negative facts") is identical with the position of the sceptic. The critic can only reason in the following way: if my conclusion were proved, then we could infer from it that the sceptic's position is erroneous, since a formal proof of T is at least the showing of adequate reasons why

¹ cp. V, 6.

we ought to assert T. Conversely, if the sceptic's position were proved, it would follow that there could be no adequate reasons why we ought to assert T, and that there must therefore be a formal fallacy in my *reductio* argument. To this I reply that so to state the matter is to attribute to the sceptic's position a formal strength far beyond anything that sceptics' arguments ever do possess. My argument for T is a simple formal one, purporting to prove that T ought to be asserted. What is meant by saying that the sceptic 'shows reasons' why T ought not to be asserted? The best that the sceptic could do would be to produce a formal proof that T unconditionally cannot be proved. If the sceptic were to produce such a proof, we would now have my formal argument implying that T can be proved, and his formal argument implying that T cannot be proved. So even if the sceptic obtained his proof, the issue would remain indecisive so long as no formal fallacy were found on either side. Now Gödel has formally proved that a certain proposition belonging to a certain system is not provable *within that system*. Even if the sceptic were to produce a proof of this formal calibre, it would not be good enough, because the condition 'within the system' is inescapable. But sceptics' arguments in fact fall very far short of such formality. In fact they are not formal at all, for besides being complex they rest upon an *epistemological premiss* claiming a radical separation between reality and the knowing mind. I do not see how such an argument could possibly be set against a simple, formal argument depending upon no empirical premiss. So, until the sceptic points out the formal fallacy in my argument, which his criticism implies to be present, I shall remain impervious to his epistemological speculations. And if, on the other hand, he should ever discover the formal fallacy, I would still lack interest in the epistemological speculations alleged to be associated with it, for then the central issue would be the formal fallacy, not the speculations. So one cannot help concluding that the epistemological hypothesis which constitutes the sceptic's position is fundamentally irrelevant anyway. Modern philosophy is suffering desperately from the vast traditional disaster of trying to make and unmake logic with the aid of empirical hypotheses. As well use a hammer to move the earth out of its orbit.

(d) *The Type objection*

This brings me to an objection which might, far more plausibly, be regarded as pointing out a *formal* fallacy in my argument. This applies to either of the arguments implied above:

- (1) There must be facts, because if there were not, there not being any would itself be a fact.
- (2) There must be negative facts, because if there were not, there not being any would itself be a negative fact.

The objection is that each argument question-beggingly assumes that what is asserted by a philosophical thesis is logically of the same kind as what is asserted by an ordinary statement. For convenience I shall consider the objection only in relation to the second of the two arguments, as being the more relevant to my subject-matter.

What the objection claims is that the words 'negative facts' in "There are no negative facts" cannot be taken to apply to negative facts of the order "There are no negative facts" (supposing this is a fact) but only to *ordinary* negative facts such as "There are no flying horses". Now the alleged fallacy of classifying both orders of facts (or statements) together under the heading 'negative facts' could not be regarded as a *formal* fallacy unless the difference laid down by the objector is claimed to be *logical*, in the sense that the difference has to be made *if logical paradoxes are to be avoided*. This, then, is the force of the word 'logically' as occurring in the statement of the objection. It is true that logical paradoxes might be involved in this case, since "There are no negative facts" is reflexive if it is taken to be capable of expressing (*qua* negative proposition) one of the negative facts which it is about. We are here concerned, then, with *reflexive* paradoxes, and the standard logical theory for the avoidance of these is the theory of types or some modification of it, whereby it is laid down that the apparently reflexive sentence is meaningless if taken to be about itself, i.e. the sentence is set apart (if meaningful) in a class which can never overlap the class of sentences which it is about. But the objection to my argument consists in making just such a distinction, and it therefore presumably rests upon the theory of types or some modification of it. This theory and weakened forms of it were discussed, and in my opinion adequately

refuted, in Chapter I. In fact one of my stated purposes in dealing with the theory of types at that early stage was to anticipate objections of the kind just considered. Nevertheless I am quite certain that many readers would have supposed the objection to have been fatal had I not dealt with it again here.

(e) *The double negation method of denial*

It might be contended that the non-existence of any negative facts is not properly a negative fact, but the positive fact making true the *affirmative* proposition that all facts are positive. This, however, would not secure the desired end. A positive fact may well be negative too, e.g. the fact making an affirmation of a difference true is also a fact making the negation of an identity true. To secure that no facts are negative, by means of the proposition that all facts are positive, we therefore have to interpret 'positive' as implying the *exclusion* of negative aspects of fact. But this dependence upon exclusion shows that the proposition "All facts are positive" used for this purpose, is in effect negative after all, and is therefore true only in virtue of a negative fact.

This raises the more general question as to whether there is *any* way in which the conception of double negation can be used to avoid PN.

Admitting negative facts, it might be supposed that they are all reducible to a certain special form, and that this form is free of PN. The standard P, O, and D theories each reduce negative facts to a certain special form, but these forms have each been found to involve a paradox which either is PN or is tracable to PN. The 'double negation' theory supposes negative facts to be either reducible or restrictable to the special form *doubly negative facts*, and such facts might be supposed free of PN because a double negation is equivalent to an affirmation. Hence e.g. the fact that it is not the case that some crows are not black is usually taken to be the same fact as the fact that all crows are black. (Here 'is black' is taken to be no part of the definition of 'crow'.)

This would apply, of course, only in cases in which there is some double negation to which a given affirmative proposition is equivalent. It would not apply if the alleged double negation were meaningless. If all double negations were meaningless, then the supposed restriction of negative facts to doubly negative facts

would amount to the claim that there are no negative facts, and this we have seen to be impossible. Thus we are concerned only with the case in which an affirmation is equivalent to a *significant* double negation.

The significance of a double negation can be accounted for in two ways: either it is due to the fact that the negated negation is also significant, or if this is not so it can be attributed directly to the fact that the equivalent affirmation is significant. On the former alternative the affirmation *negates* a significant negation, and if (as we are supposing) the affirmation is a fact, then the double negation is a genuinely *negative* fact F_1 , excluding from existence a second negative fact F_2 . In other words, PN occurs because the *negated* negation is a *fact*, viz. F_2 . This alternative is not, therefore, free of PN. Consequently this way of accounting for the significance of a double negation must be avoided, and the burden of explanation must be made to fall wholly on the second alternative. This means that the equivalence of an affirmation to a double negation is nothing but a verbal definition, ensuring that the words occurring in the doubly negative sentence mean nothing more nor less than whatever is meant by the words occurring in the affirmative sentence. Such a definition would be entirely pointless, since in spite of the employment of the word 'not' it would fail to introduce any conception of negation, for such a conception would be the addition of something over and above what is so far contained in the meaning of the affirmative sentence. This destroys all hope of reducing negative facts to the so-called 'doubly negative' facts, which on this view are *purely* affirmative facts with no point of connection whatsoever with negative facts. And if the word 'negative' in 'negative fact' were similarly reduced to emptiness in order to avoid the implied two uses of the word 'negative', we would again be driven into the impossible position of implicitly claiming that there are no properly negative facts at all.¹

(f) *The paradox itself as a denial*

The standard methods of denying negative fact seem therefore to fail. But apart from these, one might regard the very contra-

¹ A parallel dilemma occurs in the case of obversion, considered near the end of section 3.

dicatoriness of negative fact proved in the previous sections (PN) as constituting a *reductio ad absurdum* of the existence of negative fact. One feels that there simply must be some way out of such a scandalous contradiction. But on examination each 'way out' has been found either to involve an infinite regress or to be made possible only by overlooking the NC test. Now the infinite regress signifies that that, from which escape is sought, reappears as the end of each step taken in the process of escape. But this not only shows why the escape has to be continued unendingly, it also shows that no effective escape is ever made, since the *n*th step is no improvement on the 1st. In other words, this is not really an alternative at all. On the other hand, to overlook the NC test is obviously to fall into contradiction again. If anyone claims that one contradiction is better than another, the answer surely is that they are all reappearances of the original contradiction of non-existence, showing that this initial contradiction has not after all been avoided.

By pointing to actual examples of the paradoxical non-existence, the next chapter will show still further that this particular contradiction cannot be used as a *reductio ad absurdum* of the existence of negative fact. It will also become clear that the whole programme of denying negative fact is misguided, since PN springs from the same root as NC, so that PN must hold for reality to the same extent that NC holds, and NC doubted in proportion as PN is rejected. In other words, PN will be seen to affect NC itself, raising the whole question whether an orthodox logic can stand.

CHAPTER V

THE PARADOX OF THE PAST

1. THE IMPLICATION OF CEASING TO BE

Let us suppose that your pet dog Fido died yesterday, and let us suppose, as we usually do in the case of animals, that death implies the coming to an end of the individual that dies. Fido's death therefore implies that at one time (yesterday) he existed, but that now he does not exist. Ordinarily no one would dream of suggesting that a situation of this kind is impossible.

When we say that Fido does not exist, we cannot but be referring to Fido himself. The corresponding fact about Fido, viz. his non-existence, (a) is clearly a *present* fact, and (b) seems to be a fact about Fido himself. As to (a), the fact of Fido's non-existence obviously does not obtain during the period of Fido's official existence, but only afterwards, and in particular now; if the fact of Fido's non-existence did not obtain now, he would still be in existence, which contradicts the hypothesis that he died yesterday. As to (b), I said only that the fact 'seems' to be about Fido himself, because this is the point normally attacked by those who try to avoid the Paradox of the Past. Yet to common sense it is obvious that it is Fido himself, and not some wraith or replica of Fido, who does not exist. In fact the wraith could still exist (if you believe in wraiths), and conversely if the wraith does not exist, this would not ensure that Fido does not exist. It was Fido himself, a particular *individual* being, who existed; when he ceased to exist, this event surely happened to that very same individual being, Fido himself; it was therefore Fido himself who passed from existence to non-existence, and thus the subsequently continuing fact of Fido's non-existence, which at the outset concerned Fido himself, must continue to be a fact about Fido himself. But unless Fido still exists now, there can be no such present fact about Fido himself—viz. his non-existence. So as well as not existing, Fido must still exist.

Arthur Prior¹ treats the existence of Fido as there being at least one fact about Fido. But *prima facie* the paradox still follows

¹ *Time and Modality* (Oxford, 1957), Ch. IV.

from this view, since the non-existence of Fido is now the fact that there are no facts about Fido, and this fact (that there are no facts about Fido) would be a fact about Fido. Prior's eventual rejection of this outcome arises from his attack on what he calls the 'sempiternal' existence of individuals like Fido whose official existence in time is limited to a definite period. The sempiternal, or eternal, existence of individuals is a consequence of the usual logical treatment of time as a relation of succession, for an individual existing at a particular time then becomes an object concerning which certain non-temporal (and thence eternal) facts obtain. As against this Prior maintains that there are no facts concerning an individual before and after the time of its official existence, that properly the individual is not nameable during such times. This appears to be plausible in the case of an 'individual' whose official existence is (tentatively) dated for the future, so long as there are insufficient facts now to guarantee either 'its' existence then or 'its' non-existence then. But since, in the case of a past individual, no doubt at all can be cast upon the possibility of individualisation, it would seem that no way is open for a parallel denial of existence and nameability in this case.

2. SOLUTIONS IN TERMS OF DESCRIPTIONS

Nevertheless Prior tries to maintain the same position in regard to past individuals as he does in regard to future 'individuals', and to this end he uses a form of the theory of descriptions. There is no Bucephalus (the horse rode by Alexander), but only some x which once Bucephalized. Now if x were an ordinary individual, it would, like the non-existent and unnameable 'Bucephalus', be itself non-existent and unnameable now. Thus any x of which we can say now that it once Bucephalized (or contributed to Bucephalizing) must be a relatively simple and atomic being capable of enduring unchanged from the time of Alexander to the present time. For the explanation of historical facts we seem to have to fall back upon eternal atoms of some kind. Now as Prior has shown,¹ an atomism of this kind is implied by the attempt to *maintain* sempiternity with respect to the *future*. But in this case Prior rejected the alleged sempiternity on the ground that it led to this atomism which, he said, is probably bad

¹ Ibid. pp. 29-30.

physics and certainly bad logic. By the same argument, therefore, his own attempt to *deny* sempiternity with respect to the *past* should also be rejected.¹

However, let us concede the required atomism. Although these ultimately real atoms are *necessary* for the historical existence of 'Bucephalus', they are not sufficient. For 'Bucephalus' to come into existence, certain facts of relationship between the atoms must also obtain, and for 'Bucephalus' to cease to exist at least one of these facts must cease to be. What are we to say about this fact which once was but now is not? It is non-existent and unnameable because past; we have neither the right nor the ability to think or speak about 'it'. Is 'it', like 'Bucephalus', a combination of other sempiternal facts? But then again, what about the facts of *their* combination, one of which must cease to be when the combination itself ceases to be? Evidently we have a P-regress, and the atomic theory fails to explain what it purports to explain.

According to a more ordinary account in terms of descriptions, the past is nothing but a series of descriptions which have successively been uniquely satisfied and then no longer satisfied. A particular past event is not *that individual* event, it is only *an* individual event uniquely satisfying *that* description. At the time we could have referred to *that individual event*, but now we can only refer to that description and to the fact that it was once satisfied (by some individual event). Consequently we have to reckon with the ceasing to be of the fact of the description's satisfaction, and this leads to the same type of regress as before. For just as we cannot refer now to the individual event, equally we cannot refer now to the individual fact of the description's satisfaction (since this too has ceased to be). In lieu of this we must introduce a *description* of this fact, and claim that this description was satisfied during the appropriate time, then ceased to be satisfied. So now we have still another fact that has ceased to be, and so on *ad infinitum*.

3. THE CONCEPTION OF HISTORY

These accounts in terms of descriptions aim at avoiding the

¹ More recently (*Philosophy*, Jan. 1960) Prior has supported his position by claiming that there are predicates which do not presuppose the existence of the subject of predication. This characteristically leads to his positing a 'being' distinct from 'existence', which I believe is a retrogression to the pre-descriptions era of Meinong (see III 2).

admission now that *individual* past events ever occurred, but once this aim is formulated it is seen to be in direct conflict with our conception of history. For although our *knowledge* of history is expressible without the use of singular propositions, viz. by saying only that *an* event of a certain description occurred, no one supposes that the past itself is comprised of nothing but descriptions which are somehow satisfied without being satisfied by this or that particular event. On the contrary, we take the descriptions to be only indications of the individual events which actually occurred. An historical fact is not merely the fact that *an* event of a certain kind occurred, for this would mean that the past would have been just the same for two quite different series of individual events, so long as the same set of descriptions applied to the members of each series. But it is evident that two such series of individual events would have constituted two entirely different histories, two different pasts, and in that case the past would not be determinate.

The conception of a uniquely referring description is useless for the solution of the problem. When we say that a certain description is a uniquely referring description, we mean that there is one and only one individual entity to which it applies, and this presupposes that we are empirically given a class of individual entities, of which one and only one happens to fit the description. But in the case of the past, the class of individual events which once occurred is precisely what we are supposed *not* to be given now, and it is just their absence which has evoked the attempt to explain the past in terms of descriptions.

The principle of the identity of indiscernibles is useless for a similar reason. It is a principle applying to the nature of the world *qua* consisting of a class of empirically given individuals, it is not a principle concerning the nature of certain descriptions as such. If the veins and contour of an oak leaf have a certain geometrical pattern, we are assured by this principle that we can always proceed to sufficient detail in this pattern to ensure that nothing else in nature will conform to it. But given some such geometrical pattern apart from nature, we could by no means predict that just one object would have that pattern. The probability of none or many instantiations would obviously be far greater.

Nor can we avoid the difficulty at this point by remarking that,

in the case of the descriptions constituting history, we in fact know that each applied only to one event. The fact that *only one* event satisfied the description X is no more effective than the fact that *an* event satisfied the description X, for singling out a particular one event from all the possibilities. Each is *an* event, and each is *only one* event. The impression of uniqueness is conveyed only by surreptitiously allowing the expression 'only one' to possess not only the numerical sense of unity, but also a reference to the one individual event which (as we unofficially know all the time) actually occurred.

We might attempt to introduce the mark of uniqueness into the description itself by supposing that there are not only uniquely referring descriptions, but also 'uniquely individuating' descriptions. The theoretically exhaustive description of an individual event would have to include every description which applies to it, in particular it would have to include any uniquely individuating description applying to it. Perhaps a uniquely individuating description is identifiable with the theoretically exhaustive description. But if the individual event is now understood to be utterly non-existent, I do not see how even its exhaustive description could be confidently asserted to be uniquely individuating. We might be right in supposing it to apply to *only one* individual event. But this 'one' is either the numerical *one* which allows for various possibilities, or else it is the individual *one* which presupposes unique selection from the class of individual events *presupposed given*. Presumably the argument chooses the first of these two alternatives and declares that the possibilities are reduced, in these special circumstances, to one. If, however, this *one* is divorced from the individual event (as it is bound to be, if the existence of the latter is not to be implied), I see no reason for supposing that the past *is* determinate. It is only because we identify the past with the *individual* events which occurred that we do suppose the past to be determinate. If we jettison the individual events, we have no further need to insist upon the determinateness of the past, and we have no need to resort to such a nightmare as the uniquely individuating description. Conversely if we insist upon the determinateness of the past, this is only because we have already tacitly accepted the past as consisting of individual events rather than descriptions. Thus the various devices which culmi-

nate in the theory of uniquely individuating descriptions do not properly affect the validity of this argument; instead they stage a veiled attack upon its premise (viz. the determinateness of the past), which is normally accepted as so obviously true that only a *veiled* attack upon it could gain any credence.

It is unfortunate that the word 'history' is used to refer either to historical knowledge or to the series of events constituting the subject-matter of historical knowledge. This encourages a confusion between the actual events and our knowledge of them, lending plausibility to attempts to account for the actual past in terms of descriptions. Taking the word 'history' to refer to the series of events, it is clear that our idea of history is the idea of something determinate precisely because we have already accepted the implicit actualness of the events of history. Why are we so certain of the determinateness of history? The answer is: because we are so certain of the present existence of the events of history. Their very negation as they lapse into the past ensures *their* negation as individuals, therefore points to *those individuals* as logically possible objects for any subsequent thought, therefore as existents relative to any subsequent thought.

4. A WHITEHEADIAN SOLUTION

But surely there must be *some* way of distinguishing the past from the present. It is true that, in a sense, an individual event of the past has not ceased to *exist*, rather it has ceased to *live*. It has become irrevocably *fixed*, both internally and in relation to other past events. In it, time and change have suddenly come to a stop, and if this has destroyed its life it has also preserved its structure, to the surprise of those who think that the past has faded out of existence. As against the life and activity affecting a present event the past event has 'perished but become objectively immortal', and if it still exists it does not exist 'in the full sense'.

What are we to say of this additional factor of life affecting only the event as present? Is it an added individual something which ceases to be when the event becomes objectively immortal? But this would lead to a paradox, regarding this added individual something, exactly parallel to the former paradox regarding the individual event itself. The life of any particular present event is thus to be regarded not as an individual entity peculiar to that

event, but as an inseparable part of, or as a functioning of, that universal life or 'creativity' affecting all present events. Nevertheless, any particular event as it becomes objectively immortal must obviously break off from the universal life, and the universal life on its side must withdraw from that particular manifestation of itself. When the event was living in the present there was a bond, a fact of relationship, between itself and the universal life; now the bond is broken, the fact of relationship no longer exists. Thus for every event which passes from the full existence of life in the present to the Plutonic existence of objective immortality, there is an individual fact of relationship, an individual manifestation of the universal life, which then existed but now does not exist.

This attempt to escape the admission that an individual being ceases to be is therefore unavailing. To interpret the individual being as a fact of relationship or manifestation only puts it out of sight; it does not alter the fact that some individual being is supposed to cease existing. The attempt to dispose of the difficulty only recreates it in a more complex form, which is for that reason more easily overlooked. There is no point in inventing some kind of D theory which only obscures, without removing, the need to admit that passage in time requires the existence followed by the non-existence of an actual individual being. We might just as well admit that the events themselves lapse out of existence, so long as we also admit that just because of this lapsing they also exist. The paradox remains, and remains as a *contradiction*, that any event which has ceased to be is preserved. In our experience of events in time examples of the paradoxical non-existence are therefore inevitable.

5. RATIONALISATION BY THE IDEA OF MEMORY

It is only natural that there should be various devices of thought and language which aim at concealing the contradiction. Foremost among these is the idea of memory. This allows us to locate the respect in which the past is preserved in a private mind well out of the way of real events, and therefore leaves us free to suppose that the past event itself has lapsed in such a way as to be in no sense preserved. "When faced with a contradiction, draw a distinction."

In the first place, however, this does not really obviate the two-sidedness of the past. For as plain men we not only suppose a past event to have lapsed irrevocably out of existence, but in spite of the introduction of memory we still go on believing that there are present facts about individuals located in the past. To insist that the past event has lapsed irrevocably out of existence looks like removing the aspect of preservation from the idea of the past, but in fact it makes doubly sure of retaining it, for it is precisely the present fact of the non-existence of that individual past event which makes the event present. If preservation is memory, then in the remembered events themselves there is a kind of memory which makes it philosophically redundant to superimpose another memory located in a mind experiencing the events from outside. As some psychologists are aware,¹ memory is inherent and natural; it is only forgetting which is a faculty of the extraneous mind. The preservation of the past is a matter of logic and of the real nature of events as occurring in succession.

But secondly, if we do superimpose an artificial memory from outside, we superimpose with it a reduplication of the problem we sought to avoid. In such a memory we cannot have the past event itself, since this would imply the preservation of the past event; we can only have an image representing the past event. The question then is, how the memory image can still seem to refer to the event, although the event is not there to be referred to. My memory of a past event seems to be secured by the fact that I witnessed it; so long as I witnessed it I seem to be able to reach back into the past and recover it. But what about the event of my witnessing it? Do I have to reach back into the past for this too? But if *this* event had also gone in such a way as to defy preservation, it would simply not be, there would be nothing to reach back to; instead of explaining memory, this event of witnessing would in that case only duplicate the problem of how memory is possible. My having witnessed the past event explains how memory is possible only on the supposition that the witnessing event is preserved even though the witnessed event is not. Thus our very means of circumventing the contradiction that the past is preserved rests upon a concealed form of the same contradiction.

¹ Notably Bergson and Freud.

6. RATIONALISATION BY THE IDEA OF MIND

The prevalent assumption that memory is free of contradiction is supported by a wider belief concerning mind as such, viz. that things absolutely impossible in reality are nonetheless perfectly possible in the mind. This belief is clearly the *sine qua non* of epistemological explanation, but its source is not, I think, to be found in the field of philosophy, but in the ordinary experience of being able to think and imagine all sorts of things which do not exist. In this ordinary experience the mind is contrasted with an *external* reality, but as regards reality *as such* it must be presumed that since reality includes everything it must include mind.¹ This difference between two conceptions of reality, a limited and an unlimited reality, results in an uncomfortable conflict between the attitude of epistemology, which is normally counted a branch of philosophy, and the attitude of metaphysics and ontology. For since epistemology takes over from experience the ordinary conception of a contrast between reality and a mind, like an external observer, viewing reality from *outside*, epistemological conclusions can be valid only for a *limited* reality. Therefore the more ultimate philosophical questions concerning e.g. the meaning of 'exists', or the nature of reality as a whole, are quite beyond the reach of epistemological argument, and a philosophy such as the Kantian which aims at deducing a metaphysics from epistemological considerations runs the gauntlet of radical equivocation on terms such as 'reality' and 'existence'. The thoughts and imaginings which are unreal in relation to an external reality are, in themselves and in the wide sense, real.

Although this conclusion seems plain and obvious it has been contested by an important idealist tradition which assumes that the idea of mind can serve as a fundamental term for philosophical explanation. Bradley's conception of the *ideal content* of the judgment shows the contradictoriness of this, for on the one hand there certainly has to *be* an ideal content if the structure of judgment is to be explained in this way, yet on the other hand the ideal content is, in contrast with the psychological content, denominated universal and hence *nothing*.² Of course there is a valid distinction in our experience between an idea as capable of external reference

¹ cp. IV 8 (b).

² *Principles of Logic* (London, 1883), Ch. I.

and the same idea as a psychological entity. This is Descartes' distinction between the objective reality and the formal reality of an idea. But because the idea as a psychological entity is rather obviously real, we have no right to interpret the other aspect of the idea as a mysterious something which is at the same time nothing.

In Russell's 'robust sense of reality'¹ and in Whitehead's *Ontological Principle*² the use of the idea of mind or ideal content as a fundamental term of explanation is implicitly denied. Either the unreal is explicable in terms of the real, or it is not explicable at all. To explain the phenomenon of non-being or unreality it is not enough to suppose merely that the mind imagines something which is not there, or makes a false judgment. This 'explanation' becomes clear only when the illegitimate term *mind* is replaced by the actual structure of reals which alone makes sense of the 'explanation'. This structure is usually given nowadays in terms of *signs* or *symbols*, though Descartes gave it with equal validity, I think, in terms of resemblance or lack of resemblance between the idea and an external object. (The trouble with the semantical account in terms of symbols is that it is accompanied by an empiricism that wholly omits the factor of awareness on the grounds that it is unreal.) Once the structure underlying our ordinary conceptions of non-existence is made plain in terms of reals, the inadequacy of any such explanation is revealed, for the explanation is seen to depend upon absence of correspondence, i.e. a certain symbol has no object corresponding to it. This *assumes* that non-existence (of the object) is already given, and cannot show how non-existence is possible.

At the level of ordinary thinking and experience, there is a contrast between a mind and external reality; we say loosely that something (e.g. unicorns) may not exist in the external reality but exists in imagination (the mind). This gives an explanation, sufficient for practical purposes, of how non-existence is possible or conceivable. But it fails to take non-existence seriously. Non-existence implies not only that something (e.g. unicorns) is absent from one realm (external reality) but present in another realm (imagination), but that something (e.g. living unicorns of flesh and

¹ *Introduction to Mathematical Philosophy*, p. 170.

² *Process and Reality*, Pt. II, Ch. I., section 1.

blood) is wholly absent from reality as such. At this point there is no mind outside reality in terms of which we can make the total absence conceivable. The primary explanation has to be given in terms which are unequivocally real, or not given at all.

Thus in ultimate explanation the procedure of epistemology has to be inverted. We have a practical familiarity with particular minds, but the idea of mind remains logically difficult. It cannot serve as a term of explanation, but instead itself stands in need of explanation. It is no use treating it as a universal cure for all troubles. If, for instance, change is shown to involve logical difficulties, it is no solution to call in the mind and say that change is therefore *illusory*; the fundamental problem of change is that, *qua* passage from existence to non-existence, it involves *non-existence*, so an *illusion* of change, which logically is the *non-existence* of change, is just as problematical. It is therefore primarily the mind and mental phenomena which stand in need of explanation, and their explanation is not given satisfactorily except in expressly real terms. The beginnings of an explanation of mind in terms of reality were hinted at in the last section, for the Paradox of the Past occurs in the setting of *real* events, but one aspect of the Paradox is the preservation of the past by a kind of *memory* inherent in the real events themselves.

7. THE STATUS OF EVENTS IN TIME

The conclusion so far is that in our experience of events in time there is actual contradiction, due to the present non-existence of past individual events which have ceased to be. Yet even so we can deny that the contradiction affects reality by claiming that the time process as a whole, in which the contradiction occurs, is not real but a mere appearance distinguishable from a reality free from contradiction. This Bradleian type of solution depends upon taking NC to be more certain than the reality of our own experience.

In general such an assumption may be justifiable, but in the present circumstances it is not. For we have not only to consider absolutely whether a contradiction can possibly occur in reality, we have also to consider the *relative* certainties of NC and PN. Now these certainties are equal. They are both *logical* certainties arising from the meaning of negation. As regards NC, our cer-

tainty of it depends upon an *a priori* intuition of the nature of negation. There are three alternatives regarding the truth of a pair of mutually contradictory propositions *p*, not-*p*; either both are true, or one is true, or neither is true. Now on the first alternative we have both of the *facts* *p* and not-*p*. But the meaning of negation as occurring in the negative fact not-*p* is that there is no positive fact *p*.¹ Thus the first of the three alternatives reduces to the truth of not-*p* only, and this is a special case of the second alternative. It follows that never is more than one of *p*, not-*p* true, and this is NC. (For obvious reasons a *reductio ad absurdum* argument is avoided here, but I do not think it necessary to avoid the initial assumption of the three alternatives, even if this does involve special cases of EM.) But as regards PN, this too follows from our *a priori* intuition of the nature of negation. As in the argument for NC, this intuition ensures that a negative fact not-*p* implies that there is no opposite positive fact *p*.¹ But since, as has formerly been shown, a negation of existence has ultimately to relate to the individual existences negated, the opposite fact *p* is implied to exist. This is PN.

Therefore in whatever respect and in whatever field NC holds, just there will PN hold. If NC holds for reality itself, equally will PN affect reality itself. Consequently we cannot confine PN to appearance while still supposing that NC holds for reality, and the whole *raison d'être* for the distinction between appearance and reality breaks down. A Bradleian approach could work only for paradoxes less fundamental than PN. PN and NC always go together. If the Paradox of the Past is valid for the time process this is only because NC also holds for the time process, and if NC does not hold for the time process but does hold for a supra-sensible reality beyond it, then this supra-sensible reality turns into mere appearance again through being infected by PN (*because* NC holds for it). Therefore in the Paradox of the Past we can detect neither any reason for denying that time is real nor any reason for positing a supra-sensible reality beyond time.

¹ See IV, 5.

CONCLUSION

(A) If the arguments of Chapter II were valid, they showed that a conventionalist view of the *ultimate* principles of logic could not be upheld, but this leaves open the possibility that a particular principle such as NC might be claimed to be a convention provided it were denied ultimacy. If, however, the proof of NC in the last section was valid, then NC is known intuitively, and is true because negation has the nature which it does have. This leaves no room for claiming that we could, if we had so chosen, have held NC to be false; in other words it leaves no room for a conventionalist view, or any variant of the conventionalist view, of NC. This conclusion is strengthened by another fundamental argument. If NC were replacable by a different convention, then the meaning of 'not' as determined by it would not be *negation* as we understand it, and thus there would be no implication of an exception to NC. Instead this would indicate the existence of an exception to a principle expressed by the same words as NC, but having a different meaning owing to the changed meaning of the word 'not' occurring in it. A parallel argument has in fact been put forward in regard to EM.¹

These arguments seem conclusive. Yet they might have been discredited had it been possible to produce plausible exceptions to NC or EM at the level of ordinary logical discussion, i.e. prior to the emergence of radical contradictions such as PN or the reflexive paradoxes (I shall argue below that PN has no tendency to reintroduce conventionalism). In my opinion, however there are no such plausible exceptions. By way of illustration consider the proposition

This penny is both circular and noncircular (p)

which Nagel² puts forward as a plausible exception to NC. Following Aristotle's qualification of NC by the phrase 'in the same respect', Nagel points out that NC has to be presupposed in defining the 'respect' in which the penny cannot be both circular

¹ W. Kneale, 'Are Necessary Truths True by Convention?', *Aristotelian Society*, Supp., Vol. XXI.

² 'Logic Without Ontology', in *Readings in Philosophical Analysis*, pp. 193-4.

and noncircular. But surely Aristotle was mistaken in introducing the qualification, and surely it is unconditionally false (at the ordinary level) to assert 'This penny is both circular and *not circular*'. The trouble is that the predicate 'is noncircular' is equivocal. To make p an exception to NC 'is noncircular' would have to mean 'is not circular', but to make p plausibly true 'is noncircular' would have to mean 'has a shape, e.g. the elliptical shape, other than the circular shape'. It is logically possible for an object to have two shapes neither of which is the other, e.g. a cylinder may be oblong and circular, and an ellipsoid elliptical and circular. In general, an object can have more than one property. If P is one property, another may be called 'non-P' on the ground that it is other than P, but no plausible exception to NC can arise unless it is clear, for reasons independent of this nomenclature, that non-P is in addition *incompatible with P*.

Having admitted that incompatibility exists in a given case it is not enough, of course, to contend that NC is used in the process of deciding that the incompatibility exists. A person who says that a penny has a diameter of $\frac{11}{16}$ of an inch and also a diameter of $\frac{12}{16}$ of an inch is clearly admitting the compatibility of the two predicates, as against the usual supposition of their incompatibility, and thus he is not contradicting NC but only the usual supposition that here we have a case of the application of NC; and conversely *if* the incompatibility is admitted, no-one could *then* go on plausibly to assert both predicates. The process of decision in this example is not a process of deciding that there shall be no exception to NC (this is assumed all along), it is a process of deciding whether the two properties are incompatible. Where no exception to NC is mooted, we have no need to invoke either convention or ontology to defeat the exception. Moreover wherever convention is invoked, ontology would give a better explanation of the felt necessity to suppress exceptions.

A more likely case of a plausible exception is to be found elsewhere. Suppose that P were the property 'red all over' and non-P a property such as 'grey all over', i.e. a property clearly incompatible with P. A normal person might see an object X as P while a colour-blind person might see X as non-P, and from this it might be inferred that X is both P and non-P. In view of the incompatibility we now have that 'non-P' implies 'not P', and

hence the contradiction that X is both P and not P. As a matter of fact, however, few philosophers do proceed to this point. To avoid contradiction they immediately conclude that the properties P and non-P (seen as if on X) are in part the products of subjective conditioning, so that the inference that X is *itself* both P and non-P is not forthcoming. But even in the case of the remaining ones who admit the inference, it is a mistake, I am sure, to represent their essential position, as is sometimes done, by saying that X is both P and not P. What they mean, surely, is that our consciousness of the incompatibility of non-P with P applies only in respect of the sense experiences non-P and P, not in respect of the actual qualities. On account of this incompatibility within sense experience we could never *see* both P and non-P as if on X simultaneously, and so at the level of sense 'non-P' would imply 'not P'. But so far as X itself and its actual qualities are concerned, these philosophers should not deny that X can be both P and non-P, and so at this level they should not say that 'non-P' implies 'not P'. It is just for this reason that they are able to assert, on the grounds of the phenomena of normal and colour-blind vision, that X in fact is both P and non-P. And now there is no contradiction because at this level we cannot replace 'non-P' by 'not P' and obtain 'X is both P and not P'. In other words neither of the two offered solutions implies an exception to NC, and if the second solution sometimes appears to do so, this is only because its exponents express themselves in a loose manner. If indeed they really meant definitely to maintain not only

(a) that X is both P and non-P, but also

(b) that X is not both P and non-P (and hence that 'non-P' implies 'not P'),

then in (a) they would be denying the very incompatibility, expressed in (b), which is the primary source and condition of the whole problem. Similarly in (b) they would be rejecting the inference (a), the admission of which distinguishes their solution from the first.

(B) If, to the objective necessity of NC, we add the proved objective necessity of PN, this still brings us no nearer to conventionalism. Owing to the resulting conflict of necessities conven-

tionalist language could perhaps be used to describe the way things appear, but it could not be a description of the way things are. What we have in fact is over-necessitation, not under-necessitation.

This suggests that the fundamental issue concerns NC rather than EM, for an exception to EM is naturally representable by maintaining the existence of a 'middle' or third alternative, which increases the degree of freedom, giving under-necessitation. To weaken necessitation is really to move within orthodox logic. If EM in the form 'p or not-p' is replaced by 'p or N_1p or N_2p ', then ' N_1p or N_2p ' is a function, say N, of p, so that the new law becomes 'p or Np '. This gives 'N' the same meaning as 'not' and identifies the new law with EM, and orthodox logic is not contravened.

With under-necessitation there is no real problem, but with over-necessitation the problem is devastating, and this is the situation brought about by PN. If we admit that PN cannot in fact be avoided, it is nevertheless clear that before we know this we will spontaneously seek every available means to avoid it. This would explain why, as shown in Chapter V, our practical reaction is to divide off a subjective world of imagination from the real world, locating in the former all those things which do not exist in fact—yet exist. The division turns out to be a subterfuge. There is no escape from being, and the significance of negation is not a supposedly simple non-being, nor yet subjectivity, but a real conflict between being and non-being.

The problem began at the stage of PNE (Chapter III), which according to official philosophy is nothing but a superficial verbal puzzle. Unfortunately the official methods of solving the puzzle only disguise it, and a more subtle stage sets in, viz. the stage of PN, in which the 'puzzle' has become a veritable problem. This eventually leads back to PNE, without hope now of any easy solution. In fact it appears that there is no solution at all. Theories of negation and non-existence survive only by failing to recognise some essential aspect of negativity. A new-found order arises only to collapse into a profound unknowingness in which what is not is indistinguishable from what is.

In these circumstances the most that I have been able to do is to present the problem, principally by showing how the standard

solutions fail. The various arguments are perhaps to be viewed not in relation to particular solutions rejected, but as showing up various aspects of the problem itself. At all events I have no solution in the ordinary sense to offer, and the uniformity of the criticism suggests that any solution within the framework of normal philosophical argument would fail. Consequently an altogether different approach is called for if enquiry is to proceed beyond this point.

The problem of over-necessitation is a more general one than that of PN. PN can be stated in the form: Whatever is negated thereby *is*. Hence for any p , not- p implies p , and in so far as p implies not-not- p (by NC) and not(not- p) implies not- p (by PN), we have the converse too, viz. p implies not- p . But the mutual implication of opposites can be brought about by other means than PN, e.g. by the arguments of a reflexive paradox, though these hold only for propositions p having certain special forms.¹ Whenever we have this mutual implication of opposites it follows that both are true. This remains so, even if we begin by choosing the solution that both are false, for falsehood is effectively definable only in terms of the negative fact making the contradictory of the false proposition true. Over-necessitation consists in the admission of the truth of both opposites. It is with this more general problem of over-necessitation that I shall now be concerned.

It might be thought that dialectical philosophy deals adequately with the problem of over-necessitation. An argument which proceeds from an admittedly true premise to a conclusion explicitly contradicting the premise certainly *admits the existence* of over-necessitation, but this is not the same as saying that it *adequately expresses* over-necessitation. If p expresses a fact (state of affairs) and p logically implies q , this gives no indication of a relation of succession in time between the two states p and q , but it does

¹ It may seem strange that I paid so scant attention to the reflexive paradoxes after showing that the standard sort of solution fails. The reason is that even if, as I am inclined to believe, there is no ultimate solution of an orthodox kind, the problem is nevertheless dwarfed by PN. It must be remembered that I rejected the theorem that a contradiction implies every proposition, and it is only by the help of this theorem that the well known reflexive paradoxes have been supposed to have the highly general paradoxical result that every proposition is true. One reflexive paradox however is exceptional, because it proves the result *directly*, viz. Curry's paradox (*Journal of Symbolic Logic*, Vol. 7 (1942), pp. 115-7). But I feel that PN concerns a level of intuitive thought more basic than this.

directly indicate their co-existence (simultaneous truth). If p is now made the same as not- q , it becomes impossible for us normally, thinking as we do in accordance with NC, to think p and q as co-existing. This impossibility compels us to think the two opposite states p and q as successive. In an ordinary argument in which p is not the same as not- q we can still think the two states as co-existent, and their successiveness is not forced upon us; but because, by contrast, the case of an argument involving over-necessitation forces the successiveness upon us, the impression is conveyed that over-necessitation possess a logic which is peculiarly associated with succession. Two conclusions occur here: (a) that the successiveness is necessitated by the nature of the over-necessitation and so is an adequate expression of the over-necessitation, (b) that the movement implied by the successiveness is logical in nature because necessitated, and therefore that it is not contingent and not essentially temporal. But both of these inferences turn out to be gratuitous additions to the bare premise that the successiveness is necessitated. For in the first place, as we have seen, the successiveness is necessitated by *our enforced reaction* to the over-necessitation, not by the internal nature of the over-necessitation, which implies co-existence rather than succession; and in the second place a successiveness which occurs because co-existence is displaced, and so as the negation of co-existence, is pre-eminently temporal whatever else it may be.

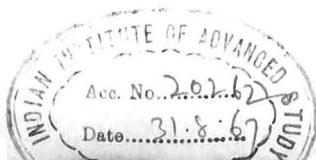
It is clear that *we* cannot normally think contradictory opposites otherwise than by disjoining them, and their separation and succession in time is therefore something demanded by *us*. It is equally clear that what over-necessitation consists in is the *conjunction* of opposites, their co-existence in time, and that the succession of the opposites in time is therefore something *not* specifically demanded by the over-necessitation itself. The conclusion is that the idea of a dialectical movement expresses not the nature of over-necessitation itself, but the fact that a self standing outside the over-necessitation needs to rationalise it. The theory that the sequence of historical events can be shown to be broadly necessary, on the basis of the conception of the identity of opposites, seems therefore to be unwarranted.

I very much doubt whether Hegel's arguments from Being

to Nothing and vice-versa¹ are valid. Yet such arguments are worth studying because they demand a sustained effort to comprehend the conjunction of opposite states, which may well result in a better intuitive grasp of over-necessitation. The value of dialectical philosophy lies in the fact that it concentrates attention upon over-necessitation by pointing to it as a root conception. Naturally, however, a valid argument involving over-necessitation is more likely to lead to an adequate intuition than an invalid one. On the other hand speculation is futile because it takes the orthodox categories of reasoning for granted.

The problem of over-necessitation is immense, unsolved and almost untouched. Let no-one underestimate it.

¹ *Science of Logic*, by Hegel, trans. Johnston and Struthers (London 1929), p. 94.



INDEX OF MAIN SUBJECTS DISCUSSED

- Abstract entities*, I, 5, 7, 8
Abstract System
 Pure abstract system impossible, I 3
Being
 Dependent being (non-substance), I 5 (b)
 Different meanings of to 'be', I 5
 Ideal being, I 7
Categories
 Types compared with categories (see *Being*), I 1, 6
Contradiction
 'A contradiction implies every proposition', II 5 (b)-(g)
 Distinction between appearance and reality on grounds of contradiction, V 7
 Law of non-contradiction (see under this heading)
 Realistic implication of contradiction, I 2
Descriptions, Theory of, III 1, IV 6, 7 (b), V 2, 3
Dialectical philosophy, Conclusion (B)
Difference theory of negation (see under *Negation*)
Existence
 Existence not a property, I 5 (b)
 Existential import, II 2, 9
 Paradox of non-existence (PNE), III esp. 1, 6
 Presupposition of existence, III 2, IV 7 (c)
 Two meanings of existence, I 8, II 9, IV 7 (c)
Fact
A priori proofs that there are facts (including special case of negative facts), III 3, 4, IV 8 (a), (b)
Hypothetical propositions, II 2, 9 (b)
Implication, Paradoxes of, II 5 (f)
Law of Excluded Middle (EM), Conclusion (A)
Law of Non-contradiction (NC)
 Existential or factual form of NC test, IV 5
 Intritive proof of NC, V 7
 NC test, IV 1
 No easy exception to NC, Conclusion (A)
 Presupposed by the Rule of Inference, II 5 (g)
Lewis Carroll's Paradox, II 6
Liar Paradox, I 4
Logical Empiricist Theory of Logic, II 7
Mind
 Fallacious explanation in terms of mind (See Subjectivistic and mentalistic explanation), V 5, 6
Negation
A priori proof that there are negative facts, III 4, IV 8
 Double negation, IV 8 (e)
 Negation as an alternative, IV 3 (end)
 Negation as an operation, IV 7 (a)-(c)
 Negation as a relation of difference, IV 4, 7 (d)
 Negative facts, III 4, IV 8 (a)
 Paradox of Negation (PN), IV
 Paradox of Negation, statement of, IV 1
 Paradox of Negation unavoidable, IV 7, esp. 7 (e)
Non-existence
 Indispensability of non-existence to negation, IV 5, 6
 Irreducibility of non-existence, IV 6
 Limited non-existence, IV 6 (end)
 Meaning of non-existence, IV 7, esp. (c)-(e)
 Paradox of Non-existence, III, esp. 1, 6
Ontological Argument, I 5 (b)
Opposition regress, IV 3
Over-necessitation, Conclusion (B)
Past, Paradox of the, IV, esp. 1
Proposition regress, IV 2
Reflexive paradoxes, I 4, Conclusion (B)
Relations, regress of, II 6 (a), (e)
 'Rule' theory of logic, II 4, 5 (a), 6 (c)
Russell's Paradox, I 3 (end), 4
Subjectivistic and mentalistic explanation, II 8, III 4, 5, IV 8 (b), (c), V 5, 6
Types, theory of
 Aristotle's Type argument, I 5 (c)
 Criticisms, I 2 (1st), 4
 Instance of an argument appealing to Types, IV 8 (d)
 Statement of, I 4
 Types compared with Categories, I 1, 6
Universals, III 5
Vacuous occurrence, II 3, 9 (a)