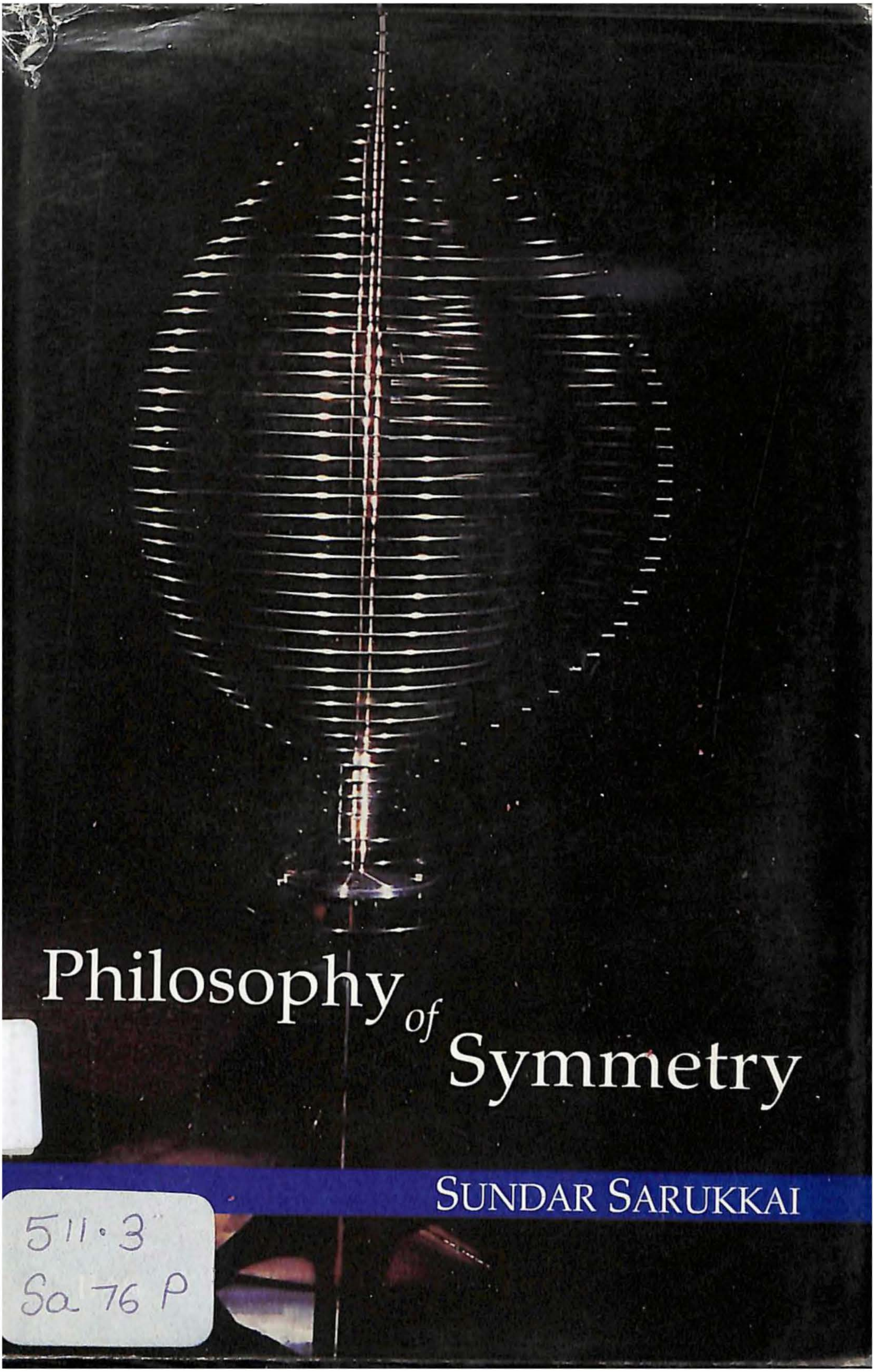

Philosophy of Symmetry

SUNDAR SARUKKAI

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Preface

Symmetry is a term that is used extensively in both technical and common language. It is used across many disciplines ranging from science to the arts. The idea of symmetry is central to science, particularly modern physics. In art, symmetry was a necessary aesthetic principle in architecture and sculpture of all ancient civilisations. Given the breadth of this notion, it is understandable that there is no single concept which can encapsulate its scope. Considering the importance of the idea of symmetry and its occurrence over a whole range of human activity, it is important to explicitly clarify its philosophical foundations. This is the task of this book.

Part One begins with a general overview of the various manifestations of symmetry in nature, science and art. Symmetry is manifested in a wide range of objects, from molecules to galaxies. In science, symmetry plays many roles: for example, to classify crystals, illuminate the nature of spacetime, describe quantum objects and explain the fundamental laws of conservation in science.

Part Two of this book uses metaphysical categories to explicate the nature of symmetry in science. This allows us to consider the meaning of symmetry in objects, relation between change, invariance and symmetry, relation between symmetry and form, metaphysical structure of groups, the special nature of conserved properties and the link between symmetry, conservation laws and causality. Through this analysis, we find that symmetry should be considered as a first-order property of objects and systems.

There is also another dimension to symmetry, its phenomenological one. There seems to be something unique in the phenomenology of symmetrical objects. Balance is a term that sometimes captures this uniqueness. Our experiences with symmetrical objects give us a phenomenological idea of balance, whether in balancing a stick at one point or building a paper plane by folding along the axes of symmetry. The experience of balance is not only tactile; it is also visual and auditory. Tasty food, for example, generally manifests a balance of different tastes. There are also other terms which in our common usage captures the idea of symmetry. These are simplicity, harmony, elegance, unity and so on. Symmetry, from ancient times, has also been intimately associated with the notions of beauty and truth. All these terms suggest that we can attempt to understand the idea of symmetry in art by drawing upon theories of aesthetics. Part Three offers a discussion on the phenomenological and aesthetic aspects of symmetry leading to the conclusion that symmetry in art should be understood as an aesthetic property.

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Balan Nambiar has been gracious enough to allow me to use the photo of his remarkable sculpture made in steel, a piece titled *Valampiri Shankha*, for the cover of this book. I am extremely thankful to him. This sculpture reflects essential principles of symmetry drawing upon both scientific and aesthetic aspects. In this respect, it is quite similar to what I have tried to do in this book and is therefore an ideal representation of the material in this book. The photograph is by Namas Bhojani and this piece was commissioned by Texas Instruments, India. I thank them for permission to use this photo on the cover.

Finally, this work would not have been possible without the emotional and intellectual support of Dhanu.

PART ONE

Universality of Symmetry

Symmetry is manifested in diverse ways in the natural and social world. Figures and objects that occur in the natural world exhibit complex symmetries. Symmetry has been an important principle in arts and architecture. Arguably, it is the central principle in modern physics and is closely linked with some fundamental laws of nature.

Although symmetry is so universally manifested, its fundamental philosophical foundations are unclear. What ideas and concepts underlie the notion of symmetry? What is common in the ideas of symmetry in natural objects, arts and music, and its use in the fundamental formulations of the physical and life sciences? Is symmetry a primary term or is it derivative of other properties and concepts?

These and similar questions will be addressed by formulating a 'philosophy of symmetry'. This formulation will be in two parts: metaphysical and phenomenological. However, before we begin to understand symmetry, it will be useful to see the wide-ranging manifestation of symmetry across many domains.

1. SYMMETRY IN NATURE

Symmetry is everywhere around us. Broadly, we can discern two kinds of approaches to symmetry: one is the phenomenological, which understands symmetry in terms of the 'experience' of symmetry through a symmetrical object, and the other is in terms of specific actions that leave invariant some aspect associated with the object. For example, if we rotate a perfectly round pebble we notice that nothing seems to change under this transformation. This is one way of looking at symmetry, one that is privileged in science. However, even if we do not rotate or otherwise transform an object, we seem to grasp some characteristic of the symmetrical object, perhaps articulated in terms of the balance and harmony of its shape or pattern. A trivial symmetry possessed by all objects lies in the action of moving an object from one place to another. Under this change, the object is unchanged. But this is only a simplistic idea of symmetry and its essential nature will remain unclear until we clarify what changes, what remains the same and their relations to the idea of symmetry.

The natural world consists of objects that manifest various kinds of symmetries. Many objects of everyday experience, like trees, plants and leaves reflect symmetries. Many flowers have a shape that is, most often, perfectly symmetrical. Mainzer (1996, 521) gives the example of inflorescences, which exhibit a high degree of rotation and reflection symmetries, as well as the giant sunflower (*Helianthus maximus*) where 'the small blossoms are arranged in logarithmic spirals, whereby two sets of spirals occur with opposite directions of rotation' thereby exhibiting spiral symmetry. The blossom of the *Gladiolus debilis* exhibits bilateral symmetry in the distribution of its colour (Hahn 1998, 48). Given the predominance of symmetric forms in the plant world, it is reasonable to believe that symmetry plays a central role in the evolution of these objects.

The animal kingdom too extensively exemplifies the property of symmetry. The bilateral symmetry of the human form is found in more than 95% of all types of animals. While it is true that higher organisms do not exhibit symmetry in the distribution of the inner organs, nevertheless the form of most of these creatures is bilaterally symmetric. The starfish and sand stars are examples of creatures which are clearly symmetrical. The common starfish is pentagonal symmetric although other forms are also present.[1] Snails embody spiral symmetry in their shells, manifesting either left-handedness or right-handedness. Creatures such as sponges, rotifers, pterobranchia, echinoderms and jellyfish reflect rotational symmetry. The honeycomb is a classic example of a structure that is highly symmetric. Sometimes even the colours and patterns on insects, birds and animals show complex symmetries, suggesting that symmetry is manifested not only in the form of the creature but also in the distributions of patterns and colours.

Viruses have complex symmetric structures; for example, the adenovirus exhibits the symmetry of icosahedrons (Mainzer 1996, 517). In the case of dynamic change like cell division and reproduction, the plane of splitting is normally the symmetry plane of the cells (Hahn 1998, 37). Even in multi-cellular organisms, cell division generally follows mirror symmetry. Fundamental biological entities manifest complex symmetries. The DNA with its helical structure illustrates this clearly. It must be noted here that this wide manifestation of various symmetries suggests that these are not accidental forms; rather, they are a consequence of natural laws of evolution and formation.[2]

Naturally occurring (as also synthesised) chemical molecules exhibit a wide range of symmetries. Benzene, which is a common example, has hexagonal symmetry. Most naturally occurring carbon compounds have a high degree of symmetry. Polymers reflect 'frieze' symmetries. Naturally occurring proteins are symmetric, although they occur as left and right-handed forms. Haemoglobin has a 'two-fold axis of rotation of its molecular chains' (Mainzer 1996, 507). Even the orbital structures of molecules, such as sigma and pi orbitals of benzene, are symmetric. The ubiquitous water molecule is classified by the symmetry group C_{2v}.[3]

It must be mentioned that many of the important, naturally occurring molecules like fructose, dextrose, tartaric acid, proteins, nucleic acids etc., exhibit asymmetry of the left and right forms in their natural occurrences. This asymmetry is extremely important in various bio-chemical processes. For example, right-handed glucose tastes sweet while its left-handed form does not. The differences between the left and right-handed forms are not always this benign: consumption of left-handed phenyanaline leads to insanity while the right-handed form does not.[4] All these examples suggest that somehow symmetry is a fundamental property of nature and has causal powers associated with it.

Crystals are a paradigm example of symmetry in naturally occurring solids. The study of crystalline structures has significantly contributed to the classification of symmetries. Naturally occurring inanimate formations also show striking symmetries. Symmetry plays an explanatory role in understanding why these forms have turned out to the way they have. The presence of symmetry ranges from the microscopic domain to the cosmic scale. The beautiful forms of galaxies showing spiral and spherical symmetries, for example, once again reinforce the pervasive and fundamental nature of symmetry. Thus, nature in its biological, chemical and physical domains exhibits various kinds of symmetries. The purpose of this book is to understand what these symmetries mean, and how and why they seem to be so central to so many entities of our world.

2. SYMMETRY OF FIGURES

Symmetry is most clearly perceived in idealised geometrical figures. Consider a circle. The circle has many symmetries. If we rotate the circle around its centre we will find that the form of the circle is unchanged. This suggests that the circle is rotationally symmetric for rotation through all angles. If we place a mirror along any of its diameter we find that the form of the circle is unchanged and hence it is also mirror symmetric. It is not true that every change in a circle will be associated with a symmetry. As an example of a non-symmetric change, consider making a dimple on the circle. Now the circle no longer looks like one and thus we would say that the circle is not 'dimple-symmetric'.

Different figures have different symmetries. For example, consider an equilateral triangle. If we rotate this triangle about its centre point, we note that it is only rotations of 120 degrees (and integral multiples of it) that gives us back the 'original' triangle. The hexagon is similarly symmetric under rotation of sixty degrees. In general, a regular polygon of n sides will be symmetric under rotation about its centre through an angle of 360/n degrees. A point to be noted about symmetry of geometrical figures is that the idea of symmetry is dependent on points or axes around which the symmetry is manifested.

The symmetry of natural forms like the snowflake mentioned earlier are symmetric not only with respect to their form but also with respect to some point or axis of symmetry.

The symmetries possessed by these planar figures can be used to classify different figures. Elements that have the same symmetries can be said to belong to the same family or class. Groups are mathematical structures which characterise these symmetries. (Groups will be discussed in more detail in the section on mathematics and symmetry.) For example, the symmetries of the regular polygons are described by cyclic groups. Along with rotational symmetry, figures could also have reflection symmetry. Dihedral groups exhibit both rotational and reflection symmetries. The classification of central symmetries of planar figures (in the Euclidean (flat) space) can be entirely given by these cyclic and dihedral groups. More complex figures, especially those that form lattices, possess translation symmetries in addition to the rotational and reflection symmetries.

Like planar figures, regular three-dimensional figures are also classified by symmetries. The cube is a simple example. Crystals are three-dimensional ordered figures. These figures also exhibit rotation, reflection and translation symmetries. Just as in the case of planar figures, groups corresponding to these symmetries classify the different three-dimensional figures. The idea of symmetry has played a central role in understanding crystal structures. It also helps in classifying the properties of these solids based on the kinds of symmetry they possess. Although these perfect symmetries are most clearly exemplified in ideal figures we have to remember that when we talk of symmetries of natural objects, like the snowflake or crystals, we are most often talking about the symmetry properties of the shape or form of these objects.

3. SYMMETRY AND SCIENCE

A fundamental reason for a philosophical analysis of symmetry is the central role accorded to symmetry by science. The use of symmetry to classify two and three-dimensional regular figures was briefly described above. The discipline of crystallography is indebted to the notion of symmetry. This discipline not only allowed a description of crystals but was also instrumental in the development of spectroscopy, wave mechanics and many other branches of science. The analysis of molecular structures, Pasteur's experiment on tartaric acid that showed the left and right handed nature of certain molecules, analysis of protein structures etc., are some of the important developments related to crystallography. Symmetry considerations offered a cogent theoretical framework to describe and explain these experimental observations.

Even though symmetries were essential to classify crystals, it is only in particle physics that the idea of symmetry comes to occupy a seminal position. The development of gauge theories and particle physics placed symmetry as a foundational principle of nature. The relation of conservation laws to symmetry emphasises this foundational nature of symmetry. It can also be argued that the impetus to relativity theory was based on symmetry considerations. Quantum mechanics (and further on, quantum field theory) engages with the idea of symmetry in an essential manner. Even classical Newtonian physics illustrates fundamental symmetry principles.

Considering the importance of symmetry in all these disciplines of science, it might seem to imply that there is a common understanding of symmetry in all of them. Although the use of groups is common in all these disciplines where the idea of symmetry is prevalent, there is really no common conceptual grounding in all of them. Also, there seems to be some important philosophical ideas that are manifested in the diverse expressions of symmetry in these disciplines. The task of this book is to explore these philosophical foundations. Before I do that, it will be useful to summarize the various symmetry considerations that appear in physics. I restrict myself to physics not because I believe that chemistry and biology can be completely reduced to physics but because symmetry considerations in chemistry and biology are very similar to that of physics. In particular, group theory, which describes symmetries, is the common mathematical tool used in these different disciplines.

In the earlier examples, symmetries were associated with a figure or the form of objects. While this is useful in the study of crystals, the idea of symmetry in classical and modern physics is not restricted to the form of objects alone. A simple example is the time-reversal symmetry exhibited by Newton's law, which states that force is equal to mass times acceleration where acceleration is the second derivative of position with respect to time. Changing t (standing for time) to –t keeps the equation invariant if the force is time independent. We can thus say that time reversal (change of t to –t) is a symmetry of Newton's equation under certain conditions. But note that the ideas of change and invariance in this case are not with respect to a geometrical figure or the form of an object. We can only say that the form of the equation remains the same under the transformation $t \rightarrow -t$. But then we will have to distinguish the idea of the form of an equation and of an object or figure. Most of the important symmetries in physics deal with expanded notions of form, change and invariance.

Time reversal symmetry of Newton's law has observable consequences. For a system that strictly obeys this law, time reversal symmetry would imply reversibility of processes that occur over a time period. There is also a larger set of transformations that keeps Newton's law invariant. Newton's force law incorporates acceleration, which is related to position. Position itself is defined with respect to some frame of reference. Suppose there is another frame of reference that is moving with uniform velocity along the common x-axis of both the frames. Then it can easily be seen that Newton's law is invariant under the change of coordinates from one frame to another. This larger set of transformation is referred to as the Galilean transformation (Rosen 1995, 77).

This theme of invariance under various sorts of space and time transformations is central to symmetry considerations in science. These are dynamic transformations in the sense that changes take place in the position and time coordinates. Under such transformations, certain features are left invariant, such as the equation of motion. There will also be observable consequences corresponding to this like the statement that Newton's law will be the same in every frame moving with constant velocity. Conservation of certain observables, like momentum and energy, is also a consequence of symmetry. Thus, although the idea of symmetry in modern physics seems to be indebted to mathematical forms, its significance is captured in the corresponding physical observations.

Relativity is based on symmetry considerations. In fact, van Fraassen (1989) goes to the extent of saying relativity is symmetry. The claim that 'physics' must be invariant in different frames of reference implies that more complex transformations of space and time must keep invariant (covariant) the laws as well as the observational consequences of dynamic equations. Lorentz transformation is a specific form of transformation of the space and time coordinates such that the equations of special relativity remain covariant. In a particular classical limit (when the velocity is much smaller than the velocity of light), the Lorentz transformation reduces to the Galilean transformation. Special relativity is invariant with respect to 'global' Lorentz transformations, thereby meaning that the 'laws of nature are invariant with respect to them only if the same transformation is applied to all four points of 4-dimensional space' (Mainzer 1996, 351). Global here refers to the constancy of velocities of the different frames of reference.

In relativity, the connection between symmetry and nature arises in the following way. Euclidean space is 'flat' space. Minkowski space is a four dimensional space, also flat, which considers space and time coordinates as belonging to the same 'kind' but with the crucial difference that the time coordinates carry a negative sign while the space coordinates carry a positive sign (or vice versa, according to convention). Minkowski spacetime is invariant under global Lorentz transformations. Thus, if we believe that space and time form a continuum and that they are Minkowskian in character, then the global Lorentz transformation actually reflect the symmetries of this spacetime. In other words, just as objects exhibit symmetries as discussed earlier, the spacetime 'object' too has its symmetries.

The general theory of relativity goes one step further. While global

Lorentz transformation was restricted to constant velocity with the implication that all of Minkowski spacetime changes in the same way, local Lorentz transformations allow for non-uniform change of spacetime. Physically, this is the case of comparing physics in accelerated frames of reference. Generally, the physics will not be the same in accelerated frames of reference. But if we make a modification to the field equations to account for this, we get Einstein's general theory of relativity.

The symmetry corresponding to transformations of space and time coordinates is exhibited in quantum mechanics also. For example, the wave function of a particle under Galilean transformation transforms into another function which differs from the original one only by a complex phase factor (Landau & Lifshitz 1977, 52). Since observables in quantum mechanics are bilinear forms, that is, involving products of the wave function and its conjugate, the Galilean transformation leaves them invariant.

External symmetries are important in studying composite systems like atoms and molecules. The movement of the electron around the nucleus of an atom will possess rotational symmetry. This has important consequences in spectroscopy and in understanding molecular bonding. In this context, a well-known result is that of Wigner who described the essential connection between symmetries and the quantum numbers of the spectra (Mainzer 1996, 388).

Quantum systems exhibit many interesting symmetries. Three very important ones are those of charge conjugation, parity and time reversal. These are discrete symmetries in contrast to continuous symmetries discussed above, that is, transformations do not range over all possible values. Typically, a discrete symmetry is defined as a symmetry operation which 'if applied twice to any physical system, will leave that system unchanged' (Emmerson 1972, 33). Parity (P) is a simple operation which replaces space coordinates by their negative value, i.e., x by –x, y by –y and z by –z. In general, parity is a symmetry of nature. But like all symmetries we will consider, there are certain systems that do not exhibit this symmetry. It is well known that electromagnetic and strong interactions conserve parity but weak interactions do not (ibid., 46 – 47).

For every particle with charge q, we can conceive of a particle with charge –q with all other properties like mass and spin remaining the same. Such a particle is called the anti-particle of the original particle.

This operation of replacing a charge by its opposite value is called charge conjugation (C). If charge conjugation is an exact symmetry of the world, then it is reasonable to expect a world of antiparticles similar to our world made of particles. But the universe shows a distinct asymmetry between particles and antiparticles: the amount of antiparticles occurring naturally is much less than the particles. For example, there are innumerably more electrons than positrons (anti-electrons) in our universe. This suggests that the symmetry between particles and antiparticles is not manifested in our universe. It must be mentioned that for strong interactions this remains an exact symmetry but is violated in weak interactions.

Like parity, time reversal (T) involves changing t to –t. Again, it is not clear if this is an exact symmetry of the world although it might be so for strong interactions (ibid., 55). Note that for discrete symmetries, there are no corresponding conservation laws as for continuous symmetries (Itzykson & Zuber 1980, 21).

These three discrete symmetries, P, C and T, are important for one reason: the combined operation of PCT is always an exact symmetry. That is, every process of nature, whether they belong to strong or weak interactions, always obeys PCT symmetry taken together (Emmerson 1972, 56).

These examples, except for charge conjugation, have to do with changes in space and time coordinates. These are usually referred to as external symmetries. But one of the most important cases of symmetry is that of internal symmetry where the transformations are not changes in spacetime coordinates but of some 'internal' parameters. These internal symmetries have come to occupy a central role in modern science.

A simple but important example of internal symmetry arises in quantum mechanics. Isopsin symmetry is an important symmetry that has had great influence in theories that followed. The proton and neutron are 'similar' to each other, except that the proton has a unit positive charge and the neutron has no charge. Heisenberg suggested that one could consider the proton and neutron as belonging to one 'family' (the doublet) and the difference in mass can then be derived/explained through the breakdown of the symmetry. The symmetry of this 'doublet' is referred to as SU(2) symmetry. This idea of family resemblance is used to construct more fundamental symmetries in particle physics, as in the grand unified theories.

An extension of this is the SU(3) model, which was used to predict quarks. For example, the triplet of pions forms a family. In the case of quark model, the up, down and strange quarks belong to a triplet family. These kinds of symmetries are idealised symmetries because nature only manifests them 'approximately'. (This is similar to exact symmetries of geometric figures as against symmetries of objects in nature, which only have these shapes with small deviations.) The notion of 'internal' in the SU(2) and SU(3) cases refers to the symmetry under rotation in the two and three dimensional *internal* space of these doublets and triplets respectively. (It may be noted that the 'spin' of elementary particles is also an internal characteristic of the particles.)

Similar to global and local transformations of spacetime coordinates, these internal symmetries can also have global and local transformations in the internal space. If the SU(2) group is dependant on spacetime coordinates, then the corresponding symmetry is a local, internal symmetry. In general we may note four kinds of symmetries: external, global; external, local; internal, global and internal, local (Kosso 2000, 83). These symmetries are symmetries of nature – of objects, spacetime and events in both the macro and the microscopic world. Science believes that there are observable consequences of these symmetries, as well as causal roles that can be ascribed to them.

There are two other internal symmetries that need to be mentioned: permutation symmetry and supersymmetry. Particles in the quantum world are either fermions (those with half integer spins) or bosons (those with integer spins). They have a fundamentally different property under permutation. The famous Pauli principle says that no two fermions can occupy the same state whereas any number of bosons can do so. The final project of unification of the forces and particles in nature must also attempt to unify fermions and bosons. Like the case of the proton-neutron doublet, we can consider a doublet of a fermion and a boson. Supersymmetry corresponds to the transformation that changes a fermion to boson and vice versa. As a consequence, it is postulated that for every fermion there is a corresponding bosonic partner. For example, an electron has a corresponding boson 'selectron'. We can note the similarity of this to the particle-antiparticle pairs: for every particle there is an antiparticle derived through charge conjugation.

As in the earlier cases, there can be both global and local supersymmetry. What is most interesting is that local supersymmetry necessarily involves introduction of gravitational fields naturally into the model. (This is similar to the local symmetries of general relativity.) Thus this allows the possibility of unification of the four fundamental forces of nature into 'one' model. This local supersymmetry theory is also called supergravity. Of course, physics has not stopped with this construction but has gone on to develop string and superstring theories. Even in these models, the idea of symmetry as described here holds. Thus it is clear that symmetry is central to the formulation of science and we can agree with Mainzer's (1996, 477) statement that 'current high energy physics and the physical cosmology derived from it are closer to the Platonic ideal of exact symmetry in nature than any previous developmental epoch in the natural sciences.'

3.1. *Broken symmetry and asymmetry*

In general, the symmetries described above are exact mathematical symmetries. When we say the Lagrangian (or equivalently, the equation of motion) is invariant (or covariant) under some transformation, it is usually the mathematical symmetry and this is an exact symmetry. But the natural world does not usually manifest this complete symmetry. This is similar to the difference between a real circular object and the idealised geometrical circle. In fact, the real world predominantly manifests inexact or approximate symmetry. I believe that these approximate symmetries should be understood as deviations from symmetry rather than independent of symmetry considerations.

We have already noted the diverse expressions of symmetry in physics –from symmetries of objects like crystals, symmetries of spacetime, discrete symmetries like P, C and T, and internal symmetries. In many physical examples, these symmetries will not hold 'completely' and there are consequences that arise from the deviation from symmetry. These are most powerfully exemplified in gauge theories.

We must distinguish between symmetry violation and broken symmetry. For example, when we say that parity is violated in a process, it means that the expected symmetry of parity is not to be found in that particular process. This is not broken symmetry, unless we know that the system had parity symmetry but lost this symmetry due to some reason.

Consider a round drop of water, say formed at the mouth of a tap. As it forms, the gravitational pull tends to elongate the surface downwards. The exact symmetry of the drop is broken by gravity. In the case of animals, gravity too acts to break the full symmetry.[5] These examples suggest that it is easier and correct to understand many such phenomena as broken symmetry rather than asymmetry.

In the case of internal symmetries, we considered the example of proton and neutron as belonging to a single family. Proton and neutron differ not only in their charges but also in their masses. But physicists think it more fruitful and elegant to consider them as belonging to the same family, described as a doublet. The mass difference is then explained through electromagnetic interaction, seen as a consequence of this perfect symmetry being broken. All grand unified theories work on this logic. It is first presumed that nature has a particular 'larger' symmetry which allows different particles to be grouped together into one family. Grouping them in this way is to gloss over their differences. Then the differences are explained by saying that this exact symmetry is broken. This view rests on the belief that nature is fundamentally symmetric and essentially ordered. The highest state of symmetry is postulated to be at the moment of Big Bang and over time various symmetries get broken.

It is to be remembered that the idea of broken symmetry is based on the idea of loose identity. There is also an important consequence of broken symmetry which I will discuss in the following section.

3.2. *Functions of symmetry in science*

According to science, symmetry is a fundamental 'property' of nature, manifested in natural objects and in natural processes. As Weinberg notes, symmetry 'is the thing that actually *drives* the dynamics' (Crease & Mann 1986, 187). The idea that nature is inherently simple is a powerful motivating force for theorists. Symmetry, as a central principle, allows this formulation of simplicity. Weinberg considers Lorentz invariance as the most important symmetry of all because it 'is not only a symmetry which governs the form of the equations, it tells us what the equations are about.' He goes on to say that 'the identity of the particle is fixed by its symmetry properties. The particle is nothing else but a representation of its symmetry group' (ibid., 187).

First of all, it is important to note that symmetry is not an 'accidental' property of nature. Even in the case of natural objects which show various symmetries, it can be argued that the particular forms and shapes that they have are due to a prior symmetry principle. In the case of biological organisms, Hahn argues that symmetry is an important evolutionary factor. This means that symmetry plays a causal role that explains why these organism manifest the symmetry they do. Thus symmetry plays a functional role in these organisms.

In the case of inanimate objects that show a wide range of symmetries, like crystals and molecules, symmetry is an organising principle that explains not only why the shapes of these entities are the way they are but also how symmetry influences the dynamics associated with them. For example, chemical bonding used to describe chemical reactions can be deduced from symmetry considerations. Nature seems to subscribe to a least action principle. Simply put, this means that natural processes tend to take a path of least (actually extremal, which could be either minimum or maximum, but in most cases it is always the minimum) energy. If we analyse why natural objects have the shapes they do, we can find this least action principle at work. This principle is closely aligned with the idea of symmetry. The spherical case is a good example, for not only small objects but also huge planets, stars and galaxies possess (approximate) spherical shape. The sphere has a high degree of symmetry. When nature tends toward natural formations that show a high degree of symmetry, the influence of various factors such as gravity modify the shape associated with these highly symmetrical shapes.

In terms of theoretical analysis, the principle of least action is of fundamental importance. It is this principle that helps us derive the equations of motion, both in classical and quantum physics. These equations of motion are derived from the Lagrangian (or equivalently, the Hamiltonian). The Lagrangian in the classical formulation is given by the difference of the kinetic and potential energy terms. This Lagrangian will have some symmetries like invariance under Lorentz transformation. These symmetries will also be the symmetries of the equations of motion.

Moving from objects to processes, we note that symmetry considerations have various phenomenological and observational consequences. The most important of which is the relation between symmetry and conservation laws. In modern physics, this is seen as a consequence of Noether's theorem. Noether's theorem states 'that to any continuous one-parameter set of invariances of the Lagrangian is associated a local conserved current' (Itzykson & Zuber 1980, 23). From this current, we can find a conserved 'charge' by appropriate integration. This theorem holds both for external and internal symmetries.

Loosely put, what this theorem states is that when there is a (continuous) symmetry in the system, something is conserved in its processes. Conservation of momentum is a common example. We know that when two objects collide and there is a change in velocities or mass, then the total momentum before collision is the same as after collision. This sameness of an initial quantity and the final one is called as conservation of that quantity. In the case of collision, conservation of momentum occurs if there are no forces like friction. Conservation of momentum is a consequence of invariance under translations. Similarly, if the system is invariant under rotation, the quantity called angular momentum is conserved. Also, if the system does not depend explicitly on the time parameter then we can immediately say that the processes of the system will conserve the total energy.[6]

While these examples of conservation of linear and angular momentum are consequences of external symmetries, conservation is true of internal symmetries also. For example, the conservation of charge is an extremely important principle that is part of the most basic natural processes. As before, we can understand this by saying that the total charge before a process is the same as after the process. For example, in neutron decay the neutron disintegrates into a proton, an electron and a neutrino. The initial electric charge of the neutron will be the same as the sum of the electric charges of the proton, electron and neutrino. In general, knowing the electric charge of the initial configuration, we can make a reasonable guess as to what the charges of the final constituents should be since they are constrained by the equality of the initial and final total charge. The principle of conservation of charge can also be understood as a consequence of some symmetry. For example, conservation of electric charge is a consequence of the global phase invariance of electrodynamics.

Charge in modern physics is not restricted to electric charge. From Noether's theorem, we know that for every continuous symmetry there is a corresponding charge which is conserved. The word 'charge' is used in a variety of ways and exhibits similarity with electric charge in that it is conserved. For example, baryons (heavy fermions like protons) are given a baryon charge and leptons (light fermions like electrons) are given a lepton charge. It is believed that in many processes involving changes of baryons and leptons, their respective charges will be conserved.

These conservation laws are a consequence of the symmetries of nature. They are manifested over a wide range of processes: from collision of billiard balls to elementary particle decays. They also tell us which processes are possible and which are not, as illustrated in selection and superselection rules. The idea of symmetry plays a descriptive, explanatory, causal and predictive role. It is thus rightly a first principle, a primary property of nature – at least, according to science.

Even in the case when symmetry is broken, there is a very interesting consequence. In gauge theories (like the unified theory mentioned earlier), it is recognised that perfect symmetry is not manifested. So this symmetry needs to be broken in order to explain the observations regarding the elementary particles. The symmetry is explicitly broken in the model by introducing new terms in the Lagrangian. Gauge particles are those which carry the force of interaction of the four fundamental forces. These particles get masses in the theory through the breakdown of symmetry. This theoretical mechanism is needed because perfect symmetry implies massless gauge particles. This seems to suggest an essential role to symmetry not just as a principle but also as a dynamical mechanism which is somehow involved in the creation of particles and ultimately matter!

4. SYMMETRY AND GROUPS

It is difficult to talk about symmetry in science without invoking the appropriate mathematical terminology. The discussion in the previous section was to exhibit the different ways in which symmetry is understood in science. The emphasis there was to list the various symmetries of nature in order to support the claim that symmetry is indeed an essential component of natural objects and processes. There was also reference to various groups that classify these symmetries. In this section, I will give a brief introduction to groups. In Part Two, I will describe the distinction between sets and groups in the context of metaphysics and in Part Three, I will exhibit some common structural similarities between groups and the Gestalt principles.

Sets are common mathematical entities. Sets are a collection of members which have some criteria for membership in a particular set.

For example, a set of mammals will have as members creatures that we recognise as mammals. Groups are also sets but with some difference. First, groups are sets with an operation defined on them. This means that not only groups have members but they also have, necessarily, an operation defined over that collection of members. For example, the set of all positive and negative integers (including zero) is a group under the operation of addition. The group members also have to obey certain other conditions.

1. The closure property: if a, b are two members of the group, then a • b should also be a member of the group. The group operation is denoted by •. For example, for the set of all integers mentioned above, we can take any two elements, say, 5 and 7. Since this is a group under the operation of addition, then 5+7 (= 12) should also be a member of the group, as it is.
2. There is an identity element, which is unique, in the group. That is, there is a member of the group, e, such that a • e = a = e • a for all members of this group. In the case of the above group, this identity element is 0, since a + 0 = a for all a.
3. Every member a of the group has an inverse, denoted by a^{-1} such that $a \bullet a^{-1} = e = a^{-1} \bullet a$. In the above example, – a will be the inverse for every a, since a – a = 0, and 0 is the identity element.
4. The elements of the group obey associativity law. That is, a • (b • c) = (a • b) • c.

There are various kinds of groups. In the example of the equilateral triangle discussed in the last section, it was mentioned that the triangle is invariant under rotation of 120 degrees. As we can easily see, the triangle is also invariant under rotations of 240 and 360 degrees (and integral multiples of 120 degrees which are essentially the 'same' as these three angles under rotation). The set of these three angles forms the C_3 point group. (Point because in these rotations the central point does not change.) All planar and higher dimensional symmetric figures can be classified by groups. For example, the dihedral groups describe rotation and reflection symmetry. These are discrete groups because not all rotations are possible. In the case of the equilateral triangle, the only rotations that leave the triangle invariant are the three angles mentioned above.

The circle has many symmetries, actually infinite. If we rotate a circle around its centre by any amount, however small or big, the circle remains invariant. The symmetry of the circle is an example of a continuous symmetry. This rotational symmetry of the circle is described by the group U(1) whose members are all of the form e^{ix} . We can easily check that for any two members, e^{iy} and e^{iz} , $e^{iy} \bullet e^{iz} = e^{i(y+z)} = e^{iw}$. Also the identity is 1 and inverse is e^{-ix}.

Continuous groups are very important in modern physics. These are called as Lie groups. For example, the group SO(3) is the set of all rotations in three dimensional Euclidean space. Although these are abstract definitions of groups, we can find representations for them. These representations obey the group equations. For example, the elements of SO(3) can be represented in terms of 3x3 matrices.

It may be mentioned here that there is an interesting connection between groups and shapes. The U(1) group has a manifold structure, that is, the group is 'like' a manifold, meaning that its elements are like the points of a manifold. In this case, the manifold is the circle. The SO(3) group has the manifold of a sphere in our usual three dimensional space. The important complex group SU(2) has the manifold structure of a four-dimensional sphere.[7]

5. GENERAL SYMMETRY PRINCIPLES

So far, I have given examples of various kinds of symmetries that occur in nature. There are also a few general principles that try to explain the nature of symmetry. These principles are generalisations of some of the common aspects of symmetry.

Rosen (1995, 104) formulates the symmetry principle as follows: 'The symmetry group of the cause is a subgroup of the symmetry group of the effect ... Or less precisely, the effect is at least as symmetric as the cause.' Note that this means that the symmetry of the effect can in principle be larger than the symmetry of the cause. As a modification, of particular relevance when symmetry is broken, Rosen, quoting Birkhoff, adds, 'nearly symmetric causes need not produce nearly symmetric effects' (ibid., 130). That is, '*approximate* symmetry of a cause might appear in the effect as exact symmetry, as approximate symmetry, or as badly broken symmetry' (ibid., 133).

In the case of processes and natural laws, Rosen offers a more general principle of symmetry that captures the evolution of the processes: 'For a quasi-isolated physical system the degree of symmetry cannot decrease as the system evolves, but either remains constant or increases' (ibid., 145).

And a modified 'special symmetry evolution principle' stating: 'The degree of symmetry of the state of a quasi-isolated system cannot decrease during evolution, but either remains constant or increases' (ibid., 146).

The Curie symmetry principle is similar to Rosen's formulation but does not recognise symmetry of causes as a subgroup of effects. This principle, while stating that symmetry elements of causes 'must also occur in the effects,' also states that the 'effects cannot contain more elements of symmetry than the causes' (Mainzer 1996, 511). There is another statement that if effects 'possess a certain dissymmetry,' then the cause will also manifest it.

Similar arguments are offered by van Fraassen. He points out that there are two forms of arguments when we consider symmetry principles. The first one he calls the 'symmetry requirement: problems which are essentially the same must receive essentially the same solution,' and the second being that 'an asymmetry must always come from an asymmetry' (van Fraassen 1989, 236 and 239). The similarity of these two formulations with the ones described above can be noted.

6. SYMMETRY IN ART

It seems clear that symmetry is of fundamental importance to the scientific description of nature. In the case of arts, the situation is not so clear. Ancient and medieval cultures have exhibited a high degree of engagement with the idea of symmetry in the fields of arts, architecture and even music. But the fundamental problem in enquiring about the nature of symmetry in these fields, unlike in science, rests on an ambiguity concerning the meaning of symmetry. In the case of scientific discourse, symmetry is understood in terms of invariant transformations. The description of symmetry by groups further gives a semblance of uniformity to the notion of symmetry in all its manifestations in nature. Whether we are talking of symmetry of crystals, or of patterns on an insect, or of processes we are working within the same formulation of invariance using mathematical groups. However, this does not mean that there are no philosophical issues in this approach. All that it shows is that there is some conceptual and methodological homogeneity in the study of symmetry in science.

In art, the situation is somewhat different. Some common manifestations of symmetry occur across many domains: patterns used in ornamental art; the use of certain proportions (such as golden section) in architecture; ideas of balance and harmony in painting; repetition of certain harmonies at specific intervals in music and so on. In none of these cases we can understand symmetry in terms of invariant transformations. In these fields we do not even begin to believe that we could describe objects of interest through frames of reference with respect to which transformations can be performed on these objects. Thus we tend to come across terms like balance, harmony, simplicity, unity and elegance that somehow seem to keep referring to the idea of symmetry. Also, since we do not understand objects of art through a mathematical description, the description of symmetry through groups is also not possible – thereby negating the simulation of a belief that symmetry is one clear idea manifested in different ways in nature.

But this does not mean that symmetry was never an important concern in arts. It is well known that the earliest cultures across the world, whether they were ancient Indian, Chinese, Greek or the Navajo cultures, created art forms that exhibited a strong sense of symmetry. In Indian art and architecture, symmetry principles played an important role. It is well known that the fire altars used in Vedic rituals were built based on considerations of symmetries. Vatsyayan (1983, 27) notes that the different designs of the altars were 'conceived in the likeness of the human body.' Both man and bird images were used to delineate the final form of these altars. These shapes embody geometrical forms and the importance of these geometrical motifs is captured in the comment that these motifs guided the 'destinies of Indian art for centuries' (ibid., 33). The form of the square and the circle (and combinations of these) figure prominently in the *Nāṭyaśāstra*. These two figures symbolise the 'coming together of two opposites' (ibid., 42). The square is the 'perfect form suggesting order' whereas the circle 'is the continuum of time' (ibid., 42). The example of *Sricakra*, with its complex conjunction of symmetrical figures is another example of the importance of symmetry in ancient Indian thought.

In the *Nāṭyaśāstra*, the representation of the body and the many postures described in it also exhibit preference for symmetry. The navel of the body is the 'centre of the wheel (*cakra*) of the Vedic and Upanisadic image' (ibid., 52). (Note the similarity with Leonardo da Vinci's famous drawing of the human figure inscribed in a circle.) The various postures of the body, described in the *Nāṭyaśāstra*, used in sculpture and theatre (including dance) can also be inscribed in the form of symmetrical and proportional relations within a circle. One of the central positions so described exhibits a highly ordered symmetry of the various 'parts' that define the postures (ibid., 54).

The geometrical figures are also used as symbolic representations. For example, the equilateral triangle (the 'most' symmetric form of the triangle) in Vedic thought represents *Puruṣa, agni* as also *Viṣṇu.* The various Indian dance forms illustrate this engagement with manifestations of symmetrical figures. As Vatsyayan notes (ibid., 57):

> *Bharatanāṭyam* is a series of triangles in space, *Kathakali* a square, *Manipuri* a spiral or an intertwined serpent and *Kathak* an axis. *Orissi* evolves its distinctive basic motif of the tri-bhanga which is also a symmetrical geometrical figure from the *vaiśākha sthāna.*

The human body was also used as a measure in Indian architecture. In the case of temples, 'the analogy of the human body is consistently followed in the structural plan of the temple' (ibid., 74). In Indian architecture and sculpture, the principles of composition were determined by the square and the circle.

Both in the Indian and Western traditions, the idea of symmetry is essentially related to proportion and measure. The word 'Symmetry' is derived from the Greek roots *sym* meaning 'with' and *metros* meaning 'measure' (Hahn 1998, 9). Both Indians and Greeks placed an inordinate emphasis on proportions in art, architecture and music. The Golden Section, which is nothing but a particular value of a proportion, was long considered the 'aesthetic standard' (Mainzer 1996, 41). Even in ethics, Aristotle lays emphasis on the proportional as being what is just (ibid., 48). The ideas of proportion, harmony, balance, simplicity and unity are generally found wherever symmetries are. In both Indian and Greek traditions, symmetry (and the above terms that stood for it) has been associated with beauty and truth. These issues continue to find expressions in art from ancient to modern times and will be the subject of more detailed discussion in Part Three.

7. SO WHAT IS SYMMETRY?

In this brief overview of symmetry, there are various insights into its complexity. In its many manifestations, symmetry seems to play multiple roles and in so doing gets associated with many important scientific and philosophical concepts. The remaining part of the book will explore some of these issues in more detail thereby clarifying the relation of symmetry to many of these concepts. Some of the key terms that are associated with symmetry and an outline of their relation can be listed as follows.

1. **Property**: Is symmetry a property *of* natural objects, both microscopic and macroscopic? What kind of a property could it be? Is it a property of the first-order property of shape or form? Colour symmetry immediately suggests that the idea of symmetry is more than a property of shape. Various other conceptual ideas associated with symmetry preclude the restriction of symmetry to shape. But even if we don't use these arguments and stick to shape alone, we will see that symmetry is not a property which is derivative of shape and in general is not a second-order property.
2. **Conserved properties**: Is symmetry a property of certain events and processes? Conservation of charge or energy, for example, occurs in a process in which the initial and final energy or charge is the same. These events are characterised by the property of possessing appropriate symmetries although it is more often phrased the other way: when symmetry, then conservation. All conserved events seem to exemplify symmetry as a property.
3. **Causal role**: Symmetry seems to behave as a causal agent. It has a causal role that can be used to explain the formation of shapes, evolution of organisms and also the dynamics of processes. The creation of masses through symmetry breaking, conservation laws etc., exhibit a causal logic of symmetry.
4. **Transformations**: In many examples of symmetry (especially in science), symmetry is related to transformations. What exactly is the character of this relationship? Symmetries by themselves are not transformations. We call transformations associated with symmetry as symmetrical transformations. For example, rotation is an action, a transformation. Under this transformation, we may recognize/infer/observe a symmetry. Sometimes, symmetry is

confused and equated with transformations. But when we say that a circle has rotational symmetry, the idea of symmetry seems to 'belong' to the circle rather than to rotations. Also transformations by themselves do not say anything about symmetry. The condition of symmetry arises as a consequence of transformation *and* invariance.

5. **Invariance**: Symmetry has been understood in terms of invariant transformations. Particular symmetries specify invariance of particular transformations. But invariance is a loaded concept. Invariance of what? In the examples discussed above, invariance can be with respect to the form of an entity (snowflake); with the structure of a pattern (ornamental groups); or even the form of equations (as in most of modern physics – invariance (covariance) in the 'written form' of the Lagrangian, Hamiltonian and the equations of motion). The problem of invariance is the problem of understanding similarity and recognizing criteria for comparing initial and final states in order to claim that there is invariance. Thus, this issue is related to the metaphysical problems of identity. As noted earlier, broken symmetries, symmetry breakdown and approximate symmetries are dependent on some idea of partial identity.
6. **Law**: As we have seen earlier, conservation laws are a consequence of symmetry. But what is this nomic role of symmetry? What is the relationship between the form of a law and symmetry?
7. **Symmetry principles**: We have seen some examples of symmetry principles. These principles, including general statements on beauty, truth and simplicity, occur extensively in any discussion on symmetry. Nature illustrates a high degree of simplicity and also exhibits a wide range of symmetries; so perhaps symmetry is related to some idea of simplicity. Theories that build on symmetrical principles have an economy of expression and are seemingly elegant. The least action principle that explains fundamental natural processes also exhibits this simplicity and elegance. But how do we analytically understand simplicity with reference to nature? If mathematical symmetries are simple and elegant, should nature also be so? What is the correspondence between the simplicity of theoretical models and the simplicity of nature? Also, the relation between symmetries in cause and effect is intriguing.

8. **Mereology**: One of the immediate consequences of symmetry is the natural relation of symmetrical objects with the issue of parts and wholes. Given a slice of a symmetric object, the whole is easily imaginable. This is easy to see in ornamental and mosaic symmetries. Knowing that something is symmetric, we need only be given a part and we can confidently construct the whole. Symmetry has the ability to fill in the blanks and to give us a sense of the whole from its parts. This also has phenomenological consequences.
9. **Criterion for kind** (as ordering): When we form a set, we use some criteria for membership. Symmetry offers a criterion for the construction of families. The classification of planar and 3-dimensional objects is an example. Classification of manifolds (and shapes) through symmetry is very important in mathematics. The classification of elementary particles into families that obey some symmetry considerations is also central to modern physics.
10. **Proportion**: This, as we saw earlier, is one of the dominant senses of the idea of symmetry in the realms of art and philosophy. Although this term is phenomenologically loaded with associated ideas of beauty, harmony etc., the link between symmetry and proportion needs to be further examined.
11. **Harmony and balance**: This is related to the point about simplicity. As already mentioned, symmetry is very closely aligned with the notions of harmony and balance. Balance is a fertile image for symmetry. Symmetrical objects and events are usually considered to be 'balanced' and 'in harmony'. Balance of tastes is a simple phenomenological example of symmetrical distribution of tastes. But all these terms have to be sharpened to be of further use.
12. **Perception**: Do we perceive symmetry? Like we perceive colours? There seems to be an observational content to symmetry. Some of the terms in this list like balance, harmony, simplicity, and the relation of parts and wholes are also those that can be phenomenologically accessible. To understand symmetry we have to further look towards phenomenology of perception. The interesting relation between perceptions of form and Gestalt 'laws' of perception also suggests a reason as to why symmetries are described by groups.
13. **Epistemology**: In the use of symmetry as models for theories, its

presence in natural laws, in making possible predictive results and in its explanatory capacity, it seems clear that symmetry has an epistemological role.

14. Aesthetics: Related to points 7, 10, 11 above. There is an aesthetic component to symmetry – both in the case of natural and art forms as well as in the construction of theories and experiments, suggesting its essential role as an aesthetic property.

So what is symmetry? This list conveys the complexity of the idea of symmetry in its most general sense thus suggesting the possibility of a fertile philosophical analysis. The philosophical discussion of symmetry offered here is in two parts: metaphysical and phenomenological. The discussion of symmetry as an aesthetic property is found in the latter.

NOTES

1. For some wonderful pictures relating to the symmetry of these and other similar creatures, see Field & Golubitsky (1992). See also, Hahn (1998, 168).
2. For a comprehensive study of patterns in nature, see Ball (2001).
3. For more on symmetry in chemistry, see Hargittai & Hargittai (1986) and Hoffmann (1990).
4. Mainzer (1996, 510). See also Close (2000).
5. For a more detailed discussion, see Hahn (1998).
6. Mainzer (1996, 297). See also Rosen (1995, 151 – 152).
7. See Schutz (1980).

PART TWO

Metaphysics of Symmetry

Metaphysics offers an analysis of various categories that are useful in understanding symmetry. As the listing at the end of Part One indicated, there are indeed enough reasons to believe that the idea of symmetry is quite complex and a metaphysical analysis could elucidate its nature. I will discuss the relevant metaphysical categories here and analyse the idea of symmetry in the context of these categories. The metaphysical categories that are relevant to our study are: object, event, properties, change, form, identity, quantity, kinds, causes and laws.

1. OBJECTS

Symmetry is manifested in diverse ways across a range of objects. In a sense, to be clarified further, it indeed seems to be the case that symmetries are properties of these objects. Thus, to begin this metaphysical analysis of symmetry it would be useful to clarify the notion of an object.

1.1. *What is an object?*

In general, we can distinguish between two types of objects, concrete and abstract. Concrete objects are typically those that are spatiotemporal, existing in space and time, examples being ordinary material things. There are different formulations of what it means to be abstract and the common one is that abstract objects are non spatiotemporal. There are philosophical problems in this distinction between the concrete and the abstract, and thus a more concise explication of concrete and abstract needs to be given. Symmetry is manifested in both concrete and abstract objects.

Lowe (1998, 34) distinguishes two approaches to the question 'What is an object?' and calls these the Semantic and Metaphysical approaches.[1]

In the metaphysical view 'the term 'object' properly applies to any item which enjoys determinate identity-conditions and hence to any item falling under some sortal concept supplying a criterion of identity for its instances' (ibid., 34). He considers a particular book and a particular boy as examples of objects in this sense.

Even the simple example of a particular book as an object suggests that the recognition of an object as a book involves certain criteria of identity that will enable us to sort books as one kind and this depends on some criteria of identity for objects. When we consider the semantic view, there is inflation in the kinds of entities that can be called objects. A common example is that of a grin: The grin on John's face is broad. This statement seems to imply that there is 'something' which is 'the grin on John's face'. One can then consider this sentence as implying 'John is wearing a broad grin' but this quantifies over grins. Paraphrasing is an attempt to remove the reference to grins. For example, we may paraphrase the above sentence to 'John is grinning broadly' but paraphrasing is symmetrical and does not tell us which of these two formulations should be taken as correct. Lowe also notes the important point that such paraphrasing is also possible for objects like books. One of the ways to deal with this problem is to claim that only those singular terms that have a criterion of identity can refer to objects, thus immediately ruling out grins as objects. Two responses against this are relevant to our discussion. First, formulating criteria of identity means moving into the metaphysical domain and away from limitations of meaning. Second, criteria of identity cannot be provided for all kinds of objects. The identity of such objects, as the example of persons given by Lowe, does not consist in giving criteria 'outside' them.

The metaphysical answer is that 'to be an object is to be an entity possessing determinate identity conditions' (ibid., 37). Note that this is a statement about objecthood and not about the existence of entities that may not have such determinate identity conditions. This view will imply, for example, that subatomic particles, which manifest wave-particle duality, cannot be seen as 'objects'. This is not a statement about existence but about what is it to be an object. Thus, there can exist entities like grins and waves but these are not to be seen as objects.

Lowe extends this metaphysical analysis of objects further by adding the condition of (1) determinate countability and (2) determinate identity conditions. This generates four types of entities. Those that have both determinate countability and identity conditions he calls 'individual objects', such as a book. Entities that have (1) but not (2), like quantum particles, are quasi-objects. Quasi-individuals are those that have (2) but not (1), that is determinate identity conditions make identification possible but they are not countable, like mass and energy. We are then left with entities which have neither (1) nor (2), which he calls as non-objects, an example being 'the particular sphericities of individual spherical objects' (ibid., 58).

It may seem that the identity conditions for objects are actually those of its properties. There are two ways of dealing with this problem. We can subscribe to an ontology that differentiates objects from the 'sum' of all its properties. This is equivalent to taking the position that properties are themselves not objects. Lowe argues that a particular red apple that belongs to the kind apple exemplifies properties such as redness or sphericity only 'in virtue of possessing particular instances of those properties' (ibid., 157). That is, properties are adjectival rather than 'objectual'. Such a position invariably needs an ontology of substance to uphold it. Substance is that which does not 'depend for its existence upon anything other than itself,' in terms of identity dependence. Such a substance is a (concrete) particular and excludes universals, clearly suggesting that universals in this view are Aristotelian rather than Platonic.

Lowe's position then is that particulars and universals can be objects provided they possess determinate identity-conditions. And also both can be concrete or abstract. The identity conditions of concrete objects, since they exist in space and/or time, are 'necessarily temporal in character' while abstract objects will have 'timeless identity conditions' (ibid., 158).

1.2. *Form and objects*

We have noted that substances are concrete particulars. These substances may be composed of other concrete parts as components. Such substances which are not mereological sum of these component parts are called as composite substances. This brings us to the important notions of form and matter that commonly occur in understanding objects. There is an intrinsic relation between form and identity.

> *What* a composite substance is composed of ... may be called its *matter*, and *how* it is so composed may be called its *form* ... It is the form of a substance, rather than its matter, that must be preserved through qualitative and relational changes in that substance – such changes being *events* in

> which that substance participates and through which it persists identically. Thus *sameness of form* (or sameness of 'structure' or organization') is the equivalence relation on a substance's components which grounds its diachronic identity: and precisely what this equivalence relation is will depend on what *kind* of substance the substance in question is. To state what this equivalence relation is for composite substances of a given kind is precisely to provide a *criterion of identity* for such substances (ibid., 168).

A composite substance is made of parts but its composition is not the mereological sum. Thus these substances instantiate a hierarchy of kinds. The important point to note here is that 'the kind which determines a substance's 'form' for the purposes of providing its criterion of identity (that is, the form whose sameness must be preserved throughout the persistence in time of that particular substance) is the *highest* kind in the hierarchy of kinds instantiated by that substance' (ibid., 168).

We can immediately note the relevance of the above formulation to the idea of symmetry in objects. The symmetries of some natural objects are typically those that keep the form invariant. The bilateral symmetry of animals is a symmetry of the form of the animal although its internal parts are not arranged symmetrically. When we say that a snowflake has 60 degrees symmetry under appropriate rotation, we are essentially talking about the sameness of form. Form does indeed provide the criterion of identity over change (transformation) and the symmetry of the object is a statement about the persistence of the object over time. If this is the 'highest kind' instantiated by that substance, then symmetry – which provides a stricter criterion of identity – is fundamental to the object. Symmetry is also used, as we saw in Part One, to classify objects into certain kinds. The idea of symmetry in natural objects indeed manifests this priority of the exactness of form.

But Lowe has an expanded definition of form, namely sameness of form is also equated with sameness of structure. In the case of composite substances, as in his examples of clock and horse, the diachronic identity of these objects is based on the sameness of the structural components of a clock or a horse. When we talk of the symmetry of natural objects and for the most part they are composite substances, we seem to be restricted to the form, that is, the shape of the object. Symmetry transformations of these natural objects do not change the internal structure but this is incidental to the definition of symmetry. Symmetrical transformations by necessity, in the examples discussed, are structure preserving. When a snowflake is rotated through 60 degrees, its form is invariant (the exact nature of this invariance will be clarified later) although the position of its constituent parts, say particula- water molecules, will have moved in spacetime. In any case, the invariance of the form does not change the structural relation. That is, symmetry of the form that is privileged in the symmetries of nature already supplies a stronger criterion of diachronic identity having as a consequence structural sameness. This is made clearer in those objects/figures that are ideal geometric forms like a circle or an equilateral triangle.

The symmetries relevant to the above discussion are those which are associated with concrete objects. We have seen earlier that they are manifested even in events. And in particle physics the idea of symmetry is essentially inspired by and implicated within the domain of mathematical forms. To understand these formulations, I will consider symmetry in the context of abstract objects.

1.3. *Abstract objects*

The metaphysical formulation of objects allows both concrete and abstract objects. Since objects are defined through determinate identity conditions, it implies that abstract entities which possess these identity conditions should also be objects. There are many abstract entities – mathematical entities like sets, properties and even propositions. Some of them lack determinate identity conditions, like the property of colour or even that of a grin. As noted earlier, this does not imply that these entities do not exist but only that they should not be given the status of objects.

There are different ideas of abstractness. One is that abstract entities are not spatiotemporal, in contrast to concrete entities, which if not existing in both time and space, exist at least in time. Lowe calls these as abstract$_1$ entities, examples being numbers and universals. Abstract$_2$ entities are those that cannot exist independently of some other entities, for example, colour. Abstract$_3$ entities are derived from abstractions from concepts such as Fregean extensions.

It is important to note that the lack of spatiotemporal existence (as for abstract$_1$ entities) in itself cannot preclude objecthood. As Lowe correctly notes, to exist spatiotemporally is 'just to have certain sorts of properties and relations – spatiotemporal ones' (ibid., 212). This implies that $abstract_1$ entities can be objects and Lowe, like others who prefer an ontology of abstract objects, takes recourse to causal powers that such objects can have. These causal powers are seen as playing an explanatory role and in this case certain universals can be considered as abstract objects – for example, kind universals like horse possess the required determinate identity conditions. They also play an essential part in '*natural laws* governing the behaviour and composition of all particulars which instantiate those universals' (ibid., 219). Note that $abstract_2$ entities do not have to exhibit the lack of spatiotemporality. They are only those that cannot exist independently from other entity or entities. Events are $abstract_2$ objects according to Lowe.

Consider the case of symmetry. Sets are abstract objects. Groups are sets along with an operation and the elements of this group obey some conditions as described in Part One. Symmetries are described by groups. Transformations are related to events, $abstract_2$ objects. Symmetries play a causal explanatory role – an important role, which can allow us to see abstract entities as abstract objects. So what is the exact relation between symmetry and abstract objects? Is it similar to the relation between symmetry and concrete natural objects discussed above?

What kind of abstract entities occur in our understanding of symmetry? Groups are one such class of entities. But there are others too. Geometrical figures that are symmetric are also abstract entities.

Consider a circle, which has infinite rotational symmetry as also reflection symmetry. A circle is a geometric object and is not located spatiotemporally. We may have a pictorial representation of the circle but that does not make the circle a spatiotemporal object. The analogy with sets is useful here. Sets are abstract objects though we write sets in a particular way: {a, b, c...}, for example. The abstract circle need not only be represented in the usual pictorial form on the page. It can also be defined by the equation $x^2 + y^2 = r^2$. This equation itself embodies the symmetries of the circle. For example, we can easily see that changing x to $-x$, y to $-y$, individually or both together, leaves the equation invariant. The rotational invariance is also easy to see if written in a matrix form. The set of these matrices will form one representation of the corresponding group. (Note that the equation in itself is a relation and not an object but the circle defined by that equation is an object. We may look on this as the equation 'referring' to the object called the circle.)

Now what is common to the mathematical circle and an object which may have circular symmetry? In the case of natural objects, the circle seems to be instantiated concretely in some sense, unlike the mathematical circle. But what both these entities have in common is the property of circularity. This property may be instantiated in particular circular objects which are spatiotemporal. So both the mathematical circle and a circular object have a common property and this allows us to view circles as being present in the natural as well as in the abstract world. Symmetry of the circle is a property of all particular circles, both in mathematical and natural objects.

Do shapes of natural objects, usually seen as a first order property, always have a correspondence with an 'equivalent' mathematical shape? This question is important because I believe shapes have been invested with an undue priority in considering them as primary properties. Since all natural objects have shapes, this priority reflects the belief that shapes are somehow a part of the object. There are reasons to be suspicious of this position. Shapes of objects (form, contour, boundary are the other terms we may use) are *abstractions* from the object. The form or the boundary delineates the extensionality of the object. This abstraction of the shape, therefore, must really be seen on the order of abstract entities. While all shapes are abstracted from the object, certain shapes can have some equivalent mathematical formulation. For example, the circle is also defined by the equation given above. An irregular shape may have no such mathematical form that may refer to it but can always be so defined through some patching of more regular shapes.

Thus, mathematical abstraction as against perceptual abstraction is more exact and describes or maps every part of the form. Our abstraction of shape when we see an object is more general and only recognises an 'overall' picture, as well illustrated when we look at a tree. Perceptual abstraction in general has the characteristic that the form is seen as part of the object whereas mathematical abstraction makes form the ultimate end.

When we usually talk of the symmetry of an object, we are talking about the symmetry of its form. This form is actually an abstraction and shares something essential with a corresponding mathematical form. Symmetric objects have forms that are symmetric. But it is only in the case of physical objects that we separate form from matter. The mathematical circle has only its form, which is also defined through other mathematical ways. Consider a simple example of a physical object that has the shape of an equilateral triangle. We can claim that the shape of this object is *actually* a mathematical equilateral triangle, provided this can be defined mathematically. But how can we define this equilateral triangle purely mathematically, like the circle for instance? And how can we extend this to more complex forms?

Symmetry is an answer to these problems. We can define a mathematical form by defining its symmetries. That is, once we list the mathematical symmetries of an equilateral triangle, any mathematical form that has all these symmetries will be 'equivalent' to this form. Symmetries can mathematically classify all equivalent mathematical forms. In fact, the classification of manifolds is possible through the classification of the symmetries of the manifold. In other words, given the symmetries, we can know what the form is. So now since the equilateral triangle can be classified by its symmetries, we can believe that the form of a physical object which has equilateral form is nothing but the mathematical form.

As a consequence, shapes are not the primary property of objects. The form of these objects is an ideal form, which is defined through its symmetries. Therefore symmetries are the 'entities' that are primary for particular descriptions of objects.

To rephrase it in another way: Lowe argues that identity with respect to form supplies the determinate identity conditions for composite substances. But this form, as a mathematical form, derives its identity not because it continues to 'look like' the old form but because of the symmetries of that form. Thus, symmetry supplies the identity conditions for most mathematical forms and so is prior to what we call as form.

What about irregular forms that cannot be mathematically defined in an exact manner (like the equation of a circle) or that do not possess any exact symmetries? Obviously the shape of these irregular objects is still an abstract shape, if not a mathematical one. (The real distinction is between the abstract and the mathematical. While mathematical entities are all abstract, not all abstract entities are mathematical. Then the question is whether all shapes are abstract or are they, more precisely, mathematical? The reason for the reduction from the larger space of the abstract to that of the mathematical is that it allows us a 'metaphysical ordering'.)

In the case of irregular forms, I would suggest that they can always be decomposed as a 'sum' of regular forms, or they can be patched together with regular forms like what is done in triangulation of spaces. These regular forms are those that can be defined by their symmetries and this patching also allows us to see irregular forms as 'composition' of mathematical ones.

In metaphysical analysis, the primacy given to shape does not seem to be sensitive to the mathematical formulations of it. And in what sense is a mathematical description of use in metaphysics? We need look no further than sets and the important role it plays in metaphysics. My use of mathematical description of shapes is similar to the use of sets in metaphysics.

1.4. *Shape or symmetry?*

Consider the example of the triangle. Armstrong (1997) asks, 'What is it to be a triangle?' And his answer is: 'It is to be a thing anchored by boundaries having just three parts, each of which is a straight line' (ibid., 56). His query is placed in the context of determinables and determinates and the whether determinate shapes can be seen as universals like the example of length. Length is a determinable and a particular length, say one meter, is a determinate. There is a problem in extending this analysis to shapes.[2] As Armstrong notes, 'the unity of the class of shapes is a much messier affair than the lengths, durations and masses...' (ibid., 55).

In the case of triangles, the straight lines are three 'non overlapping particulars' and since the idea of length is inherent in the lines, we are able to use length as determinable. These three lines of the triangle are related to each other in that they intersect and form angles. So, for Armstrong, the triangle can be described 'in terms of properties of the three boundaries and relations of the three boundaries to each other' (ibid., 56). And a similar analysis follows for shapes with more boundaries thus allowing Armstrong to exhibit a more complex relationship between determinables and determinates in the case of shapes.

But mathematics offers a way of classifying shapes and thus suggests how 'unity of the class of shapes' is possible. This is through the use of symmetry which gives a criterion to put a class of shapes into one family. Consider the equilateral triangle. The symmetries of the triangle capture the property of 'triangularity'. We should remember that our picture of a triangle with lines and angles is just one representation of the triangle.

Imagining triangles outside this picture is indeed difficult considering the fact that we are so immersed in this way of picturing a triangle. But the mathematical descriptions (through symmetries or equations where possible) are not limited by this graphic vision.

The case of a circle is simpler to understand. We do not need a picture of a circle to understand everything about a circle. It is a graphical representation of the equation of a circle. The symmetries of the circle are already given in its equation. So when we talk of circularity, it would not be completely dependent on the way we draw it. This is true in general for continuous manifolds like a sphere. It is also true for higher dimensional figures like a 3-dimensional cube or even a 4-dimensional one. In fact just as there are five platonic solids in three dimensions, we can construct analogues of these platonic solids in four and higher dimensions (Mainzer 1996, 162).

The key point here is the difference between an abstract shape and realisations of that shape. The relevant question then is how much metaphysical investment can one put on one particular realisation, namely the visual boundaries? Or following the dictates of *a posteriori* (scientific) realism, which philosophers like Armstrong hold, shouldn't the metaphysical property of shapes go beyond the particular *visual* realisation of shapes?

Let me illustrate it further by considering the theory of manifolds. We can begin with this definition of a manifold M: 'A set (of 'points') M is defined to be a *manifold* if each point of M has an open neighbourhood which has a continuous 1-1 map onto an open set of R^n for some n' (Schutz 1980, 23). Consider the example of a sphere. (In metaphysics it is common to talk of sphericity as the property of the sphere.) First of all the sphere is a manifold – here we are referring to only the surface of the sphere. The equation for a sphere (similar to that of a circle) is $x^2 + y^2 + z^2 = r^2$. This has only two degrees of freedom because the third can be found once we know any two values of x, y or z. This sphere is denoted as S^2. There is more than one mapping from S^2 to R^2 (plane). One is the stereographic map of the sphere to the plane. The nature of the mapping immediately suggests that the sphere has a different global topology from R^2 – that is, we cannot find a single map that is good for the entire surface of the sphere (ibid., 26 – 28). The power of this formulation lies in capturing the essence of the global topology (what we might call as the sphericity of the sphere). As Schutz notes, these remarks also apply to the surface of a bowl or a wine glass, which are deformations of S^2. As we can see, shape is not a term that has any significance in this description. The global properties are captured by maps called diffeomorphisms. This gives us a class of manifolds that share a common global topology. Examples of diffeomorphisms are the mappings from a smooth crayon to the sphere; surface of a teacup and a torus (the exemplar of a torus is a doughnut) (ibid., 29 – 30). Thus we have an illustration of how to form classes of shapes that are defined through diffeomorphic equivalence. The metaphysical priority to shapes inflates the ontology of shapes whereas mapping of manifolds generate 'kinds' of shapes. But also note that shape as it occurs in metaphysics rarely occurs in this mathematical analysis.

Although shape is not significant, symmetry considerations are. Not only do certain manifolds have symmetries, the groups that describe symmetries are themselves manifolds. Here are some examples of manifolds that are related to transformations and groups: The set of all rotations of a rigid object in three dimensions (subset of which are those rotations that leave such objects 'invariant'); the set of all pure boost Lorentz transformations (that occurs in the symmetries of relativity theory); and important for our purposes, Lie groups, which are also manifolds (ibid., 28 – 29). Lie groups are groups of continuous transformations. Every continuous symmetry is described by a Lie group. Since Lie groups have a manifold structure, it is entirely reasonable to believe that the shape of this manifold is nothing but the symmetry of the group.

I hope that by now we can believe that at least for symmetric figures, shape is an ill-defined term and symmetry should take its place. In the case of figures like triangle or for surfaces that are not exactly symmetric a similar conclusion holds but is expressed somewhat differently. For an irregularly shaped manifold, we can cover the manifold with a symmetric figure and thus give an effective description of this manifold.

One may, as a final recourse to save shapes claim that shapes can be accessed phenomenologically. But there are two problems. First, the recourse to phenomenology at this late stage is unwelcome and arbitrary. Secondly, it is not clear that we 'perceive' shape instead of something like balance, symmetry etc. It is not even clear that shape is not something that is essentially abstracted mentally as mentioned earlier.

The upshot of this discussion is the following:

1. Shapes of concrete objects are abstract entities. (Here is a source of confusion – shapes as universals are necessarily abstract entities. But here the notion of abstractness is that even particular shapes are not concrete and are 'abstracted' entities. That is they do not 'belong' to the object like mass and charge do.)
2. Abstract objects have the 'same' shapes as their natural counterparts. As a contrast, consider the example of mass or colour. There are presumably no red sets or green numbers. But abstract shapes have the same shape as their counterpart concrete objects. A particular mathematical circle has the 'same' circularity as a concrete object that is circular and which presumably has the 'property' of a circle. This means that a particular shape and general shapes are properties that are common to both concrete and abstract particulars. What other kind of properties is common in this manner?
3. Shapes can be classified in terms of topology for most of the examples considered earlier. Symmetrical shapes can be classified with respect to symmetrical groups. We have already seen that Lie groups, for example, are themselves manifolds.

These are some of the consequences of a metaphysical reflection on objects and their relation to symmetry. There is yet another mathematical object that has to be analysed in further detail – groups. But before I do that, more comments on point 2.

1.5. *Tropes and symmetry*

Campbell (1997, 125) in a critique of universals suggests a trope structure for properties. Drawing upon D.C. William's theory of tropes, Campbell claims that properties are not universals but particulars. Abstract particulars are called tropes. His definition of abstract is with opposition to concrete – similar to Lowe's abstract$_1$ entities. Recollect our earlier point that shapes are abstractions from concrete objects. Campbell's notion of abstract is equivalent: 'an item is abstract if it is got before the mind by an act of abstraction, that is, by concentrating attention on some, but not all, of what is presented' (ibid., 126). The shift to tropes as against universals understands properties as always being instantiated in a particular entity. Since these properties, whether as universals or tropes, are abstract, the particularity of instantiation of these properties leads us to consider them as abstract particulars, which are called tropes. Tropes are fundamental and Williams calls them 'the very alphabet of being' (ibid., 127).

Tropes appear as 'terms of the causal relations involved' and also 'as the immediate objects of perception' (ibid., 130). Campbell points out that we never see an entire cat – perception of the cat does not involve seeing its back or its interiors. Trope theory claims that what is perceived is not the object but the tropes of the object. This does not mean that all tropes can be perceived. A concrete particular is the 'maximal sum of compresent tropes' where by compresence we mean tropes 'present at the same place' (ibid., 132). Trope theory suggests that a property should be seen as a set of resembling tropes. This set has as its members all the instances of the property. The tropes that are members of a particular set resemble each other. This (seemingly) dilutes the difficulty of comparing resemblances between concrete particulars by considering the resemblances of tropes. In particular, Campbell notes that the problems of co-extensive properties and of 'imperfect community' (finding common characteristics of resemblances in a family, for example) are avoided by the trope approach.

The problem of shape is not easily disposed of even by subscribing to trope theory. As Campbell notes, 'form and volume are not tropes like any other' (ibid., 136). This is because other tropes like colours need form and volume over which they are spread. But since tropes are particulars, this means that whether other tropes are present or not, shape and size are inevitably present and conversely, they are found only in the company of other tropes. Campbell concludes that 'geometric features' like shape and size are 'essential to ordinary tropes' but cannot in themselves be counted as 'proper beings'. Thus he does not consider form and volume as 'tropes in their own right' (ibid., 137). In other words, red tropes, for example, are not classified independently of the shape and size of the particulars in which they occur.

In this context, we need to clarify what Campbell means by 'geometric features' that are in general not seen as real tropes. Campbell seems to imply that all geometric features are like this. Does this mean that lines, angles and manifolds are not real tropes? While he considers tropes as abstract particulars, mathematical entities are abstract, both as particulars and universals. We noted earlier that shape is the same for concrete objects as well as their equivalent idealized and/or mathematical objects. These shapes are abstract but they are not located in any spatiotemporal region because they are properties of abstract forms. In the case of abstract objects then, it seems that their properties cannot be tropes. 'Geometrical features' is only one example of mathematical properties. Sets give rise to other mathematical features.

In the context of symmetry, the trope approach seems to be more immediately sensitive to symmetry than the approach of universals. If we consider symmetry only as a property of the shape (and this is a limited view of it) then since shapes themselves are not real tropes, and moreover since tropes themselves do not have a particularised property,[3] symmetry cannot be a trope. But there are compelling reasons to believe that symmetry is a good candidate for being a trope. Symmetry seen as a property of objects (this has to be argued and will be done in a later section) can be a trope in itself. Since tropes are what are perceived, a phenomenological response to symmetry in terms of harmony, proportion or balance, for example, can be explained by the tropic nature of symmetry. Moreover, since the problem of co-extensive properties is manifested in the case of symmetry and tropes avoid this problem, it is possible that symmetry itself should be seen as a trope.

1.6. *Quantum objects*

The relation between symmetry and objects is not complete without a brief mention of quantum objects and the role of symmetry in classifying such objects. Within this approach is the genesis of the idea that symmetries are more fundamental than objects. French (2001) discusses the historical developments in the growth of the idea of quantum objects, in particular with relevance to structure, objectivity and groups. Individuality of quantum objects has long been considered a tricky issue. Cassirer argues that electrons are not individual objects and believes that quantum physics supports the 'shift away from particles as substantival 'things'' (ibid., 5). The debate on individuality of quantum particles is too long and complex for me to deal with here; I will therefore restrict myself to a few comments on the group-theoretic formulation of objects. The beginnings of this view can be traced to the classic paper by Wigner in 1939 on the irreducible representations of Lorentz group, which had a correspondence with the equations of quantum mechanics. This approach was finally instrumental in classifying quantum particles.

The association of these representations with objects imply that what we get are 'kinds' of objects and not individual, distinct objects. Thus, we will have the class of electrons through this classification and not individual electrons which are marked out in some way through this group representation. Castellani notes that the group-theoretical considerations are necessary but not sufficient for unique determination of quantum particles (ibid., 16).[4] The basic point here is that if intrinsic properties are derived through symmetry considerations then this is equivalent to 'constituting' the object as a set of invariants (Castellani 1998, 10). The implications and limitations of this approach are still quite open; nevertheless, it suggests the importance of symmetry as a fundamental principle in ontology.

2. SETS, GROUPS AND CLASSES

The consideration of sets is central to metaphysics. The most basic formulations of kinds and classes seem to be naturally associated with the idea of sets. One of the views of property is that a particular property is nothing but the set of all its instances. There are also fundamental problems in the metaphysical analysis of sets. One, well known, is that one can construct any set one likes. Given some criteria of membership, we can construct a set corresponding to it. This becomes a problem when we consider the ontological status of sets.

A set is an abstract object and satisfies determinate identity conditions. The equality of two sets is given by the axiom of extensionality. This is a one-level criterion of identity that states 'if x and y are sets then x is identical with y if and only if x and y have the same members' (Lowe 1998, 41 – 42).[5] Symmetries are described by groups and groups are first of all sets in the sense that they collect elements together. But there are also some important differences between groups and sets.

2.1. *Sets and groups: some differences*

Can we supply identity criteria for groups? For example, we can extend the axiom of extensionality to groups and thus supply an identity criterion for groups. But there is an added rule here that is needed to not only equate the members of the groups but also the operation that is defined on the groups. And this leads to another problem – what does it mean to give an identity condition for operations? An operation is typically a two-place relation (and in the context of groups, they need not be symmetric – that is, for two elements *a* and *b* of a group, *a* • *b* need not be equal to *b* • *a*). So if we consider a group as consisting of elements and an operation, then any identity condition must supply 'identity' for this operation. Note that we can also have relations in sets, that is, a relation between two elements of a set. For example, we could have a relation of ordering in a set of elements that is defined by a > b > c > d for the set {a, b, c, d}. But the role of the operation in groups is not of this 'kind'. It is a relation that is actually defined by the constitutive rules of the group, the rules which make the group a group. As we have seen earlier, these are the rules of closure, identity, inverse and associativity.

We can construct both sets and groups. To construct a set, we choose a criterion (or some criteria) and collect all members which satisfy this criterion. In constructing groups we may begin with say two elements following some criterion *and* then impose the rules associated with the operation. The closure property, for example, will necessitate that a third member of this group could be the result of *a* • *b*, if this result is not equal to either *a* or *b*. The identity element is specific to the nature of the operation. For addition it is 0 and for multiplication it is 1. (Note that 0 and 1 are not necessarily numbers. They can be representations of 0 and 1, as in the form of matrices of appropriate dimensions.) Thus the membership of the group is constrained by the operation and certain rules with respect to this operation.

In an important sense, the rules of a group can be seen as rules that restrict the members of a set. Here, we can use the ideas of internal and external relations. The elements of a set will have internal relations, those that constitute the criteria of membership to the set. For example, we may construct a set based on resemblances and resemblance is itself a two-place symmetric relation. This is an internal relation in the sense that it is also the criteria to collect the elements together in the first place. The group operation may be thought of as external relation but it is not clear that this is necessarily so. For, membership to the group depends essentially on this operation. As an example, consider the equilateral triangle whose invariant transformations form a group. The set of elements of this group are angles whose values (in degrees) are (120, 240, 360). The group operation is addition of these angles which corresponds to rotation of the triangle. It is the case that (120, 240, 360) is a set – but of what? We may say that it is a set of three angles which are the first three multiples of a given angle, namely 120 degrees. Instances of these sets could just be a collection of angles that have these three values. As a set this has nothing to do with the equilateral triangle per se. The many instances of this set could be instantiated by angles made by two lines of various lengths. This collection is indeed infinite and random since any combination of the distribution of these angles are instances of this set.

The role of the group operation is very important. First of all, the group operation restricts the range of the elements of the group. In fact, it gives a criterion for aggregation that explains why the three elements *together* constitute a *single* set. Note that in the case of 'pure' angles, even the meaning of addition is unclear. What does it mean to add two different values of two different angular forms? Whereas when we consider the same set as a group, the group nature gives us a criterion for them *not* to be seen as a set but as an aggregate that belongs to each other in some sense.

In order to understand the difference between sets and groups we will have to consider the many properties of groups. What follows will be a brief description of some of the important characteristics of a group which will illustrate the difference between sets and groups.

The rules of membership to the group (always in the context of a particular group operation) not only restrict the membership to a group but also the kind of groups we can form. While it is the case that the examples of groups we have considered earlier correspond to certain properties, it should be remembered that we can, in general, define a group abstractly. Particular forms of these groups are particular realisations of the abstract groups. Given a group there can be many realisations of it. All these realisations will exemplify the structure of the group which they realise.

The distinction between sets and groups, in the context of membership, can be phrased along the difference between internal and external relations. The criteria for membership to a set can be 'external' and 'accidental' whereas for a group, the membership should obey certain internal and essential relations defined with regard to that group. For example, it is well known that certain properties of an object are not its 'essential' properties and are called as 'mere Cambridge properties'. Xantippe's becoming a widow just because Socrates died or somebody becoming an uncle merely because his sister gave birth to a child are mere Cambridge properties. Sets (as against groups) may be seen as mere Cambridge collection because we collect elements together that may not have an essential relation between the members. Note that this does not mean that the elements may not share some property, say being a planet in the solar system. We should also note another common problem that arises in the case of sets, namely, whether we should make an ontological commitment to the existence of a set over and beyond its elements. If we do not want to make such a commitment then we can say that a set is supervenient on its members and since, at least according to Armstrong, supervenience is an 'ontological free lunch', we make no extra commitment to a set beyond the collection of its members.

Is it different in the case of groups? A group is a collection of its elements. But it is also more than this collection. There is an explanation for why certain collections can be called groups. This explanation is necessary, internal and essential to the elements of the group. That is, the group structure is something more than the collection of its elements. Consider the part-whole relationship in a set. Given a set, we can choose *any* sub collection of its members and this will be a subset of the original set. So we can form different subsets of the original set. For example, if {a, b, c, d} is a set, then its subsets can be {a, b}, {b, c}, {c, d} and various other combinations of singlets, doublets and triplets. The first point to note here is that the subsets belong to the same *kind* sets as the original set. That is, each one of these subsets is a set in itself. This may also be viewed as the partitioning of a set into a collection of subsets. Later, I will discuss the partition in the case of groups and also whether we can make an ontological commitment to them.

2.2. *Classes*

Let us consider sets and classes first. A set is a collection of elements. The collection of all elements into 'one' set gives rise to the plural-singular transformation, leading to the question whether this set exists over and beyond its members. The way we talk of a collection and of a group of things in singular manifests this common tendency to singular quantification. Lewis (1991, 65) notes that plurals 'are the means whereby ordinary language talks about classes.' In the case of sets or classes, Russell's paradox is well known. Even though we may talk of non-self-membered classes in the singular, we know that it is false. But this example does not repudiate the common shift to singular quantification whenever we talk of a collection. When we form a collection, we are indeed bringing together, in a sense to be clarified further, the many different elements.

Mereology is the standard name for this bringing-together. The word composition or fusion is also commonly used. This language is indebted to the notions of part and whole. Given a collection of things we say that the elements of that collection are a part of it. Then loosely we can say that the whole (collection) is a sum of its parts. The fusion of the parts into a whole is called mereology. While mereology has its detractors, philosophers like Lewis and Armstrong take it to be unproblematic. Lewis writes that mereology, for him, is 'perfectly understood, unproblematic, and certain' (ibid., 75).

To understand mereology, we have to have some definition of a part. Let me use Lewis' definition here (ibid., 74).

> '*x* is a *part* of *y* iff everything that overlaps *x* also overlaps *y*; or iff everything distinct from *y* is also distinct from *x*; or iff *y* is a fusion of *x* and something *z*.'

From this, Lewis goes on to state three 'basic axioms' of mereology:

> '*Transitivity*: If *x* is part of some part of *y*, then *x* is part of *y*.
>
> *Unrestricted Composition*: Whenever there are some things, then there exists a fusion of those things.
>
> *Uniqueness of Composition*: It never happens that the same things have two different forms.'

Note that these are entirely metaphysical definitions and the mathematics of set theory is avoided in these definitions. Also note that fusion and class are to be distinguished here. Armstrong (1997, 185) calls fusion as aggregate and like Lewis, he claims 'that to *every* class there corresponds its aggregate.' The aggregate is just the total collection of its elements but the class depends on how we divide this collection into specific parts. For Armstrong, this division is predicated on some principles and is eventually related to states of affairs. Armstrong subscribes to the unrestricted composition principle but calls it 'unrestricted mereological composition'. Since this composition only supervenes on its parts, there is no extra ontological commitment entailed by the aggregate or fusion. Lewis (1991, 81) phrases it thus: 'mereology is ontologically innocent.' Or equivalently, 'if you are already committed to some things, you incur no further commitment when you affirm the existence of their fusion' (ibid., 81 – 82).

Unrestricted composition allows us to construct a fusion whenever there are some things and it 'doesn't matter how many or disparate or scattered or unrelated they are' (ibid., 79). Because both plural quantification and mereology are 'innocent', we have no further ontological commitments. But the case of sets, in contrast to classes, is different. Given a collection of elements, we have a set, a singleton by itself. Then we can construct set of singletons and sets of this set and so on.

It is important to distinguish between parts and members. This is also related to the distinction between classes and fusions. A class has members. For example, the class of all cats will only have cats as members. But a cat may have many other parts, such as whiskers. These are parts of a member cat but are not members of the class of cats. Membership is not the relation of part to whole because a 'member of a member of something is not, in general, a member of it; whereas a part of something is always a part of it' (ibid., 43).

Lewis suggests that classes also have their subclasses as their parts. That is, subclasses are not its members. As examples: 'class of women is part of the class of human beings, the class of even numbers is part of the class of natural numbers' (ibid., 4 – 5). We can distinguish between members and parts by noting that a whole can be divided into parts in many ways just like a class into its subclasses but 'a class divides exhaustively into members in only one way' (ibid., 5). Also, mixed classes and individuals and classes, although allowed by unrestricted composition, are largely ruled out. Individuals by themselves are never part of classes. There are one-membered classes called unit classes or singletons. Singletons have no subclasses, no proper parts and thus are mereological atoms. Every member of a class has a corresponding singleton, as does every singleton and every set. Proper classes which are 'classes that are not members of anything' do not have singletons. Singletons are also unique to a member. Thus a class has as many singletons as its members, in a one-one correspondence. Singletons are parts of classes because they are subclasses. But there may be other subclasses other than the singleton. This structure allows us to view a class as the fusion of all its singletons.

Not all classes are sets; a class is a set if and only if it is a member of something. And since there is a one-one correspondence between member and singleton, this implies that 'something is a *set* iff either it is a class that has a singleton, or else it is the null set' (ibid., 18). Since a proper class is one that has no singleton, this allows him to sidestep Russell's paradox because non-self-member class is a proper class that has no singleton and so cannot be a member.

The crucial thing to note here is that a class is not just a fusion of its members *but* a fusion of the singletons corresponding to these members. Since classes are 'mereological sums of the singletons (unit classes) of their members,' a class {a, b, c, d} is identical with {a}+{b}+{c}+{d}, where the + stands for mereological addition. This also gives us a distinction between fusion and class since a fusion has no 'unique decomposition into parts' whereas a class has in terms of singletons.

The relevance of the above discussion to the analysis of groups is immediate. Lewis' formulation of class as not being the fusion of its members is based on the recognition that class and members belong to different 'kinds', and a part should belong to the 'same' kind as the whole. Since singletons are also sub-classes, they can be parts of a class whereas members cannot. In the case of groups, we can, in the same spirit, demand that the parts of a group belong to the same 'kind' as groups, that is, they should be *subgroups*. Unlike sets, where a singleton of every member is a legitimate subset, a singleton of a member of a group is *not* a subgroup.

Lewis also identifies some fundamental problems in understanding the nature of singletons. We do not exactly know what these singletons are nor how they are related to their elements. We do not know where classes are located – outside space and time or where the members are? Is there some characteristic that distinguishes one singleton from another? And so on. I will not discuss the problem of singletons further but now turn my attention to groups.

2.3. *Groups and classes*

First of all, note that a group has elements just like a set. It is a set with an operation defined on its members. There is nothing very mysterious about the presence of operation within sets. The notions of field and ring in algebra are precisely those that define operations on a set of elements. What does it mean to have an operation defined over the members of a set? Here is one definition that uses the idea of mapping: 'In general an *n*-ary operation in a set *S* is a function $f = f(a_1, \ldots, a_n)$ of *n* arguments $(a_1, \ldots, a_n)$ which are elements of S and whose value $f(a_1, \ldots, a_n) = b$ is a unique element of *S* when *f* is defined for these arguments. If, for every choice of $a_1, \ldots, a_n$ in *S*, $f(a_1, \ldots, a_n)$ is defined, we say that the operation *f* is *well defined* or that the set is *closed* with respect to the operation *f*' (Hall 1959, 2). Another operation is a mapping of a set into another which 'assigns to each *x* of the set *S* a unique *y* of the set *T*' (ibid., 2). In particular, one can map a set into itself. (This can be a one-to-one or many-one mapping.)

There is an important consequence of the mapping of a set into itself. If there are two maps of a set into itself, then we can construct a third map which is the composition of these two maps. It is well known that one-to-one (also written as 1-1) mappings of a set into itself in general form a (noncommutative) group. 1-1 mapping is a permutation of the elements of the group. Permutation rearranges the members of the set. Given two permutations of the members, we can easily find their composition which is also yet another permutation. Although not generally commutative, this composition is associative. The identity permutation is the case of no permutation where the members map only to themselves. Inverse is also easily defined. So the set of all permutations of the members of the set forms a group called the symmetric group.

An interesting consequence follows from Cayley's theorem which can be stated as: 'Every group *G* is isomorphic to a permutation group of its own elements' (ibid., 9). Thus 'every group of finite order *n* is isomorphic with a subgroup of S_n,' where S_n is the symmetric group of degree *n* (Rosen 1995, 64).

So, given a set, a collection of members, the idea of groups is not far behind and is canonically associated with it. The permutation group and Cayley's theorem immediately reflect this claim. Before we consider other groups, we can reflect more on this connection between sets and groups. Consider Lewis' formulation. A class is nothing but a fusion of its singletons. That is, the class {a, b, c, d....} is identical to {a} + {b} + {c} + {d} + ... where the + is mereological fusion. Given a class, the permutation of its members is allowed naturally. This is equivalent to saying that {a, b, c, d ...} is identical to {b, d, a, c ...} or any permutation thereof as long as no member is lost or added. This can be adduced from the fact that mereological addition of singletons is *independent* of the order in which the singletons are 'added'. Note that merely given a class and not using pair ordering (which is set theoretical), we cannot necessarily assume the possibility of permutation. But the singleton decomposition of a class allows for permutation. This permutation is not a transformation of a singleton to another, that is, it is not {a} being transformed into {b} because this, given the individual membership or essence in Bigelow's term, is not possible. Thus, given a class, we are given its permutations and all possible permutations give an associated 'group' of that class.

But this obviously does not mean sets are the same as groups! It merely means that there is a canonical relation between sets and groups. And so it allows us to understand groups using the language of metaphysics of sets. At least up to a point. Given that groups are entities that are central to modern physics, it becomes more important to understand the nature of groups.

Among the many differences between groups and sets is the issue of parts. Firstly, note that subclasses are part of classes. Not only that, the 'parts of a class are all and only its subclasses' (Lewis 1991, 7). Moreover, Lewis adds two further thesis: 'Reality divides exhaustively into individuals and classes' and 'no class is part of any individuals' (ibid., 7). This does not mean that there are no mixed mereological fusions of individuals and classes but only that they can be divided exclusively into individuals (that which has no members but is itself a member) and classes. The basic lesson in this division is that individuals and classes divide reality exclusively and that parts of classes can only be subclasses, and, in particular, individuals are not parts of classes. (This excludes null set as being a part of classes.)

The thesis that only subclasses are parts of classes suggests that a part of a class can only be that which is of the same 'kind' of class as the class itself. We may justifiably ask whether groups can be seen as classes once we are given the group elements. Say {a, b, c, d} is a group. Then is this also a class? My point is that once it is known that the elements a, b, c, d obey the group rules then this collection is equivalent to a set or class of these members. Since in general group elements are mathematical entities, can we say then that groups cannot be classes? There are two responses to this. One is that groups more than sets are 'physical' as so well exemplified in physics. Second, Lewis in an example of a class talks of 'the class of even numbers' as 'part of the class of natural numbers' (ibid., 4 – 5). What exactly constitutes a class *vis a vis* mathematical entities is not clear. Gideon Rosen (1995, 619) in his analysis of singletons says that classes 'are identified with objects we are supposed to have independent reason to believe in – items with a place in an ontological view developed independently of the demands of mathematics.' Jubien (1989, 96) argues that there is no intrinsic difference between a set and class (as an extension) and suggests, for example, that set of dogs and class of dogs are the same thing.

Lewis distinguishes between class and set by saying that a set has a singleton, whereas a proper class is one that has no singleton. Similar arguments can be offered for groups. Let us assume that groups have singletons of them. We should remember here that an important reason for wanting proper class not to have a corresponding singleton is to avoid Russell's paradox. As Lewis (1991, 18) notes, 'the class of all sets that are non-self-members' cannot be a set but is a proper class. Now it is the case that for groups there is no equivalent formulation of 'group of all groups that do not belong to themselves'. First of all, unlike sets, there is no way to form 'group of groups' like 'set of sets'. Note that if we are given a group, we can find its subgroups. This is not the same as group of groups. While sets always allow for a set formation whose elements are other sets, in general it is not clear how it would be possible to have a group of groups all of whose elements obey the group rules. That is, if we are given two groups G and H, it does not follow that {G, H} is itself a group. The minimum condition for even bringing them together would be to check if the group operations of G and H are the same. Let us assume that it is so. Then we have the possibility that we can construct a *set* whose elements are those of G and H with the common operation. For this set to be a group then we have to check whether the group rules hold. Typically we can envisage an immediate problem. While G and H are closed within themselves, the element $g \bullet h$, $g \in G$ and $h \in H$, need not belong to this set {G, H} in which case {G, H} is not a group. Thus, although we can form a 'set' of groups, it is not in general possible to form a group of groups. Also, there is nothing called a null group – this is excluded immediately by the definition of groups. That is, if there are no members, there is no identity, no group compositions that can be used to check for group membership. Thus the category of groups avoids the two basic problems that beset sets – null sets and

Russell's paradox. This implies that the problem of accepting singletons for classes is not really a problem for groups seen as classes.

All classes are not sets. Proper classes differ from sets. Proper classes are needed to avoid Russell's paradox. This paradox cannot occur for groups. So there is really no need to distinguish between classes and groups. This does not mean that all classes are groups but only that all groups are classes.

2.4. *Membership*

A perennial problem in accepting classes or sets is to explain how and why we allow members, however disparate and scattered, to form into one class or set. If we accept unrestricted composition, that is, 'whenever there are some things, no matter how many or how unrelated or how disparate in character they may be, they have a mereological fusion' (ibid., 7), then there is really no strong condition that will limit membership to sets or classes. Gideon Rosen (1995, 622) notes that it 'is central to the applications of set theory that there be classes whose members are wildly scattered and heterogeneous.'[6] Lewis believes that mutilating mereology is unduly drastic and instead suggests that we can restrict the 'making of' singletons. Rather than discuss the merits of either approach, I want to argue that such a problem does not arise in the case of groups.

Given the operation or the identity element or one member along with an operation, we cannot have unrestricted membership to a group. Set membership, subscribing to unrestricted composition, comes at no cost. For Lewis and Armstrong, at no ontological cost. But group membership is *really* a membership and the rules of inclusion are as much rules of exclusion. We may naively believe that restricted membership does indeed come with a cost. Perhaps an ontological one? If there is such an ontological cost, and I tend to believe that there is one, then it is manifested in the properties related to groups. In physics, groups correspond to certain symmetries and I will argue later that symmetry is a genuine property of objects and systems. But for now, let me discuss the notion of restricted membership.

We have seen two important consequences of this restriction – there is nothing called a null group or non-self-member group. We have also seen that in general it is not possible to construct a group of groups without many special conditions imposed on this 'fusion'. It is clear that in the case of groups, unrestricted composition does not work. Is this only mathematical? To answer this we will have to consider what it means to restrict fusion. Note here that unrestricted mereology does not imply all classes have to be 'large'. Classes are always restricted in some sense. When we consider the class of all cats, the membership is restricted to cats and not dogs. But there is something else that restricts membership into groups – something which is 'internal' rather than 'external'.

Class membership is external. When we form a class of cats, for example, we conceivably select all those members which are cats. Consider a class of all cats. Supposing a new cat is born after we have collected (at least in thought!) all the cats. Then this new cat, which has 'just' been born, must be allowed membership into the class of cats. The relationship is external in the sense that the membership of this new cat does not in any way depend on the other members of the class. In other words, the other cats have no say in whether this new cat should be admitted into the class or not. It is the membership relation, external in this sense, which grants membership to the just-born cat. In general, set composition is dependent on such 'external' relations.

There is a potential for confusion here. A collection, a class, a mereology, will 'have' more than one relation. For example, Lewis thinks there are 'intrinsic' characters of classes. He is concerned not with the relation of membership to the class – this relation cannot be an important concern for anybody who subscribes to unrestricted composition – but with the character of the relation between the member and its singleton. When Armstrong (1997, 176) in the context of numbers talks of relation, he is also essentially talking about the relation, an internal one, that holds between a property and the aggregate. The criterion of external relation described above is a relationship that distinguishes membership based on whether the relation is sensitive to the members of the class or not. Even Bigelow's argument that coextensive properties are the source of sets does not sufficiently capture the internality of the relation. The internal relation can thus be described as follows: If membership to a class depends on the prior members of the class who decide on the new members, then there is an internal relation among the members of the class. Let me start with this 'weak' formulation for it seems to allow resemblance as an internal relation also.

To strengthen the above points, let me use the idea of generation, an idea that is central to groups also. Lewis, following Goldman, briefly discusses relations that generate. Suppose we start with an 'ancestor' that generates the relations of 'ancestral of membership'. Lewis (1991, 39) notes that Goodman 'stipulates that the ancestral of membership *is* a generating relation in a system founded on set theory.' Lewis argues that classes generated 'via the ancestral of membership is not ... a legitimate sort of unmereological composition' (ibid., 40). His argument is based on the fact that a member of a class is first a member of its singleton and this is not composition but the class as fusion of singletons is mereological. If I understand him right, just having an ancestral member does not explain composition. I think the basic point for Lewis is that we do not have sufficient reasons to postulate compositions that are not mereological.

Now consider groups. Given one element of the group, in principle all or some of the elements can be generated from it. Consider a group {a, b, c, d}. One of these must be the identity which is usually written as *e*, so let us write this group as G = {a, b, c, e}. Given *a*, for example, then we ask what is the composition of *a* with itself – written as a^2. Closure rule of groups implies that a^2 must be a member of the group, i.e., it should be one of *a, b, c* or *e*. Suppose it is *b*. Then we compose *a* and *b*, i.e., *ab*. What is this equal to? And so on. It may so happen that just one element is sufficient to *generate* all the elements of the group. Such a group where a single member, called the generating element, generates all the other members is called a cyclic group. For example, if {a, b, c, e} is a cyclic group and *a* is the generating element, then this group is nothing but {a, a^2, a^3, a^4 = e}. This is one possibility. Note that the presence of the identity element will reduce all other combinations of *a* into one of the group elements. That is, $a^5 = a$, $a^6 = a^2$ and so on. In general, there will be more than one element that can generate all the other elements and one can have a *minimal set* of generators such that all the members of the group can be got from just this minimal set. Even in the case of 'infinite groups' or continuous groups, we can find generating elements. Examples are Lie groups: U(1) has one generator, SU(2) has three and so on.

Thus, the idea of generating elements is central to groups. The group rules restrict membership to the set. All members of the group are generated from one or a minimal set of elements of that set. So now how do we understand fusion of these elements and these elements alone which gives us a new 'kind' of sets called groups?

2.5. *Singletons and partition of groups*

Let me approach the question of fusion through parts. Given a group whose membership is given, we can ask what are the parts of the group? This is a legitimate question because a group is first and foremost a class. Given a group, note that no new elements can be added – because all possible compositions of its members must already belong to that group. (If a cat is just born then too bad, it missed the group-bus!) It is in this sense that internal relations define group membership.

We can say that groups are a special kind of class that have an internal or generating relation among its members. Actually we have to note a clarification here. A group can be abstract. There can be many different realisations of this abstract group. This is similar to the traditional conception of class as extension of a term. These realizations and groups as seen in physics, are closely related to physical structures. While I talk of the group in abstract here, this point about its realisations must be kept in mind.

So if we consider a group as a special kind of class, then what about its parts? The thesis of Lewis states that classes can have only subclasses as its parts. Classes and subclasses belong to the same 'kind' as the whole. In the case of groups, it is reasonable to expect its parts also to belong to the same kind – that is, be subgroups of groups. So we can import Lewis' formulation of parts of classes for groups.

What about singletons? Here is where the major problem arises. Is a group a fusion of its singletons? Let me look at the notion of fusion from the opposing direction, namely partition. The equivalence of a class as a mereological sum of its singletons is equivalent to the partition of a class into its singletons. In the appendix of his book, Lewis (1991, 123) talks about partition in this manner: 'Suppose that x_1 and x_2 are a *partition* of x: that is, they are distinct, and their fusion is x.' This sense of partition is not any different from the partition of a class into its singletons. In the case of sets and classes, this partition into singletons seems 'natural' – mainly because of the independence of the members from each other in contrast to the internal, generative sense described earlier. But given a class, like a group, with *necessary* relations between its members, is it obvious that such a partition is possible? One way of answering this is to look at how groups are partitioned.

Partitioning of groups is an essential part of group theory. We have the idea of a subgroup, namely a subcollection of elements of a group which is a group in itself. All groups have two trivial subgroups – the group that consists only of the identity element and the whole group itself. Any other subgroup is called a proper subgroup. Contrast now with sets. If we ask that the proper partitioning of a group must be a partition only into subgroups, that its parts should be of the same *kind* as the whole, then the group structure itself gives us the criterion for division. Not every subset of a group is a subgroup. Thus if {a, b, c, d} is a group it may not be the case that {a, b} is a subgroup in itself, whereas if this is a set then {a, b} is always a subset. The group structure imposes constraints on how we can form the possible subgroups. For example, there is a result which states that for a finite order group (say of order n, where order is the number of elements in this case), if its proper subgroup is of order m, then m is a divisor of n, i.e. $n = ms$ for some integer s (Rosen 1995, 34). Thus, for example, we can immediately say that a group of order four cannot have a subgroup of order three.

Therefore the ideas of part, whole and mereology have to be further clarified in the case of groups. It is unclear as to what it means to consider a group as a 'sum' of singletons. Given a group, the only singleton that is itself a group, a subgroup of order 1, is that of the identity element. Consider a group {e, a, b, c} where e is the identity element. Then the only singleton subgroup is {e}. (If {a} is also a subgroup, then a should be the identity.) Now if we look at {e, a, b, c} as a class, neglecting its group character, then we can identify it with {e} + {a} + {b} + {c}. In the case of sets, each of these singletons is a set and thus we can conceivably claim that this is a whole-part relation. But in the case of a group, there are no singleton subgroups other than {e}. And we cannot ignore the group character and decompose the group as a sum of singletons because the parts no longer belong to the same 'kind' as the whole.

As a consequence, we can reject the equivalence of a class with the sum of its singletons in the case of groups. In general this may also suggest that groups are more than the sum of its parts if its proper parts are to be seen as subgroups. Can we make this claim? To understand this issue further, let me describe some more characteristics of subgroups and their relation to the group of which they are subgroups.

Given a proper subgroup H of a group G, we can form the set aH, where a belongs to G but not to H. That is, remove those elements of G which are in H and then choose any a that is in the remainder and allow it to act on all the elements of H. Since groups in general are not commutative, the order of the group action is important. The set aH is called a left coset of H. (Ha is called the right coset.) It is easily seen that aH is by itself not a subgroup of G, since aH does not contain the identity element. Moreover, H and aH have no common element. Given G, we can take the union of H and aH. If there are more elements of G which do not belong to this union, we can construct another coset bH, where b does not belong to H or aH. In this way we can continue to construct cosets until we get back the full G (of some finite order). Thus we can write (Rosen 1995, 44)

$$G = H \cup aH \cup bH \cup \ldots \cup kH$$

This decomposition is unique and has some resemblance to the decomposition of sets. But here too the problem of whether only the subgroups are the proper parts of a group persists. As noted earlier, aH, bH etc. are not subgroups of G. Also, for the same group G and subgroup H, we can decompose it in terms of right cosets. The left and right cosets in general, will be different partitions of G. But for an invariant subgroup, these are the same. The invariant subgroup is defined as follows: 'If H is a subgroup of G and $g^{-1}Hg = H$ for all g in G … then H is called an *invariant subgroup* (also *normal subgroup*)' (ibid., 40). It may be noted that for an abelian group (group all whose elements commute with each other) all subgroups are invariant subgroups. Also, for an abelian group, the left and right coset partitions are the same.

If H is an invariant subgroup of G, we note that the collection of H, aH, bH etc. form a group structure among themselves. That is, the rules of a group are satisfied for this collection under the operation of coset composition. (Note that the group operation defined on G is used for creating aH and so on). This collection of H, aH, bH and so on is called a factor or quotient group and is written as G/H. Thus G/H = {H, aH, bH, ...}. This group has sets as its elements. The union of these sets is a unique partition of G. Obviously this partition depends on the subgroup H. For different subgroups of G, the way G is broken up is different. Thus there are as many ways of dividing G as the number of subgroups it has. But in all these different ways of dividing the group, the aggregate remains the same.

Here is then a fundamental difference between partition of classes and sets, and that of groups. For Armstrong, a class is the aggregate 'plus a strict way of dividing' this aggregate into parts. Division through singletons is one way. What the coset decomposition does is to give us a strict way of dividing a group into its parts. What the coset decomposition and the structure of factor group show is that the criterion for dividing a group G is based on its subgroup (H) and is more a relation of 'division' or more appropriately 'proportion', as well exemplified in the symbol G/H. This also suggests that just as a class can be decomposed into a mereological sum of its singletons, the sets that are formed from the coset decomposition play the role of singletons in the division of groups. That is, the group cannot be broken into its smallest constituents – the singletons comprising of one member alone – but in terms of larger fundamental units that are generated from the coset decomposition. For example, if we consider the order 4 group C_4 whose elements are e, a, b, c then its only proper subgroup is H = {e, b}. The unique decomposition of C_4 with respect to H is (ibid., 44),

G = {e, b} U {a, c}

The primary 'singletons' in this case are {e, b} and {a, c} rather than {e}, {a}, {b} and {c}.

It is also pertinent to note the use of 'class' in group theory. But first, the following definitions (Hall 1959, 10):

> An arbitrary set of elements in a group is called a *complex*. If *A* and *B* are two complexes in a group *G*, we write *AB* for the complex consisting of all elements *ab*, *a* ∈ A, *b* ∈ B, and call *AB* the product of *A* and *B*. If *K* is any complex in a group *G*, we designate by {K} the subgroup consisting of all finite products $x_1 \dots x_n$, where each x_i is an element of *K* or the inverse of an element of *K*. We say that {K} is *generated* by *K*. It is easy to see that {K} is contained in any subgroup of *G* which contains *K*.

The complex gives us a way to understand the correspondence between an element and subgroup. This should immediately remind us of a member-singleton relation. Given a group *G*, take as the complexes sets which have only one element. For each of this complex K_i, we can find $\{K_i\}$ and from the above construction we note that this subgroup is always {e}, where *e* is the identity element. This is the only singleton possible. Thus, {e} is generated by all the elements/members of the group. While this perhaps does not shed any light on the mystery of the member-singleton relation, at least we see that groups have an internal structure that allows us to generate a singleton from its members. This means that no other singletons are possible as subgroups. This we knew even before introducing complexes. But what we didn't know was that not only are members of a group generated from each other but that the identity singleton is also a generative relation.

Consider a particular way of partitioning a group. Take a set of elements S in a group G. Form the set $g^{1}xg$, $x \in S$ for a particular g. If there is another element $y \in$ G, such that $g^{1}xg = y$, then x and y are said to be conjugate to each other. Conjugacy is an equivalence relation and the conjugacy class of the group is 'a subset of elements of a group that consists of a complete set of mutually conjugate elements' (ibid., 39). In particular, the identity is always a class. Now, if we take S to have only one element, that is take the collection of individual members then for each member we can form a conjugacy class. For example, if G = {a, b, c, e} then there are four elements a, b, c and e and the conjugacy class of a, for example, will be given by the set consisting of elements $a^{-1}aa$, $b^{-1}ab$ and so on. Then the partition of the group is given by $G = C_a + C_b + C_c + C_e$ where the C_i's are conjugacy classes. In general, we partition G (for S having one element) as

$$G = C_1 + C_2 + ... C_s$$

The C_i's are disjoint classes and every element of G belongs exactly to one class. It would seem that here we have a natural partition of G as a partition over 'classes' or in Lewis' terms, a fusion of subclasses C_i's. Each of the C_i's actually corresponds to the minimal class which partition the group. Does this imply that the conjugacy classes play the singleton role?

Let us look at it in another way. Unrestricted composition implies an independence thesis. Since any mereological fusion is allowed, the members of a class are, in general, independent of each other – in the internal sense that the members by themselves are not used to restrict membership. In the case of groups, there is also a fusion of members but there is a restriction that is internal. The independence thesis allows the class to be seen as a fusion of singletons. Restricted composition does not.

We can also adduce another argument. Given a class, we see that the class is not a fusion of its members but of the singleton of these members. The fusion of members is just an aggregate whereas the fusion of singletons is a class. Even if it is not explicit, it is clear that singletons belong to the same kind as the class and hence the part-whole relation in this is also the same-kinds relation. If we accept this, then we can also expect that in special classes, namely groups, their proper parts must also be of the same kind. But in general it is not possible to partition a group only in terms of subgroups, but in terms of equivalence classes. (If it so happens that every element of the group is conjugate only with itself then we have a partition into singleton classes. But this is in general not the case. Because once a group G has a subgroup H, then we can form $g^{-1}Hg$ for $g \in G$ and the result is that $g^{-1}Hg$ is also a subgroup of G for any $g \in G$.)

In the context of Lewis' formulation, singletons form the equivalence classes. In general for sets, if there is a relation xRy for *x*, *y* belonging to the set, then a set can be partitioned by the equivalence classes. Now consider the relations of equality. In his book, Halmos (1960, 28) notes that if R is the equality relation in a set then the equivalence classes are nothing but the singletons. This implies that the partition of a class into singletons is based on the relation of identity holding in the class. Resemblance, in a loose sense, is identity. Thus the view of class as a fusion of singletons seems to be based on the relation of equality presumably holding in all classes. It is only in this case that a class can be partitioned into singletons. This means that for classes like groups the correct formulation of fusion is through partition that is sensitive to the internal relation holding in that class. Here it may be argued that the idea of relation is borrowed from set theory while Lewis is attempting to avoid set theoretical principles in his formulation. I think the idea of relation in terms of ancestral and his own formulation of mapping and partition suggest that this is not a basic problem. We can therefore conclude that, in general, a group is not a fusion of singletons (but abelian groups, for example, are an exception) – implying that *not all classes have to be fusions of singletons.*

The above analysis suggests that singletons are not *necessarily* the *atomic* parts of *all* classes. What it basically implies is that the subclasses (in the case of groups) for a given partition are not the fusion of their singletons but are themselves to be seen as basic units, and the larger class is a fusion of these complex units. What are the implications for the property of unithood that Armstrong uses to explain the relation between members and their singletons? Unithood, in such cases, is more complex, something on the lines of molecular units and not atomic units.

2.6. *Ontological commitment to groups*

Finally, should we make an ontological commitment to groups? Let me answer it this way. The problem with sets is that they allow disparate and unconnected members into the set. This more than anything else makes one hesitant to accept their existence. In other words, if membership is open to all and sundry, then the club that is formed is really not a club. By restricting membership, there seems to be a genuine 'object' – the club – the exclusive club if you like. Once rules of membership are given, is there more to the club than its members? There is no clear answer at this point. But when we consider the property of symmetry and the property of groups that represents symmetry, then we may have a better reason to consider the possibility of making an ontological commitment to groups.

John Bigelow (1993, 73) considers sets as 'higher-order properties of their members.' Following his claims that 'mathematics deals with universals, with physical properties and relations' (as in the examples of ratios and proportion), he situates sets also as universals. And partly because they 'play a significant role in physical theory', he considers sets as physical. We have already discussed the notion of singletons and the problem of finding the relation that relates a member to its singleton. This relation for Lewis was mysterious; for Armstrong it is nothing but the reflection of a state of affairs. Bigelow offers a different response. Individuals are characterised by their essence. The essence of an individual gives the uniqueness of that individual. An individual's essence is something which it should have and is not shared with other individuals. But this does not mean that the individual cannot have more than one property that would uniquely mark the status of the individual as such. As is usually the case, we have to avoid external and accidental properties and relations that an individual could have. Thus the individual essence must include properties that the individual could not lack. But, in general, conjunctive properties, even if they are internal and 'essential', cannot be an individual essence since for these shareable properties, 'the conjunction will in principle be shareable too' (ibid., 85). And an individual essence cannot be shareable. So Bigelow suggests that there is a property which cannot be shared but must also be a primitive and not analysed as conjunction of other properties. This property is the 'thisness' of the thing or what is also known as haecceity. If haecceity is allowed then it will be an individual essence of the individual and Bigelow suggests that it is this individual essence that is 'identical to the unit set of that individual' (ibid., 86).

Also for Bigelow, like for Quine, sets are universals. Since universals are always instantiated in particulars, if we look upon set membership as a relation of instantiation, then sets could be seen as universals. So when we say an element belongs to a set, we are perhaps really saying that the member instantiates the set. When we consider a set that consists of some elements, then we tend to believe that something is 'common' to the members that allow them to be collected into one set. Just as a singleton is 'an individual essence of its only member' this collection of members suggests that sets are 'plural essences'.

Bigelow also makes the important point that the discovery of coextensive properties is an important project of mathematics. He uses this insight to note that when there are coextensive properties then there must be something they share in common. And what 'they share is their extension, which is a set' (ibid., 92). Thus, he argues that historically the search for coextensive properties in mathematics naturally leads to the formation of set theory – so coextensiveness 'is the source of sets'. But to get a set from coextensiveness needs a further analysis of properties.

There is a significant difference in Bigelow's privileging essence and haecceity, and Armstrong's notion of a set. We saw earlier that the relation of a member to its singleton, for Armstrong, is that it manifests the unit-making property called unithood. He rejects the existence of haecceities because he does not believe 'that there is something about each different particular that makes them internally and secretly different' (1993, 99). He also does not view sets as universals. The notion of similarity underlying universals is based on 'genuine identity' whereas in the case of sets this need not be the case.

We can further understand the relation between sets and groups by considering the nature of the closure property. Armstrong says that the arithmetic + is same as the mereological +, and we can use state of affairs to connect 7 + 5 and 12. But what we want in the case of groups is something else. Given a, b $\in$ G, then under group composition, we can form aRb. The closure rule is not about aRb but is rather a check to see if aRb also belongs to G along with a and b. So once we are given a group under some composition principle of its members, then for understanding the metaphysical nature of a group the particular form of composition is unimportant. This is similar to the analysis of classes. Once we have the class of cats then the analysis is not about the resemblance that affords membership to particular cats in the class of cats. So to analyse group structure along the notions of set and classes, fusion and parts, it is enough to know that the members of a group are 'generated' from some two other members. (Remember that it can also be generated from one member as in the case of cyclic groups.)

Bigelow's formulation of sets affords another way to view this more complex system of membership. Although he thinks that coextensiveness is the 'source of sets', he argues that we need to find a link between coextensiveness and sets. This link is furnished by the 'connection between properties of things and properties of their properties' (Bigelow 1993, 93). The argument goes as follows. Members of a set have properties that are similar and which they share, and also properties which they don't, thereby suggesting the difference between them. Sharing of some properties allows them to be taken into *one* collection but having different properties allows them to be distinct members. His example of points on conic section illustrates this – the points on the ellipse have (properties) that they share with points on the hyperbola – that is being points on a conic section. But being points on ellipse also involves having other properties that have to do with being an ellipse and these properties are not shared with the points of a hyperbola.

Further, Bigelow distinguishes between second-order property – a property of an object and not of another property – and second-degree property which is a property of a property. And what distinguishes second-order property from other, for example, first-order properties of an object is that they stand in 'distinctive entailment relations to properties of its properties' (ibid., 94). Thus, he concludes, sets 'emerge' from coextensiveness by this entailment between second-degree and second-order properties. Properties having something in common imply that the instances of these properties have something else in common – namely 'membership in the same set'. Bigelow also points out that random sets do not fit into this view. In the context of groups we need only note that there is no possibility of random groups due to the restricted membership rules.

Bigelow's view of sets seems more amenable to groups. The members of a group have some properties in common, and are related by the rules described earlier. That is, all members obey the group laws and thus can be given membership to the set. There are also properties which they do not share – the property of having unique inverses (i.e. the inverse of two members is not the same; note that this does not imply that a member cannot be its own inverse). An example may make this clearer. Consider the invariant rotations of an equilateral triangle which form a group consisting of angles (in degrees) {0, 120, 240}. The members of the group share the common property that they are the angles of rotation that maintain the invariance. They also have a common property that the composition of any two members (in this case addition) will yield another member of this collection. This is not a property per se of the members themselves – it is a property of being members of this group. The coextensiveness lies in the property that these angles maintain invariance.

Consider rotations of an equilateral triangle. Let us say that we rotate only in units of 10 degrees. So all these angles – 0 degrees, 10 degrees, 20 degrees share a common property that in this instance they are all rotations of an equilateral triangle. Among these there are three angles 0, 120 and 240 degrees which have a further property (a second-degree property of rotational angles) that these three rotations maintain the invariance of the triangle. The second-order property is a property of the object and in this case it is what we may call the oriented form of the triangle. Then the special subset, the group, emerges from the entailment between these second-order and second-degree properties.

Approaching groups via properties sidesteps the trickier problem of composition, group rules and membership. As in the above example, if we construct the set of all transformations (or compositions in general) that retain invariance of a second-order property then we know that this set will be a group! That is, we do not have to check for closure rule or other rules of a group. So if properties to do with invariance of an object, the entailed set *has* to be a group.

Armstrong has two problems for Bigelow's view of sets as universals. He thinks that there is a distinct difference between sets and universals because in the case of universals (at least for a sparse theory) it is 'quite a feat for two things to instantiate the very same universal' whereas 'nothing is easier than for two things to be members of the same set' (Armstrong 1993, 99). Universals are related to 'genuine identity' while for sets there need be no identity. Secondly, random sets have, generally, no universals (in Armstrong's sense).

Note that both these objections fail to hold for groups. It is indeed quite a feat to get group membership and no two things can arbitrarily be made members of a group once we are given some members of a group along with its operation. In this sense, we can say that there cannot be a random group. So, conceivably groups could be universal in the way Armstrong understands it.

3. CHANGE

The first section of this Part was concerned with the nature of objects and identity. Since groups are the mathematical 'objects' corresponding to symmetry, a detailed discussion on the nature of sets and groups was given. In this section, we will discuss the metaphysics of change. Any discussion of symmetry must consider the notion of change in all its complexity. We have to remember that the general formulation of symmetry in science was equated with invariance under change. I will begin this discussion with some preliminary remarks on change.

When we normally say something is changed, we usually mean that some property has changed. For example, when a green leaf turns brown we say that its colour has changed. People change – both in their attitudes and in their ageing process. More drastic changes are also common: paper when burnt turns into ash; a caterpillar changes into a butterfly and so on. How can we understand all these different types of changes? What is it that changes and equally important, what is it that remains unchanged and which allows changes to be made visible?

To begin with, consider this definition of change, what Lombard (1986, 80 & 81) calls the Ancient criterion of change (ACC):

An object x changes if and only if

- (i) there is a property, P,
- (ii) there is an object, x,
- (iii) there are distinct times, t and t', and
- (iv) x has P at t and fails to have P at t' (or vice versa).

There are various consequences of ACC. The object must exist at least at t and t' and 'must *survive* the loss of the property it changes with respect to' (ibid., 82). In the case of symmetry, we are interested in changes that are invariant. To phrase it another way: an object has a property at t, fails to have it at t', and regains it at t''. Thus the idea of change is more restricted in this case.

ACC is too broad a definition of change. It is well known that there can be changes of a particular property of an object that are 'external' in nature. A simple example is that of becoming an uncle. One may say that I have changed, that is, have a new property associated to me, just because a baby is born to my brother. As is well known, we have to sharpen the distinction between this kind of change and a change that happens to me 'internally', like my hair turning grey. Changes such as becoming an uncle are called as 'mere Cambridge changes'. These types of change are referred to as relational as against non-relational changes. Lombard distinguishes between these two not necessarily in the language of change but as alteration. Thus, objects which change non-relationally seem to be 'altered' in some way, which is not exhibited by becoming an uncle.

Lombard argues that when an object undergoes relational change, it must necessarily imply that another object has undergone non-relational change. For example, he notes that Xantippe's becoming a widow because Socrates died is a relational change for Xantippe but this change occurs only because Socrates died – a non-relational change of Socrates. Even if the immediate correlate is a relational change, it is the case that down the line something has changed non-relationally. He also notes that *all* changes in *abstract* entities will be relational since by definition their non-relational properties are essential to them and any change in these would change the abstract object itself.

What kinds of changes are possible? Usually when we talk of a change, we are referring to changes that are somehow like each other, i.e., belong to same 'kinds'. For example, the green leaf turning brown is a change from one colour to another. We do not expect a colour to change into a new shape. Lombard notes that change 'must involve the having of a property and the subsequent acquiring of another, contrary property' (ibid., 112). To formalise this, he considers a quality space as a set consisting of simple, static, properties and if an object undergoes a change by losing one of the properties of the set, then it must gain another property which should be of the same kind. Belonging to same kind only means that they are contraries. So, we can say that an object changes 'if and only if it first has one and then another property, where those properties belong to the same quality space (and where the successive havings of them is not what that object's persisting for some period of time consists in)' (ibid., 113 – 114). The definition of events follow form this: 'events are non-relational changes in objects; when an object changes non-relationally in a certain respect, there is an event that is that object's changing in that respect' (ibid., 114). If we look at it graphically, an object moving in this quality space is an event. Lombard also notes that the classification of properties into specific quality spaces is one that is *a posteriori* and suggests that scientific theories will arbitrate this issue.

What about changes of parts of objects? If a part undergoes some change, can we say that the object itself undergoes some change in a non-relational way? Lombard subscribes to a strong version of a thesis that answers the above question in the affirmative – 'any event which is a change in an object (is identical with) a change in any other object of which the first is a part' (ibid., 121). Also note that since an event is the change of an object, one can locate the event in the object. But do we really have to insist that a change in a proper part of an object should be identical with a change in the object? He dilutes this condition and suggests that the 'subject of an event' is only the 'minimally involved object' that undergoes change. Thus the location of an event is the location of the proper part which undergoes change. Note that this does not refute the above thesis but only specifies the location of an event.

The questions – what is an event, what is the identity criterion for events, where is an event located, can multiple events be located at the same place etc., are similar to the metaphysical questions about object, properties, identity and so on. Since I want to restrict myself to a brief discussion of change and events, especially in the context of symmetry, I will very briefly describe some of the answers developed for the above questions.

Events could be universals. As universals, they will be instantiated in particular events. Why should we think that events exist? Lowe, for example, claims that events exist because they are indispensable in 'singular causal explanations'. For Lowe, events are not abstract entities. They are concrete objects because they have temporal properties and relations. In contrast, Chisholm considers events to be abstract universals. They are instantiated in their repeated occurrences. Lombard (1998, 282) notes that if events are seen as abstract universals then it is incompatible with the view that events are changes.

Against the view of events as abstract universals, Quine suggests that they are concrete particulars. As Quine (1960, 170) says, 'physical objects

... are not to be distinguished from events.' Like objects, events in this view are not repeatable. But this leads to the problem of 'distinct' multiple events, like change of colour and shape, to be the same event. Lombard believes that the inability of an object to 'change in two different ways at the same time' is reason enough to argue against the view of events as concrete particulars. The third view of event is in between the two described above, namely, events as abstract particulars.

3.1. *Change and symmetry*

Consider the idea of change in the context of symmetry. Let me consider the example of the equilateral triangle. We say that this triangle is invariant under some particular rotations and reflections.

When we rotate a triangle, something changes. To understand this further, let us see how Lombard (1986) considers motion as change. He believes that when an object moves, it changes non-relationally. If this is so, we will have to show that in moving the object lost some property and regained another property. But the typical problem in considering motion is that it is always a change with respect to background space. We could understand this change by saying that the object occupied one place at one time, lost that property of location in moving and gained another property of being located in another place after some time. But this is not enough if places are only relative to other places. In this case, change incurred during motion is relative change. Usually we talk of a position of an object as being defined with respect to a frame of reference. But theory of relativity suggests that motion is relative since a moving object in one frame of reference can be at rest in another frame of reference. The case of 'pure' rotation complicates this further.

A change is always over an interval of time and 'maintenance of a state' is usually described as non-change.[7] Non-change is also over an interval of time. Newton's law in fact suggests that the motion of an object with constant velocity is actually non-change, thus it is a state. Corresponding to change of properties, we might say that properties change only when there is a cause. Something makes change possible. Otherwise it will be spontaneous.

Let us first consider symmetries of figures. As before, consider the simple example of an equilateral triangle. We say that this object is invariant under certain transformations, say rotation. Rotation is a transformation and it involves change. What is the distinction between transformation and change? Conceivably when one transforms something, a change has taken place. When we talk of transformations of an object in the context of symmetry, it usually means that the object is not transformed into another object but its states and properties are transformed. So transformation in this context is merely an agent that causes some change.

Woodward uses the term 'interventions' as synonymous with transformations. Intervention, for him, is to be specifically tied in with a causal process and he defines it thus: 'an intervention on some variable *X* with respect to some second variable *Y* is a causal process that changes *X* in an appropriately exogenous way, so that if a change in *Y* occurs, it occurs only in virtue of the change in *X* and not as a result of some other set of causal factors' (Woodward 2000, 199 – 200). In principle, the intervention can be in thought; as Woodward says 'an idealization of an experimental manipulation.' The relation between interference (I) and properties is as follows: the intervention is a manipulation of a property X. Thus the effect of I on X is to possibly change the property. Since Woodward is emphasising the causal connection, he is really not considering the change in X due to I but rather what changes occur in Y due to changes in X. A feature of this is dependent on the fact that the intervention changes the 'value' of X and this change is entirely due to the intervention. For this to be so, it must be possible to have a well-defined notion of change for X. Note the similarity to the static space defined by Lombard.

Although Woodward uses the terms 'intervention', it is clear that this is equivalent to the idea of transformations without any necessary link to a further causal connection (although there may be such causal links). Transformation is a change of some sort and intervention is a change of some X. To be consistent with terms used in symmetry, I will use the term transformation.

Transformation in general can be related to change and non-change. Non-change under transformation can be called as identity transformation – such as transformations which retain the state as it is. But since states and changes are not temporally pointlike but involve intervals of time, to say transformation keeps invariant the state or property does not necessarily imply that there has been no change at *all* points in the interval. In fact it is this possibility of change that occurs in a time interval and the non-change as noticed after this interval that gives us an idea of invariance and symmetries.

At this point, I would like to distinguish between 'idealised' transformation (in mind, in principle etc.) from 'real' transformation. I will discuss the former separately when I consider changes of mathematical terms. Real transformations are changes that occur in an object – here it is unimportant as to what causes these changes. Real transformation, for example, is the rotation of the equilateral triangle. Now we can ask what property changes under rotation and what property is gained. It is after analysing this that we can look at the notion of invariance.

First of all, for all rotations we need to specify some point or axis or axes about which rotations take place. It is possible that under rotations the spatial location of the object (like a sphere) does not change but those of its parts do. In general, if the rotation does not involve deformation, then the part-whole structure is retained. Suppose we rotate an object. It seems at the outset that no property is lost – the object is still the same, mass and colour, for example, are unchanged and in general the form or shape is also the same. But the moment we have a frame of reference, then we can notice one particular change, namely the way in which the form is *oriented* with respect to that frame of reference. Say we have a frame of reference outside the triangle and we rotate the triangle with respect to that axis. Then we see that the form is indeed oriented differently and thus we may claim that a particular orientation of the form has been lost and another one gained after rotation. So rotation does indeed cause a change to occur.

Is this change relational or non-relational? If the change is noticed only with respect to some frame of reference, is it not relational? We don't need to invoke abstract frames of reference in this analysis. I have a book in front of me on the table. I rotate it in some suitable manner. I can notice that the orientation of the different parts of the book with respect to me has changed. The side closest to me has gone elsewhere. So I can say that a change has occurred in the distance and in general, the orientation of the different parts of the book. But still the question is whether this change is entirely relational with respect to me or whether it has any intrinsic character.

Transformation of something means that something is changing (I include identity transformations as being a subset of transformations).

Whether this change is 'real', has a causal role to play or not, whether there is no effect etc., can only be answered after further analysis. It may seem that transformations are defined within a context. Take Salmon's well-known example of males taking birth control pills (Woodward 2000, 207). Taking the pills may not have any causal effects in the *context* of birthing by males but taking the pills does definitely involve changes in the male who is taking it – for example, changes in the chemicals present in his stomach or blood stream after taking the pill.

3.2. *Centre of mass and oriented form*

We would like to have a way of describing the exact changes involved in rotation. There is a canonical way of doing this. The problem with using frames of reference is the apparent implication that change occurring in rotation is relational. But there is a *natural* frame of reference. Once we are given an object, we know that there is a 'point' defined by the object which can function as the origin of any frame. From physics we know that for every object with some mass and shape, there is a point called the centre of mass. Every object, extended and massive, has a centre of mass.[8] This can be extended to systems of objects also. Every figure has a central point. This centre of mass is a unique point once we know the mass distribution in the object. In principle we can calculate it for any object. It also has phenomenological implications. In general the point of stability of an object is its centre of mass. If we take a uniform stick we can balance it at its centre. We can balance a homogenous disk on our finger only at one point which is its centre, the centre of mass. If one part of the stick is heavier than the other, then if I want to balance it I tend to move away from its centre and towards the heavier side, that is, towards its centre of mass. In physics the centre of mass is what allows the possibility of point representations. Much of physics is very dependent on the idea that the effect of a force acting on an extended body is entirely equivalent to the same force acting at its centre of mass.

Centre of mass (henceforth CM) seems to be an essential property of any object. It may seem that this is a second-degree property of two other properties namely mass distribution (not total mass) and the shape of the object. (We can distinguish mass and density as follows: 'mass is an inclusive property and density is an exclusive property' (Johansson 1989, 45)). But this conclusion would be hasty because it is only the location of the centre of mass (i.e. its determinate value) that depends on the mass distribution and shape. We have to answer whether CM is a genuine property of the object and not one dependent on mass and shape. We may argue thus: Given an object, mass and shape are accepted as first-order properties. Particular masses and shapes are determinates of the corresponding determinables. In the same way, given an object there is a centre of mass. Is the fact that it has a centre of mass a property of other properties or is it a genuine first-order property of objects? Or is it a property of a relation between mass and shape? An intriguing point to note here is that an object which has no mass but only shape also has a centre. Without further discussion on whether CM is a first-order or higher order property, let me only note that it is a property of every object.

The basic point then is that the CM is a canonical point of origin that can be used to fix a frame of reference. So if we consider an equilateral triangle with equal distribution of mass on the three lengths, then the centre of mass gives us the axis about which we can rotate the triangle. In fact, in many cases where symmetry is seen to be manifested, the transformation will be with respect to some appropriate centre. Once we fix the centre thus, without taking recourse to arbitrary frames of reference, then we can understand change in the context of rotation.

When we rotate the triangle around its centre, we note that the position of the parts change – each point moves to another location with respect to this centre. To specify this change more rigorously, let me introduce the notion of *oriented form.*

The equilateral triangle has a specific form. Any general rotation, even around an arbitrary axis, will keep the form the same. Now consider the form of the triangle around its central point. The form defined with respect to this point can be called the oriented form. When a rotation, of say, 10 degrees, is performed about this point, we notice that all the points shift from their original position. In particular, the three vertices shift from their original positions. The relation between the central point and the regions or parts of the triangle can be defined as the oriented form. This relation may be specified, for example, by the angle made by the line connecting the origin and the vertices with respect to the axes. Under rotation the vertices change location. Changing location is general and is true for any frame of reference. But specific to the canonical frame, we can say that the relation between the vertex and the origin given in terms of an angle changes value. This relation is an internal relation – it is a necessary consequence of the structure of the object. Rotation changes the relational property corresponding to the relation between the origin and the vertex. Or equivalently, rotation involves change in the oriented form. The oriented form is itself specified by the triplet of values of the relation between the centre and the three vertices that define the triangle.

For example, consider an equilateral triangle in the 'standard' configuration of one vertex A along the y-axis and two base vertices B and C. We can specify each of these vertices in terms of the angle made by the line joining the vertex and the origin with either the vertical or horizontal axis. If we consider the angle made with respect to the horizontal axis, then the vertex A in this configuration is specified by an angle of 90 degrees. Similarly, the vertices B and C correspond to angles of 210 and 330 degrees. We can also equivalently consider the angles made with respect to the vertical axis in which case the angles specifying A, B and C are 0, 120 and 240 degrees respectively. Let me use the latter specification although it does not matter which ones we use.

Now we can define the oriented form as the form that is specified by these three angles. The three vertices specify the triangle and these angles specify the oriented form of the triangle. The oriented form is a particular 'collection' of these three angles. In the language of sets, we can say that the oriented form is defined by the triplet corresponding to {0, 120, 240}, with respect to anticlockwise measure along the vertical axis. This particular triplet is one configuration that describes the oriented form of the triangle with respect to the centre. Now rotate the triangle by 10 degrees in the anticlockwise direction. Then the triplet is given by {10, 130, 250}. This triplet defines the shifted form, i.e., a new 'value' for the oriented form. This formulation actually tells us how to describe change in the rotation of the triangle about its centre. That is, the oriented form in general for the triangle is given by {a, b, c}. After some rotation, it goes to say {d, e, f}. This change in oriented form actually reflects the change in the triangle caused by rotation.

As is easy to see, under a rotation of 120 degrees, the oriented form is specified by the triplet {120, 240, 360} and since rotation through 360 degrees is the same as through 0 degrees, we can write this as {120, 240, 0}. Now we see why the oriented form needs a triplet structure. If it is a triplet, then we know that {0, 120, 240} is the same as {120, 240, 0}. Thus we can confidently claim that under rotation of 120 degrees, the oriented form gets back the 'same' value. So when we say that this triangle is invariant under a rotation of 120 degrees, actually what is invariant is *not* the triangle per se, but its oriented form. And what also changes value under rotation is not the triangle or its form but only its oriented form.

For rotation of 240 degrees, the oriented form changes value to {240, 360, 480} which is the same as {240, 0, 120} and thus the oriented form is invariant under 240 degree rotation as well. Similar triplets can be easily written for angles calculated along the horizontal axis.

This approach holds good for any planar polygons and also for solids. In general, once we fix the central point, then we give the relation between the point and the vertices of the two or three-dimensional solids in terms of a set of angles. The number of elements of the set will be the number of vertices. This general n-plet corresponds to a particular determinate value of the property of oriented form. Under rotation there is a change in the values of the angles belonging to the n-plet. If after a certain amount of rotation the values reach the initial configuration, then there is an invariance of the oriented form and this invariance is usually called the invariance of the form.

Invariance is of secondary concern in this formulation. Any form which is not symmetrical can also be described as above. Take an arbitrary triangle and choose its centre. Specify the angle between the vertex and the centre with respect to any axis through the centre. There will be a triplet formed, say {a, b, c}. This triplet is a particular value of the determinable property – 'oriented form'. Now rotate the triangle about its canonical centre. Then we get a new set {d, e, f}. Thus, in rotation, a specific value of the oriented form is lost and another gained. It is only when {a, b, c} is equal to {d, e, f,} that we say there is invariance because a lost property is regained.

Why invoke the central point in this analysis? We could have done the same with any chosen axis. Or we could have used something other than angle to specify the relation. I think the fact that there is a natural centre given to us, once we are given the figure or object, is sufficient reason to privilege that centre. Also the symmetries, in general, for these figures are with respect to these central points or axes. For a general axis 'outside' the figure, there need be no invariance under rotation. But this does not mean oriented form as specified by the values of the triplet (for a triangle) cannot be constructed. Also, angles are the correct relation to specify change under rotation. Distance between origin and vertex is not helpful since under rotation, lengths remain the same for any angle of rotation.

This kind of spetification is useful to describe symmetries other than the rotational ones also. Consider the example of a square. For a standard configuration, we can specify the oriented form as the set {135, 225, 315, 45}. We can note that rotations of 90, 180 and 270 degrees only permute the elements of this 4-plet. Thus we can immediately note that rotations of 90, 180 and 270 degrees are symmetric transformations, whereby some change has occurred after which the initial state is regained. We can also consider other transformations like reflection along the same line. A reflection about the centre takes {135, 225, 315, 45} to {315, 45, 135, 225} which is once again a permutation of the same elements – thus, this is a symmetry transformation. So also for reflection about the x and y axes.

To summarise: objects and figures have a canonical centre and with respect to this centre one can give a description of oriented form, a relational property that holds between the central point and the form or shape of the object/figure. This is an internal relation specified by a set structure. Rotation is a change in this set and thus gives us a measure of recognising change in property and gaining another property. For arbitrary and continuous shapes, we can either use discrete points or topological considerations to define the oriented form.

However, in the context of symmetry, these kinds of transformations are only of one kind, although a dominant kind. This analysis will explain one way to understand the notion of change in symmetry transformations. Translation symmetries that occur in patterns can also be understood along this line.

There are other kinds of transformations related to symmetry. We will have to consider these types of transformations in order to understand the meaning of change in these transformations. Symmetries are also related to transformations that occur in a system as a whole. A system in this case can be seen as a structure with various parts. Transformations of parts by themselves are possible. For example, one leg of a table may rotate while the other parts may not undergo any change. Symmetries are also related to changes that occur in an event. For example, two objects collide and scatter. There is a change in the momentum properties of the two objects before and after collision. Examples of some of the other kinds of transformations are spacetime transformations, transformations corresponding to symmetries in particle physics and so on. In these cases we can perceive a priority given to some notion of form. These are generally represented by mathematical structures. Transformations correspond to 'changes' in this structure. To motivate an analysis of this, let me consider what we could mean by mathematical change.

3.3. *Mathematical change*

Consider an expression of the form $y = x^2$. We can see from the 'form' of this mathematical expression that it is invariant when x is changed to – x. The idea of change in this case is clear: values of x change to – x. Invariance is only a possible consequence of this change and does not depend on the change per se. For example we could change x in any way we like, say x to sin x. Since the analysis of symmetry in science is so dependent on invariance of mathematical expressions, it is useful to understand what it means to talk of change in mathematical entities.

We know there are certain mathematical entities which cannot undergo change. Let us try and hold onto the idea of change as being within the same 'kind' or the same quality space. If so, then a number cannot change. 1 cannot become 2 or 1.1 or any other number. But numbers can be operated upon, like 2 can be multiplied by 5. But we know that the operation here is a two-place relation, and is not a transformation of any property of two. Change is a change of a property of an object and no property of 2 is changed by multiplying it with 5. This doesn't seem to be the case for a variable. But then, what is the property of a variable other than 'being a variable'? Transforming x to – x does not also seem to change its intrinsic property of 'being a variable' because – x is also a variable. We can then consider a 'larger' entity like a set. A set has more than one property, its cardinality, for example. We can ask if it is possible to understand a change in the entity called set. Of course, there is a basic problem in trying to look for such analogy because change, as we understood it, was explicitly temporal. Change takes place over an interval of time. If mathematical entities are abstract and do not exist in space and time, what could we mean by change? And why stretch this connection anyway? The latter question is easier to answer because we do have a notion of change and invariance in mathematical structures.

They also have deep physical consequences, best exemplified in the symmetries corresponding to some changes.

The first question is answerable. If we consider the members of a set as physical objects and if these objects undergo change, what could the corresponding change in that set be? If mathematical entities and structure map physical reality in some sense, then changes in physical reality could help us get an idea of corresponding change in the mathematical entities and structures. In this discussion, let me stick primarily to this motivation. For example, if we have a finite set of all green cats and one green cat for some reason turns blue then it loses its membership in the set and the set now has one member less, implying that a property of the set has indeed changed. This is an example of change in the structure. I believe that changes in the mathematical terms can usefully be defined for such structural expressions.

This is of course well known. In all considerations of symmetry, it is the transformations of certain mathematical expressions that are generally considered. And not too surprisingly, these expressions are usually analogous to forms and indeed the phrase 'formal similarity' is commonly used.

Consider spacetime transformations. Galilean transformations are one class of these where a transformation along the x-axis consists of changing x to $x + vt$, where v is the velocity. In relativity, we consider the more complex Lorentz transformations. What is the meaning of spacetime transformation? Obviously the spacetime coordinates are changed. But these are a change of what? Let us say we have an object specified by some location with respect to some frame of reference. Then a change in space coordinate, for example, is the change of its frame of reference. That is, instead of 'looking' at an object say from a distance of one meter, we are looking at it from another distance. So if we call a particular frame of reference as an 'object' then a space-time transformation is nothing but a change of the property of location of this 'object'.

The case of 'internal' transformations can also be understood in a similar manner. We can consider an electromagnetic field and change its value at every point. Here we are changing not the spacetime values but the value of the field itself. In the case of quantum theory, we have seen the example of neutron-proton doublet. These two particles are put into one set. Then we allow transformation on this set – multiplication by matrices is one realisation of the rotation of this doublet. The effect of a particular rotation is to create another doublet, a set that consists of mixed states of those two particles. In the quantum case, the linear superposition of states that form the wave function allows us to 'mix' objects in this form. The grand unified theories follow the same logic. Internal (also called gauge) transformations are changes in the value of the appropriate function, or doublets, triplets etc. In each of these cases, what changes is the state of a system. Following ACC, we can say that a system has a property of being in a state S_1 at time t_1, and after an interval of time loses this property and gains the property of being in state S_2 at time t_2.

Discrete transformations are significantly different. Permutations and C, P and T are examples of discrete transformations (more on this in Part One). These transformations are not continuous changes. In permutation, we interchange one object at a particular location with another at a different location. For indistinguishable particles, the system does not seemingly undergo any change (but not in all quantum systems). Two identical parts are interchanged while keeping the same structure. Although the individual parts have changed in their location this change is a relational change. Charge conjugation replaces a positive charge by its equivalent negative charge. There is no continuous process by which a charge q (say positive) gradually loses its value, becomes zero and then becomes -q. What is envisaged is an ideal 'intervention' that changes q to -q.

So transformations, i.e., occurrences of change, belong to a wide spectrum in the many cases of symmetries. In particular what is important for symmetry considerations is invariance under change. What exactly is this invariance?

3.4. *Invariance*

Woodward (2000, 205) distinguishes two types of changes: one, of the background conditions and two, of changes that occur 'in those variables that figure explicitly in the generalization itself.' These are similar to external and intrinsic changes. External is with respect to the form under question. Thus for Newton's law of gravitation, a change in the colour of an object is a background change whereas a change in relative distance is intrinsic to the form of the law. Given such a generalization, invariance is that which continues to hold even when other conditions are changed.

The idea of symmetry (in science) is essentially related to transformation and invariance. Transformation is change, a property lost and another gained. Invariance is not non-change but regaining that property which was lost. This is a very specific idea not of change per se but of a *sequence* of changes. An object loses property P at t_1, has property Q at t_2 and regains property P at t_3. Of course, it is not clear that there are only two changes or events that are needed because P may change to Q which may change to R and so on before we regain P. But what is clear is that at least two events need to occur to get back the lost property. In the example of the equilateral triangle, changes were related to change in the specification of the 'oriented form'. And for a particular rotational value, say 120 degrees, we regain the same triplet that specified the original oriented form. We can look at it in two ways. Rotation proceeds through changes in angles till we reach 120 degrees. Or, we could conceivably say, that one rotation of 120 degrees gives us the same original property. The latter view seems to be privileged in most discussions of symmetry – that is we consider one particular change that keeps some property invariant. But this view cannot strictly be right because invariance has to be related to that change which changes a property which is then regained. The need to specify it thus is also equivalent to making a distinction between external and intrinsic changes. So for Newton's law change in colour is not an invariant because it does not cause any change which is not invariant.

There is confusion about what undergoes change and what is invariant. Assume that we rotate the equilateral triangle by 120 degrees. We cannot really call this invariance unless we have noticed that there were changes of some property in this rotation of 120 degrees. So if we say that the triangle is invariant under 120 degrees rotation we are not making explicit the point that we know lesser values of rotation has created some change, which, as we saw earlier, is a change in the determinate values of the oriented form. So in all cases of symmetry, whether of objects, systems, or processes, we can identify change which is non-relational. In the case of spacetime symmetries this becomes a more problematical claim. But if one subscribes to a realist view of space then changes associated with spacetime transformations can be described in a similar manner.

Let us assume that we now have reasonable belief that in the case of objects and figures, invariance is always associated with change of a property and then regaining that property. One other problem arises at this point. When we say we have regained a property, what do we mean? What is the identity condition that is presupposed here? That is, how do we know that the property P that was lost is the same one that has been regained? The 'same' here needs to be clarified. It seems obvious that a lost property is not strictly regained. Rather another property has been gained that has the same identity as the first, lost property. Say a green leaf turns brown. After some more time, it turns back to green. Can we say that it has regained the lost property of green? Consider the example of a round rubber ball. It has the property of having a spherical shape. Now make a dent on the ball by pressing with the thumb. The ball no longer has the property of being spherical. Remove the thumb and the ball regains its original spherical shape. Is this regained shape the same as the one before making the dent?

This question of the identity of properties which underlies the notion of invariance is crucial. All examples of symmetry have to manifest some criteria for the identity of properties before and after some change has occurred. Also, a loose sense of these criteria allows for the very important ideas of approximate and broken symmetry.

What could these criteria of identity for properties be? Given the range of entities that exhibit symmetry, we can guess that there will be different criteria for different 'kinds' of symmetry. Let me first start with objects which lose a property and then regain it. Obviously the object cannot lose a property and then regain it without a chain of other properties (or at least one other property) occurring in the process. It seems reasonable to claim that it is never the same property that is regained but the regained property shares some identity with the original, lost property. Of course, if we accept that temporal slices of objects exist, then the identity of properties is nothing but the instantiation of the same property in two different temporal parts. It might also seem that the 'real' criterion of identity in the case of symmetry is the identity of the object that undergoes change. When we talk of a symmetry of an object we are essentially claiming that after change has occurred it still remains the same object when the lost property is regained. This may seem to imply that the talk of symmetry subscribes to a substratum approach. In this view the identity condition can be based on the identity condition of objects. But then for the substratum approach, how can we distinguish between absolute non-change and losing and gaining the same property? Strictly speaking, one should, for no-change can be the 'same' as a sequence of changes. This is indeed a source of confusion for if we observe an object any time after it has regained a property, then we will not be able to say whether it had undergone any change at all. (Of course, there is a similar problem with change in general if one does not know what the initial property was.)

What the special case of losing and regaining the same property suggests is that symmetry is a modal term – that invariance should always be checked in the realm of possibility. Indeed symmetry arguments are a classic example of the kind of question Lewis asks in arguing for possible worlds: 'What if...' While we can ask a similar question for possible change, we do not ask it *after* we notice a change. But in the case of symmetry, every invariant change is actually a possible change. In fact, this modal character is manifested in all theoretical formulations of symmetry.

Let us look at other types of invariance associated with symmetry. Systems have invariance of some properties and this invariance is reflected in the symmetries of the system. Consider total momentum invariance, what is usually referred to as conservation of momentum. Say we have two particles with momentum P_1 and P_2 (remembering that these are vectors). The momentum of a system of these two particles is $P_1 + P_2$. Now assume they collide. Momentum of each of the objects change, say to P_3 and P_4 respectively. Total momentum invariance is the statement that $P_1 + P_2 = P_3 + P_4$. So if we were measuring the property of the total momentum of this two-particle system, then before and after collision, there is absolutely no change – although the individual momenta have changed. If we consider the full system, no change has taken place. This is similar to changes in the parts of a structure such that the whole structure remains the same.

Momentum invariance is a tricky example. If we ask, following our views on invariance, what property was lost and regained, we run into problems. For, at any time before the collision the total momentum was $P_1 + P_2$. For all times after the collision, the momentum was $P_3 + P_4$. And since $P_1 + P_2 = P_3 + P_4$, there is really no time at which there was a change in the property of the total momentum – strictly not even at the moment of collision! Thus the mark of symmetry cannot rest within the property of total momentum because this property is not lost and regained. Let us look at it in terms of individual objects. Collision changes the momentum of each of the two particles. So a change has occurred. What is invariant is not the regaining of the same property (that is having the same initial momentum) but the invariance is with respect to the sum of the two momenta.

We can rephrase this in a better manner by using the language of events. There is an event of collision. There is a property held by the particles before collision. After the collision there is another property of the two particles. The conservation of momentum says the total momentum remains invariant. Similar problems arise for all events which leave some quantity invariant, like charge conservation. In such cases, it does indeed seem that the inability to point to a property which changes and is then regained suggests a problem for the general definition of invariance described above. But this is not true.

The invariance and symmetry illustrated in conservation of momentum is not the invariance of the individual momentum of the objects which collide. Rather, the symmetry involved in this case is that of space. It is the symmetry of space that leads to the invariance of total momentum. So what is the transformation and invariance here? Interestingly, the transformation is that of individual momentum – that is, the properties which we recognise as being changed are the momentum of the individual particles. Invariance is not of these individual momentum but of the total momentum, that is, not of the properties that really undergo change! And finally the symmetry associated with this process is not that of the particles or the system but of the background space having a particular property. Such a mix is characteristic of symmetries and invariance of processes such as those that obey some conservation principle.

In the case of collision we can note these points. Transformation and invariance are not with respect to the individual particles because after collision the momentum of each is changed. First of all, what we are looking at it is the system, the system of two particles about to collide. The act of collision is the transformation. After this transformation, individual momentum change but the momentum of the system remains the same. Two important points should be noted here: one is the association of invariance with system rather than parts of a system and the other is the possibility that there is indeed a change occurring in the total momentum. The first point actually reinforces the possibility of looking at mathematics in terms of structure and suggests that symmetries in science actually reflect some important structural elements of mathematics.[9] The equilateral triangle also manifests this connection with system. Consider the three vertices as three particles. Under rotation of 120 degrees, each one of these vertices has indeed changed position. What remains invariant is not the location of these vertices but the structure of the system as represented by the oriented form.

The second point is also tenable. Consider a collision where one object hits another, comes to a complete stop and transmits all its momentum to the second object. In this process, we can argue that there is a moment at collision where both the particles are at rest or at the least undeterminable. If this is so, then we can look at the total momentum as the property which is lost and then regained.

Therefore, the best way to understand symmetries is to look at how these symmetries and invariances are discovered or explained, particularly in terms of their mathematical formulation. In fact, symmetries in modern physics are often found by searching for invariance of mathematical forms. It is not an accident that symmetry of forms and shapes is very similar to 'symmetry' of mathematical forms. A brief example gives an indication of how this process works.

Consider Newton's force law: $F = m\, d^2x/dt^2$. The 'form' of this equation remains invariant under change of x to –x; as also for a Galilean transformation of x to x + vt. Both these invariances are a consequence of the form of d^2x/dt^2. Under the above transformation of x, the force law remains invariant. So to derive symmetries all that we need to look at are the mathematical equations and invariance of the form of these equations!

In general we introduce the notion of covariance instead of invariance. Covariance captures the invariance of the form but is not invariant because the values may change but form remains the same. Similarity of form is called covariance whereas identical sameness is called invariance.[10] Under changes in spacetime values, we expect the physics to be the same, that is, we expect the laws and equations of motion to be invariant or covariant under these transformations. Without making a jump into symmetry, we can at this point understand the notion of change and invariance of these mathematical expressions as invariance of 'written form'.[11]

4. PROPERTY

Here are some definitions of symmetry:

1. 'A symmetry is where some alteration makes no difference' (Lucas 1984, 116). An example that follows is the radial symmetry of a starfish.

2. Van Fraassen (1989, 243) notes that 'symmetries are transformations ... that leave all relevant structure intact – the result is always like the original, in all *relevant* respects.' But soon after he talks of symmetries *of* space, symmetries *of* figures, 'relativity *as* symmetry' and so on.
3. 'Symmetry is immunity to a possible change' (Rosen 1995, 2).
4. In the context of quantum mechanics, here is one description: 'A symmetry principle assigns to every physical state a corresponding state such that the physics of the system remains unaltered' (Emmerson 1972, 27).

Generally all attempts to define symmetry emphasise invariance from change. They also seem to imply that objects 'have' symmetry just as some of them may 'have' colour. There is also the implication that symmetries are transformations, meaning nothing more than change, at the most perhaps a special kind of change. But in the same breath we are pointed towards symmetries of space, that is space having or possessing something called symmetries. In the case of systems, classical or quantum, symmetries seem to have something to do with the invariance of their structures. Is there a common conceptual thread in all these ways of articulating symmetry?

Kosso (2000, 83) writes: 'A symmetry of an object or a law of nature is a transformation that leaves some specified feature of the object or law unchanged. Symmetry is invariance under transformation.'

So symmetry is something 'of' an object as also 'of' a law of nature. But it is also a transformation, i.e., change. But how can change be *of* an object? What other terms stand for *of* an object? We usually talk of colour *of* an object, mass *of* an object, charge *of* an object and so on. Kosso's definition suggests that we can talk of change *of* an object or for example rotation *of* an object. While mass, colour and charge are properties of an object, rotation is not a property of an object per se. Thus a particular change is not of an object but only of the appropriately 'changing object'. And it is also not any change, but only that change which 'leaves some specified feature of the object unchanged'. We have already seen earlier that this is not exactly correct. Strictly speaking, it is change by which a feature is lost and then regained.

So on the one hand the expression 'symmetry of an object' suggests, correctly, I think, that it is a property of an object but on the other hand claiming that symmetry is a special kind of transformation or change suggests that it is not a property of the object. The second sentence of

Kosso's definition makes it more confusing. Symmetry is invariance under transformation. This means that symmetry is synonymous with invariance. But then why call something symmetry if it is nothing but invariance? Is symmetry referring to some specified feature that is invariant, or to the object in which one particular feature is invariant or to nothing but the process of invariance?

If it is about the process of invariance, then this process must be independent of the features of the object or system. Clearly symmetry cannot be another name for invariance in this independent sense. Can changes be 'independent' of the object? A change is a change in a property or properties of an object. This means that change is always with respect to some properties which the object has. This idea of change only refers to the change-in-object, not change in itself. Since properties are instantiated in a particular change of properties is change in particulars. But also, change of a property is not a property of an object – properties of an object need not change. What we need to do is to clearly distinguish between change, object and invariance. So it seems to be the case that while change in general is 'external' to the object, certain kinds of changes bring out some invariant feature of the object.

So we have this double-sided nature of symmetry: the invariant feature of an object is something that belongs to the object, indeed is a property of the object. This invariant feature may, in many cases, not be apparent. But this invariant feature is *exhibited* by making appropriate transformations. But then we should not equate the invariant features with the transformations that make these features apparent.

For example, I apply pressure on a rubber ball. Let us call this transformation as deformation. Once I remove my finger I find that the rubber ball, which had lost the property of being spherical, regains this property. Thus the rubber ball is invariant under deformation (if we assume that there is no continuous pressure being applied). So what is the symmetry in this case? We can say that symmetry is just the statement of invariance under deformation. But not all balls made of different substances bounce back to the original shape after deformation. So this invariance has something to do with the nature of the rubber ball – the property of elasticity. It is this property of the ball that makes invariance possible. So we can say that 'the invariance of the shape of the rubber ball under deformation' is what we call symmetry. But 'invariance of the shape of the rubber ball under deformation' is nothing but a description of a property of the rubber ball, in this case a property called elasticity. Therefore, symmetry is not a transformation, not an invariant transformation but a property of the object or system. The transformations are only agencies that throw light on this property. We can understand this agency in two ways. One is by looking at it as some physical action of change. Other is to view it in terms of an explanatory role that explains how it is that an object has something called symmetry.

Consider a simple example. Let us say we have a red ball. The colour red is a property of this ball. In a dark room we do not see the colour, perhaps not even the ball. Switching on the light – creating a transformation – allows us to see the colour. The transformation makes apparent a property of the object, its colour. But the colour should not be confused with the transformation. Similarly for other properties like charge.

The simple conclusion is this: we should not mistake a transformation with its effects. Throughout the discussion, it may be noticed that we were always flirting with some notion of causation. Soon we can bring this into the open. Symmetry is intrinsically related to causation and laws, thus supplying the notion of necessity to it. For all these reasons we can claim that symmetry is a property of objects and systems. I will now discuss what kind of a property it could be before moving on to causality.

4.1. *Nature of properties*

We have reached the position where we can say symmetry is a property. What kind of a property is it? Can it really be a property of the object or is it a property of some other property of the object? Similar queries have to be dealt with when we talk of symmetry of a system or a process.

To begin with, let me follow Armstrong's formulation of properties. When we talk about an object, we normally talk about the properties which the object has – like its mass, shape and colour. In our common usage we do make an ontological commitment to these properties. But it is not clear what kind of entities they are. What is the relationship between an object and its properties? How would we know what kind of properties there are?

We can note the following features about properties. One property can be present in, had by, many objects. One object can have more than one property and usually has many properties. A property, like mass, is common to many entities. We have different objects like red ball, red shirt, red building etc., which all have the same property of being red. We write this by saying that a property *F* is instantiated in a particular *a*. This is equivalent to saying that a particular *a* has the property *F*-ness. Moreover, a particular *a* can instantiate many properties *F*, *G* ... Further, if we introduce state of affairs into our ontology then we say that *a*'s being *F* is a state of affairs.

To understand properties is then to explain how one property can be instantiated in many particulars and also how one particular instantiates many properties. There are various explanations available. One may be a nominalist and reject the existence of properties. One can be a universalist and claim that properties are universals – a universal being that which is instantiated in particulars and particular being that which is not instantiated in any other particular. One can be a trope theorist and claim that properties are abstract particulars, that is, they are abstract but each property is unique in its instantiations. In this view, there is no common redness that is present in different red objects, rather what there is a collection of red ball, red shirt and so on. Universals are abstract entities. There are two possible types of universals, platonic and aristotelian. Platonic universals do not exist in space and time while the latter exist in the spatiotemporal particulars in which they are found. Armstrong offers the idea of properties as universals because it explains best the problem of One over Many: the problem of how many different objects seem to have the same property.

Let me note some typical problems that arise in the theories of properties. Firstly, there seems to be no way to know what the properties of an object could be. While mass, colour and shape, for example, seem to be properties that seem unproblematical, one can, in principle, have an infinite number of properties associated with a particular. One can try and get around this problem by distinguishing between genuine properties (or sparse or natural properties) and relational properties. We can also form hierarchy of properties, such as properties of objects, properties of properties and so on. What is important to note here is that even when philosophers disagree on how to understand properties, some of them look to science, particularly physics, to tell us what genuine properties an object could have. Thus, Armstrong accepts properties that are scientifically *a posteriori,* defined by what physics calls as properties, like spin and charge. Another important problem that arises in this context is to understand the meaning of instantiation.

Suppose properties are universals. First of all, there are no disjunctive or negative universals. That is, if F and G are distinct universals, then there are no universals of not-F, not-G and F or G. Armstrong qualifies this by saying, in the spirit of *a posteriori* realism, that if physics necessitates these universals, then they can be allowed as such. Conjunctions of universals are more acceptable for him. So if F and G are (distinct) universals, 'then F & G can be a universal, provided always that a particular exists at some time which is both F and G' (Armstrong 1997, 31). Conjunctive universals (and corresponding properties) is one example of complex universals. We can also have structural properties and structural universals which involve a combination of properties and relations. Also, for Armstrong, not all properties are universals. It is only what he calls as first class properties that are universals. Second and third class properties are not universals. Colour properties and generally perceptual properties, he considers as 'second class'. He draws upon the possible validity of 'micro-reductive' physics and suggests that the true property-universals are only those that are instantiated in the fundamental particles. All other properties will supervene on these.

We can formulate the nature of properties using the idea of determinables and determinates. Determinables specify the kind of properties such as the general kind of shape, mass and colour. Objects do not possess properties in this general form. A given particular has a particular shape, mass or colour. The 'absolutely specific' lengths, masses etc., are the determinates. We can list the following relations between determinables and determinates (ibid., 48 – 49):

1. If a particular has a determinable property then it has some determinate property corresponding to the determinable.
2. Having a determinate property 'entails having the corresponding determinable.'
3. A given particular can have only one determinate belonging to the same determinable. So a particular cannot have two different lengths or masses at a given time. (Armstrong notes that tastes do not obey this condition.)
4. The relationship between determinables and determinates is not that of a genus/species relation.

5. Determinates of a given determinable resemble and can be ordered, whereas determinables cannot.

Given this formulation, properties that are lowest determinates are the right candidates for universals, examples being 'exact sameness of length, exact sameness of mass' (ibid., 49). The strict identity criterion present in these examples allows them to have the status of universals, although not all determinates should be seen as universals – second-class properties being an example. If we then say that determinates form a class for a determinable, then we need to answer how different determinates belong to that class. If colour is the determinable, then specific colours like red, orange and green will belong to the class of its determinates. Belonging to the class implies a criterion for membership and resemblance seems to be the natural criterion. But in what sense do the determinates resemble each other? Armstrong takes the view that this resemblance 'is constituted by partial identity, where the greater the resemblance the greater is the degree of identity' (ibid., 51).

4.2. *Role of properties*

One particularly useful way of talking about properties, especially if we are to believe in property as an entity, is to ask what role do they play? One answer to this is that properties play an explanatory role in ontological classification. This would be, for example, 'explaining a purported fact or solving a problem' (Oliver 1996, 11). (The problem here refers to problems in ontology.) Then we can ask whether all properties play a similar role. Oliver argues that a reason to believe in properties is because we 'associate some role with the category of properties and argue that it must be played' (ibid., 12). Further, he notes, that if 'there is a property role worth playing then there are entities which qualify as properties' (ibid., 14). The first point that follows is the distinction between a particular and property discussed earlier – a property is had (or instantiated) by particulars. Consider some of the roles a property could play. One such is Armstrong's view of properties as universals that is addressed to the problem of One over Many. The relation between property and predicates has been a source of much philosophical discussion. While there are many views on this, we can say here that properties stand for meanings of expressions. Thus, 'the most that can be said is that properties are the meanings, in some sense or other, of predicates and/or abstract singular terms' (ibid., 16). A third role that Oliver notes is the relation between properties and causality, manifested in a statement such as, 'a cause has its effects in virtue of its properties' (ibid., 17). We have already seen how the category of properties is a way to understand similarity and identity between objects. Lewis argues for a restricted set of properties – natural properties – which allows for 'similarity in intrinsic respects'.

Oliver identifies three types of candidates that can play the role of property. I will discuss them briefly. We have already seen the basic outline of Armstrong's theory of properties as universals and universals as aristotelian rather than platonic. Since aristotelian universals are located in particulars, this implies that a universal, say a particular red colour, is 'wholly' present in all the particulars that instantiate that particular red colour. That is, all those things which have the same red colour have the universal of that redness completely 'present' in all of them. Also, since a particular can have more than one property it implies that two aristotelian universals can at the same time occupy the same place. The basic problem that arises in this conception is that it violates the notion of 'thinghood' – that is, two different particulars which have the same properties cannot be distinguished. The other important consequence of the aristotelian position is that there can be no uninstantiated universals. As for the roles played by the universals, there are two such: 'grounding objective resemblances and grounding causal powers' (ibid., 30).

Another type of candidate for properties is sets. In this view, a property is nothing but a set of all the particulars which instantiate that property. Instantiation is the relation of set membership. Thus we will say that the property of the colour red is the set of all those particulars which has this redness, for example, a set consisting of the red ball, the red house, etc. As Oliver notes, if the particulars over which a property ranges are the only actual particulars then properties will have wrong identity conditions. Lewis modifies this to say that 'a property is the set of its actual and possible instances' (ibid., 22). But this view necessitates the shift into the formulation of possible worlds. Also, set theoretic constructions come with a baggage of problems. One is that there is really no unique identification of which set we should take to stand for a property.

The third candidate for understanding properties is tropes. The significant difference between trope theory and that of universals is that tropes are different for each particular. Thus a trope of redness will be instantiated in each red particular but these tropes are all distinct from each other. Properties are abstract particulars in this view. Although tropes of redness are not the same trope, nevertheless they are exactly similar to each other. A particular is seen as a bundle of compresent tropes, that is a bundle of tropes corresponding to various properties. Viewing particulars as mereological sums of tropes does not lead us to the problem of identifying distinct particulars (as in the case of universals) because tropes are distinguished by the particular they belong to. This implies that an aristotelian trope, like the aristotelian universal, can occupy the same place with other tropes at the same time but unlike.the universal·it 'cannot be wholly present in more than one place at the same time' (ibid., 36).

While these are different ways of understanding what a property is, we still have the worry that there are altogether too many properties for our liking. If properties are sets of actual and possible particulars, then we have an inflation of properties. If we ground properties in terms of similarities and causal powers, the number of properties can be reduced. The latter position gives us a theory that has 'sparse' properties while the former creates 'abundant' properties. Lewis offers a similar view of 'natural' properties. The charge and mass of subatomic particles are perfectly natural properties while colours are less so and a particular colour less natural than the colours. Natural properties are objective and most importantly, they allow us to understand objective similarity.

4.3. *Symmetry as property*

With this brief summary of properties as understood in metaphysics, let me address the question of symmetry. Earlier, I suggested that we should look upon symmetry as a property. Now this suggestion can be dealt with in detail. First of all, in physics, symmetry is best understood as a property of objects, systems and processes. Since a dominant view in metaphysics, including those of Armstrong and Lewis, privileges scientific realism as the arbitrator of what properties there are, we must, therefore, accept symmetry as a candidate for property. But we can come to this position even if we do not subscribe to this form of realism. Symmetry is also closely associated with causality; it plays an explanatory role describing why some objects and systems are the way they are.

At this point, it is useful to remember that there is a phenomenological experience of symmetry. There are various synonymous terms that capture the phenomenology of symmetry – balance, harmony, order, stability and so on. Symmetries such as bilateral, circular, those of regular polygons etc., are phenomenologically accessible in the sense that we are able to recognise something about these objects, something intrinsic and special to them. We can perhaps even argue that we do not have to make transformations and see if some of them are invariant or indeed even what feature is invariant. We may not be able to specify or quantify what exactly these transformation features are but this is irrelevant to saying we are seeing or experiencing something called symmetry. The analogy with colour is helpful: we may not be able to name the colour we see nor even know about wavelengths but we do have a phenomenological experience of colour. The recognition of symmetry in patterns, with no idea of translational or other invariance, is another important reason for accepting a phenomenological experience of symmetry. More on this issue in Part Three.

Symmetry is a property. Let us see if symmetry first of all satisfies the criteria for calling something a property.

1. Is symmetry instantiated in more than one particular? Yes. All particulars that have a form such as square, rectangle or circular have some symmetry associated with each one of them. We can give a name to each of these specific symmetries like we give names to specific colours. Group theory has already given us a classification that we can use. The crucial point is that each of these particulars have a specific symmetry.
2. Is symmetry a property over and beyond other properties like shape, mass or colour? We need more discussion on what kind of a property symmetry is before we can answer this question with some confidence. I will be arguing in the affirmative to this question.
3. What about the identity conditions in the case of symmetry? In particular, the problem of coextensive properties seems to occur in this case. One particular instantiates both the property of shape and the property of symmetry in the case of many symmetries. In the symmetries of patterns and colour symmetry, it is more complex coextensive properties that seem to be involved. Of course we might say that there is no coextensiveness if we claim symmetry is really a property of shape. But once again this issue cannot be resolved until I show that symmetry is a property of the object.

4.4. *Shape or symmetry? A lesson from physics*

As mentioned earlier, metaphysicians who accept property place the burden of finding properties on physics. In physics, especially modern physics, it is well known that symmetry plays a central role. In particular, physics believes that objects and systems have unique symmetries associated with them and in fact, many interesting results are obtained as a consequence of an object or system 'having' symmetry. In the way physics views symmetry, we can and must say that symmetry is a fundamental property of objects, systems and processes. In particular the symmetries of spacetime is an important category for physics. And for those subscribing to naturalism, spacetime is all that there is. Physics studies the symmetries of spacetime and gives us a classification of these symmetries. These symmetries are a property of spacetime.

Given the importance accorded to physics by some philosophers in cataloguing properties, it is relevant to ask how physics understands properties. The common examples of properties used by philosophers are mass, shape, colour and charge. Does physics consider these as properties and if so, in what sense? Obviously we must accept that the way in which physicists understand properties will be different from philosophers because their concerns are quite different. But even if this is so, since we look to physics to give us properties, we have to have some idea of how to see some terms as properties. Consider this simple example: given an object its mass seems to be necessary to describe and explain certain physical processes. The language of physics makes commitment to properties in statements of the form 'a particle has mass m', 'a particle of mass m' etc. Mass also comes to be the defining property in dynamics. Motion of particles is sensitive to the amount of mass an object has, as clearly illustrated in Newton's law. Although the role and meaning of mass gets murkier as we go to relativity, field theories and quantum theories, it is clear that mass plays a property-role in physics as defined by physics itself.

Similarly for charge. In physics, we say that an electron has a unit negative charge. All electrons have this same charge. An electron also has other properties like mass and spin. Other particles like proton have a different charge. In these cases, physics not only uses the language of properties for charge but also makes an ontological commitment to it. Making an ontological commitment, for physics, is to ask properties to play a specific role, namely observability. It does not have to be 'direct' observability but in general a correlation with other observations. In physics we speak of observability of particulars, properties, relations, processes and so on. The distinction between observability of particulars and properties (and relations) is generally quite clear, at least in macrophysics.

Shape is a more difficult case because, in general, shape plays an insignificant role in physics. This statement has to be qualified. The representation of objects as points replaces extended objects, with some mass m, with a point-object of the same mass. Shape is really removed out of the consideration of dynamics of individual objects. This reduction to point is made possible because of the notion of centre of mass, which acts as the point at which the whole object seems to be concentrated. Shape of objects seems to play no significant role in much of physics. While shape is seemingly unimportant for physics, symmetry is not! That is, under reduction of extended objects to points, we notice that certain properties are carried over. Mass and charge, for example, are properties of the point representation as well as the object with a shape. The same values of mass and charge are supposed to be present in the point-object. What is lost in the reduction is shape. But symmetry of the extended object is also carried over to the point-objects. So at least as far as physics is concerned, symmetry is a property that is on par with fundamental properties like mass and charge. So symmetry is prioritised over shape.

Even in the quantum domain, we do not talk of the shape of particles. Even when systems of particles interact, shapes are usually ignored except where they play an essential part in the interaction. This leads to a conundrum: shape seems to be a genuine property of objects but for physics shape seems to be irrelevant. In most cases, especially in fundamental theories, shape seems to have no explanatory or causal role in the physics of individual objects. Does this mean that philosophers, following physicists, jettison shape as a primary property? We must remember here that shape is also a problem for philosophers to handle, as discussed in an earlier section. If we subscribe to the strong view that only those entities which are properties for physics are properties for metaphysics, then we have to reject shape as a primary property.

Although shape is seemingly ignored by physics, there is yet another property which plays an explanatory and causal role even in the case of individual objects – the property of symmetry. This is the case in quantum physics as well. For even if we do not speak of the shape of the electron, we can speak of its symmetry as manifested, for example, in its spin. Along with the reduction of objects to points, yet another reason for the unimportance of shapes lies in the observation that mass and charge possessed by a given object are values whereas shapes are not. In fact, in the project of mathematising the world, of which point reduction is one part, shapes properly belong to classes and not as individuated values. Even in classical physics, shapes are replaced by the property of symmetry. For physics, a spherical object's property that is of any value is not that of having sphericity but of having spherical symmetry whose consequences can be observed. This spherical symmetry is taken over even when the object is represented by a point, that is, after its shape has been erased. This priority given to symmetry in physics might suggest that we accept symmetry as the genuine property of objects. But since symmetry is so closely linked with shape, we have to pause before we take symmetry as a primary property.

There is a deeper problem. All objects are not symmetrical, only some of them are. But all objects whether symmetrical or not have shape. So it does seem that shapes come first and symmetry later. But the arguments above still holds good. As long as reduction of extended objects to points is the first step in physics, shape is not a member of its set of properties except in special circumstances. Also the fact that all objects are not symmetric should not bother us. For not all objects have charge but for those which do, charge is a genuine property. Also, all objects which have charge also have mass but this does not make charge secondary to mass. We can make a similar claim for symmetry.

We have already seen the problematical nature of shapes. Shapes are essentially abstractions. And physics' removal of shapes in many important cases suggests that it does not see shape as an essential property. But perhaps all abstractions are like this? Consider abstracting away mass. If we 'remove' the mass of an object, we are left with a shape that has no mass. The shape remains the same but the object has lost the mass it had. In such a scenario, the physics would be drastically modified and thus is significantly different when compared to abstraction of shape. While the abstracted shape and the 'real' shape of the object are the same, if we abstract mass and charge away the object loses an essential property. That is, the abstracted entity is not the 'same' as the quality possessed by the concrete object, in stark contrast to shapes.

There is yet another pointer from physics. When describing quantum particles, physics carries over some important concepts from classical physics. In particular, properties such as mass and charge are seen to be properties of quantum particles but shape is completely ignored. The question of the shape of these particles does not even arise in describing these particles. Along with mass and charge what is retained is the symmetries. As the brief discussion earlier indicated (see 1.6), symmetries are in fact what quantum objects are all about!

It is also not an accident that among all fundamental physical laws, shape does not occur in any of them. For physics, shape of an object does not have any explanatory or causal role to play, roles which are played by symmetry. Of course, in many of these laws, the elimination of shape occurs through reduction to pointhood particles – as in Newton's gravitation law and in Coulomb's law. But the reduction itself is possible because shape is not a genuine, intrinsic property *necessary* for physics. Note that shape of individual objects is what is thus eliminated. One can argue that in chemical reactions, for example, shape is very important. But in these cases, it is the structure that is central and not the individual shapes per se.

But we may respond by saying that when we carry over symmetry to the point-object what we are doing is merely carrying over shape or at worst, some property of shape. But this cannot be right for the point-object carries no property of shape if the shape has no symmetry! So physics does grant symmetry a more basic position than shape.

In the earlier section on objects, we have considered the way in which mathematics deals with shapes, which in some respects runs counter to the philosophers' view. Shapes in mathematics, in general, belong to classes, to equivalence classes. For example, topological classification of spaces would consider a circle equivalent to any shape which is a deformation of the circle, say an ellipse. These equivalence classes capture the essence of shape not through independence of individual shapes but through some other characteristic. This characteristic 'essence' which captures the essence of shape, in topological classification for example, is through a group structure! For example, the circle and all its equivalent deformities (such that the 'hole' character is maintained) are classified by the same homotopy groups. This implies that in many important cases what really lies behind shapes is the property of groups and as we know, groups are intrinsically tied to symmetries. Here is another instance that suggests we should accord priority to symmetry over shape.

Groups and shapes often occur together. Diffeomorphisms, discussed earlier, are one example. Homotopy groups give a topological classification. Homology and cohomology groups are used to distinguish boundaries and solid objects. We know that what distinguishes a boundary from a filled space is that the boundary by itself does not have a boundary. Some shapes are such that they are only boundaries, like a circle or a hollow sphere. This argument is used to classify spaces and this classification once again has a group structure as described by homology and cohomology groups.

The upshot of this is that shapes are 'reducible' to some other essential terms. These essential terms, which characterise the nature of a shape, are given in terms of topological invariants, index etc. These terms are generally invariants, specific to each shape and are the same for shapes belonging to the same equivalence class. The invariance here is of course linked to invariance that is characteristic of symmetries and arises from the group structure.

But mathematical categories are not metaphysical ones. The problem of shape is exaggerated only when we look to physics to supply us the list of properties – the implication being that we should not accept those properties which physics does not consider as properties. Suppose we do not look to physics to supply us the list of properties and accept that shape is a first-order property of objects. Then what is the connection between shape and symmetry? In particular, is symmetry a property of shape rather than the object? One way of answering this is to look at phenomenological experience of symmetry. More on this in Part Three.

Why would physics (and in general science) privilege symmetry over shape? Science would claim that it privileges symmetry over shape because that is the way the world is. But I think we can peel away this claim and see the metaphysical inclination of science underneath. Namely, the privilege given to 'order'. Shapes, as individuals, are random in the 'visual' sense. They are also not amenable to quantification like various other properties are. Symmetry functions as a quantitative measure of some property of the object or system.

Related to this is the importance given to explanation in science. Science would like to believe that 'order' in objects, systems or processes must be caused by something. This is dependent on the principle that systems left to themselves become more disordered. In thermodynamics, this principle is the law of entropy – under no ordering intervention, disorder only increases, i.e., entropy continues to increase. This is the way nature is! So anything which exhibits some kind of ordering has to have a reason (generally a causal relation). This implies that ordered objects and systems have within them the reason for the order. Arbitrary shapes are disordered. But symmetric shapes are not. Symmetry is actually the measure of the 'order' of these figures. Thus symmetry is what is retained in point-objects. So even though shape can be discarded, symmetry has to be retained because the symmetry may actually indicate some other prior causes.

But even this is not the end of the story. It seems that symmetry is privileged over shape for a more important reason – the formation of shapes can be explained through the idea of symmetry. If we look at the natural world, we see myriad objects with many different shapes. We can perhaps believe that all these different shapes are accidental properties of the objects similar to what we may believe about the mass of an object. (This is with reference to natural objects and not artefacts.) Since it seems that any arbitrary shape is possible, it is reasonable to posit shape as a primary property of an object. But is this really so? Can we not equally say that considering the number of natural objects, it is extremely remarkable that so many of them have symmetrical or approximately symmetrical shapes? Why, if shape is independent of some ordering mechanism, are so many exact and approximate symmetrical shapes found in nature?

The answer is simple. Shapes do in fact reflect the effect of some order – for example, through the forces acting on an object. Hahn calls symmetry a principle of evolution because symmetry principles decide on the design of the objects (and the universe itself!). In the context of shapes, as long as shapes are explained by use of some laws then shapes are not the primary properties of objects. A common example is a pebble rounded in a manner that is explained by the nature of water flow around it. The shape of the pebble reflects 'something' of that which caused the shape – the water flow. The symmetry of the pebble reflects the symmetries of the water flow in the river. Thus if shape can be explained with the use of some prior properties, then it is shape that is hierarchically 'lower' than these properties. Symmetry is one such prior property.

4.5. *Symmetry as first-order property: An analogy from motion*

Bigelow and Pargetter (1989) offer an interesting analysis of vectors. Earlier, I briefly discussed the problem of change associated with motion. It is clear that motion involves change of place but it is not so clear as to what the changing properties are because locations may not be intrinsic properties of an object. The authors discuss two contrasting doctrines of motion, what they call the Ockhamist doctrine and the flux doctrine. The Ockhamists looked at motion as nothing more than the 'occupation of successive places at successive times'. The doctrine of flux held that motion involved more than just a change of locations but also possesses an intrinsic velocity, thereby giving a vector quality to a moving object. This velocity vector is an intrinsic property according to the flux doctrine.

The basic difference between these two views lies in ascribing a new property to motion, namely velocity, over and beyond the property of changing locations. Both these doctrines accept that motion implies changing locations with time. The flux doctrine says that the moving object has a (first-order) property of instantaneous velocity over and beyond the change of locations in contrast to the Ockhamist view. However, the Ockhamist view does not imply that there is nothing called velocity and would argue that the sequence of positions characterising motion is enough to characterise velocity. In other words, the first-order property in motion is the 'having of a position' and velocity is a second-order property of positions. Thus, in this view, the role of velocity is not to explain why there is a change in positions because all that is primary is the sequence of positions.

The doctrine of flux, on the other hand, by introducing instantaneous velocity as a first-order property of the moving object, answers why there is a change in positions. The direction and magnitude of the velocity explain the change in positions. Thus, the important consequence of this position is that 'first-order properties of position are explained by another first-order property of instantaneous velocity' (ibid., 292). Velocity is not to be thought of as second-order property of positions but is independently a first-order property. But it is also true that velocity is indeed related to change in positions. To claim a first-order title to velocity, the independence of velocity from sequences of positions should be established. Bigelow and Pargetter accomplish this by some clever arguments. First, they demonstrate that it is possible to have instantaneous velocity without an Ockhamist sequence of positions. Using the example of the motion of an image, they also argue that Ockhamist sequence of positions is possible without instantaneous velocity. Thus they exhibit the independence of first-order and second-order (derived from sequence of positions) velocity. Finally, what is needed is a link between these velocities.

The authors argue that laws of nature provide the required link between the velocities in the Ockahmist and flux doctrines. Vectors such as velocity, momentum and force feature in laws of nature and play an indispensable explanatory role. Also, the Ockhamist would potentially face a problem in distinguishing a homogenous disc when it is static and when it is spinning. What differs in both these cases is their causal powers. This causal power of the spinning disc is the presence of instantaneous velocity in each part of the disc.

This analysis of velocity in the context of motion is an apt formulation to understand the claim that symmetry is a first-order property and not a property of the first-order property of shape. Call O-doctrine the view that says shapes are first-order properties and symmetry is a property of these shapes. This is similar to the Ockhamist doctrine of motion. Call F-doctrine the view that along with shapes as first-order properties there are also other first-order properties, namely, symmetries. The O-view would not accept symmetry as a first-order property and instead would look at symmetry in terms of sequence of changes in shapes (or more strictly, changes in oriented form) and more generally, as changes in some property that is eventually invariant. The F-view would dispute this and claim that along with these sequences that characterise change there is also another first-order property that explains these sequences.

The first point to clarify is what is symmetry a property of? Just as instantaneous velocity was a first-order property of *moving objects* so also we should look for a category that accommodates symmetry as a property. This category is invariance and symmetry would be a first-order property of *invariant objects* (and systems which are invariant in some respects) just as velocity would be a first-order property of moving objects. The O-view of invariance would claim that the transformation connected to invariance is nothing but a sequence of positions or some other change of a first-order property. The F-view would say that along with such a sequence there is also another first-order property called symmetry. This property of symmetry in fact explains this invariance that occurs in the

O-view. Also, it is related to laws of nature and has causal power. Thus, following the arguments of Bigelow and Pargetter we can claim a first-order property status for symmetry.

It is indeed true that symmetry (that has to do with forms) is exhibited through a sequence of change in form. Thus, just as in the case of motion, we have to show the independence of change of shapes and symmetry. We could, for instance, consider this example. If we keep a mirror about a symmetry axis then there is no moment at which there is sequence of change in shape yet there is a recognition of symmetry. But most of all, we would, in a manner similar to the example of motion, look to the possession of causal power, explanatory power and association with law of nature to argue that symmetry is indeed a first-order property like instantaneous velocity. As argued in sufficient detail earlier, symmetry is the property that is associated with laws and explains, among other things, the shapes of various objects. Point-reduction carries the symmetry of the object but not the shape and this is a good reason to believe that symmetry is independent of shape, at least in science. The above arguments can be extended to symmetries associated with systems and processes, and also for symmetries that are not associated with shape.

5. CONSERVATION LAWS AND CONSERVED PROPERTIES

There is a set of properties, closely associated with symmetry, which exhibit a unique nature: they are conserved. How do we understand conserved properties in metaphysics? What is conservation? Consider a process or event. For simplicity let us say that only two objects are participating in that event and let the event be that of collision of these two particles. Before collision, let us say the two particles are moving (or at least one of them is). Each of these two particulars has the properties of mass and momentum. After the event of collision takes place, there is, in general, a change in the values of the momentum. The conservation of momentum says that the total initial momentum has the same value as the total final momentum. Similarly, consider an event in which two particles having charges q_1 and q_2 interact and end with two new values, q_3 and q_4. Then conservation of charges implies $q_1 + q_2 = q_3 + q_4$. Energy and spin are entities which are also conserved. These conservation laws are not only empirically sound but are also fundamental laws which are embodied in many chemical and fundamental particle interactions.

In all these examples, the conserved quantity has to do with properties. Charge, momentum, energy, mass and spin are properties. Conservation of these properties is not limited to properties of subatomic particles. Energy, momentum and charge conservation are manifested in processes involving macroscopic objects.

Conservation is generally not in relation to a single object, except in special cases, like conservation of angular momentum of a single object. The example of an ice skater who changes the speed of revolution by extending and dropping her hands is well known. The conserved property here is angular momentum. As the skater brings her hands closer to the body, she spins faster. This process of her spinning faster is explained by the fact that the total angular momentum has to be the same. In order for it to remain the same, the angular velocity increases when moment of inertia decreases. Therefore, the skater spins faster. The invariance of angular momentum explains this phenomenon of the skater and symmetry (rotational symmetry of the system) explains why angular momentum is conserved. Spontaneous decomposition is another example where a single object generates two or more objects. Here, energy, momentum, charge and spin (if applicable) are all conserved.

In the process of conservation the following holds: (a) There is no *net* change in some property or properties; (b) Equivalently, in the case of two objects, what is lost by one is gained by the other. This is true not only for scalar values like charge but also of vector quantities like momentum and spin. (c) In the case of more than two objects, what is lost by one is distributively gained by the other objects.

Could this special nature be a property of these properties? That is, is there a property, say 'conservation', that is a property of mass, charge etc.? In what sense can 'conservation' be a property of charge? Given an object with some charge, what is the meaning of saying that 'conservation' is also a property of charge? It seems that this property of conservation is hidden and is manifested only when the object undergoes a change in its charge. Like symmetry, this seems to be a property that is made 'visible' only under certain conditions of change. Like symmetry, conservation is also related to invariance. But what exactly is the relation between a property and the conservation of that property?

It is clear that conservation is not a property of a property such as charge, energy and so on. We can rule out conservation as a property of mass-energy, for example, primarily because an object of some mass does not necessarily have to undergo collision or decomposition, the processes in which the conservation of mass-energy is manifested. Similarly for charge: given two objects, each with some charge, there is no *necessary* condition that they undergo change in the values of the charges so that the total charge is a constant. That is, it is not a property of charge that makes charge conservation happen.

Since conservation is seen in events, perhaps conservation has something to do with events rather than properties themselves. We may perhaps claim this: a property of events is that certain properties are conserved. But why only certain properties? *What is it in the events which decides which property must be conserved?*

Physics understands conservation as follows. Every event in which some property is conserved necessarily has to have a symmetry. That is, it is the particular symmetry or symmetries of an event that 'causes' conservation of a property associated with that symmetry. Conservation of momentum is a *necessary* consequence of the translational symmetry possessed by the colliding system. Energy conservation is a consequence of temporal symmetry. Charge conservation is a consequence of an internal (gauge) symmetry of the system and so on, and these are all *necessary* consequences.

Therefore, we may say that some events have the property of being symmetric with respect to something such as space or time. Any such symmetry in the system explains conservation of some property. These conservation laws are the foundational laws of physics. What is important to note here is that only certain properties are conserved. Mass, charge, spin are common examples. It is also these properties that some philosophers call sparse properties. *Note that there is no shape conservation.* Does this go to suggest, once more, that shape should not be accorded a privilege over symmetry? Does this imply that the real natural properties or essential properties that we must accept are only those which are conserved under some appropriate conditions? That is, can only conserved properties be natural properties? I think the answer is yes to all these questions.

In analysing the nature of causality, Dowe (1999, 270) suggests the 'conserved-quantity' (C-Q) theory of causality. Two definitions of conserved quantities and their explicit relation to causality is given as follows:

1. A causal interaction is an intersection of world lines which involves exchange of a conserved quantity.
2. A causal process is a world line of an object which manifests a conserved quantity.

A conserved quantity is one, like the examples of charge and momentum described earlier, which is conserved in a process. Dowe notes that conservation processes are symmetric but neglects to link the notion of an underlying symmetry which is the cause of such conservation laws. We are interested in asking what it means for an underlying symmetry to cause conservation of certain properties of particulars that participate in a process. Even Woodward's analysis of invariance understands it as a property of laws but does not consider how we should understand laws that are a consequence of some invariance.

It may be useful to first understand the nature of laws before we seek to clarify the link between symmetry and conservation laws. It can be argued that properties are necessary to explain laws. It is reasonable to say a certain property can cause a particular effect. Armstrong (1997) formulates the notion of law as follows. Let a particular *a* have a property F and a relation R to another particular *b* which has a property G. Consider the case when this state of affairs is immediately succeeded by *b*'s becoming H. He explains this sequence in terms of *a* having F and R to *b*, thereby *causing b* to have H. Thus he sees a law as a 'causal connection between state-of-affairs types' (ibid., 226). And for laws of this form, there is nothing more to them other than to be 'instantiated in such sequences'. That is, the law is exemplified only in the manifestation of the particular sequences. It is also to be noted that Armstrong believes that all cases of singular causation are instantiations of some law. Further, non-causal scientific laws supervene on causal laws. And finally, laws, in this view, play a very important explanatory role in that they explain the regularities in phenomena and processes.

But these are only certain types of laws. Armstrong uses the example of a guillotine which causes decapitation to illustrate the above structure of a law. But such a structure of law is not the one that is generally considered as a fundamental law in science. These are laws which are related to generalisations. The laws of importance in science, what Armstrong calls functional laws, are those which manifest the determinate-determinable relation through some mathematical relation.

He isolates two characteristics of such laws (ibid., 242):

1. They are determinable laws, and under each such law falls a large, perhaps infinite, class of determinate laws.
2. A great many of the determinates are likely to be uninstantiated.

An example is Newton's law of gravitation. For different sets of masses and relative distance, a value of the gravitational force between them can be found. Armstrong argues that each such calculation can itself be seen as a law but it is extravagant to believe so when we can unify all these individual determinate laws under one determinable (functional) law. Such a relation not only supplies unification but also explanation. Without entering into a discussion on the metaphysical nature of such laws, let me consider the nature of conservation laws.

As is to be expected, conservation laws seem to be functional laws. Conservation of linear momentum is a determinable and specific cases of such conservation is a determinate. There are a plethora of conservation laws for each value of the total momentum. But while this seems to be the case in general, there is also a crucial difference between conservation laws and Newton's law of gravitation, for example. Newton's law is not explained by any prior principle that necessitates the form of this determinable expression. Conservation laws, on the other hand, are a consequence of a prior symmetry. Thus it seems that conservation laws are functional laws but they also need a specific causal structure as in the example of the guillotine. Also, note that a functional law is more than a functional expression. For example, we can write the angular momentum as a product of angular velocity (ω) and the moment of inertia (I). But this functional equation is not a law like Newton's law essentially because there is no cause-effect relation involved in the terms that occur in this equation. In Newton's law there is a particular acceleration caused by a force acting on a body. The law specifies this causal relation. In the case of conservation laws, it is typically the case that there is no causal information in the law itself. For example, in the case of conservation of angular momentum, we may write the law as $I_1\omega_1 = I_2\omega_2$. This law only tells us that the original angular momentum is the same after some change has taken place, as in the example of the skater discussed earlier. There is no causal link between I or ω. And yet, this law is like a functional law in that a whole range of determinate values are unified under a determinable.

Those who believe in the reality of laws predominantly hold the N-relation view of laws as necessary nomic relations among properties. If F and G are *first-order* properties, a law is a necessary nomic relation between them and is written as N(F, G). Newton's law of motion has mass and force as the two first-order properties and acceleration as a ratio of force to mass as the nomic relation.

It has been noted that not all scientific laws can be put in this form. Conservation laws and symmetry principles have been given as example of such laws which are not N-relation laws.[12] But it is not clear whether this is really so. If we accept symmetry as a first-order property then we can rewrite conservation laws as N(F, G). Consider the example of conservation of total linear momentum. Let F be the first-order property of the system that is its total linear momentum (just like the total mass of a system). Let G be the other first-order property that is the symmetry possessed by that system. Then N(F, G), the necessary nomic relation, is the identity relation on F at various points of time. That is, the relation between F and G in this case is such that G causes F to take the same values at all times.

The status of conservation laws might be better understood if we consider symmetry as the property that causes conservation of certain quantities. Symmetry is not explicitly present in the conservation law but it is that which makes possible these laws. The symmetry could be that of spacetime and/or of the configuration of the system. Whatever be the case, what is clear is that symmetry should be seen as a primary property of the system and causes conservation of certain other properties. Shoemaker (1997) argues that properties should play a causal role for them to be accepted as genuine properties. Moreover, he notes that properties 'reveal their presence in actualisations of their causal potentialities' (ibid., 242), a point that helps us to further emphasise the conclusion that symmetry should be seen as a fundamental, intrinsic, first-order property.

NOTES

1. See also Quine (1961).
2. See also Fales (1990).
3. For example, see Daly (1997, 140).
4. See also Castellani (1998).
5. For a two level criterion for set identity, see Lowe (1998, 42).

6. For those who are troubled by such profligacy, we can attempt to restrict composition. In this context, see van Inwagen (1990).
7. For example, see Johansson (1989, 93).
8. This centre of mass does not always have to be 'inside' the object.
9. See Resnick (1997) for more on mathematics as structure.
10. See also Lucas (1984).
11. See Sarukkai (2002) for more on this aspect.
12. See Swoyer (2000).

PART THREE

Phenomenology and Aesthetics of Symmetry

A philosophy of symmetry cannot be restricted to only metaphysical analysis. Phenomenology is the other tradition that is needed to understand the nature of symmetry. A phenomenology of symmetry would have to begin from a reflection on the phenomenological dimension of symmetry. The arguments adduced earlier have hopefully shown that symmetry is indeed a 'primary' property, rather than something derivative as it sometimes appears to be. It is also the case that the manifestations of symmetry are also inherently phenomenological in nature.

Natural objects show a wide range of symmetries. The symmetries of nature are usually associated with the shapes or forms of natural objects; in the distribution of colours as seen in a wide variety of insects and animals; in complex patterns which are discovered in various manifestations of the animate and inanimate world. Metaphysics gave us an analysis of the many categories that are involved in understanding these symmetries. But there is a content to symmetry that needs a different 'kind' of philosophy, namely, phenomenology.

We experience symmetry in various ways. We usually talk of it in terms of proportion, harmony, balance, beauty, simplicity and so on. The phenomenological experience of symmetries influences the way we respond and react to them. In the case of symmetries of shape, colour or patterns, it is the phenomenology of visual perception that is most important. Perception is itself complex. It can be described with the help of science as well as through phenomenological experience and philosophical categories. One of the important examples in phenomenology has been the discussion on perspective involved in perception. Rarely do we see a 'full' object and in most cases there is nothing we can do about it. But in symmetrical objects we are able to have a sense of the whole even if we see only a part. This filling up the blanks in perception (and indeed knowledge) is one way of understanding symmetry in its phenomenological guise.

It is well known that the idea of vision has been central to philosophy in both Eastern and Western traditions. Some writers refer to the 'hegemony of vision' as something that has shaped the growth of philosophy.[1] Even science is deeply indebted to vision in various ways, suggesting that the hegemony of vision has also been important in the formation of scientific discourse.[2] Modern physics, which uses symmetry as a fundamental principle, is also indebted to metaphors and images drawn from our ideas on vision. Thus, symmetry itself is placed within the larger discourse on vision that is present in both macro and microphysics. Phenomenology gives us new insights into symmetry and its relation with vision. In particular, as I will argue later on, even the structure of groups seems to be 'correlated' to certain principles of vision as described by Gestalt psychology.

Although vision is dominant, ideas of symmetry are present in the experience of other senses also. For example, we generally consider a dish to be tasty if its various tastes are balanced. The balance of tastes and of smells is related to the judgement we make about the quality of the food we eat or the smell we experience. Our experience of tastes and smells are phenomenological. Harmony, proportion and balance are ideas that are integral to what we judge as 'good' taste and smell. These examples illustrate that ideas associated with symmetry are intrinsic to aesthetic judgements, as in the case of visual experience. When we talk of such experiences we do not have mathematical descriptions, like group theory, but nevertheless symmetry is basic to the aesthetics of sight, smell and taste.

Tactile sense is also involved in the phenomenology of symmetry. A simple example is the common experience of balancing objects like a stick or a disc. We can balance a stick at a particular point on the stick; we can balance a homogenous disc at its centre. These phenomenological experiences illustrate a tactile sense of symmetry. There is, of course, no need to consider these senses as independent of each other. If we follow Merleau-Ponty, for example, we can hold the view that the senses of our body are not independent of each other but are how somehow 'intertwined'. Such a non-reductive view of the senses allows us to understand symmetry in a broader sense.

In the ways in which we deal with objects in our daily life, we are always engaged in some phenomenological expression related to symmetry. Consider the simple example of making paper planes. Even a child who learns to do it understands intuitively that there is some notion of balance involved. In the making of a paper plane, given a sheet of paper, we fold it along a line of symmetry. Further folds follow some axis of symmetry. A child has a phenomenological understanding of balance and symmetry without knowing what symmetry is – especially in the restricted view of understanding it entirely in terms of transformation and invariance.

Simplicity and beauty are terms that occur in any description of symmetry. Even in the case of mathematical or physical theories, these two notions have been privileged in various ways. Both these notions need to be given a phenomenological reading.

1. PHENOMENOLOGY OF PERCEPTION

What does it mean to say that we see a thing? I see now a book in front of me. I see the black colour of the cover of the book, the shape and size of the book from my perspective. I also see it as one object among other objects on the table. I am writing as I am seeing the book. I read the title of the book and the names of the authors. I look out of the window and see mountains and trees but at the same time am also hearing the steady sound of crickets. In general, perception involves recognition not just of one thing but of many 'things' in the field of vision, which is not restricted to sight alone. It also involves, when I focus on the object, the distinction between the background against which the object is placed and the foreground of the object.

Seeing is obviously related to the body and, in particular, the eye. It is also an intentional act which lets me focus on a particular book and not anything else. Let us say that I am now focussed on the book. My recognition of the object that I see as a book also involves prior understanding of what a book is. Although there are many books of different shapes and colours, I see them all as books. Similarly for the different kinds of trees. One may invoke consciousness to describe all these characteristics of seeing. But if we are already in a conscious state of seeing, then what is the 'experience' of seeing?

Let me initiate this discussion by asking what is it to perceive a shape of an object. What are the assumptions in making the statement that an object has a particular shape? I begin with shape for obvious reasons since symmetries seem to be intimately related to shape or form. In the last Part, I argued that symmetry has a claim to priority over shape. Does phenomenology also tell us something similar about symmetry?

When we say an object has a particular shape, we have to note that in general this is a problem as far as perception is concerned. It is clear that shapes and sizes are not absolute but are dependent on perspectives. As Merleau-Ponty (1981, 299) notes, usually we talk of the size and shape 'when it is in a plane parallel to the frontal elevation.' For him, the frame of reference is first and foremost the body. The empiricist arguments for a definite size and shape of an object does not answer the primary question of how such a perception is possible. The intellectualist answer tends to make perceptions judgement of our faculties or reason. Merleau-Ponty answers the problem of perception of size and shape in terms of lived experience – knowledge of the size and shape of a thing 'is grounded in the activity of the body' (Hammond et al 1991, 187). He uses the fact that there is an 'optimum distance for perceiving things' (ibid., 188). It is interesting to note that this optimum distance is understood in terms of balance. Moreover, perception of a thing involves an already sedimented knowledge that allows us to recognise the object as such. Thus, rather than the object's properties causing the appropriate perception, the habits involved in perception also contribute to the experience of the perception. Further, his view that perception is not restricted to one sense alone but involves the other senses also, implies that perception and experience of shapes is much more complex than when understood in terms of vision alone. The clarity of an object increases with the synthesis of all the senses – 'unity and the reality of the thing perceived are only fully appreciated when the senses are acting in unison' (ibid., 195).

Such an analysis is useful when we discuss the property of symmetry and the ways in which we experience it. First of all, in the usual way we talk about symmetry, it is clear that sedimented knowledge is important. A scientist's view of a symmetric object is quite different from a layperson's one. The question of what shape and size an object has, given their difference in various perspectives, does not relativise these notions completely. For Merleau-Ponty, we can still hold onto an empiricist view about a particular shape and size of an object except that it is now viewed through the body's orientation towards the object. But what can this say about symmetry? What happens to the symmetry of an object under different perspectives? And how does the body even perceive and recognise symmetry?

Before we try and answer these questions, let me discuss the notion of shapes and forms in further detail. When we perceive form, we are in essence perceiving a boundary of the object. Plato distinguishes form from nature and identifying nature with matter 'makes matter prior to form, its temporary accidental arrangement' (Lang 1998, 50). For Plato, form is imposed on matter. For Aristotle, in contrast, matter cannot be essentially identified with nature. Indeed, for Aristotle, nature is 'the shape, namely the form, not separable except in definition, of things having in themselves a principle of motion' (ibid., 50 – 51). Thus nature is form and not matter. Lang notes two reasons for Aristotle's position: one, the primacy of the actual over the potential, the actual being specified by form, and two, his belief that 'form is that toward which a thing tends or grows' (ibid., 51). We should also note the constant interplay between nature and art that informs these views – natural things occur with a form while artefacts have their 'form imposed from without'. To make a further distinction, Aristotle considers 'nature' as identified with form but 'by nature' as combination of form and matter. Thus, form is that which is actual and is 'the primary constitutive principle' whereas matter is substance as potential.

As we can see, the idea of symmetry in its relation to form and in its essential embodiment in nature is close to Aristotle's position. Especially so in the light of modern physics where symmetry plays a central role not only in explaining the forms of natural objects but also as a principle of evolution of forms. Aristotle's priority of form over matter also takes into account the evolution of forms as being central to the identity condition of an object – 'any natural thing grows toward its form and toward nature in the sense of form' (ibid., 53). Matter, in the case of nature, 'runs after form' and desires for form. Further, form itself is the cause of motion for motion always is a movement towards form, towards actuality from potentiality. This relation between form and matter gives us a distinction between nature and art. In art, it is the artist who imposes form on matter. Aristotle also distinguishes between form and place; both are limits but are distinct from each other. Form is the 'limit of the thing contained' and place 'is the limit of the container' while matter is the 'body of the contained'.

One of the fundamental problems for Aristotle was the relationship between form and matter. Aristotle treats matter and form together; this is usually referred to as hylomorphism. To motivate the discussion, let me consider Lowe's (1999) analysis of a bronze statue. In what sense is a bronze statue a combination of matter and form? The statue is not form in addition to bronze. If so, Lowe argues, the bronze would have to be seen as a part of the statue, which it is not. Similarly, the form is also not a part that belongs to the statue. Lowe suggests that the way out is to consider the form of the statue 'as something belonging exclusively to the statue *rather than* to the bronze,' that is, 'identify that form with a particular property which the statue has but the bronze does not' (ibid., 8). This allows him to distinguish between shape and form because he places the shape along with the concomitant presence of the statue. Due to which, the identity of the statue over time is not the identity of shape of the statue but only the identity of the *statue of that shape*. The primacy accorded to form then offers an explanation for the identity of the statue. As a consequence, Lowe takes the position that form and matter 'are relatively independent.' This implies that it is conceivable to think of form without matter. In fact, Lowe considers the description of particles in modern physics as exemplifying this possibility. Neither does matter help in the individuation of things by giving a principle of identity for them. Even in the case of individuation, Lowe believes that form is what allows such individuation.

In contrast, Kant suggests that forms 'do not pre-exist in things' but are 'generated by the a priori forms of intuition' (Mainzer 1996, 562). Intuition therefore 'works spontaneously in order to determine the "unity of the manifold"' (ibid., 562). Mach, whose philosophical writings were influential among scientists including Einstein, considered form 'as independent of other phenomenal qualities' (ibid., 562) . Mach was influenced by Gestalt psychology, which considered form not as addition of constitutive elements but in terms of a whole, a gestalt.

The American philosopher, George Santayana (1955), discusses the idea of form and the relationship of it with beauty. He views form as an aggregate that has elements, and the manner in which the elements are combined constitutes the character of the form. For him it is important to be able to distinguish the relation of parts in a perception for it to be considered as a perception of form. But he notes that unity is a virtue of forms although unity by itself cannot be a form. Santayana relates forms to symmetry through the categories of unity, individuality, multiplicity and so on. I shall, later in this Part, discuss this relation among various terms related to symmetry.

2. FORM AND VISION

Without further discussion on the nature of form, let me focus on the relation between form and vision. As the above formulations of form show, the idea of form is intrinsically related to vision. In the case of form related to other senses, similar notions can be developed, but the privilege given to visual form continues to influence them. For example, the perception of forms, according to Gestalt psychology, is based on certain organisational principles that I discuss below. One can extend these principles to the cases of other senses but extending these principles thus only reinforces the dominance of the metaphors of vision.

The consequence of sight being the dominant sense is that the paradigm of vision, referred to as 'ocularcentric', becomes central to the preoccupation of philosophy and science. Hannah Arendt notes that 'from the very outset, in formal philosophy, thinking has been thought of in terms of *seeing*...' (Levin 1993, 2). This 'hegemony' of vision can be traced back to Plato and to the ideas of Platonism in general, a point insistently stressed by Heidegger and Derrida. Plato's metaphors of light, darkness and cave, are well known. The philosopher, according to Plato, should be able to 'see what needs to be seen even in the darkness,' and one who should not be 'dazzled by excess of light'; rather, the philosopher should be guided, not merely by the physical sensation of light, but by the 'light of reason' (Levin 1997, 12). Plato's language draws upon the imageries of light and vision, including his comment that the 'soul is *like* the eye' (ibid., 13). If Platonism shapes a predominant culture of vision then it can be argued that Platonism in mathematics implies that mathematical entities are themselves implicated in a larger idiom of vision – I will argue for this possibility in the example of groups.

The metaphors of vision have entered the vocabulary of common language. For example, the relationship between 'truth' and 'enlightenment' as also that between 'ignorance' and 'darkness' has shaped the language of science, epistemology and philosophy. The continued use of these images also serves as 'ideal models with a distinctively normative rhetorical function' (ibid., 8). Thus the rhetoric of truth, and, in general, knowledge, has privileged philosophy's preoccupation with sight. Levin notes that philosophers like Heidegger, Foucault and Derrida have a common complaint that 'the thought and culture of modernity have not only continued the historical privileging of sight but allowed its worst tendencies to dominate' (Levin 1993, 5).

It can also be argued that in the case of Descartes that although his engagement with metaphors of vision shaped his discourse there were also significant differences in his account of vision. While it is indeed the case that Descartes found darkness to be a 'nightmare', he also developed the science of optics that developed the possibility of displacing the 'priority of the eye and centrality of vision' (Judovitz 1993, 69). Also, his belief that touch is more reliable than vision because 'touch allows us to conceive ideas ... vision blinds us to the object' opens up a movement beyond the visual sense. This insight into the limitations of vision even as it seems to be the dominant sense has also been echoed by other philosophers. Although this does not repudiate their immersion in the hegemony of vision in their writings, it points to the constant tension in using these metaphorical images and in doing so, attempting to get away from their suggestive orbits. Hegel, for example, responded to this issue by conceiving of the 'generosity of vision'.[3] For Hegel, visual sensation was 'less' than the sensation of touch because vision by itself does not allow us to experience depth and space like touch can. Similarly Merleau-Ponty. While his philosophy liberally drew upon metaphors of vision, at the same time he insisted on the oneness of the senses and in his later work argued for granting invisibility the same ontological status as visibility.[4]

McCumber (1993, 236) notes that philosophy has used 'vision as a model for knowledge' and this tendency has itself given meaning to what we call vision. Derrida argues that what is presented in the thing is its form – 'form is presence itself' (ibid., 236). If a thing 'can be known only insofar as it is form,' the consequence is that matter is suppressed at the expense of form and this Derrida refers to as the 'founding opposition of metaphysics'. That is, 'the subjection of vision to form is thus only the first step toward a deeper subjection of vision to speech' (ibid., 237).

It should not be surprising that science has also actively participated in the discourse of vision. Science, seen as an activity that attempts to make visible the invisible, draws upon ideas of vision in its articulations but at the same time also tries to find a suitable language of the invisible.

It would not be an exaggeration to claim that science, more than any other discipline, has privileged the visual, even as it has enriched its engagement with the other senses. The development of microscopes and telescopes brought micro and macro objects within the visual field. Photographs, X-ray, MRI, PET scans etc. are attempts to capture a sense of the invisible through visual means. The visualisation of the writing of science is of fundamental importance to its discourse. As Ihde notes, the textlike phenomena permeates science in the use of 'charts, graphs, models, and the whole range of "readable" inscriptions which remain visual' (Ihde 1998, 167).

Why is this privilege extended to the visual in science, both in practice and discourse? A traditional answer, according to Ihde, is that vision is seen as the 'clearest' of the senses. But he rejects this view because he believes it is nothing but 'cultural prejudice' to believe that vision is the 'best' sense. In contrast, he suggests that what is special to visualisation in science 'are its repeatable Gestalt features' occurring in the visible forms in technological imaging in general (ibid., 161). This association of vision with the Gestalt will be reinforced by my observations about the role of Gestalt in the ideas of symmetry – once more suggesting that vision plays a dominant role even in the languages of the invisible.

It is only the prior sedimentation of metaphors of vision in philosophy which can explain why science too develops its ideas of form within an ocularcentric discourse. This suggests that science understands the domain of the visible through creating a form of the invisible that is strongly dependent on the idea of visual form. This is manifested most powerfully in the case of symmetries in science and the use of groups to describe them. *Thus the making visible of the invisible in the scientific discourse lies in making visible the form of the invisible based on the logic of visual forms.* The fertile engagement of science with strategies of writing, strategies that are beholden to the idea of form, reiterates the importance of form in science, especially in its discourse.[5]

3. SYMMETRY, FORM AND THE GESTALT

Symmetry, form and the Gestalt are intimately related concepts. I will discuss this relation in two parts. First, a brief summary of the Gestalt 'laws' of perception will illustrate the principles that describe how we perceive forms. Second, I will extend this to the case of groups, where I will argue that the group structure is similar to these Gestalt principles, thereby emphasising the point that symmetry, even when described by abstract mathematical structure, seems to be closely related to the principles of visual perception.

3.1. *Gestalt laws of perception*

How do we perceive forms? What are the principles of vision that make perception of forms possible? As is well known, Gestalt principles have played an important role in the psychology of perception.[6] Gestalt means 'whole' and the fundamental basis of Gestalt is the interplay between the ground and figure in perceptual process. I will discuss in brief some of the organisational principles of vision based on experimental observations by the Gestalt psychologists. Our perception of form is fundamentally based on certain principles that help to organise the field of vision, the first of which is the figure-ground perception. The famous examples of the many 'ambiguous' images where the images shift from one to another are an illustration of the need to factor both figure and background in any perception.

Another organisational principle is that of good continuation. This principle suggests that we tend to see continuous forms that are smoothly continuous in contrast to discontinuous forms. A third principle is that of proximity, which essentially states that units of a figure that are closer to each other will tend to be perceived as part of a single entity. A good example of this is an array of dots that will be seen as columns if the dots are closer to each other along the vertical line. If the dots are closer along the horizontal line then the array will be seen in terms of rows. Thus, this is a principle of organisation that explains why we see figures and forms the way we do.

A fourth principle is that of similarity: when we see a form we tend to pick out patterns that share similar elements, like colour patches in between. Finally, there is another important principle that guides our perception, namely, closure. This principle points out that our perception picks out all elements that will form a closed figure or whole. That is, our perceptual process tends to see closed figures as belonging to one entity. A simple example is that of a circle made of dots; even if one dot is missing the figure is still perceived as a full circle.

It must be mentioned here that when these principles act in cooperation the grouping is very stable but when there is conflict between these different organisational principles then the image tends to be unstable and ambiguous. It must also be noted that symmetry is a factor in both the organisational principle of form as also in the recognition of figure-background perception. One of the Gestalt principles is that of symmetry, what was also called as 'good figures', characterised so 'because they could not be reduced perceptually to any simpler components' (Wade & Swanston 1991, 35).

Although these central (and other minor) principles of Gestalt have to do with planar configurations, they point to certain necessary relations that inhere in the ways in which we perceive form. Rather than use these principles as the benchmark of perception of form, I want to restrict my analysis to exhibit the close similarity between these principles and the structures of groups that describe symmetry in science.

3.2. *Groups and the Gestalt*

Groups describe symmetries in science, including the symmetries of geometric forms, objects, events and laws. They describe symmetries of the visible and the 'invisible' world. What is special to groups that they are seen as natural mathematical structures to describe symmetry? I suggest that the structure of groups is strongly correlated to the Gestalt principles of (visual) perception. We can claim that it is possible that groups are mathematical structures that describe the (non-mathematical) Gestalt principles. In which case, groups would be the structure needed to describe the mathematics of perception of forms. But symmetry in science does not acknowledge that it is a perception of form in general. Since the structure of groups is similar to the structure of the Gestalt principles we can argue that groups actually are modelled after the principles of organisation that describe how we perceive forms. This would then imply that symmetries in science, described by groups, are actually based on a prior idea of (visual) forms and their (visual) perception.

Why are groups correlated to the Gestalt principles? Groups are sets that have an operation defined on them, and whose elements obey certain simple properties – that of closure, existence of identity, inverse and associativity. Sets are a collection of points, things, elements that are brought together into one set through some criteria of membership. As a collection of these elements, they are first of all similar to the grouping principles of the Gestalt. Note that the bringing together of a set of points as belonging to one 'set' is itself a Gestalt principle. We may ask why groups should be the mathematical structures that describe symmetry? There are two 'parts' to a group. One is its set nature, a collection of elements. The other is the operation defined on these elements. Given the operation and the elements, the group rules give us the relation between the elements of a group – we may look upon this as the rules of membership to a group. But then why is the idea of symmetry – in many cases to do with shape or form – related to a structure of set?

Symmetry, as we saw earlier, has to do with invariance of some form under some change but why should this have anything to do with sets? The explanation for this, in the context of Gestalt, is that a form can be understood as a 'collection' of points. Let me give an example. When we talk of the symmetry of an equilateral triangle, we describe it by a group that has as elements three angles, the identity 0 (or 360 degrees), 120 and 240 degrees. This idea of a group is itself based on the view of a triangle as being defined entirely by its three vertices. Like the Gestalt, the triangle is specified once three dots corresponding to the vertices are given. It is the Gestalt organising principles that offer us a reasonable belief that we are indeed seeing a triangle although we are 'only' seeing three dots. When the triangle is rotated about its centre, we keep track of these dots. The rotation of the dots is assumed to imply the rotation of the shape of the triangle also. Specification of these three dots is enough to specify the form of the triangle. The changes in the form of the triangle under some transformation, like rotation, are also specified by changes in these three dots. This principle of filling out a form from its constituent dots is central to Gestalt principles of grouping. So, we may understand the set nature of groups corresponding to symmetry as reflecting a Gestalt principle of grouping.

We can also understand the Gestalt grouping principles as related to sets. Obviously, the bringing together of points to suggest certain forms is nothing but a collection of one set of points which function in a particular way. However, groups corresponding to a symmetry should not be equated with the collection of these points that suggest a form. For example, the symmetry group under rotation of an equilateral triangle has three elements but these three elements cannot be identified with the three dots that suggest an equilateral triangle but are represented through certain angles corresponding to invariant rotation of these dots around the centre of the triangle. We may say that symmetry groups are nothing but a Gestalt principle of organisation of elements in the transformation space and Gestalt principles of organisation are first and foremost a collection of elements – that is, a set.

But groups are also more than sets. The elements of a group have certain relationship among each other. The closure property is a common 'name' in both groups and Gestalt. The closure property in Gestalt principles claims that perception fills in the gaps in a form and projects a filled form, even if the form has certain elements missing in it. Given a set of points in the shape of a circle, even if some points are removed we continue to see it as a circle. Closure property of groups states that the result of the group operation of two elements of the group should yield an element that also belongs to the group. That is, group operations cannot take an element out of the set of elements in the group. We can also understand this in the following manner: given two elements of a group, we know what another element is. This is definitely on the order of an organisational principle of the mathematical groups.

The important criterion that a group should possess an identity element is 'equivalent' to the Gestalt principle that emphasises the role of unity of perception of form. Identity element stands for the identity transformation, or equivalently, the case in which no transformation is made. This is nothing but the 'first' perception of the form. The inverse elements are those that give back the identity after some transformation. This is phenomenologically equivalent to the temporal identity of the 'first' perception. It reinforces the point that perception from different perspectives does not transform the given object into something else. In fact, this can be seen as the grounding of perspectivism itself and the fact that perception does not change the identity condition of the object but only the set of meanings that one ascribes to it. The idea of passive transformation, in which the observer moves rather than the object, is what generates what we call different perspectives as against active transformations which are the transformations of the object. Thus, a particular symmetry is nothing but the recognition that some perspectives offer the 'same' vision. The identity and inverse rules of groups are 'correlated' to the Gestalt principles of recognition of the full form even as we recognise them as being made up of individual elements.

What is the consequence of this similarity between groups and the

Gestalt? First of all, in the case of simple figures like triangles, the similarity of the group structure and the Gestalt organisational structure is clearly linked to the perception of visual form of these figures. The consequence of this is that vision in symmetry is the vision of the Gestalt – at least as far as the scientific description of symmetry is concerned. Secondly, and more striking, since groups describe symmetry of not only visual forms but also of systems, quantum particles (where the idea of form is extremely ambiguous) and so on, it seems that even the larger idea of symmetry in these cases is similar to Gestalt principles. This should be surprising: why should the general notion of symmetry continue to have Gestalt structure? Why do we model even non-visual symmetries in the same way as we do the visual ones, especially since the visual form is constructed via the Gestalt?

One response could be to say that something similar to the Gestalt principles for forms also holds good for the other cases such as colour and music where the symmetry has nothing to do with the form per se. Therefore, when we describe symmetries of quantum particles by using groups, we are in essence still within the Gestalt as far as the organisational principles go, although we are not restricted to the Gestalt of visual forms.

The consequence of making this connection between the structure of groups and Gestalt principles of perception of form is the recognition that symmetry in science is deeply immersed in the ideas of form and vision. This is indeed surprising, for the importance of symmetry in science is not in the recognition of symmetry of geometric forms but in the formulation of symmetry principles related to the microscopic world as well as in conservation laws. In the case of conservation laws, what can it mean to say that they are also in some sense indebted to ideas of form and vision? In the case of symmetry as an important principle in quantum theories, such as the unified theories, the conclusion is somewhat more startling. The use of groups in these instances suggests that the form of the invisible microscopic domain is first grasped and understood through the models of the form of the visible world. It can be argued that in the more abstract mathematical formulations of symmetry, what is privileged is the form of the mathematical equations. This form is nothing but the form of the orthographic inscriptions of mathematical symbols and equations. This is clearly manifested in the repeated reference to formal equivalence when talking about symmetries in physics.[7]

To conclude, we can see that even in the invisible world, the articulation of properties of objects of that world are strongly indebted to organisation principles similar to the Gestalt. Thus, even in domains of the unseen, there is an attempt to illuminate the world and make it a world of vision because the language of description of this world is primarily the language of vision.

4. OBSERVING SYMMETRY

I have talked of symmetry as a primary property of the object. I have also claimed that there are phenomenological experiences of symmetry. When we usually talk of properties, we tend to believe that they are in some sense observable. We do see the colour of an object. We believe we see the shape. We do not see the mass, which is considered as a primary property of objects. But obviously seeing is not the only sense used in the detection of properties. In the case of objects with charge, which is also a primary property, we do not see the charge. We smell a rose and that smell is indeed a property of the rose. We have seen that in the case of symmetry, there are experiences of symmetry in the senses of sight, touch, smell and taste. But in the case of sight, it may not be obvious that symmetry is a primary property that we see; rather, it may be construed as a property of the form. In the previous Part, I argued that this cannot be so. Phenomenologically, the experience of categories like balance, harmony and simplicity, for instance, all point to an experience of symmetry. In other words, symmetry is observable and experienced in-itself.

Since symmetry, especially in the scientific formulations of it, is intrinsically related to the notions of transformation and invariance, the observation of symmetry has been thought to be the observation of transformation and invariance. Kosso (2000), for example, holds this position in saying that observation of transformation and recognition of invariance is equivalent to an observation of symmetry. This implies that we do not observe symmetry as such but only as the 'sum' of two other observations. Although this is the definition of symmetry in general, he also notes that when we 'see' a snowflake we already 'see' its symmetry.

Let me first begin with the possibility of observing symmetry in objects. Kosso suggests that observation of symmetry in objects can occur in two ways: one is through 'seeing' and 'recognising', and the other is to observe transformation and invariance (hereafter T&I), and thereby conclude that we are observing symmetry. Both these modes of observation are fundamentally different. In the case of observing symmetry directly, as in the example of snowflake, it can be argued that we are only observing a pattern. Rather than enter into a critique of this naïve phenomenological observation of symmetry, I will restrict myself to pointing out that even in these cases, observation of symmetry is more than a recognition of the form of the snowflake. A person with no idea of symmetry whatsoever (in terms of understanding it as invariant transformations) can still experience something about the snowflake that captures the property of symmetry. One way of emphasising this point is to consider how visual forms are themselves experienced. For example, Arnheim (1954) mentions that visual patterns have a notion of 'visual balance' associated with them; '[E]xactly like a physical body, every finite visual pattern has a fulcrum or center of gravity.' When we see a snowflake, we are responding to the visual 'forces' in the figure leading to an experience of the balance of the figure. Further, Arnheim notes that in a 'balanced composition all such factors as shape, direction and location are mutually determined by each other in such a way that no change seems possible, and the whole assumes the character of "necessity" in all its parts' (ibid., 12). This notion of balance is not merely a consequence of particular symmetric figures. It is a characteristic of perception itself. Moreover, 'the function of balance can be shown only by pointing out the meaning it helps to make visible' (ibid., 27). It is not an accident that points of visual balance are most often the point about which symmetry transformations are defined.

What about the observation of symmetry as the combined observation of T & I? This process is obviously temporal in nature and necessarily involves principles of comparison, as Kosso has rightly stressed. There is also an explicit agency involved in these actions of transformation and comparison for invariance.

In the case of observing symmetry as the concomitant observation of transformation and invariance, it is not immediately clear that symmetry is a property of the object and not a property of the rules of transformation. That is, the consequence of defining the observability of symmetry through the categories of transformation and invariance does not necessarily imply that symmetry is a property inhering in the object, or even a second order property of the object. But the problem in this is that this implies symmetry is not directly observed but observed only as a consequence of a transformation of the object.

What other properties of objects are manifested only through transformation? The object in its entirety, that is, the complete form of the object in 3-dimensions, is possible only through transformation. As is well known, when we look at an object we do not see it in its fullness – for example, we do not see the 'back' of the object. It is only by transforming it, by creating different perspectives of it, that we can get an idea of the complete form.

This also suggests that transformation is not essentially connected with symmetry but has a prior engagement with the recognition of the form of the object, a recognition that is possible only through transformations. That is, transformations create a complete mapping of the form and once the form is grasped, symmetry is seen as a property of the form that is related to invariance. It is in this sense that one can say symmetry is present in nature.

The above discussion points to the importance of the metaphors of vision in the articulation of symmetry, including in modern physics and in the use of group theory. In the case of symmetry in science, this is only part of the picture. The case of symmetry associated with laws and equations, and the idea of internal symmetry expands the domain of symmetry.

4.1. *Explanation and observation*

Kosso (2000) classifies symmetry into four types: external, global; external, local; internal, global and internal, local. He argues that external, global symmetry is observable as is internal, global. Both these symmetries are 'directly' observable, whereas external, local and internal, local are 'indirectly' observable. This leads him to the conclusion that the 'observation distinction matches the divide between global and local symmetry' and not that of the external, internal divide. Local symmetries have a different empirical status in comparison to global symmetries because of the introduction of 'dynamical' symmetries in order to exhibit the invariance.

The ideas of observation in the case of global and local symmetries are significantly different. Direct observation, as discussed above, implies the observation of transformation and invariance. Kosso suggests the examples of Lorentz transformation for an external, global symmetry and the global symmetry of electric potential for internal, global symmetry. Observation in these cases corresponds to invariance of certain experimental observations. But these examples of experimental observation, such as observation of experimental results in transformed systems and the example of the pigeon sitting safely on a high-voltage wire, are primarily not observation of symmetry; rather, they are explanations of these results through invoking the idea of symmetry. Explanation through the idea of symmetry is not the same as directly observing symmetry. For example, before the symmetry principles were understood to be central to physics, there were conceivably other explanations in the case of the pigeon sitting on the electric wire. The explanation of this phenomenon, namely, the pigeon not getting electrocuted, draws upon many diverse ideas of electrostatics. The explanation due to symmetry is one link in the larger explanation. It is not clear then how an explanation (dependent on a particular theoretical formulation) can be transformed to the status of an observation or an observable entity. That is, just because symmetry explains why the pigeon is not electrocuted it cannot become an observable quantity in itself. Unlike the case of observation of transformation and invariance, which was initially defined as the observation of symmetry, observing the pigeon on the electric wire is not equivalent to observing the global symmetry of the electric potential.

This is a problem of conflating observation with explanation. That which explains is itself not observed, as is generally the case. One may then look towards causal explanations in order to situate symmetry on a more firm footing. But it is not clear that symmetry plays a causal role in the example of the pigeon.

Kosso's example of the observation of the global phase transformation again indicates a similar problem. This observation is the observation of invariance involving a double-slit interference experiment. He notes that one can change the absolute phase of the incoming electron in different ways. But 'none of the global transformations changes the outcome of the experiment; that is, none of them changes the interference pattern on the screen.' Thus the 'invariance is easily observed and the experiment as a whole amounts to an observation of the internal, global symmetry in nature' (ibid., 93). There are two problems with this conclusion. One is the problem of conflating explanation and observation mentioned earlier. The other arises when we ask transformation and invariance of what.

4.2. Invariance vs. independence

Observation of transformation and invaiiance not only suggest the possible presence of symmetry but also the independence of the parameters of transformation in the observation of invariance. For example, in the double slit experiment, if one introduces a filter that does not modify the electron beam or change a parameter which will not modify the pattern then also the interference pattern will remain invariant. Thus, transformation parameters that are not involved in the theoretical explanation of the phenomena will also exhibit invariance. But this is why Kosso says that the transformation must be observed. But transformation of what? Isn't changing filters that change phase seen as a transformation? If so, then changing filters that do not change phase is also a transformation and the same invariance is also seen. So we cannot conclude from this that the phenomenon has the symmetry under transformation of the independent parameters. This strongly suggests that, at least in the examples provided by Kosso, the presence of symmetry is an inference and not an observation, per se. As inference, it is theoretically loaded.

There are two other points that must be noted. It cannot be the case that mere recognition of transformation and invariance suggests the inherence of symmetry. There has to be something more: a breakdown of invariance is needed to postulate the invariance associated with a particular symmetry. Consider two examples. First, the case of the pigeon. Now suppose that the pigeon has flown from the ground and sits on the branch of a tree. There is a transformation and also invariance, exactly on par with the case of the pigeon sitting on an electric wire. What does this observation of transformation and invariance tell us about the observation of 'symmetry'? Symmetry of what? Thankfully, such a problem does not arise in the case of the electric wire only because there is another possibility that allows us to infer the symmetry of the electric potential, namely, the phenomenon that the pigeon can get electrocuted when it is simultaneously in touch with the wire and a grounded source. It is the possibility of non-invariance that in turn is associated with a symmetry. In the double-slit experiment, changing the phase of one beam without changing the other changes the interference pattern. It is this possibility that then suggests that the invariance of the interference pattern when both beams undergo same phase change implies a symmetry of the system. These call into question whether symmetry is empirically observable in the manner described by Kosso.

Symmetry always needs a breakdown of invariance in order for it to be recognised as such. In other words, we cannot simply place the observational status of symmetry on two other observational categories of transformation and invariance. Both of these are not by themselves independently observable and indeed, the observation of invariance is predicated on recognition of the observation of the breakdown of invariance. In the case of local symmetries, the breakdown of invariance is immediately manifested. In the case of global symmetries, invariance is predicated on a breakdown of invariance as discussed here. This suggests that as far as the empirical, observational status of symmetries is concerned, the divide is not between the global and local, as Kosso has it.

The second point refers to the idea of form and its relationship to symmetry, even in the above discussed examples. In the double slit experiment, the change of global phase keeps the pattern invariant but this needs to be qualified. A phase shift of 180 degrees, for example, interchanges the maxima and minima (the dark and white regions). Thus, what is actually invariant is not the exact position of maxima and minima but only the 'form' of the interference pattern. This engagement with form in the context of symmetry thus goes beyond form of objects and leads to the consideration of the 'form' of phenomena themselves. More importantly, the idea of form is of central importance even in the theoretical formulations, most notably in the inscriptive strategies of theories.

Transformations are distinct from operations in that they leave the form as is. For example, in tensor analysis, differentiation of a scalar leads to a vector and differentiation of this vector to a second order tensor. These operations change the 'kind' of objects they operate on but transformations of a vector leave the form of the vector the same and transformations on some tensor leaves its 'order' the same. (Of course, the primary operations of addition and multiplication leave the form invariant and should thus be seen as 'transformations' in this context.) Group theoretical transformations cannot change the form. In the case of global symmetries, like phase change (U(1) transformations) or even SU(2) transformations which leaves the form of the doublet structure invariant, even the introduction of operators leave the form invariant.

This is entirely due to the property of the exponential function, a simple example being the shift operator, which operating on a function f(x) only transforms it into another function f(x+1) rather than to a vector form. This property of the exponential function (or operator) allows group theoretical operations to remain at the level of transformation – only because it leaves the forms invariant.

In quantum field theories the idea of symmetry explicitly plays a causal role. The causal role of symmetry in these cases arises as a consequence of privileging the formal similarities of equations. In unified theories the generation of the gauge field masses (masses of bosons) arises from symmetry considerations. When we construct a theory (like the electroweak theory) explicitly on grounds of symmetry, which then creates the possibility of empirical observations like the masses of bosons, we can claim an explicit causal function to symmetry and thus postulate symmetry as an inherent property of the system. But we should remember that even in these cases the notions of form and similarity is fundamentally privileged.

5. SYMMETRY AND AESTHETICS

Very often, we are struck by the beauty of objects – whether natural or works of art. We look at an object and respond to it in some way. There are some experiences of the senses that create pleasure in us. We may call this subjective, individual experience as the experience of the beauty of that object. As is usually the case, when we look at objects with some symmetry, we are captivated by 'something' in that perception. Maybe we are responding to some sense of balance or proportion in the presentation of that object (natural objects or even paintings, for example). Perhaps it is to the sense of harmony between different elements in a given perception. Or maybe we are struck by the complex meanings suggested by simplicity in that object. Or, as is many times the case, we only respond intuitively to the beauty of an object of nature or art but do not know why we do so.

We may note that the ideas of balance, harmony, simplicity etc. are closely related with symmetry, suggesting therefore that symmetrical objects are intrinsically related to the idea of beauty. Symmetrical objects generate feelings in us that somehow respond to the nature of symmetry possessed by that object. It has also become commonplace to talk about symmetry as not being aesthetically pleasing but, on the contrary, quite boring and monotonous. To understand all these issues in detail, we may look to aesthetics to offer us an adequate formulation. In this section, I will discuss the relation between symmetry and beauty. How do we understand symmetry in the context of aesthetics? Are symmetrical objects necessarily beautiful? Can symmetry be an aesthetic property? Or is it a concept that is the basis for some other aesthetic property? Why is it thought that broken symmetry and asymmetry contribute to the notion of beauty, sometimes in a manner more significant than perfect symmetry itself?

Aesthetics is derived from the Greek word *aesthesis* meaning 'sense perception' or 'sensory cognition,' and is 'the domain of a certain form of receptive experience, or perception, or of response-dependent properties which are not necessarily unique to artworks' (Carroll 1999, 158). This perception is not a reasoned one, rather it involves the notion of 'apprehending through immediate sentiment' (Ferry 1993, 14). The discipline of aesthetics is essentially involved with the notions of sensory experience, the possibility of taste, the role of judgement of taste, the question of whether there are aesthetic properties present in an object that we have an aesthetic response to, whether there are principles of taste or only judgements of it, and, among other things, whether we need to invoke a term such as beauty. The question of beauty has been central to many theories of aesthetics, in particular, its relation to morality, its ability to evoke pleasure in the subject. Perhaps the central problem in any discussion of aesthetics revolves around a well-known problem: beauty, or in general, aesthetic experience, is entirely subjective, an experience of a particular individual, but when we talk of beauty or taste we seem to believe that they are objectively accessible, if not to all of us, at least to a significant majority of subjects. How is it possible to reconcile this 'objectivity' of an entirely 'subjective' experience? Related to this is the question of whether a term like beauty is actually a 'property' of an object or merely an idealist impression formed in our minds.

The problem of the objective and subjective in the context of beauty has been a central preoccupation for aesthetics. Plato, for example, considered beauty as something beyond its inherence in the subject. As Ferry notes, for Plato, the 'idea of the Beautiful is generally associated with the bringing into reality of an order where "measure" and "proportion" should rule' (ibid., 8 – 9). Socrates, in a similar vein, talks about the presence of harmony in objects that arise in the creation of parts in the formation of the whole. While this seems to place the property of harmony within the objects, the shift in modern aesthetics, as Ferry points out, is that the 'harmony is no longer thought of, and this is the real break with antiquity, as the reflection of an order external to man: it is no longer because the object is intrinsically beautiful that it pleases but, rather, we can go so far as to say that it is because it provides a certain type of pleasure that we call it beautiful' (ibid., 9). But even modern aesthetics, while placing the idea of beauty in the subjective domain, also considers the work of art as 'inseparable from a certain form of objectivity' (ibid., 10).

This interrelation between the subjective and objective is not unique to beauty or, more generally, aesthetic experience. The history of philosophy illustrates a continued engagement with this topic. Even perception is involved in the interplay of the objective and subjective. One way we can delineate the aesthetic from sensory perception could be to distinguish 'forms' of subjectivity. There are two streams that are commonly known in the historical discussion on beauty: those that privilege the *sensible* (Pascalian) and those the *intelligible* (Cartesian). The Cartesian approach 'locates the essence of the cogito in reason' while the Pascalian locates the essential 'in the heart or the feelings' (ibid., 26). This illustrates the struggle for the site of subjectivity as one between establishing the autonomy of the sensible or the intelligible. Platonism, for example, privileges the intelligible over the sensible. We can note a similar tension that pervades our understanding of science. The objectivity of science – although the activity is essentially a human one and therefore located in the subject – can be emphasised only by denying the Pascalian view of subjectivity. If the subjective expression is located in 'reason' then the possibility of objective access to subjective experiences is defined through the working of 'reason'. In the case of subjective experience of feeling, we also seem to make a judgement of that experience.

It is interesting to note that the metaphor of taste, which has come to be central in aesthetics, itself points to the objectification of subjective experience. There is something objective about taste – as biological taste – and the use of this term suggests that even in matters of aesthetic taste there is something objective – at least as objective as the sense of taste in the body. If judgement of taste is to be based on reason, then objectivity of beauty, as in science, will be reduced 'to a mere sense representation of truth' (ibid., 33). The connection between aesthetics and science can also be noted in a historical sense. The shift to observation over deduction in the eighteenth century gives the priority back to the object. Not surprisingly, this was an inspiration to understand aesthetics along the lines of physics. Thus, it was believed that we could expect to find principles or laws of judgement that bring together many observations in aesthetics, just as in physics. The consequence of this shift is, of course, to privilege the notion of discovery over invention in art works. According to Ferry, it is in Kant's *Critique of Judgement* that an attempt at a synthesis of the Cartesian and the Pascalian is first found.

In the consideration of beauty, we may proceed along two paths. One is to claim that there are (objective) principles of taste and the other is to claim that there are no such principles or laws but only judgements of taste. Mothersill calls the view that there are *no* principles (and laws) of taste as the 'First Thesis'. Kant holds this view as do many others. But what is a principle of taste? Such a principle 'would provide deductive support for a verdict, that is, for the judgement of taste under its normative aspect' (Mothersill 1984, 87). If 'principle' is too strong, then one could use the notion of 'criteria' of tastes. But principle, laws and criteria are mutually implicated in each other. The basic point is that principles of taste have to be normative in character. If there are such principles or laws of taste present, then we can reasonably explain why there is some objective experience of something like beauty. For example, we may say that laws of taste 'specify conditions for pleasure' (ibid., 97). Beardsley finds laws of taste in certain 'desirable features' present in a work of art. Three such features are those of 'unity', 'intensity' and 'complexity' (ibid., 98).

In contrast, Mothersill like Kant, believes that there can be no principles or laws of taste but there are judgements of taste. These judgements, at least what Mothersill calls 'genuine' judgements, make aesthetical experience, like beauty, somewhat objective. As is well known, Kant's 'antinomy of taste' was a demonstration of the thesis that judgement of taste cannot be objective because it is not based on concepts; the antithesis allows for the universality of such judgements.[8] As Ferry (1993, 85 – 86) points out, the issue that is brought to the fore by the antinomy is 'how to think aesthetic intersubjectivity without grounding it either on a dogmatic reason or on a psycho-physiological empirical structure?' Kant moves towards a solution to the antinomy by considering what differentiates the judgements of taste in contrast to other kinds of judgement. Thus he distinguishes between 'determinant' and 'reflective' judgements as follows (quoted in ibid., 86):

> Judgement in general is the faculty of thinking the particular as contained under the universal. If the universal (the rule, principle or law) is given, then the judgement which subsumes the particular under it is *determinant*. ... If, however, only the particular is given and the universal has to be found for it, then the judgement is simply *reflective*.

Thus, a judgement that has to do with knowledge, cognition and reason is a *determinant* judgement whereas the judgement of taste is a *reflective* judgement. Kant's original contribution in this matter is his shift to the notion of 'reflection'. Ferry isolates five moments of reflection. Reflection considers a particular in terms of a universal to which it possibly belongs. This placement of the particular in an appropriate universal suggests that the universal is itself given through and after reflection. The search for the universal (or concept) in reflective judgement is open-ended, thus providing a principle for reflective judgement, namely, the principle of purposiveness (that nature forms a system).[9] Finally, it is reflection that is 'at the origin of a satisfaction Kant calls aesthetic' (ibid., 87).

It is clear that in the above formulation there is a centrality accorded to the notion of system, which is the belief behind the principle of purposiveness. The idea of the beautiful, as a consequence, arises in 'the reconciliation of sensibility and intelligence' (ibid., 88). The idea of nature as system as being central to reflective judgement can be used to articulate an aesthetics of symmetry, because symmetry is fundamentally related to the idea of a system – whether in the relation of part to whole, as phenomenological experiences, as related to invariant transformations and so on. It is also not an accident that this formulation of taste leads Kant to privilege natural beauty, a position that is in constant tension with the very idea of artistic beauty.

6. BEAUTY

With this very brief introduction to judgement of taste, let me consider the experience of beauty. We often talk of beauty as something we experience and as something that is characteristic of that which is seen as being beautiful. The 'objects' that embody beauty are manifold: ranging from insects to mountains, persons, novels, poems, music, dance, painting, buildings and so on. Beauty allows gradations – we often say something is more beautiful than something else. If beauty is entirely a subjective feeling, then there is a genuine problem whether a piece of music experienced as beautiful is specific only to the experiencing subject. But in making a judgement that something is beautiful, we do expect others (at least some others) to acquiesce. Kant took this position to the extreme when he claimed that when we feel something is beautiful we demand this judgement from others, that is, if it pleases me it 'ought to please everyone' (Mothersill 1984, 213). This demand is similar to the demands of truth and takes beauty into the folds of truth and, therefore, 'final' objectivity. This is similar to certain traditional views on beauty which related it to morality and goodness. For example, Plato held the view that beauty is '(i) a kind of good (ii) which can be possessed by items of any kind and (iii) which is linked with pleasure and inspires love' (ibid., 262).

Obviously, such stringent requirements of beauty, as also in Kant's demand, are problematical. This is especially so when we ascribe beauty to certain artworks. It can be reasonably argued that our judgements on the aesthetic content of an object is dependant on various socio-cultural factors. What work of art is judged to be beautiful (by some community, if not universally) seems to depend on prior experience, recognition and knowledge about the appropriate domain of art. Santayana's position that judgements of beauty are judgements of individuals at a particular time incorporates this larger complex of factors in our judgement of beauty. There is a similarity between Santayana's and Hegel's view on beauty in this context: neither believes in principles or laws of taste.

Given the larger set of problems of beauty in works of art, are we justified in believing that there is 'something' called beauty? Mothersill argues that beauty is a 'standing concept' and is also 'indispensable'. Beauty 'picks out the concept of genuine judgements of the goodness of aesthetic objects' (ibid., 249). What does it mean to call beauty a concept? She answers it by noting that 'if j is a concept, then (1) there will be a general agreement with respect to what it *is* to be j and (2) j is indispensable, in the sense that it is not clear how one would follow the order, 'Get along without j'' (ibid., 259). Moreover, individual works of art or nature, cannot be the paradigm cases for establishing a criteria of beauty. This reverses the priority of objects over subjects in beauty. Contrary to Hume's position that there are certain objects that a 'qualified observer' will find beautiful, Mothersill argues rather that 'every qualified observer ... finds *something-or-other* beautiful' (ibid., 261). This question of what makes something-or-other beautiful leads her to consider Plato's views on beauty described earlier. In particular, beauty's link with pleasure is important. Mothersill considers this link as a causal one. Thus, 'if an item is beautiful, it is the cause (or a potential cause) of pleasure' (ibid., 279). But the reverse is not true: items may please us, like the winning result of a team, but it is not seen as beautiful.

This observation points to a potential problem. If two objects cause pleasure but only one of them is beautiful, how do we so distinguish them? If the distinction is possible because of some properties in the subject, then it seems to be accepting the position that there are principles of taste. Neither does it help to place the distinction in the subject, because, as Mothersill notes, it runs into 'circular' and 'metaphorical' problems. For instance, we can attempt to distinguish between objects of pleasure and causes of pleasure. While this is a problem particularly for the aesthetic experience of music (for example, what is the object of pleasure in music – the instruments that make the music, the music itself, part of the music, the musicians...?), in the case of objects like an insect or a painting, there is at least no real ambiguity about the objects of pleasure although we may not be sure what it is *in* the object that is the visual cause of pleasure. In the case of music, the object of pleasure gets enmeshed with various meanings of music, particular genre of music, listener's prior knowledge and experience of listening to such music, listener's sensibilities and so on. But in the case of a concrete object or a painting their identity can be understood in terms of their properties. It may seem that aesthetic pleasure arising from perception of these objects arises after recognition of the object per se. But this view is also contentious. When I say I see a beautiful object, I am not first cognising an object and then recognize that it is beautiful. Many times, the beautiful object in its presentation is instantaneously beautiful as much as it is instantaneously an object.

Even assuming that the object of pleasure is a concrete object whose identity conditions are not in doubt, we still have to ask what *is* it in the object that causes pleasure, if there is something in it that does so. Asking such a question implies that we already believe that there is something *in* the object that 'causes' pleasure in the subject. To understand the stakes in this question, we need to critically consider the causal relation between the object and the experience of pleasure. Mothersill argues that to say 'the object of pleasure is not the cause of pleasure' is misleading. That is, there cannot be an object and something else that somehow cause pleasure *vis a vis* the object. Neither is the object of pleasure an intentional object.

If some object pleases me, it is quite conceivable that I have an idea of what pleases me. Mothersill argues that the person who is experiencing pleasure usually has an idea of what causes her pleasure. Thus, it may be enough to say that the 'object of pleasure for x at t' is nothing but 'what x takes to be the cause of his pleasure at t' (ibid., 300). As is quite possible, I may be mistaken to what 'actually' causes my pleasure but in the case of singular cause, Mothersill notes, we can say with some confidence that we do know the cause of the pleasure we experience. Certainty is not necessary to say that we know the cause of our pleasure; one just needs 'ordinary justified true belief'. There is yet another reason why we can believe in the above formulation. Many times, we go back to experiencing that object which once gave us pleasure. Most times when I listen to a particular piece of music, I derive pleasure. And, when I *want* pleasure, I know what kind of experiences, what kind of objects, I need to go to. This process of intervening (as against just 'representing') to create pleasure for myself gives a measure of certitude to the above ideas.[10]

So, for Mothersill, the object of pleasure is not reduced to intentional object in each of the subjects. While a book may be experienced differently by its many readers, it is not the case that there are as many objects of pleasure corresponding to each of the reader's particular projection of the book. Since the book was the cause of pleasure to all of them (why and how may differ) the book is the object of pleasure.

There is the related point mentioned earlier: two objects may please me but I only find one of them to be beautiful. How do we explain this? As Mothersill correctly notes, this is not done by ascribing a particular property which the beautiful object has and the other doesn't. This will only imply that it is that particular property which is carrying the property of beauty but other objects with this property may turn out not to be beautiful. Mothersill draws upon Isenberg to answer the above question. Isenberg refers to the role of a critic who articulates her reasons for finding an object aesthetically pleasing. Thus the critic distils certain 'qualities' which in her view describes and explains the aesthetic nature of the object. Firstly, these properties can only be discovered through acquaintance. These qualities are called 'aesthetic properties'. Mothersill offers three definitions relating beauty and these properties. The first one is as follows (ibid., 342):

> Someone takes an individual to be beautiful if and only if the individual pleases him and he believes that it pleases him in virtue of its aesthetic properties.

Drawing upon Sue Larson, she defines aesthetic property as 'a property common and peculiar to individuals that are indistinguishable from one another' (ibid., 344). Thus the real test lies in the ability to distinguish two objects if we claim that they have different aesthetic properties. If they are indistinguishable then they have the same aesthetic property. What is important here is the claim that the same aesthetic properties cannot be had by two distinguishable items. Further, she contends that even if two objects share all properties like shape, colour etc. but have some distinguishable items, however 'small' it may be, like a scratch, then they cannot share the same aesthetic properties. Then, by virtue of the above definition, aesthetic properties in the object are the causes of pleasure. And because of this, in general, an object which pleases me (in virtue of its aesthetic properties) will continue to please me. The explicit causal link is captured in her third definition (ibid., 347):

> Any individual is beautiful if and only if it is such as to be a cause of pleasure in virtue of its aesthetic properties.

The important consequence of her view is that 'whatever is found beautiful *is* beautiful' (ibid., 349). Mothersill believes that the above definition offers a solution to Kant's antinomy of taste. She notes that to say 'O is beautiful' is only to claim a 'specific causal power' for O, that is, the ability to please 'in virtue of O's aesthetic properties' (ibid., 371). The judgement of taste is not instantaneous; rather, it needs critical reflection upon the object. This judgement is also contingent to so being confirmed. Thus, through this reflective critical study of an aesthetic object, it seems possible to have 'genuine' judgements of taste that can be held by a community without having to invoke principles of taste – the solution to the antinomy.

Let me summarize Mothersill's 'positive' account of beauty. (1) Beauty is causally linked with pleasure. Beauty is a 'kind of good' and other 'kinds' of goodness are different from beauty because of the *necessary* link between beauty and pleasure. (2) We also feel a sense of pleasure from 'objects' that are not beautiful – like hearing some good news. So a formulation is needed that is sensitive to this difference. (3) Kant distinguishes between the 'merely agreeable' from the beautiful. For him, the agreeable is pleasure taken in 'sensation', is pleasure that is 'interested' – for example, the pleasure arsing due to gratification of some desire. Mothersill does not subscribe to this for it seems to be an arbitrary distinction to distinguish (and then privilege) the pleasure arising from enjoying food and enjoying music.

7. NATURE AND ART

Nature affords us the first experience of beauty. The many symmetries described earlier are properties of natural objects. These objects do cause pleasure in us when we view them. A discussion of beauty will perforce have to consider the beauty of natural objects as well as artworks. In aesthetics, there is a prevalent tendency to compare and contrast natural and artistic beauty. The question of artistic beauty, as dealt in the many theories of art, is problematised by many human factors. It is, after all, an artist who creates a work of art. In producing the artistic product, the intention of the artist, the tradition from which the artist works from, the dominance of creative imagination etc., all add up to the complexity of 'understanding' a piece of art. But in the theory of beauty we have so far considered, all these human 'interventions' in the creation of an object of art are secondary. If we say that beauty of a work of art is to be understood in terms of its capacity to cause pleasure in a subject, then the 'meaning' becomes secondary. One may justifiably question the claim that some objects of art do indeed cause pleasure purely in virtue of the piece of art. As we saw in the development of Mothersill's thesis, the capacity to experience pleasure is itself dependant on a critical reflection that involves the role of a critic in an essential way. The critic is assumed to be able to, on critical reflection through apprehension, through bracketing all other 'interested' sources of pleasure, suggest certain aesthetic properties that will help a subject understand the aesthetic quality of a piece of art.

It might be thought that such a critical reflection is really not necessary for natural objects that we (or some of us, at least) call beautiful. The serious problems which arise in the case of art do not seem to occur in our appreciation of nature. It may also seem that natural objects do not exhibit the problem of historicity or immersion in specific traditions that create the multiplicity of artistic expressions. If we keep aside the issue of a divine creator of these beautiful objects, we tend to view natural forms and beauty in terms of certain principles or laws of formation. But if we also accept the argument that there are no principles or laws of taste, then the question of beauty in natural objects is similar to that of artistic ones. In particular, the role of the critic, in this critical reflection on natural objects, is played not only by art critics but also by scientists – a point to be discussed in more detail soon.

Kant privileges natural beauty over artistic beauty. Ferry argues that this view of Kant should not be dismissed lightly on the grounds that many of us believe in the 'superiority' of art over nature. For Kant, 'nature was beautiful when it simultaneously looked like art; and art can only be called beautiful when we are conscious it is art, yet it nevertheless looks like nature to us' (Ferry 1993, 126). Further, the role of genius, for Kant, is to 'recognize the *work of nature*' in a product of art. The artist as genius must be unconscious of the rules that created her work of art. Appreciation of such a piece of art, one that is 'disinterested' and therefore natural, places the aesthetic response to this artwork as similar to the response to natural beauty. Thus, Ferry concludes, 'artistic Beautiful thus turns out to be, in man, the exact analogue of the natural Beautiful' (ibid., 127). This does not mean that there is no distinction between fine art and nature, a distinction also granted by Kant. It is only that art must have a naturalness – understood as not being contrived. This is best captured by this claim of Kant: 'A beauty of nature is a *beautiful thing*; beauty of art is a *beautiful representation* of a thing' (Crowther 1993, 65).

Hegel's opposite position in relation to Kant is well known. Hegel holds that 'artistic beauty stands *higher* than nature' (Mothersill 1984, 384). The privilege in Hegel's view is clearly that of the mind which creates art. Beauty as such firstly 'belongs' to the mind. He goes to the extreme and says that 'even a silly fancy' in our minds is 'higher' than anything of nature. But, as Mothersill notes, Hegel's characterisation of beauty is also one that natural objects satisfy. Based on her formulation of beauty as dependent on the recognition of indistinguishability, the question of whether one can distinguish between natural and artistic objects then offers a reasonable argument against Hegel's view. (At the other extreme is Kant who claimed that the beauty of a nightingale's song is lost if we come to know that it is a human who is imitating that song!)

As I remarked earlier, one of the role of scientists is to function as a 'critic' of natural objects in that they point out certain features of natural objects which are then used to articulate the notion of beauty in such objects. Symmetry is one such term that is commonly used by scientists to describe the beauty of certain objects. But it is not immediately clear as to whether the notion of beauty in art is the same as that referred to by the scientists. It is also the case that beauty in science is not restricted to certain expressions of it in reference to natural objects; scientists routinely talk of the beauty of theories, experiments and so on. Since symmetry is central to the idea of beauty in science, I will now consider the various notions of beauty that is possible in science, both as discourse and praxis.

8. BEAUTY IN SCIENCE

The possibility of shared, objective traits of subjective experiences, as manifested in the case of beauty in art, is also illustrated in the articulation of beauty in science. First of all, note that science is a human activity and as such is essentially a subjective one. Scientists are the ones who do experiments, write theories, interpret these, and discover the principles and laws of nature. But these subjective activities are somehow taken into the objective plane. The crucial difference between science and art, in this context, is that scientific articulations are not feelings of individuals. Scientific expressions, when believed by a subject to be correct, that is, in conformity with the established traditions of science, do demand an acquiescence of others, similar to Kant's view that one's awareness of beauty necessitates a demand on others to recognise the same. The crucial difference between the experience of a feeling like beauty and an experience of scientific insight lies in the way we understand these judgements. We noted earlier Ferry's point about two kinds of subjectivity – Cartesianism, which places the subjective in the realm of reason and Pascalian, which places it in the realm of sensibilities. Scientific subjectivity is taken into the orbit of objectivity through this placement in human reason. Here, it is pertinent to remember Kant's distinction between determinate and reflective judgement that offers a distinction between judgement of science and judgement of tastes.

While these distinctions help to explain the difference between the subjective activities of science and art, it would be wrong to claim that scientific activity does not manifest subjective experiences of feelings. It would also be too drastic to jettison the ways by which we understand beauty in order to claim that *every* scientific experience is potentially objectifiable. Beauty, in science as in art, is a cause for a feeling of pleasure. In fact, science manifests a continued engagement with beauty on similar terms. Also, the observation that beauty has not become an integral part of scientific methodology suggests that beauty in science continues to reflect the problems afflicting the thematisation of beauty in aesthetics. Thus, we may confidently claim that beauty is indeed accepted as a scientific experience (and sometimes even as an ideal) but since it remains on the level of feeling, the difficulty of objectifying it in a manner suitable to scientists has led to a refusal to acknowledge aesthetic considerations as an element of scientific methodology.

In spite of this refusal, the idea of beauty plays an important role in science and occurs in the context of experiments, theories and discourse in all its many disciplines. But what can it mean to somehow talk of beauty in science, as if it is something special and distinct from arts? Let us start with natural objects. The beauty of some of these objects is a source of beauty for scientists and artists. The scientist may describe natural objects differently from artists but at the level of experiencing a sense of pleasure there can be no fruitful distinction. In the case of works of science, like works of art, appreciation and a feeling of pleasure 'caused' by *some* works may be specific to scientists just as *some* works of art are better appreciated by artists. For example, if a theory is seen as beautiful by some scientists, we cannot expect an artist or layperson who has no idea of how to 'read' the theory to experience the pleasure which it may inspire in a scientist. The flip side of this is not that clear, since when we talk of the experience of a painting or music, we do not expect that only the community of painters and musicians will experience their beauty. But before we accept that this is the case, we need to consider the role of a critic, as discussed earlier. As was noted there, appreciation of an art work is enhanced by a critic's ability to articulate, say, some aesthetic properties present in it, arising through critical reflection. Thus, while a scientist may have no idea of cubism and art history, a critic's work may help her to understand and/or experience Picasso in a more informed manner. I think one can take inaccessible scientific theories (say those which are mathematical) and open it for appreciation and reflective experience by non-scientists through the role of a critic. It would not be unimaginable to consider that a layperson will have an aesthetic response to Einstein's theory once its 'aesthetic properties' are articulated. The growing popularity of popular literature in science illustrates this amply. But there is a further role that scientists play – as critics themselves. As critics of the art works of nature. Scientific activity is a critical reflection on the represent...ion of natural objects and processes. This observation brings scientific theorising and theories of art closer to each other.

Just as much as an art critic helps in delineating the aesthetic properties of a work of art, the scientist, whose function is essentially similar to an art critic, delineates the properties of natural objects and phenomena. The scientist may not call the properties that she isolates in her role as a critic as being 'aesthetic'. But this view by the scientists, when held, is not really important if the function of the critic is to allow other subjects a way to experience an object, natural or artistic, in an aesthetic manner.

While these arguments bring the consideration of natural objects close to that of works of art, it is also the case that scientists do routinely talk of some works of science as being beautiful. Many times aesthetic considerations play a significant role in the acceptance of certain theories. Scientists also place a premium on a view that is quite popular among them, namely, the relation between beauty and truth. I will discuss these and related issues in more detail soon but first some examples of what scientists consider as being beautiful.

. collection of essays on the aesthetic aspects of science in various disciplines illustrates the inherence of the idea of beauty in scientific activity (Tauber 1996). I will not discuss the content of the book in detail but merely use it to point out that the notion of beauty and aesthetics is present in many important theories. Kohn argues that Darwin's theory of evolution had profound aesthetic influences. Darwin's 'aesthetic-emotional ambition', which was awakened on his *Beagle* voyage, was 'later transformed into high scientific theory' (ibid., 13). Darwin's two influential metaphors of 'wedging' and 'entangled bank' were central to his *Origin of Species*. Kohn argues that the 'tension between the *sublime* and the *beautiful* which 'later became *the* critical Darwinian theme' was reconciled in his two metaphors. In biology, the discipline of embryology illustrates a continuing aesthetic in its discourse. Gilbert and Faber (ibid., 129) point out that the 'visual aesthetic of embryology puts a premium on emergent form and finds expression in its focus on symmetry, order, pattern repetition, and elegance (visual simplicity).' Two other examples from the life sciences are given in this book. One is the 'aesthetic' analysis of an experiment on the replication of DNA by Meselson-Stahl which has been considered as one of the most 'beautiful' experiments in biology (ibid., 83). The other is the role of aesthetics of form in molecular biology. In this essay, Sarkar (ibid., 153) argues that aesthetic principles related to formalism played an important role in physics and in biology, in particular, in the coding of genes.

In the case of physics, Chevalley (ibid., 242) points out that Heisenberg believed that 'physics is like art.' Heisenberg's argued that different conceptual systems in physics, namely, Newtonian, thermodynamics, relativity and quantum theory, are actually like different 'styles' of art. There is also a suggestion that the overthrow of Ptolemy's theory by the Copernican one was influenced by aesthetic factors (ibid., 169). Yet another example from physics in this book is the use of aesthetic factors in the visualisation of digital image processing in astronomy (ibid., 103). In the case of theoretical physics, the importance given to aesthetics in theories by people like Weyl, Dirac and Chandrasekhar are well known. In the context of symmetry, Weyl and Wigner, for example, placed a premium on its related aesthetic factors. As Root-Bernstein (ibid., 61) notes, 'scientific aesthetic must be the same as artistic aesthetic.' He gives the example of Weyl who chose beauty as the primary criterion for a theory even 'when the facts refused to cooperate' (ibid., 62). Dirac's quote is also often mentioned in this context: 'It is more important to have beauty in one's equations that to have them fit experiments' (ibid., 62). The physicist Weisskopf says, 'what is beautiful in science is the same thing that's beautiful in Beethoven' (ibid., 62). In the case of chemistry, Root-Bernstein (ibid., 58) quotes the chemist Woodward: 'Much as I *think* about chemistry, it would not exist for me without these physical, visual, tangible, sensuous things.' (The things referred to here are crystals, odours, colours and so on.)

Mathematicians have consistently preferred (though not always articulated) aesthetic components in their formulations. G.H. Hardy is a paradigm example of one who privileges beauty: 'Beauty is the first test: there is not a permanent place in the world for ugly mathematics' (ibid., 62). Seymour Papert believes that the emphasis on the logical part of mathematics as against its aesthetic value leads to a failure 'to recognise the resonances between mathematics and the total human being which are responsible for mathematical pleasure and beauty' (ibid., 64). Looking at aesthetics in science from a Kantian perspective, Chernyak and Kazhdan claim that 'mathematics is aesthetic by its very nature … *mathematics is poetry*' (ibid., 221 – 222).

There is one point to be noted here. The nature of what is aesthetical and beautiful, on the level of subjective feeling, is similar in science and arts. But the role of aesthetics in science is somewhat limited compared to arts. For example, Root-Bernstein (ibid., 62) says that 'aesthetics in sciences, as in the arts, are based upon concepts of beauty, harmony and pattern. When simplicity, coherence and understanding replace confusion then beauty and truth emerge hand in hand.' Obviously, there is an undue weightage given, in this view, to structure, harmony, simplicity, balance and so on. It is not an accident that these are also the elements which are most often seen in conjunction with symmetry.

While contemporary theories of art may scoff at this excessive preoccupation with symmetry and related terms, the aesthetic in science will continue to privilege them. It may be thought that complexity and fractal theories, for example, may go beyond the aesthetics of symmetry but it is not really so.[11] Science does not a develop a theory of aesthetics but only works with what it thinks are its aesthetical features. The suggested equivalence between truth and beauty, so ingrained in science, will necessarily make beauty an objective idea. Also, the concept of beauty in mathematics is deeply implicated in 'formalism'. It is indeed the case that formalism in arts is reflected in the aesthetic understanding of science. The many views of famous scientists and mathematicians given above should be seen, once again, as attempts by critics to articulate aesthetic properties in virtue of which pleasure is experienced.

9. BEAUTY AS VALUE

I alluded earlier to Plato's view on beauty as a kind of good. The American philosopher, Santayana (1955, 31) talks about beauty as a value, 'that is, it is not a perception of a matter of act or of a relation: it is an emotion, an affection of our volitional and appreciative nature.' Beauty is a positive value, that is, 'the sense of the presence of something good' (ibid., 31). It is an intrinsic value of pleasure which is not 'in the consequence of the utility of the object or event, but in its immediate perception' (ibid., 32). For Santayana, 'all values must be ultimately intrinsic ... even the knowledge of truth ... is an aesthetic delight; for when the truth has no further practical utility, it becomes a landscape' (ibid., 19 – 20).

The relation between beauty and truth, so magnified in the articulation of some scientists and which, in general, can be taken as a general belief of the scientific community, invests beauty with a 'scientific' value. But what is the 'nature' of this value? It is clear that beauty is not an accessible term to be incorporated into experiments or theories like adding a chemical or performing a particular mathematical operation. It is not a value in the sense that it is not a given term that can be used in scientific methodology. Beauty arises and is recognised after the creation, after the process has been done. This is, of course, similar to the creation of a painting which, after it has been painted, we call beautiful. The artist has no particular recipe that will manifest beauty. So when we talk of the scientists' rapturous remarks on the significance of beauty, we do realise that it is a judgement of an experiment, theory, equation or whatever. When we call a painting beautiful, it is because we experience pleasure at seeing this beautiful object. And that is the end of our experience. We do not ask if the beautiful painting is true. We may say that this painting captures an insight or truth about something, but that is incidental to its invoking pleasure in us. And if we look at a painting to get an insight, then the pleasure we get from the painting is no longer 'disinterested' and hence this pleasure is disqualified from being pleasure caused by beauty. Thus, if we accept the larger wisdom (from Kant to Santayana to Mothersill) that beauty should be a pleasure that arises from disinterest, then the connection to truth and beauty can only be incidental and secondary.

The case of beauty in science seems to be caught in an activity that is purposive, oriented towards something else. To understand this, first consider what is conceivably an object of beauty for science. Consider natural objects. Say we have in front of us a beautiful insect with many colours and complex patterns. Let us say some scientists and some non-scientists consider this insect as being beautiful. When scientists talk of beauty *in* science, they are not talking about the beauty of this insect as we see it. Let us say a biologist develops a theory about the colour and shape of a pattern on the insect. Now this theory is a candidate for being beautiful – at least, beautiful for those who can understand that theory. That is, science borrows the notion of beautiful, as holding for natural (and artistic) objects, into the consideration of scientific objects. This is similar to what the artists do. The artistic object, say a painting, becomes an object about which a judgement can be made. But there is also a crucial difference. Since every scientific 'object', say experiments or theories under aesthetic consideration, has perforce to relate to the world and conceivably capture some truth about the world, they are answerable to something more than aesthetics – Weyl's strong belief aside. This relationship turns Hegel on his head. Remember Hegel's comment that 'even a silly fancy' of our mind is 'higher' than anything in nature. But for science, even for the most aesthetic of its theories, a product of a creative mind, which is definitely much more than a passing fancy, may come to nothing in the face of the judgement of nature on its truth value.

This is all the more obvious when we consider what science does with its most beautiful 'objects', say Einstein's equation of general relativity.[12] A beautiful theory is then taken up for the creation of more theories and experiments, some of which may also be seen as beautiful (but rarely is this the case). This approach is dramatically opposite the arts where one does not take a beautiful painting and because it is beautiful add other modifications to it. This seems to suggest that for arts beauty is the end whereas for science beauty is the beginning.

But there is much in common to the aesthetic of art and science. In both, there are objects that are experienced as beautiful. The objects of beauty in both these theories cause pleasure. We need a critical reflection on these objects to have an apprehension without motivated desires for pleasure. In the final analysis, aesthetic value in science can never completely reject its relation to the natural world. But keeping this point aside, we can still ask what aesthetic factors are generally involved when we talk of the aesthetic value of scientific objects. In answering this we will find that the aesthetic properties of scientific objects are very similar to those of natural objects, at least those which we believe contributes to the notion of beauty in natural objects. Symmetry is the best example of such a common property.

10. SYMMETRY AND BEAUTY

Symmetry has often been related to beauty, as much as it has also been seen as a source of monotony. As I have noted at various points earlier, symmetry is usually linked with a host of terms like balance, simplicity, harmony, beauty and so on. Natural objects, which are often seen as beautiful, invariably possess various types of symmetries. In the previous section, I concluded by pointing out that aesthetic factors which apparently play a role in recognition of natural objects as beautiful are also those that are used in the context of beauty in science. Harmony, pattern, balance, coherence, simplicity are all concepts that are implicated in the term 'symmetry'. Hardy says this of mathematical beauty: 'ideas like the colours or words must fit together in a harmonious way' (Tauber 1996, 62). Davis and Hersch in their influential book *The Mathematical Experience* suggest that the beautiful in mathematics exhibit 'harmony, balance, contrast etc' (ibid., 64). In general, given the primal scientific tendency to gather objects and events into general principles and laws is itself a move that emphasises the importance of balance and harmony. As I had argued in the beginning of this chapter, these concepts are deeply implicated in the phenomenological experience of symmetry.

Artists are ambivalent about symmetry. While Arnheim points to the importance of the notion of visual balance and harmony in works of art, postmodern art, in particular, has consciously tried to create works that break symmetry and are asymmetrical or created in forms where symmetry or asymmetry has no relevance. In the conclusion of this section, I will argue that broken symmetries, asymmetries, 'non-symmetries', in the case of art and in general phenomenological experiences, are terms that can have meaning only in terms of symmetry. Even in the case of natural symmetries, the notion of beauty as related to them is quite ambiguous. Kant notes that geometrically regular figures cannot be seen as aesthetically beautiful because they are based on concepts. He grants that we are pleased by such figures but such a pleasure is not disinterested but suggests uses. He also claims that 'stiff regularity (such as borders on mathematical regularity) is inherently repugnant to tastes, in that contemplation of it affords us no lasting entertainment' (Mothersill 1984, 128). The point about 'no lasting entertainment' is similar to the one about monotony, as in symmetric objects and figures being monotonous. In contrast, Plato considered 'simple shapes and figures' to be exemplars of beauty. Kant, while he condemns symmetry as boring, places great emphasis on beauty associated with forms. In any discussion of symmetry, as we have seen earlier, we are never far away from the pulls of form and formalism. Mothersill acknowledges that the 'manifestation of explicit symmetry is *aesthetically* pleasing' (ibid., 127). And the symmetries present in the sunflower and the seashell 'impresses with that 'purposiveness without purpose' which Kant finds essential to all natural beauty' (ibid., 127).

In Part Two, I discussed in detail the relation of symmetry to form and argued that symmetry cannot be seen as secondary to form. When we consider the aesthetics of symmetry, we are continuously brought to consider the relation of symmetry with form. Pure aesthetic judgements, for Kant, arise from the relation of 'parts and whole in phenomenal configurations' (Crowther 1993, 59). Kant considers flowers, birds etc. as objects that are 'free beauties' – they please not for any reason other than their presentation. He also considers designs as free beauties since they 'have no intrinsic meaning' (Mothersill 1984, 224). Furthermore, he says, 'in the estimate of a free beauty (according to mere form) we have the pure judgement of taste' (ibid., 225). His emphasis on design includes fine arts and he claims that the basic prerequisite for taste is 'what pleases by its form.' (However, Crowther (1993, 71) notes that Kant goes beyond formalism in saying that an artwork is always in relation to rules and standards of other works.)

Once form is privileged in this manner, it is hard to imagine why symmetry, at least 'perfect' symmetry, is banished to the other extreme. Symmetries are complex. They are more than that associated with circles and triangles. Just because one is 'bored' with circles does not mean symmetry is boring. Nor is it clear just *what* it is with a circle that causes an experience of boredom in us. Even in art, formalism has been influential. Cezanne, for example, reduced objects to their geometrical forms in his paintings. The formalists believe that art is essentially 'concerned with displaying form' (Carroll 1999, 113). An artwork is then understood as some specific relation between its part and the whole. Clive Bell's influential work emphasised the centrality of 'significant form' as that which has the 'capacity to arouse aesthetic emotion' (Crowther 1993, 57).

Santayana (1955) considers in some detail the relation of form, beauty and symmetry. Form is 'a synthesis of the seen' and 'is almost a synonym of beauty' (ibid., 47). He dedicates a section in his book to form, symmetry and their relation to beauty. He begins by saying that 'the most remarkable and characteristic problem of aesthetics is that of beauty of form' (ibid., 53). Here too the beauty of form is not reducible to the collection of its parts or elements. It is only a particular configuration of these parts that are seen to be beautiful. He then tries to argue that the balance of forms that cause beauty is related to the physiology of seeing, in terms of the 'muscular balance' of the eye. But, like Kant, he views totally symmetrical figures as boring. For example, he notes that the circle may exemplify simplicity and 'purity' but it 'lacks any stimulating quality' whereas an ellipse has a 'less dull and stupefying effect' (ibid., 57). Then when he talks of symmetry and its 'charms', he begins by suggesting that 'the comfort and economy that comes from muscular balance in the eye, is therefore in some cases the source of the value of symmetry,' especially in the recognition of bilateral symmetry. And continues, '[I]n other cases symmetry appeals to us through the charm of recognition and rhythm' (ibid., 59). While a totally symmetrical object may actually be displeasing, he finds that nevertheless this is 'often the condition of the greatest of all merit, – the permanent power to please' (ibid., 59).

Symmetry offers a 'principle of individuation' – allows us to perceive the 'unity and simplicity' of objects. That is, it is symmetry which is the condition of unity. Symmetry, in helping us to individuate objects, helps us in 'enjoying' perception. But symmetry loses its value when objects are 'too small or too diffused for composition,' where it cannot gather the unity in perception. The synthesis which symmetry makes possible must be 'instantaneous'. And explicitly, he notes that the beauty of form is 'what specifically appeals to an aesthetic nature' (ibid., 61). But while he accords such a high status to form and symmetry, he comes back to note that monotony 'deadens our pleasure' in two ways: one, actually creating a painful sensation when the repetitions are 'acute' and the other making us unconscious of them. In either case, the pleasure of the monotonous, if there is pleasure, is not the pleasure of the beautiful.

For both Kant and Santayana, there is a constant ambiguity when it comes to symmetry. The latter has a more engaging view of symmetry. It is only the 'totally' (or perfect) symmetrical objects that are displeasing. Perfectly symmetrical objects are only those of geometrical figures. Natural objects do not manifest either this 'perfect' figure nor are they only the form of these figures. Shape, colour, design, texture and the background contribute to the perception of these objects. Like beauty, symmetry too is a Gestalt. And it is conceivable that this Gestalt of symmetry contributes to the sense of pleasure that is beautiful. Moreover, Santayana's views on form and symmetry are close to that of science as reflected in his statement that perfect symmetry, although displeasing, is nevertheless of the 'greatest merit'. This translates symmetry into a principle, a value that is essentially scientific in nature. For science, the symmetries of nature are the most fundamental characteristic of it. Events of nature, in general, obey important conservation laws, all of which are seen as a consequence of some symmetries in nature.

However, the phenomenology of symmetry, as in perception, in its relation to beauty, *cannot* be the definition of symmetry as in science. Remember that symmetry in science is defined to be invariant transformations. As I had argued earlier, it is quite misleading to consider symmetry as some transformation or change. Symmetry is a property of the object which is made 'visible' by transformations that are invariant. Phenomenologically, our response to symmetry – for example, the 'instantaneous' perception of unity of an object, the recognition of balance and harmony among parts of a whole, our experience of pleasure in perceiving such an object – cannot and does not arise from making transformation and noting some invariance. When we look at a beautiful object in nature that is symmetrical, our emotional response to it is not based on whether this object or its 'oriented form' (see Part Two) is invariant under some transformations. Therefore, symmetry as described in science remains on the order of a scientific description, one which allows a way of formulating a property of the object. This is very similar to mathematical definitions of shapes in terms of the language of topology. Our experience of shapes is not in these terms. They are, both in the case of topology and group theory, linguistic constructions used to describe particular properties, those which are also amenable to phenomenological experience. My earlier argument that the structure of groups shows strong similarities with the Gestalt principles of vision is one translation that tries to exhibit the possible link between the descriptive and the phenomenological in the case of symmetry.

We can also understand this distinction in terms of determinate and reflective judgements. We have to note that symmetry as used in science is more encompassing, as we can expect from generalisations that characterise formation of principles. There are symmetries of events, symmetries corresponding to particular models, those arising from the form of the mathematical equations and so on.

Now we can attempt to understand the aesthetics of symmetry. I will begin with the ambivalence engendered by perfect symmetrical figures. These are claimed to be boring, do not cause pleasure in us, and sometimes, as Santayana says, may even be painful. What sense could we possibly ascribe to the terms 'perfect' or 'total' in the case of perfectly symmetrical objects? 'Perfect' says something about the symmetry but in the case of phenomenological experience of symmetry, what is it that allows for a gradation? In the case of scientific description, this problem does not arise because the symmetry of a figure is completely characterised and imperfect symmetries lack one or the other of the total set of symmetrical elements. Can we claim that a phenomenological experience of total symmetry is given by a 'negative' experience, such as being bored or even feeling pain? Can we say that if an object is painful or boring, then it is perfectly symmetrical? Obviously not, because many 'objects' of experience can be boring and painful but they will have nothing to do with symmetry at all. So what allows us to recognise certain feelings of boredom or pain to be associated with perfect symmetry?

This is indeed baffling. It is like saying a red flower is beautiful but if it has a particular richness of red, say a 'perfect' red, then it becoming boring or painful. This suggests that the addition of whatever it is that makes a quality perfect somehow negates the pleasure which the imperfect was capable of causing. This then should be seen as a classic instance of an 'anti-aesthetic' property which when 'added' to the aesthetic object destroys its aesthetic pleasure. It also suggests that this notion of perfect is a principle of taste, or 'non-taste' if you like. This surely goes against what we understand by beauty. For whatever else beauty is, we may at least all agree that there is no recipe for it. But then perhaps there is a recipe for ugliness? Maybe one can argue that any change, in general, to a beautiful object will upset its balance and cause it to lose its beauty? I think it can and most often is the case. So, we may have a symmetric object which is seen as being beautiful but once it attains perfect symmetry (say through some intervention or evolution) then it becomes boring and ugly. The argument about destroying beauty (quite easily, I must add) suggests that beauty is essentially unstable and even a small change can cause it to topple over into the lap of the ugly. Ugly seems to be what is stable, for changes, most often, to an ugly object will continue to keep it so. That is, although it seems easy to convert beauty to ugly, the converse is not the case. Beauty is fragile and the ugly sturdy. In the case of symmetry, it is easy to make perfect symmetry out of a given symmetry and also easy to destroy perfect symmetry. This once more suggests that the ascription of monotony to perfect symmetry is not a 'disinterested' response.

In the case of symmetry and perfect symmetry, we cannot use the beauty-ugly divide. A symmetry is always a part of its total symmetry. But there is also something intuitive which is captured in the disdain of perfect symmetry. We can understand this by pointing to one function of perfect symmetry, which is that given a part or a minimal set of parts we can envision the whole. Given a small arc of a circle, we can conceivably imagine what the circle will look like or even attempt to complete the circle. Such a process whereby 'more' is generated from minimal inputs is perhaps physiologically and cognitively preferred. (Note Santayana's reference to muscular balance and Sober's (1975) work on simplicity.) But aesthetically such a perfect symmetry is not pleasing. Why? Is it just the vanity of our minds whose imagination is insulted in having been given such a trivial task? Is the boredom and pain that seemingly arises from perceiving total symmetry a consequence of the mind being slighted, of the subject being irritated that when the mind is capable of grasping complexity why waste its energies on obvious simplicities? But if so, then the feeling of boredom or pain that is caused by perfect symmetry is not one that is 'disinterested'. If we learn to look again at a perfect symmetry, casting aside all our interests, who is to say we won't find perfect symmetries 'as' beautiful as non-perfect ones?

On the other hand, we can also explain the negative experience of perfect symmetry as follows. Our experiencing perfect symmetry leads to the recognition that the form is grasped entirely through minimal input/information. This recognition and deduction takes us away from considering the object entirely in terms of a reflective judgement. To put it simply, when we see perfect symmetry, we start thinking and reasoning, and stop feeling. It is because of this that our aesthetic judgement gets clouded. Thus, what causes displeasure or boredom is not the perfect symmetry but *our inability*, in the face of perfect symmetry, to maintain our emotional poise.

Lorand (1994) argues that while beauty is privileged in aesthetics there are many concepts that are opposite to beauty whose analysis offers a better idea of beauty itself. She lists these opposites as 'the ugly, the meaningless, the kitsch, the boring, the insignificant, and the irrelevant' (ibid., 405). She bases her argument that these are the opposites of beauty on a view of beauty which is seen as possessing 'a high degree of inner order' but an order which does not imply any principles or laws. But high 'degree' of order is intrinsically related to symmetry. Her notion of order is also confusing. For it seems that the beauty of an object lies in its being well organised, whose elements are in their 'right' places and ordering in such a manner unifies objects. This she suggests is 'aesthetic order' in contrast to 'rational order' which 'allows the observer to complete a fragmented piece since the governing laws are available' (ibid., 402). She gives the example of arithmetic progression to illustrate rational order. But is completion always in the presence of given laws? What is the law that suggests that when we see a part of a circle we imagine the whole? This could be the Gestalt 'laws' of perception but these are not 'laws' of taste and do not apply to aesthetics. Our point of contention arises because Lorand thinks that 'in the aesthetic domain the possibility of deducing missing parts based on known patterns indicates deficiency' and what gives aesthetic value is a mix of 'order and novelty'. Perhaps the general response to perfect symmetry is based on a vague belief that it have something to do with rational order in some sense. Lorand also notes that the boring is in opposition to beauty, mainly because of repetition of patterns and the inability to present us with a new view of patterns. So if we accept opposites of beauty as really giving us a degree of beauty then even mistaken claims about the ugliness or the boringness of perfect symmetry should be seen along such a gradation.

At this point, we may pause and ask whether we have any idea of what symmetry is, in the way it has been used (however sparingly) in aesthetics. Santayana's views, while helpful in a limited way, are incomplete. It seems to be clear that, generally speaking, symmetry is a property of objects like colour and shape which has some role to play in aesthetic perception. But we have to yet establish that it is or is not something more than shape and colour. In Part Two, I argued that symmetry is not secondary to form. Is it possible to make a similar claim on phenomenological grounds? Santayana says that symmetry allows us to perceive the unity of an object. But how is it that we perceive unities even without symmetry? From the critique of the aesthetic value of perfect symmetry, we can say that perfect symmetry (and in general, symmetry) incites in us the tendency to extrapolate, to complete a figure without needing the full figure for continued interrogation; gives us the ability to take a figure into mind's eye for there is nothing more the figure can yield to my gaze. These are a range of phenomenological experiences of symmetry. Harmony and balance are also similar experiences related to symmetry, as discussed earlier. So we can reasonably say that there are a plethora of phenomenological experiences that have to do with what we call as balance, harmony, extrapolation, simplicity and so on, and these constitute the experiences of symmetry. If we want to use the notion of property, then harmony, balance, simplicity etc. are all properties of something characteristic of the object, namely, symmetry.

Can this assertion be right? Is there really a property of objects called symmetry that can be accessed phenomenologically? If so, is it something more than its form, even in the context of phenomenology? We experience colour and generally say the colour is a property of the object we see. (Ignore here the issue of whether it is a primary or secondary quality.) When we see an object we see its form, maybe designs on the surface and so on. When we see a circle, we only see a circle. We do not see its symmetry. But we may have phenomenological experience of balance both in sight and in the tactile experience of balancing a circular disc. We may perhaps say that symmetry is another name for balance and it is a particular characteristic of all objects to have or not have or imperfectly have the notion of balance that can be experienced in some way or the other. This attempt of understanding symmetry as a synonym is one possible way. We may then claim that harmony, balance, simplicity etc. can all be called symmetry. But this cannot be correct for each of these terms has certain unique connotations that are lost in equating each of them with 'symmetry'. Not all balanced figures or objects need be simple, for example. Perhaps then we can say that symmetry is nothing but the collection of properties such as balance and simplicity but this also seems too contrived. What brings all these experiences together into one called symmetry? One way of responding would be to say that all objects which are seen to embody virtues such as balance, harmony and simplicity are somehow or the other symmetric. But why should we not stick to just these terms and not invoke symmetry at all?

One answer, and I believe this is the right way to approach it, is to say that symmetry, as far as phenomenological experiences go, can be dispensed with and replaced by the 'real' experiences of terms like balance but as far as the aesthetic properties go, it is the phenomenological ones that are replaced with that of symmetry. *This means that symmetry is primarily a potential candidate for being an aesthetic property.* Therefore, when we consider beauty, we are first drawn to talk of symmetry. There may be problems in this view but before I address them, let me consider another possibility of considering symmetry as a 'content' of the form.

In the case of science, symmetry can be given an explicit formulation, unlike in ordinary perception and art where there seem to be a plethora of other terms for it. In the scientific formulation of symmetry, in terms of groups, one can understand, if need be, terms like balance and harmony as descriptions arising from the prior framework of symmetry. In art and perception, it is the exact reverse, where reference to symmetry is filled with ambiguity. Symmetry in art has nothing to do with invariant transformations. It has to do with the way we perceive objects, both natural and artistic. But what I have argued so far is that the terms which stand for symmetry are actually terms that *refer* to symmetry, in a particular sense. Terms like balance and harmony capture some phenomenological experience and there seems to be no direct phenomenology of symmetry other than through these synonymous terms. Then why talk of symmetry at all?

To answer this, we have to understand in what sense these terms refer to symmetry, thereby explaining why we continue to use the word 'symmetry' as if it is something more than the experience of terms like balance and harmony. First, let me consider the relation of symmetry to form in perception and art. Given a figure, we recognise its form. Let us say we have an equilateral triangle. We cannot, in all honesty, speak of the symmetry of this object in any certain manner. It seems as if there is no artistic or perceptual concept that will allow us to talk of symmetry in this case. We can then turn to formalist theories in art. Maybe we can talk in terms of 'significant form' but as has been noted, for example by Crowther and Mothersill, this is really a circular definition. While Bell claims 'significant form' captures the aesthetic of art, we are not told what it is and how to find it given a work of art. Neoformalist theories that use both form and content may be better candidates to understand symmetry. The neoformalists would say that not only is there a form in an artwork but also content, and moreover, the form and the content 'are related to each other in a satisfying appropriate manner' (Carroll

1999, 126). One of the ways of understanding content is to look upon it as the meaning of the artwork and form as 'the mode of presentation of the meaning' (ibid., 127). If we ask what distinguishes art from anything else which have form and content, the neoformalists would say that in art, the form and content are related in a 'satisfying appropriate manner'. Of course, what is satisfying and appropriate will be a matter of discussion.

While the above points seem to be specific to artworks, they are obviously not restricted to them. Any natural object seen and responded to on the order of art will also have to incorporate these arguments about form and content. It may seem that the idea of form discussed so far is restricted to form as in shapes. But, in general, in the consideration of what constitutes artistic form for any artwork (say a piece of music), one can define artistic form as 'all the webs of relations' that can be found between all the elements that constitute an artwork. Then we can conceivably list all such relations, with each relation being an artistic form, in order to comprehensively describe one artwork. This is called the 'descriptive account'. But the descriptive account is not sensitive to the explanatory role of artistic form. As Carroll (ibid., 142) notes, 'our ordinary concept of artistic form seems to be functional.' Thus, we may define the artistic form, in the functional account, as being 'the ensemble of choices intended to realize the point or purpose of the artwork' (ibid., 143). In contrast to the descriptive account, the functional element considers only some, not all, elements and relations, which are essential to realising the purpose of the artwork. In the functionalist view, form serves the function of realising the purpose.

With these brief comments, let me consider how we can understand symmetry in both natural and art works. When we talk of symmetry of an artwork, what could we possibly mean? If artistic form is formulated in terms of formal relations (whether all or only some of them), then symmetry could perhaps be thought of in terms of formal symmetric relations. In general, this cannot be the case, for consider two elements A and B and the formal relation, A is to the left of B. This is not a symmetric relation and putting B to the left of A may change the form altogether. We might perhaps ask whether symmetry has nothing to do with form per se but is on the order of meaning, that is, symmetry is the 'meaning' that certain forms convey. But what is this meaning? In the case of art, perhaps we can isolate a theme and call that the meaning of the artwork. Again, it is difficult to see what this meaning could be other than some terms referring to the ideas of harmony, balance, simplicity etc., in which case, we come back to our earlier question: why re-name these terms as symmetry? We could look at the functional view of art – the form following the intended function of a piece of art. Here it is clear that if there is a notion of symmetry that we are searching for then it cannot be in the form unless it is somehow already factored into the function. However, given that we do not know clearly what symmetry is, we cannot conceivably create an artwork whose function is to communicate symmetry.

Here there is a difference between natural objects and artworks. It can be argued that natural objects that exhibit symmetry (in the scientific sense) come to have these symmetries due to natural 'forces' (as described in Part One). In which case, the symmetries of these objects express something about the evolution of that object and the physical principles of natural forces and their effects. If we see the patterns on a snail shell and causally relate it to some evolutionary and natural forces, then the symmetry of the shell holds some meaning about certain dynamics of nature. Now if I look upon the shell as I would an artwork, then I will have to accept that the function of the maker (say, nature) is well captured by the form (in this case, the shape). But even in this case, the perception of the shell says nothing about an idea of symmetry which is already not present in terms like balance and harmony.

So, phenomenologically, where is symmetry to be found?

The answer is actually quite simple. The idea of symmetry seems to be an important element of art because it is a part of the aesthetic experience. So the answer to the above question is simply: *In the aesthetic experience.*

When we perceive a work of nature or of art and call it symmetric, we are making an aesthetic judgement. When we experience pleasure in perceiving such an object, we are responding to symmetry as an aesthetic property. This pleasure, in the case of symmetry, is disinterested and therefore, is the cause of the feeling of beauty that it inspires in us. To clarify these points further, let me briefly describe what we mean by aesthetic experience and aesthetic property.

Our appreciation of natural and artistic objects is an aesthetic experience. Carroll notes that there are two important ways of understanding this experience, the content-oriented and affect-oriented accounts. On the former, 'attending to the unity, diversity and/or intensity of a work (or of its parts) amounts to an aesthetic experience of the work.' (ibid., 168). If a work has these features, then it can afford aesthetic experience. Beardsley who offered these categories of unity, diversity (or complexity) and intensity is committed to accepting principles of taste. The affect-oriented account says that 'aesthetic properties are what aesthetic experiences are experiences of' (ibid., 170). As we have already seen in our discussion of beauty, what is important in aesthetic experience is disinterested attention. Carroll notes that this account, because it explicitly incorporates intentionality of production, discounts natural objects as being seen as artworks. But then, if natural objects generate aesthetic experience through disinterested attention, then this exclusion is artificial. We can well proceed without having to call natural objects as artworks. But note that in either approach, the central terms used in each of the two formulations are closely linked with symmetry – unity, complexity, intensity, gestalt etc. are all elements that are somehow 'associated' with symmetry.

Earlier, I had discussed Mothersill's definition of beauty in terms of aesthetic properties. An object is experienced as beautiful in virtue of these properties. These are not properties like shape, colour and so on. Aesthetic properties are Gestalt properties (Mothersill 1984, 365). These properties, for example in music, are 'disclosed in performance' (ibid., 366). While it is the case that aesthetic experience is subjective, aesthetic properties are objective properties.[13] They are to be 'detected' in an aesthetic object.

Symmetry is an aesthetic property. It is not reducible to balance, simplicity and so on. When Kant talks about the utility aspect of perfect figures, he cannot be referring to its symmetry. It is balance, for example, that is of possible use value. So also for simplicity. And we cannot *equate* symmetry with either of these terms. Symmetry is what *is* in both; it captures a phenomenological similarity in all the terms like unity, diversity, harmony, balance and intensity. Symmetry is disinterested, in the sense that such an attention causes pleasure. We never 'see' the symmetry of an object and even in phenomenological experience it is not clear how it is accessed. *It arises only in aesthetic experience.* Symmetry is Gestalt just as an aesthetic property should be. When we refer to symmetrical figures we are talking of particular properties such as particular form, pattern or colour distribution but these can only be considered symmetrical in the scientific view of symmetry.

(Even the scientific view of symmetry as invariant transformations can be seen as an expression of an 'aesthetic' experience. If aesthetic properties, as in music, are disclosed through performance, then we can reasonably expect that symmetry as an aesthetic property is disclosed through transformations.)

Aesthetic properties are 'properties of an individual disclosed to us only through acquaintance with that individual' (ibid., 352). Symmetry, seen as an aesthetic property, is not restricted to symmetry of shapes. It is a property in virtue of which we have an aesthetic experience; a virtue which may cause the experience of beauty. This does not imply that we are talking of specific symmetries. There is nothing like specific symmetries for phenomenology. This phrase is a carry over from the scientific formulation of symmetry. Even our experience with the mirror is not an experience of reflection symmetry. Reflection gets the status of symmetry only under a specific formulation. Our pleasure that arises in perceiving reflection (and not the reflected object) is a pleasure that is caused by an aesthetic property which we may call symmetry.

In taking this position we are not committed to principles of taste. We are not claiming that symmetry is always a source of pleasure related to beauty. This confusion may arise if we give in to seeing symmetry or beauty in terms of shapes. Obviously, a symmetrical figure, beautiful in being part of one object, may not be in another object. Symmetry as an aesthetic property is that which explains the common aesthetic element in the various 'properties' such as unity, intensity, balance, design and so on. So when I say that symmetry is a source of pleasure, I do not mean that this symmetry is that which is *necessarily* embodied in the geometrical figures which may be a part of the perceived object.

We can extend this point and claim that even if a symmetrical geometrical figure is an object of pleasure in the aesthetic experience, the cause of pleasure is *not* the symmetry of the object as defined scientifically. The perception of this figure need not involve any idea of transformation and invariance. However, beauty is caused by the presence of 'symmetry' in the aesthetic sense. If you ask what this symmetry is, then I can answer by saying that the very many ideas that are inspired in perceiving that object like unity, harmony etc. are all pointers to this aesthetic property of symmetry.

Symmetry need not be the only aesthetic property in an object that pleases. Mothersill (ibid., 354) points out that while aesthetic properties are 'context dependant' there can be more than one aesthetic property in each individual. Scientific and mathematical symmetries are the residues of aesthetic symmetry, when pleasure is taken into the domains of reason.

Since symmetry in aesthetics cannot correspond to a determinate concept, it satisfies Kant's criterion for reflective judgement. Since it is not utilitarian, its experience is an experience of disaffected acquaintance. However, we have to consider the gradation of beauty as in 'degrees of beauty' that is naturally associated with this view. It may seem that the view of symmetry as an aesthetic property runs counter to Mothersill's formulation which says that if two items are distinguishable, they must 'necessarily have different aesthetic properties' (ibid., 379). Even if symmetry is identified as the cause of aesthetic pleasure in distinguishable objects, it does not follow that it is the 'same' characteristic that causes pleasure. For example, in one pleasing object, our experience of symmetry may be very different from the experience of symmetry in another object. We may not be able to specify what 'exactly' we mean by symmetry in both these cases, but may still 'feel' that symmetry is the cause of pleasure in both these objects. We may even find an expression for this symmetry but it will only be one expression, one way of reading the 'formal' relations among these terms or as generating a set of meanings.

It is also the case that we can meaningfully talk of degrees of beauty. One way of dealing with this, as suggested by Mothersill, is to order pleasures 'according to their intensity, duration and 'fecundity'' (ibid., 380). A greater beauty 'affords a pleasure that is more intense than a lesser beauty ... overflows its limits and persist ... activate the creative imagination' (ibid., 380).

The confusion about symmetry and its relation to beauty, especially in the case of perfect figures, is a confusion about the order of pleasure caused by symmetry. In the case of perfect symmetry, the most favourable conclusion we can draw is that it affords lesser intense pleasure. It may not inspire the creative imagination. It may be dissipated soon after we experience it, leaving no residue of its beauty. None of these are reasons to say that perfect symmetry is not a cause of pleasure. On the other hand, philosophers and critics join hands to claim that asymmetry and broken symmetry are a source of beauty in contrast to symmetry. But, it must be remembered that the ideas of asymmetry and broken symmetry are intrinsically and necessarily tied in with the prior formulation of symmetry, both in science and art. Is it possible to have an experience of asymmetry without having first a prior experience of symmetry? Asymmetry, for phenomenology, can only arise in the recognition of the lack of symmetry and is, for example, an experience of the *deviation* of the form from its nearest symmetrical form. Nevertheless, we need not discount asymmetry's claim to 'greater' beauty all the same. We may instead say that asymmetry and broken symmetry afford pleasure that is more intense, lasting and creative. While this point can continue to be debated, one needs to *first* acknowledge symmetry as an aesthetic property.

NOTES

1. See Levin (1993) and also Levin (1997).
2. See Ihde (1998).
3. For more on this, see Houlgate (1993).
4. For more on this, see Jay (1993).
5. A more detailed discussion on this issue is available in Sarukkai (2002).
6. See Rock (1975).
7. See Steiner (1998) for a discussion on the anthropocentric concerns that drive mathematical formalism. See also Sarukkai (2002).
8. See also Matthews (1997, 16).
9. See also Matthews (1997, 8).
10. The use of intervening and representing should remind us of Hacking's (1983) argument on similar grounds for scientific realism.
11. See Field & Golubitsky (1992).
12. It has been suggested that Eddington's experiment, which is accepted as having provided the first proof of General Relativity, did not actually demonstrate conclusive proof of the theory. It was Eddington's belief in the 'beauty' of Einstein's theory with the concomitant belief that a theory with such beauty has to be true that led him to proclaim that Einstein's theory had been proved by his experiment. See Collins & Pinch (1993) for more details of this story.
13. See also Carroll (1999, 199).

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