ARTICLE No. 9.

Lunar and Solar Eclipses in Hindu Astronomy.

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Lunar and Solar eclipses played an important part in the superstitions of all the ancient nations of the world, amongst all of whom the eclipses of the sun and the moon had a terrible import, being supposed to presage dreadful events. By the common people of the Romans, as also by the Hindus, a great noise was usually set up with brazen instruments, and loud shouts during the eclipses of the moon. The Chinese, like the Hindus, supposed eclipses to be occasioned by great dragons on the point of devouring the sun and the moon, and it was thought by the ignorant, nay the common people, that the monsters, terrified by the noise of the drums and brass vessels, let go their prey. The eclipses were looked upon with such a feeling of mysterious awe and apprehension that in 2159 B.C., the Chinese Royal Astronomers, Hi and Ho, were executed for their failure to predict a solar eclipse beforehand. The sudden occurrence of an eclipse without previous prediction was supposed to be attended with a cataclysm and was recognised as an event of serious portent. In the Mahābhārata[1] we are told that when at the time of the battle between the gods and the demons (asuras) the sun became crimson (āditye lohitāyati), both the gods and the demons raised a hue and cry. There is another reference to a solar eclipse in the Mahābhārata,[2] Indra observed that the sun was rising in the east and the moon was entering the sun and the dawn was growing crimson, and further the tithi was a new moon, hence Indra thought that it predicted a terrible war between the gods and the demons which was to take place on the morrow.

The cause, however, of eclipses, notwithstanding the superstition of the people generally, was well understood by the Hindu astronomers. The Hindus were at a very early date well acquainted with these facts relating to an eclipse. They had rules for calculation of the various phases of both the lunar and solar eclipses, the times of beginning, middle and end, as set forth in their various astronomical works. Even in the Ṛg Veda which probably dates from 2,000 B.C. at the latest, we get references to an eclipse and the calculation of its duration. The fortieth hymn in the fifth mandala of the

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[1] *Mahābhārata, Ādi-parva,* chapter 19.
[2] *Mahābhārata, Vana parva,* chapter 213.

Ṛg Veda is very important in this connection.[1] It shows that an eclipse of the sun was then first observed with any pretensions to accuracy by the sage Atri. The last verse in the hymn which, after describing the eclipse, says "Atri alone knew him (the sun) and none else could." This observation of the solar eclipse is noticed in the Sāṃkhyāyana (24.3) and also in the Tāṇdya Brāhmaṇa (iv.5.2 ; 6.13), in the former of which it is said to have occurred three days previous to the viṣuvan (the autumnal equinox). The observation thus appears to have attracted considerable attention in those days. It seemed to have been a total eclipse of the sun, and the stars became visible during the time, for the expression "*bhuvanāny adīdhayuḥ*" in verse 5 of the fortieth hymn in the fifth mandala is interpreted in that way.[2] There are several references to the solar eclipse in the Brāhmaṇas. In the Tāṇdya Brāhmaṇa of the Soma Veda there are references to an eclipse in five places (iv. 5. 2 ; iv. 6. 13 ; vi. 6. 8 ; XII, 11. 14, 15 ; XXIII. 16. 2) ; in two of these places (vi. 6. 8 and XII. 11. 14, 15) it is mentioned that darkness having enveloped the sun, Atri by his power removed the darkness and in the remaining three places it is mentioned that the gods removed it. In the Śatapatha Brāhmaṇa of the Śukla Yajur Veda there is one reference to the eclipse (v. 3. 2. 2) where it is said that darkness having prevailed upon the sun, Soma and Rudra cleared it. In the Gopatha Brāhmaṇa of the Atharva Veda there is one reference to an eclipse (VIII. 19) where it is mentioned that darkness having come upon the sun, Atri removed it. Hence it can be inferred that Atri knew how to calculate the duration and the different phases of the eclipse.[3] There are in addition to those mentioned before three other references to an eclipse in the Mahābhārata. In the Sabhā-parva, chapter 39, it is stated that Rāhu devoured the sun [4]; in the Bhīṣma parva, chapter 2, it is observed that in the Kārtikī full-moon tithi the moon became invisible and pale and also crimson in the lotus-coloured sky [5]; evidently by this a lunar eclipse is meant ; the third reference is in the same parva, third chapter, where a lunar and a solar eclipse are mentioned [6].

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1 *Ṛg Veda,* 5-40-6.

स्वर्भानोरधयदिंद्र माया अवो दिवो वर्त्तमाना अवाहन् ।

गूळ्हं सूर्य्यं तमसापव्रतेन तुरीयेण ब्रह्मणाविंददत्रिः ।

2 *Vide Orion,* pp. 159-160 and Tilak's interpretation of the passage.

3 *Vide Bhāratīya Jyotiḥ S'āstra,* pp. 62, 63, by S. B. Dikshit.

4 राहुग्रसदादित्यमपर्वणि विशांपते । Sabhā parva, 79 chap. verse 19.

5 अलक्ष्यः प्रभया हीनः पौर्णमासीं च कार्त्तिकीं ।

चन्द्रोऽभवदग्निवर्णश्च पद्मवर्णे नभस्तले ॥

Bhīṣma parva chap. 2.

6 चन्द्रसूर्य्यावुभौ ग्रस्तौ Bhīṣma parva, chap 3, verse 32.

Having gone through the references to solar and lunar eclipses in the Vedic and Paurāṇic literature we come to the period of the Siddhāntas. The Jyotiṣ Vedānga and the Sūrya Prajñapti of the Jains, which belong to an earlier date, do not give any detailed method for the calculation of an eclipse. Āryabhaṭa, the first among the systematic writers on Hindu Astronomy, who was born in 398 Saka or 476 A.D., rejected altogether the Paurāṇic idea of the demon Rāhu devouring the sun or the moon at the time of an eclipse. He said that Rāhu and Ketu were no other than the ascending and descending nodes. He tried to give a mathematical interpretation of the whole matter and suggested a method for calculating the eclipses. Varāhamihira (born, saka 427 or 505 A.D.) gave a crushing retort to the Paurāṇic idea of Rāhu and Ketu. "Some argue," says Varāhamihira, "that Rāhu is a demon; formerly Viṣṇu finding Rāhu drinking nectar cut off his head with Sudarśaṇa chakra, but Rāhu having taken some nectar did not die and was converted into a planet." If Rāhu has become a planet, why then does he not, like the sun and the moon, cast his shadow? The Purāṇas say that Rāhu has got his shadow. Why then is not that shadow seen in the sky? The Purāṇas reply, "By virtue of a boon from Brahmā, Rāhu has become black, hence he is not seen in any tithi other than the full-moon or the new-moon." Varāhamihira says, "the āchāryas describe Rāhu as divided in the upper and lower parts of his body, some describe him as a serpent and some other as formless darkness."[1] Varāhamihira does not accept these old ideas. He asks, "if Rāhu has got any form and moves among the stars, why does he then devour the moon and the sun at a distance of six signs? If you argue that Rahu's motion does not conform to any principle, why is it then possible to calculate his motion? If you say that Rāhu has got only a head and a tail, why does he then devour the moon and the sun at a distance of six signs and why not at a distance of two or three signs? If Rāhu has got the form of a serpent and devours the sun or the moon with the head or the tail, why does not the serpent-like body cover the space of the Zodiac between the head and the tail?" Thus has Varāhamihira tried to controvert the current popular belief and has established his theory of the eclipses; "at the time of a lunar eclipse the moon enters the earth's shadow and at the time of a solar eclipse the moon enters the sun. The earth's shadow moves in the seventh sign from the

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[1] Utpala has quoted a passage from *Vaśiṣṭha Siddhānta* to say that the Rāhu is serpent-like in form; it remaining at a distance of six signs from the sun and the moon covers them by virtue of a boon received from Brahmā. Debala says, "Rāhu is dark, and like a cloud it overtakes the sun and the moon at a new-moon and a full-moon respectively."

sun and in a full-moon the moon arrives there. The moon and the earth's shadow both move in the east; but the moon moves faster. Hence the moon enters the earth's shadow by the east side. At the time of a solar eclipse, the moon and the sun move in the same sign; but the moon moving in a plane lower than that of the sun and at a more rapid rate covers the sun from the west. For this reason an eclipse never begins in the *western* half of the moon or in the *eastern* half of the sun." Why does not a lunar eclipse take place at every full-moon? Varāha says, "as the shape of the earth's shadow is larger at the beginning and smaller at the end (*i.e.*, the shape is a cone), the moon, moving in the seventh sign from the sun, passes off either to the north or to the south of the shadow. If the moon does not pass a long way off, it then enters the earth's shadow by the east side." A lunar eclipse is the same in every place. But a solar eclipse is visible in some places and in other places it is not. The reason is this, the moon moving below the sun overtakes the latter like a cloud from the west side, hence the solar eclipse is total in some places and in other places it is partial, and in some other places it is not at all visible. Just as a man below the sun cannot observe the sun's disc when covered by a cloud, but another man situated in a different place can see either the half, quarter or the whole of the sun's disc, similar is the case with a solar eclipse. Varāhamihira gives another proof and he says, "The Earth's shadow which covers the moon is much larger than the moon, hence the horns of the moon when half-eclipsed are seen flattened (*i.e.* form very obtuse angles, *vide* Siddhānta Śiromaṇi, Chapter VIII, verse 7); but the moon which covers the sun is much smaller than the sun, hence the horns of the sun when half-eclipsed are seen pointed (*i.e.*, form very acute angles, *vide* Siddhānta Śiromaṇi, Chap VIII, 8). Rāhu, the demon, is not the cause of an eclipse, this is the truth." Why then is this popular belief? Varāhamihira says, "Rāhu is the name given to the node [1] of the moon's path and at a new-moon or a full-moon unless the moon is near one of the nodes, an eclipse can never take place. Hence the popular belief connects the Rāhu with an eclipse." Varāhamihira then goes on to describe ten kinds of eclipse [2]. If the eclipse begins in the right side of the moon's or sun's disc, it is called *sabya*; if in the left, it is then *apasabya*; if the disc becomes dark for a while and immediately clear, it is called *leha* (lit. licking); if half or one third or one fourth of the disc is obscure, the eclipse is called *grasana*; if the eclipse begins at one side of the disc and then the whole disc being obscured,

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1 Head of Rāhu is the ascending node and tail is the descending node.

2 Sabyāpasabya lehagrasana nirodhābamardanārohāḥ
Āghrātaṃ madhyatamastamohantya iti te daśagrāsāḥ

the middle is seen as a black mass, it is called *nirodha*; if the whole disc is totally obscured for some time, it is called *abamardana*; if after release the disc is again obscured, it is called *ārohaṇa*; if one side of the disc is seen partly obscured just like a glass partly obscured by the moisture of breath, it is called *āghrāta*; if the middle is obscured, but the sides are clear, it is called *madhyatama*; and lastly if the middle is slightly obscured and the rest very thickly obscured, the eclipse is called *tamohantya*. This description of the ten kinds of eclipse surely presupposes minute and careful observations carried on for a considerable length of time. Not that Varāhamihira observed them all, but, as Utpala has shown by quotations, Varāha got some of the nomenclatures from Kāśyapa and Pārāśara Saṃhitās.

Varāhamihira has also described ten kinds of release of an eclipse and has given separate nomenclatures for them.[1] If the release is in the south west, it is called dakṣiṇahanu; if in the north-west, it is called vāmahanu; if in the south, it is dakṣiṇakukṣi; if in the north, it is vāmakukṣi; if in the south-east, it is dakṣiṇapāyu; if in the north-east, it is vāmapāyu; if the eclipse begins in the east and ends in the same side, the release is called sañchardana, and if the eclipse ends in the west, it is called jaraṇa, if the middle of the disc becomes first clear, it is called madhyavidaraṇa; if the middle is obscured while the end of the disc is clear, the release is called antyavidaraṇa. The above description is for the release of a lunar eclipse; but it is also intended for the release of a solar eclipse, the only difference being that in place of the east side of the moon, the west side of the sun will have to be taken and similarly all the opposite sides are to be taken in the case of the release of a solar eclipse.

After Varāhamihira came Brahmagupta (520 Śaka or 598 A.D.) who in his Brāhmasphuta Siddhānta went into further details and gave more precise scientific methods for the calculation of solar and lunar eclipses. He followed in some places the old Sūryasiddhānta and tried to give a clear exposition of the whole matter.[2] But the methods are more clear and worked out in greater details in the present Śūrya Siddhānta which we shall discuss at a considerable length later on. Lalla (560 Śaka or 638 A.D.), in his Śiṣyadhīvṛddhida, tried to combine the methods of Āryabhata and Brahmagupta. But he has committed some mistakes in the calculation of the Ākṣa and Āyana Valana (mentioned later on), and has not given the detailed working for the calculation of a solar eclipse. He has, however, explained clearly the process of finding the parallax in

[1] Hanukukṣipāyubhedā dvirdviḥ sañchardanaṃ ca jaraṇaṃ ca madhyāntyayośca vidaraṇamiti daśa Śaśisūryayormokṣāḥ

[2] Brāhmasphuta siddhānta, Golādhyāya, Āryā 34–38.

latitude and longitude without which the calculation of an eclipse is impossible.[1]

Further details of scientific explanation and mathematical calculation of an eclipse are obtained from Bhāskara's Siddhānta Śiromaṇi and the Sūrya Śiddhānta. That the cause of eclipses, notwithstanding the superstition of the people generally, was well understood by the Hindu astronomers, is shown by the following extracts taken from the Siddhānta Śiromaṇi of Bhāskara[2] :—

"The moon, moving like a cloud in a lower sphere, overtakes the sun, hence it arises that the western side of the sun's disc is first obscured, and that the eastern side is the last part relieved from the moon's dark body; and to some places the sun is eclipsed, and to other places he is not eclipsed. At the change of the moon it often happens that an observer placed at the centre of the earth, would find the sun, when far from the zenith, obscured by the intervening body of the moon; while another observer on the surface of the earth will not, at the same time, find him to be so obscured, as the moon will appear to him to be depressed from the line of vision extending from his eye to the sun. Hence arises the necessity for the correction of parallax in celestial longitude and parallax in latitude in solar eclipses, in consequence of the difference of the distance of the sun and the moon. When the sun and the moon are in opposition, the earth's shadow envelopes the moon in darkness. As the moon is actually enveloped in darkness its eclipse is equally seen by every one on the earth's surface, and as the earth's shadow and the moon which enters it are at the same distance from the earth, therefore, there is no call for the correction of the parallax in a lunar eclipse. As the moon moving east-ward enters the dark shadow of the earth, therefore its eastern side is first obscured and its western side is the last portion of its disc to emerge out of darkness, as it advances in its course. As the sun is a body of vast size, and the earth insignificantly small in comparison, the shadow made by the sun from the earth is, therefore, of a conical form, terminating in a sharp point. It extends to a distance considerably beyond that of the moon's orbit. The length of the earth's shadow and its breadth at the part traversed by the moon may easily be found by projection."

Now let us discuss the method of calculating the occurrence of the eclipses of the moon as described in the Sūrya Siddhānta. To find the day on which a lunar eclipse takes place we compare the longitudes of the moon and her node on the day of the moon's opposition with the sun, when the eclipse

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1 Śiṣyadhīvṛddhida, Sūryagrahaṇādhyāya, page 34, edited by Pandit Sudhākara Dvivedi.

2 Siddhānta Śiromaṇi, Golādhyāya, chapter VIII, verses 1-6.

is expected to occur, and if at the time of the opposition the difference of the longitudes of the moon and her node is within about $7\frac{1}{2}$ degrees, an eclipse is bound to take place. In the Sūrya Siddhanta the sun's mean diameter is assumed to be equal to 6,500 yojanas and the moon's mean diameter to be 480 yojanas. But on account of the variable distances of the sun and the moon, their apparent diameter are greater when near than when more remote, and a correction is applied on the hypothesis that the apparent magnitudes vary with the daily motions which are in the inverse ratio of the distances. The mean daily motions of the sun and the moon are given by the division of the revolutions made by each in a Mahāyuga by the number of days in that Yuga, given in chapter I of the Sūrya Siddhānta. Thus, the mean daily motion of the sun$=\frac{4{,}320{,}000}{1{,}577{,}917{,}828}$, this reduced to minutes $=59{\cdot}13616'$; and the mean daily motion of the moon$=\frac{57{,}753{,}336'}{1{,}577{,}917{,}828}=790{\cdot}56'$. The daily motions of the sun and the moon on the day of an eclipse are called their true daily motions. Then the Sūrya Siddhānta finds the sun's diameter at the Moon and their diameters in minutes. "The diameters of the Sun and the Moon multiplied by their true diurnal motions and divided by their mean diurnal motions become the Sphuta or rectified diameters." (Rule 2, chap. IV.) That is, the sun's rectified diameter is $\frac{6500 \times A}{59{\cdot}13616'}$ and the moon's rectified diameter is $\frac{480 \times B}{790{\cdot}56}$, where A and B are taken to denote the true diurnal motions of the sun and the moon. "The rectified diameter of the Sun multiplied by his revolutions (in a kalpa) and divided by the Moon's revolutions (in that cycle), or multiplied by the periphery of the Moon's orbit and divided by that of the Sun, becomes the diameter of the Sun at the Moon's orbit. The diameter of the Sun at the Moon's orbit and the Moon's rectified diameter divided by 15, give the numbers of minutes contained in the diameters of the discs of the Sun and the Moon respectively" (Rule 3). That is, the diameter of the Sun at the Moon's orbit$=$

$$\frac{6500 \times A}{59{\cdot}13616} \times \frac{4{,}320{,}000}{57{,}753{,}336}$$

$$= \frac{6500 \times A}{\frac{4{,}320{,}000}{1{,}577{,}917{,}828}} \times \frac{4{,}320{,}000}{57{,}753{,}336} = \frac{6500 \times A}{\frac{57{,}753{,}336}{1{,}577{,}917{,}828}}$$

$$= \frac{6500 \times A}{790{\cdot}56} \text{ Yojanas} = 8{\cdot}222 \times A$$

The circumference of the moon's orbit is reckoned to be 324,000 Yojanas, and the number of minutes of arc in the same circumference is $360 \times 60 = 21600$. Hence, 15 Yojanas correspond with one minute of arc, and the above diameter of the sun, divided by 15, gives the apparent diameter in minutes of arc $= \frac{8{\cdot}222 \times A}{15} = {\cdot}54813 \times A$.

Therefore, the mean apparent diameter of the sun's disc $= {\cdot}54813 \times 59{\cdot}13616$ (where $A = 59{\cdot}13616) = 32{\cdot}40685'$ nearly.

The rectified diameter of the moon, divided by 15, gives the apparent diameter of the moon's disc in minutes $= \frac{480 \times B}{790{\cdot}56 \times 15} = {\cdot}04048 \times B$ and the mean apparent diameter of the disc of the moon $= \frac{480}{15} = 32'$, where $B = 790{\cdot}56$.

Next, the Sūrya Siddhānta finds the diameter of the earth's shadow at the moon. "Multiply the true diurnal motion of the moon by the earth's diameter (or 1600) and divide the product by her mean diurnal motion; the quantity obtained is called the Sūchī. Multiply the difference between the earth's diameter and the rectified diameter of the sun by the mean diameter of the moon (or 480) and divide the product by that of the sun (or 6500); subtract the quotient from the Sūchī, the remainder will be the diameter (in Yojanas) of the earth's shadow (at the moon); reduce it to minutes as mentioned before (*i.e.* by dividing it by 15)." (Rules 4 and 5.)

That is, the Sūchī $= \frac{1600 \times B}{790{\cdot}56}$ Yojanas $= 2{\cdot}024 \times B$ nearly.

The diameter of the earth's shadow at the moon is $\frac{1600 \times B}{790{\cdot}56} - \left\{ \frac{6500 \times A}{59{\cdot}13616} - 1600 \right\} \frac{480}{6500}$ Yojanas, and by the division by 15, the Yojanas are converted into minutes which

$$= 106\frac{2}{3} \times \frac{B}{790{\cdot}56} - 32 \times \frac{A}{59{\cdot}13616} + 7\frac{57}{65}.$$

Now make $A = 59{\cdot}13616'$ and $B = 790{\cdot}56'$, the mean motions of the sun and the moon; therefore, the mean diameter of the earth's shadow $= 106\frac{2}{3} + 7\frac{57}{65} - 32 = 82\frac{106}{195}$ minutes or 82 minutes nearly.

A similar method for finding the diameter of the earth's shadow is given in chapter X of the Pañcha Siddhāntikā, *i.e.*, the old Sūrya Siddhānta included in Varāhamihira's Pañcha Siddhāntikā.

"The earth's shadow is always six signs from the sun. When the place of the moon's node is equal to that of the shadow, there will be an eclipse (of the sun or the moon) or, when the node is some degrees within, or beyond, the place of its shadow, the same thing will occur." (Rule 6.)

The longitudes of the sun and the moon being computed for the mid-night preceding, or after conjunction or opposition, proportional parts are to be applied for the changes of their places in the interval between. The moon moving like a cloud in a lower sphere, covers the sun in a solar eclipse; but in a lunar eclipse the moon moving eastward enters the earth's shadow, and the shadow obscures her disc.

The Sūrya Siddhānta next proceeds to find the magnitude of an eclipse. It says that the quantity of the eclipsed part of the diameter will be $=\frac{1}{2}(D \pm d)-\lambda$, where D is the diameter of the coverer, d the diameter of the body eclipsed, λ the latitude of the moon at the time of syzygy (*i.e.* when the sun, the earth, the moon, and the node of its orbit are in one line). If this quantity be greater than the diameter of the disc of the body to be eclipsed, the eclipse will be total; otherwise it will be only partial. But there will be no eclipse when λ is greater than

$$\frac{D+d}{2}. \quad \text{(Rules 10 and 11.)}$$

To find the half duration of the eclipse and that of the total darkness we are to find the halves, separately, of the sum and difference of the diameters of that which is to be covered and that which is the coverer. We are then to subtract the square of the moon's latitude from the squares of the half sum and the half difference and to take the square roots of the results. These roots, multiplied by 60 and divided by the diurnal motion of the moon from the sun, give the *Sthityardha*, the half duration of the eclipse and the Mardārdha, the half duration of the total darkness, in Ghatikās (respectively). (Rules 12 and 13.)

This can be explained thus: Suppose V_1EN represents a portion of the ecliptic, and M_1MN a portion of the moon's path cutting the ecliptic at the ascending node N. Let E and M be the centres of the earth's shadow and of the moon, at the instant of opposition, that is, at the time of the full moon, then EM will be the latitude of the moon, at that time (say H), and $EM=\lambda$.

Then suppose V and M_1 are the places of the centres of the earth's shadow and of the moon respectively, at the beginning of the eclipse, *i.e.*, at the moment of the first contact of the moon with the shadow, then V_1E (where V_1 is the foot of the perpendicular from M_1 on the ecliptic) is equal to the difference between the longitudes of the moon at the first contact and at the time of full moon. Let us assume the moon's latitude, λ, to be unchanged for a short time. Then in the triangle M_1EV_1, $M_1E^2=EV_1^2+M_1V_1^2$; but $EM_1=\frac{D+d}{2}$, sum of the radii of the earth's shadow and the moon, and $M_1V_1=\lambda$. Hence $EV_1=\sqrt{\left(\frac{D+d}{2}\right)^2-\lambda^2}$. Let u be the relative diurnal motion of the moon from the sun, S the *sthityardha* in Ghatikās.

$$\text{Therefore, } \frac{S}{60}=\frac{\sqrt{\left(\frac{D+d}{2}\right)^2-\lambda^2}}{u}, \text{ [60 Daṇdas=1 day]}.$$

$$\text{or } S=\frac{60}{u}\times\sqrt{\left(\frac{D+d}{2}\right)^2-\lambda^2}.$$

If M be the half duration of total darkness,

$$M=\frac{60}{u}\times\sqrt{\left(\frac{D-d}{2}\right)^2-\lambda^2}.$$

The diurnal motions of the sun, the moon, and her ascending node, multiplied by the Sthityardha (above found) and divided by 60 give their changes in minutes. Then to find the first exact Sthityardha, subtract the changes of the sun and the moon from their places and add the node's change to its place; from these applied places find the moon's latitude and the Sthityardha. This Sthityardha will be somewhat nearer the exact one, from this find the changes and apply the same method of calculation and repeat the process until you get the same Sthityardha in every repetition. This Sthityardha will be the exact first Sthityardha. To find the second Sthityardha, or that for the end of the eclipse, the proportional changes in the places of the sun and the moon are now to be added to their places at the opposition, but the change in the place of the moon's node is to be subtracted from the place at the opposition. From these corrected places, the moon's latitude is again to be computed and substituted for λ in the above formula, for a nearer value of S, at the last contact. The same process is to be repeated until the exact second *Sthityardha* is obtained. In like manner, the first and second Mardārdhas are determined by repeated calculations. (Rules 14 and 15.)

The middle of the lunar eclipse takes place at the time of the full moon. If this time be denoted by H, then $H-$each 1st Sthityardha=the time of the first contact with the shadow and $H+$2nd exact Sthityardha is the time of the end of the eclipse. Similarly, $T-$exact 1st Mardārdha and $T+$2nd exact

Mardārdha are the times of the beginning and end of the total darkness. (Rules 16 and 17.)

If the diurnal motion of the moon from the sun (*i.e.*, the relative daily motion) in longitude be l, S the first Sthityardha, and m the time at any moment elapsed from the beginning, or first contact, the difference in longitude at that moment, from that at the middle of the eclipse would be in minutes of arc $=\frac{l}{60}(S-m)$. This is called the Koṭi in minutes or the perpendicular of the right angled triangle of which the moon's latitude is the base and the distance between the centres of that which is the coverer and that which is to be covered is the hypotenuse. (Rule 18.)

In an eclipse of the sun, the Koṭi in minutes, multiplied by the mean Sthityardha and divided by the apparent Sthityardha becomes the Sphuta or apparent Koṭi in minutes. (Rule 19.)

The eclipsed part in minutes=(half the sum of the diameters of the coveror and that covered)—(the distance at any moment between the centres of the coverer and that covered) $=\frac{D+d}{2}-\sqrt{\text{Koti}^2+\lambda^2}$ where D and d are the diameters and λ is the latitude of the moon which is called Bhuja. (Rule 20.)

A similar method is employed for calculating the eclipsed part at a given time between the middle of the eclipse and the end, in which case the second Sthityardha is used for finding the Koṭi or the perpendicular of the above right angled triangle.

Given the quantity of the eclipsed part, to find its corresponding time, suppose n denotes the minutes of arc of the eclipsed part of lunar eclipse. Then Koṭi

$$=\sqrt{\left\{\left(\frac{D+d}{2}\right)-n\right\}^2-\lambda^2}.$$

In a solar eclipse, the Koṭi

$$\frac{\text{Apparent Sthityardha}}{\text{Mean Sthityardha}}\times\sqrt{\left(\frac{D+d}{2}-n\right)^2-\lambda^2}.$$

From the Koṭi, find the time in Ghatikās in the same way as the Sthityardha is found. (Rules 21, 22 and 23.)

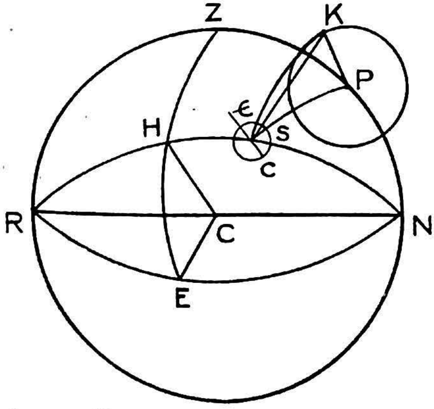
It is remarked in the Sūrya Siddhānta that the phase of an eclipse cannot be exactly understood without their projection, and the Hindu method is explained by finding what are termed the Valanas, two angles whose sum or difference constitutes the so-called rectified Valana, or "variation of the ecliptic." As an entire variation, it is equal to the angle between a circle of latitude through the place of a body on the ecliptic, and the circle of position through the same place; the circle of position being defined as the great circle, passing through a planet, and through the north and south points of the horizon.

The rules for calculating the Valana, ākshа, and āyana, altogether agree with those of the old Saura Siddhānta as contained in the Pañcha Siddhāntikā.[1]

The Sūrya Siddhānta has given two rules for finding the Valana used in the projection of eclipses. The rules are as follows :—

"Find the zenith distance of the circle of position passing through the body, multiply its sine by the sine of the latitude of the place, and divide the product by the radius. Find the arc whose sine is equal to the quotient; the degrees contained in this arc called the degrees of the (Āksha or the latitudinal) Valana; they are north or south according as the body is in the eastern or western hemisphere of the place. From the place of the body, increased by three signs, find the variation (which is called Āyana or solstitial Valana). Find the sum or difference of the degrees of the variation and those of the latitudinal Valana, when those are of the same name or of contrary names; the result is called Sphuta or true Valana. The sine of the true Valana divided by 70 gives the Valana in digits.". (Rules 24 and 25.)

This can be explained thus: Let $RZPN$ represent the meridian, P the pole of the equator, N the north point of the horizon REN, ZHE the prime vertical. Suppose S is the place of the body to be eclipsed and through S the circle of position NSR is to be drawn through the north and south points, N and R of the horizon. The object of the Valana is then to determine the position of the short arc ϵSC, as it would appear to an observer at a given place. Now ϵSC is perpendicular to SK, the circle of latitude through S. Then rectified Valana is angle KSN, angle between the circle of position SN and the circle of latitude SK.



The āksha Valana and āyana Valana are the two parts of rectified Valana KSN. The āksha Valana is $< PSN$ and āyana Valana is $< KSP$.

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[1] For details see the Sanskrit Commentary by Sudhakar Dvivedi in the Pañcha Siddhāntikā, edited by Thibaut and himself, Chap. X.

From triangle PSN, $\sin PSN = \dfrac{\sin PNS . \sin PN}{\sin PS}$.

Here $\sin PNS$ is measured by ZH, the zenith distance of the circle of position (Z say). PN=latitude of the place$=l$ (say), $PS = 90° -$ declination.

$\therefore$ sin āksha Valana$=\dfrac{\sin Z . \sin l}{\cos d} = \dfrac{\sin Z . \sin l}{R}$ in the text.

If R be the radius of the sun's diurnal path, on the day of the eclipse, and, consequently, the sun having a supposed declination d, the cosine of the arc d would be the radius of the diurnal circle.[1]

Let the place of the body be increased by three signs.[2] Then the āyana Valana is angle KSP. From $\triangle KSP$

$\sin KSP = \dfrac{\sin PKS . \sin KP}{\sin PS} = \dfrac{\sin (L+90°) \sin \omega}{\cos d}$, where L is the original longitude of the body and it becomes $L+90°$ when the place of the body is increased by three signs, KP is the obliquity of the ecliptic, $=\dfrac{\cos L . \sin 24°}{R}$, where $\omega = 24°$ nearly.

Thus the sphuta or true Valana is found by addition or subtraction of the two parts, as may be found necessary.

To mark the sine of the Sphuta Valana in the projection of the eclipse it is reduced to the circle whose radius is 49 digits in the text.

Therefore, reduced sine of the Valana : 49=sine of the Sphuta Valana : R.

Hence, reduced sine of the Valana=sine of the Sphuta

$$\text{Valana} \frac{\times 49}{R} = \frac{\text{sine of the Sphuta Valana}}{70}, \ (R=3438).$$

$$\left[\frac{3438}{49} = 70 \text{ nearly.}\right]$$

Bhāskara in his Siddhānta Śiromaṇi has almost followed the method given in the Sūrya Siddhānta for calculating a lunar eclipse in its various aspects. However, he has explained the two Valanas at a greater length, particulars of which are given in Chapter VIII, verses 30 to 43, Golādhyāya.

The use of the Valana is this that, in drawing the projections of the eclipses, after the disc of the body which is to be eclipsed is drawn, and the north and south and the east and west lines are also marked in it, which lines will, of course,

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[1] See Brennand's Hindu Astronomy, page 282.

[2] The angle called Āyana Valana is obviously the same as the angle of position of modern Astronomy.

represent the circle of position and its secondary, the direction of the line representing the ecliptic in the disc of the body can easily be found through the Valana. This direction being known, the exact directions of the beginning, middle, and the end of the eclipse can be determined. But as the moon resolves in its orbit, the direction of its orbit, therefore, is to be found. But the method for finding this is very difficult, and consequently instead of doing this, the direction of the ecliptic is determined by means of the moon's corresponding place in it and then ascertain the direction of the moon's orbit.[1]

We have already found the ākshа Valana

$$=\frac{\sin Z \sin l}{\cos d}\ldots.(B)$$

where l is the latitude of the place and āyana Valana

$$=\frac{\sin(90^\circ+L)\sin\omega}{\cos d}=\frac{\cos L}{\cos d}\ldots.(A)$$

where L is the longitude of the body.

Lalla, Śrīpati, and others used the co-versed sin L instead of cos L and the radius for the cos d in (A) and the versed sin Z in the place of sin Z and radius for the cos d in (B). Hence, the Valanas found by them are incorrect. Bhāskara, therefore, in order to convince people of the mistakes made by Lalla, Śrīpati, and others, in finding the Valanas, refuted their methods in several ways. Bhāskara says, " As the versed sine is like the sagitta (śara) and the sine is the half chord, therefore, the versed sine of the distance of the ecliptic pole from the meridian will not express the proper quantity of Valana as has been asserted by Lalla etc; but the right sine of that distance does so precisely. The Āyana Valana will be found from the declination of the longitude of the sun added with three signs or 90°. Those people who have directed that the versed sine of the declination of that point, three signs in advance of the sun, should be used, have thereby vitiated the whole calculation. Āksha Valana may be in like manner ascertained and illustrated. But it is found by the right sine and not by the versed sine."[2]

We have already quoted Bhāskara to show the necessity for the correction of parallax in celestial longitude and parallax in latitude in calculating solar eclipses in consequence of the difference of the distance of the sun and the moon, and we have also given the reason for the correction of parallax not being necessary in lunar eclipses. As the spectator is elevated above

[1] Vide notes by Pandit Bāpū Dev Śāstri in connection with verses 30 to 60 in Chap. VIII, Golādhyāya Siddhānta Śiromaṇi edited by Wilkinson and himself.

[2] Chapter VIII, Golādhyāya, verses 55 and 56.

the centre of the earth by half its diameter, he, therefore, sees the moon depressed from its place. Hence the parallax in longitude is calculated from the radius of the earth, as is also the parallax in latitude.[1]

The Hindus by means of the rising signs determined the place of the horoscope or the point of the ecliptic just rising, at any time, in the Easter Horizon—the point called the Udaya Lagna, and by similar means they found the culminating point of the ecliptic. The point 90° along the ecliptic, from the point of it just rising, *i.e.*, Udaya Lagna was called Trivoṇa or Triva lagna, which among modern astronomers goes by the name of nonagesimal. This point on the occasion of a solar eclipse was of great importance for its connection with parallax. The calculation of parallax in all its aspects has been described separately.[2] There we got the following results :—Udaya or the sine of the amplitude of the horoscope$=\dfrac{\sin L\times\sin 24^\circ}{\cos l}$, where L is the longitude of the Lagna or horoscope, and l the latitude of the place.

The sun's parallax is found with the help of the moon's parallax. The moon's parallax in longitude, on the occasion of a solar eclipse, involves a series of complex calculations, which are divided into several steps.

The true time of conjunction of the sun and moon differs from the apparent time by the relative parallax of the sun and the moon expressed in time.

The moon's horizontal parallax is estimated to be $\dfrac{1}{15}$ of the mean daily motion in her orbit$=52'\ 42''$ approximately when the daily motion is $13^\circ\ 10'\ 46{\cdot}7''$. Similarly the sun's horizontal parallax is reckoned to be $3'\ 56''$ and the relative horizontal parallax to be $48'\ 46''$.

The moon's parallax in longitude from the sun, expressed in Ghatikā$=\dfrac{D}{\text{Chheda}}$, D is the difference of longitudes of the nonagesimal and of the sun, Chheda$=\dfrac{(\sin 30^\circ)^2}{\text{Dṛggati}}$, where Dṛggati$=$cosine of the zenith distance of the culminating point $=$sine altitude of nonagesimal. Parallax in latitude $=\dfrac{\text{Dṛkṣepa}}{70}$, where Dṛkṣepa is the sine of the zenith distance of the nonagesimal.[3]

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[1] *Siddhanta Śiromaṇi, Goladhyāya*, Chapter VII, verse 11.

[2] Vide "*Parallax in Hindu Astronomy*" by the present author published in the *Bulletin Calcutta Mathematical Society*, March, 1928.

[3] *Sūrya Siddhānta*, Chapter V, verses 6–11.

Also we got $p=\frac{P \sin Z}{R}$, *i.e.*, the common parallax=the greatest parallax (P) multiplied by the sine of the zenith distance and divided by the radius.[1]

Again Spaṣṭa Lambana or the parallax in longitude $=\frac{P \times \sin \text{ zenith distance}}{R}$ and the Nati or the parallax in latitude[2] $=\frac{P \times \text{sine of the latitude of the nonagesimal}}{R}$.

If the Moon be to the east (of the nonagesimal), it is thrown forward from the Sun, if to the west it is thrown backward (by the parallax). If the Moon be advanced from the Sun, then it must be inferred that the conjunction has already taken place by reason of the Moon's quicker motion; if depressed behind the Sun, then it may be inferred that the conjunction is to come by the same reason. Hence the parallax in time, if the Moon be to the east of the nonagesimal, is to be subtracted from the end of the Tithi or the hour of ecliptic conjunction, and to be added when the Moon is to the west of the nonagesimal. The latitude of the Moon is north and south distance between the Sun and the Moon, and the Nati also is north and south. Hence the S'ara or latitude applied with the Nati or the parallax in latitude, becomes the apparent latitude of the Moon from the Sun.[3]

The Sūrya Siddhānta says, "The amount of the parallax found is north or south, according as the nonagesimal is north or south of the zenith. Add the amount to the Moon's latitude, if they are of the same name; but, if of contrary names, subtract it. The result is the apparent latitude of the Moon.[4]

The apparent time of conjunction having been found, by applying the parallax in longitude to the computed true time of conjunction and for this apparent time the moon's apparent latitude having been calculated, by applying the parallax in latitude to the true latitude, the method of procedure afterwards differs little from that employed for the calculation of lunar eclipses. An indication is given in the following verses[5]:—

"In the solar eclipse, with the apparent latitude of

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1 Vide notes by Pandit Bapu Dev Śāstri under verse 12, Chap. VIII, of *Golādhyāya*.

2 *Ibid.*, under verses 16–20, Chap. VIII, *Golādhyāya of Siddhānta S'iromaṇi*.

3 *Siddhānta S'iromaṇi, Golādhyāya*, Chap. VIII, verses 27, 28 and 29; B. D. Sastri and Wilkinson's, edition.

4 *Sūrya Siddhānta*, Chap. V, verse 12, B. D. Sastri and Wilkinson's edition.

5 *Ibid.*, verses 13-17.

the Moon, find the sthityardha (or half duration) the Mardārdha (or half the total darkness), etc., of the eclipse, as before mentioned; the valana or deviation of the ecliptic; the eclipsed portions of the disc at any assigned time are found by the rule mentioned in connection with the lunar eclipses. Find the parallaxes in longitude (converted into time) by repeated calculation at the beginning of the eclipse, found by subtracting the first sthityardha from the time of conjunction, and at the end, found by adding the second sthityardha If the Sun be east of the nonagesimal, and the parallax at the beginning be greater, and that at the end be less than that at the middle, add the difference between the parallaxes at the beginning and middle, or at the end and the middle to the first or the second sthityardha; otherwise subtract the difference. It is then when the Sun is east or west of the nonagesimal at the times of both the beginning and the middle, or of the middle and the end; otherwise add the sum of the parallaxes (at the time of the beginning and middle, or of the end and the middle) to the first or second sthityardha. Thus you have the apparent sthityardhas and from these the times of the begenning and the end of the eclipses of the Sun. In the same manner, find the apparent Mardārdhas (and the times of the beginning and end of the total darkness in the total eclipses of the Sun)."

We have given in details all the discussions on Lunar and Solar eclipses by the Hindus from the vedic times down to Bhaskara. We conclude by mentioning a remarkable achievement of the ancient astronomers regarding the recurring of the ecilpses after a certain period or cycle. This cycle was called Saros by the Chaldeans and was current among all the ancient nations of the world. We have already seen that it is upon the position of the Moon's node at the time of conjunction or opposition of the Sun and the Moon, that a solar or a lunar eclipse depends; and if the Sun, the Moon, the node of its orbit and the earth are very nearly in one straight line, an eclipse must happen. The same eclipse will return after 223 lunations or 6585.78 days or 18 years 10 or 11 days according as five or four leap-years occur during the time. The same observations apply to all other eclipses which happen when the moon is near her node, within what are called Lunār Ecliptic Limits. These all return after periods of the same length, exactly in the same order and under similar circumstances; so that a complete list of eclipses that occur in one such period or cycle is sufficient to form a list of eclipses extending over several centuries, past or future.[1]

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[1] *Vide* page 5 of Dr. D. N. Mallik's "*The Elements of Astronomy.*"

Calculation of eclipses forms an important part of observational astronomy. It is, therefore, no small credit for the ancient Hindu astronomers that they worked out the details to such an approximation. The skill shown by them in finding the valanas and the lambana and nati (parallaxes in latitude and longitude) are really commendable.