



**INDIAN INSTITUTE OF
ADVANCED STUDY
SIMLA**



Library

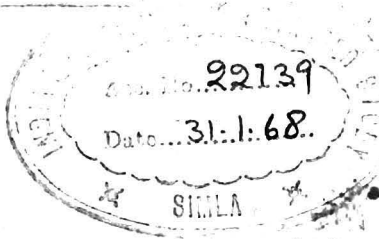
IIAS, Shimla

PH 001 Un 3 J



00022739

Reprinted from the
Journal of the University of Poona,
Libraries Section, No. 23, pp. 139 to 152.



10/10/83

PH
001
Un 3 J

28

Mathematical Representation of Some Pāṇinian Sūtras

by

M. D. Pandit

1. Introductory and Scope :

It will be at once acknowledged by one who studies Pāṇini's grammar that Pāṇinian description of the language is more comprehensive than that of any other grammarian, pre-Pāṇinian or post-Pāṇinian. Moreover, no *sūtra*-system other than Pāṇini's has equalled the Pāṇinian *sūtra*-system in matters of brevity as well as accuracy. The possible reason seems to be that the Pāṇinian grammar is based upon certain principles which in turn have paved way for certain types of techniques, useful and most convenient for a certain kind of linguistic description. These principles or techniques could again be either borrowed from other sciences or be the product of the genius of the author. If, therefore, the techniques employed by Pāṇini or the methods of statement adopted by him are critically studied and analysed, it may be possible to find out the underlying principles. The present paper aims at discovering the underlying principles of some Pāṇinian *sūtras* which can be most conveniently represented mathematically.

2. Mathematical representation of the Pāṇinian sūtra 1.1.71 :

The *sūtra* 1.1.71, *ādir antyena sahēṭā* is one of the most important *sūtras* in Pāṇini's grammar and lays down the technique of forming the *pratyāhāras*. What the *sūtra* states is that any non-*it* sound in the initial position together with any *it*-sound in the final position signifies or includes itself and all the sounds which come in between itself and the final *it*-sound¹. Thus the non-*it* sound, say *a* (of *Māheśvara Sūtra* No. 1) in the initial position and the final *it*-sound, say *C* (of the *Māheśvara Sūtra* No. 4) will form a *pratyāhāra* or abbreviation *aC* which will represent all the sounds including *a* between the limits *a* and *C* (excluding *C* and all other *it*-sounds in between *a* and *C*); the sounds represented by the *pratyāhāra* *aC* will then be: *a, i, u, ṛ, ḷ, e, o, ai* and *au*. Thus we get various *pratyāhāras* like *aK, aN, aM, yaN, haL, jaŚ, jhaL* etc. which stand for different sounds enclosed within their respective limits.

This technique of *pratyāhāra*-formation or phonemic abbreviations or sets gives us in all 305 sets, out of which only 40 sets are used by Pāṇini.

2.1. Implications of the sūtra, ādir antyena sahēṭā, 1.1.71 :

The technique of *pratyāhāra*-formation, defined in 1.1.71 looks apparently very simple and does not provoke any more curiosity. Yet it

1. cf. BD. on the *sūtra* 1.1.71 : *antyeṇṭā sahita ādir madhyagānām svasya ca saṁjñā syāt*; for a study of the Pāṇinian *it-saṁjñā*, cf. M. D. PANDIT, "Pāṇinian *It-Saṁjñā*—A Symbolic Zero", BDCRI, 25, pp. 78-94.

contains certain implications which drive us to guess a certain underlying principle.

2.1.1. Though the *sūtra* 1.1.71 specifies that the final sound of a *pratyāhāra* should be an *it*-sound from amongst those in the *Māheśvara Sūtras* (cf. the words *antyena itā*), it does not specify the category in terms of *it* or non-*it* of the initial sound of the same; the word *ādih* in the *sūtra* is without any specification. There are, therefore, two alternatives; (a) either the first sound of the *pratyāhāra* is also an *it*-sound, or (b) it is a non-*it* sound. If the first possibility is accepted, it would mean that a *pratyāhāra* is composed of two *it*-sounds. In that case both of the *it*-sounds will be reduced to zero according to the *sūtra*, *tasya lopaḥ*, 1.3.9. This would mean that a *pratyāhāra* contains a big zero which is impracticable, impossible and illogical. This circumstance, therefore, leaves us with no other alternative but the second one viz. that the initial sound of a *pratyāhāra* is and should be a non-*it* sound. A *pratyāhāra* with two *it* sounds, like *KC* (the *it*-sounds of the *Māheśvara Sūtras* 2 and 4 respectively) or *MV* or *VL* etc. is, therefore, not permissible. The initial non-*it* sound in the *pratyāhāra* is, therefore, to be included in the list of sounds represented by it while the *it*-sounds are to be excluded.

2.1.2. A *pratyāhāra* must end with an *it*-sound; it must not end with a non-*it* sound. This is explicitly laid down by the words *antyena itā* in the *sūtra*. A *pratyāhāra*, therefore, with two non-*it* sounds like *ah* (the non-*it* *a* of *sūtra* No. 1, and the non-*it* *h* of *sūtra* No. 5 or 14) or *bakha* or *chasa* etc. is not allowed in Pāṇini's grammar.²

2.1.3. A *pratyāhāra*, once formed, represents only the non-*it* sounds and excludes all the *it*-sounds; cf. BD's remark on the Pāṇinian *sūtra* 1.3.2 : *pratyāhāreṣy itām na grahaṇam anunāsika ityādinirdeśāt*.

2.1.4. No non-*it* sound, included within the limits and signified by the *pratyāhāra*, must be left out of consideration. A *pratyāhāra* does not merely represent only the initial non-*it* and the final sound preceding the *it* with which it is composed. It represents the initial as well as all the non-*it* sounds enclosed within the limits. What this means is that while forming a *pratyāhāra*, we must traverse each and every non-*it* sound coming in between the limits; we cannot avoid any non-*it* sound on the way and jump to the next non-*it*. This is the significance of the word *madhyagānām* (= 'falling in between') in the commentary by BD. Kāśikā explains the same point by the word *tanmadhyapatitānām*. Thus, a *pratyāhāra* like *aK* (*a* of *sūtra* No. 1 and *K* of *sūtra* No. 2) signifies all the non-*it* sounds in between, viz. *a*, *i*, *u*, *ṛ*, and *ḷ*. Not only that it does not signify only some sounds, but also that it does not signify only the sounds *a* and *K* of which *it* is composed.

2. Some other systems of grammar, however, allow a *pratyāhāra* with two non-*it* sounds; they have not employed the technique of *it-samjñā*. Cf. for example, the Sāraswata and the Hemacandra schools of grammar which do not lay down the *it*-sounds in the enumeration of phonemes; they, however, have used the technique of *it-samjñā*, in the case of the terminations, *ūgamas* and *ūdesas*.

2.1.5. The last, and a very important, assumption in the *pratyāhāra*-formation is that the process of forming a *pratyāhāra* is always in the forward i.e. positive direction. We cannot form a *pratyāhāra* in backward or negative direction. We must always accept the first non-*it* sound as the starting point from which we must always go in forward i.e. positive direction, on the path determined by the order of succeeding sounds. Thus, a *pratyāhāra*, like *haK*, with the non-*it* *h* of either *sūtra* No. 5 or 14 in the initial position and the *it* *K* of *sūtra* No. 2 in the final position, is not allowed, since the *pratyāhāra* is always formed in the positive, fixed direction.

2.2. Principle Underlying the Sūtra 1.1.71:

The rule of *pratyāhāra*-formation, viz. *ādir antyena sahētā* 1.1.71 together with its assumptions, especially assumptions nos. 4 and 5, gives us a clue to find out the principle underlying it. Assumptions Nos. 4 and 5 above, viz. of compulsory journey through each and every point on the way and of positive or forward direction suggest the concept of a straight line in mathematics, which is also based on compulsory journey through each and every point in a positive, fixed direction. If this parallelism between the assumptions of the mathematical straight line and the Pāṇinian *sūtra* of *pratyāhāra*-formation is correct, we may say that the Pāṇinian *sūtra* 1.1.71 is based on the same principle on which the mathematical concept of a straight line is based. The representation below of the Pāṇinian *sūtra* 1.1.71 in the form of a diagram of a straight line with the phonemes as different points on it to be traversed will make the point clear. (See Fig. 1).

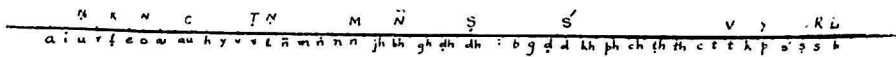


Fig. No. 1

Imagine a straight line *aL*, with the non-*it* *a* as the starting point and the *it* *L* as the end, on which are situated all the phonemes from *a* to *h* in the same order as in the *Māheśvara sūtras*. The *it*-sounds, marked in capital letters, which are not to be included in the regular list of Sanskrit phonemes and which amount to zero, are marked above the line, exactly above those non-*it* phonemes which precede them. Now, to obtain a *pratyāhāra*, say, of all vowels only, we start from the starting point, non-*it*, *a* and walk straight upto the point *au*; the zero, *it*-sound *C* above the line will mark the end of the journey and we get the *pratyāhāra* *aC* comprising of all vowels. So also with other *pratyāhāras* like *haL*, *yaN*, *jaS* etc. We must walk through each and every point on the way until we reach the end. The end will be represented by the *it*-sounds above the line. Moreover, we cannot reverse the direction which is determined by the order of phonemes succeeding the initial, non-*it*, *a* of the first *Māheśvara sūtra*, viz. *aiuN*.

This method of *pratyāhāra*-formation applies to the *pratyāhāras* of the phonemes in the *Māheśvara sūtras* as well as to those of the declensional and conjungational terminations which are enumerated in *sūtras* 4.1.2 and 3.2.78 respectively; the *pratyāhāras*, *suP* and *tiN* stand as abbreviations for the

respective groups of terminations of nominal declension and verbal conjugation.

We can even get the required *pratyāhāra*, say *aC*, by yet another, longer or indirect, method. First prepare the *pratyāhāra* *haL* in the way given in the *sūtra*, comprising of all consonants; then detach or subtract it from the biggest *pratyāhāra*, viz. *aL* comprising of all sounds. And we get $aL - haL = aC$ or contrarily, $aL - aC = haL$ in the same way as in the straight line ABCD (see Fig. No. 2 below)



Fig. No. 2

$AD - BD = AB$, or $AD - AB = BD$.

We unconsciously resort to this indirect method of *pratyāhāra*-formation frequently when the sounds to which the given rules do not apply are to be excluded. Take the example of the *sūtra* *hal antyam*, 1.3.3 itself. The *sūtra* means that the final sounds included in the *pratyāhāra* *haL*. (which are all consonants) are to be termed as *it*. On the basis of this rule we understand by implication that the sounds other than those from the *pratyāhāra* *haL* (which means that the sounds in the *pratyāhāra* *aC*, which are all vowels) are not to be termed as *it*. We have arrived at this conclusion by the simple process of subtraction of the smaller *pratyāhāra* *haL* from the bigger one, viz. *aL*; thus $aL - haL = aC$. If, therefore, $haL = x$, $aC = a$, $aL = y$, so that $y = x + a$, and it is substituted by *p*, we transform the statement "*aC* is not *it*" in symbols as follows: -

aC is not *it*

i.e. *a* is not *p*

i.e. $(y - x)$ is not *p* (since $a = y - x$)

i.e. $aL - haL$ is not *it*.

Here the example of the *pratyāhārās* *aC* and *haL* that is taken is of two mutually exclusive groups, so that the non-*haL* group contains only one *pratyāhāra* viz. *aC*. There are innumerable cases, however, where the non-*x* group (i.e. that other than given) contains many *pratyāhārās*. Take, for example, the *pratyāhāra* *jaŚ*; the non-*jaŚ* group will contain two *pratyāhārās* *aŚ* (i.e. *aŚ-jaŚ*) and *khaL* (i.e. *jaL-jaŚ*).

One cannot represent the *sūtra* 1.1.71 in the form of the diagram of a circle and the phonemes as points on the circumference, because first, the comparison is too odd and inapplicable. A *pratyāhāra* along the circumference of the circle would mean that one can even start with the final phoneme *h* of the *Māheśvara sūtra* No. 14, go forward to the *it*-sound *K* of *sūtra* No. 2 (viz. *r!K*) and compose a *pratyāhāra* like *haK* comprising of the sounds *h*, *a*, *i*, *u*, *r*, *l*. But no such circular *pratyāhārās* are found used by Pāṇini. Secondly, like the straight line, the *Māheśvara sūtras* have also fixed, starting and ending points which the circumference of a circle will not have.

The underlying principle of the Pāṇinian *sūtra* 1.1.71, therefore, seems to be that of a straight line in mathematics.

3. Mathematical representation of the Pāṇinian Sūtra 1.1.9 :

An another Pāṇinian sūtra which could be represented mathematically, or more accurately, graphically, is *tulyāsyaprayatnam savarṇam*, 1.1.9. The sūtra states that two sounds having the same place and internal effort of articulation are to be taken or regarded as 'similar' or 'homoorganic'. This sūtra thus give us the principle on which the phonetic similarity of sounds could be determined.³

It is well-known that the ancient Indian Sanskrit phoneticians have determined the *sthāna* i.e. place and *prayatna* i.e. effort of articulation of sounds. The *sthānas* or places of sounds are in all eight,⁴ they are : (1) chest (2) throat (3) head (4) root of the tongue (5) teeth (6) nose (7) lips and (8) palate. A monophthongal sound in Sanskrit has thus one, single place of articulation at a time. The diphthongs however, have two places of articulation at the same time; the diphthongs *e* and *ai* have throat and palate and the diphthongs *o* and *au* have throat and lips as their places of articulation cf. BD. on 1.1.9 : *edaitoḥ kanṭhatālu*; *odautoḥ kanṭhoṣṭham*. The only monophthong—and that too a semi-vowel—that is given as having two places of articulation at a time is the semi-vowel *v* which has teeth and lips as the places; cf. BD. on 1.1.9 : *vakārasya dantoṣṭham*.⁵ This raises the total number of places of articulation of all Sanskrit sounds to (8+3=)11.

The effort that goes into producing the sounds is mainly of two kinds : (1) the *āntara* or *ābhyantara prayatna* i. e. the internal effort and (2) the *bāhya prayatna* or what Pāṇiniya Śikṣā calls as the *anupradāna*⁶ i. e. the external effort. Now since only the internal effort is of use in deciding the phonetic similarity,⁷ we shall not take into account the external effort here. The internal effort is of four types : (1) *sprṣṭa* (2) *īṣat-sprṣṭa* (3) *vivṛta* and (4) *saṁvṛta*; cf. BD. on 1.1.9 : *ādyah caturdhā/sprṣṭeṣatsprṣṭavivṛtasamvṛtabhedāt*.

3. This was the principle used to decide the phonemes of the Sanskrit language from amongst the innumerable phones that could be produced by the tongue with the help of other physical organs; for a discussion, cf. M. D. PANDIT, "Some Linguistic Principles in Pāṇini's Grammar", *Indian Linguistics*, 24, 1963, pp. 51-53.

4. cf. Pāṇiniya Śikṣā 13 : *aṣṭau sthānāni varṇānam uraḥ kanṭhaḥśiras tathā jihvāmūlaṁ ca dantāś ca nasikoṣṭhau ca tūlu ca*.

5. The sound *v* raises many problems. For a phonetic description of Sanskrit sounds, cf. S. K. CHATTERJI, "The Pronunciation of Sanskrit Sounds", *Indian Linguistics*, 1955, p. 81 f.

6. cf. Pāṇiniya Śikṣā, 10b, *prayatnānupradānataḥ*

7. Cf. BD on 8.2.1 : *bāhyaprayatnāś ca.....savarṇasaṁjñāyām anupayuktāḥ*. This is not the place to discuss the question : Why the external effort is left out while considering the *savarṇya* of sounds ?

Thus we have the two criteria, viz. *sthāna* and *prayatna* by which we have to describe a Sanskrit sound so far as its phonetic similarity with other sound is concerned. In mathematical language, we may say that a Sanskrit sound could be marked down on the graph with the help of these two co-ordinates viz. *sthāna* and *prayatna*. The following figure will illustrate the point clearly.

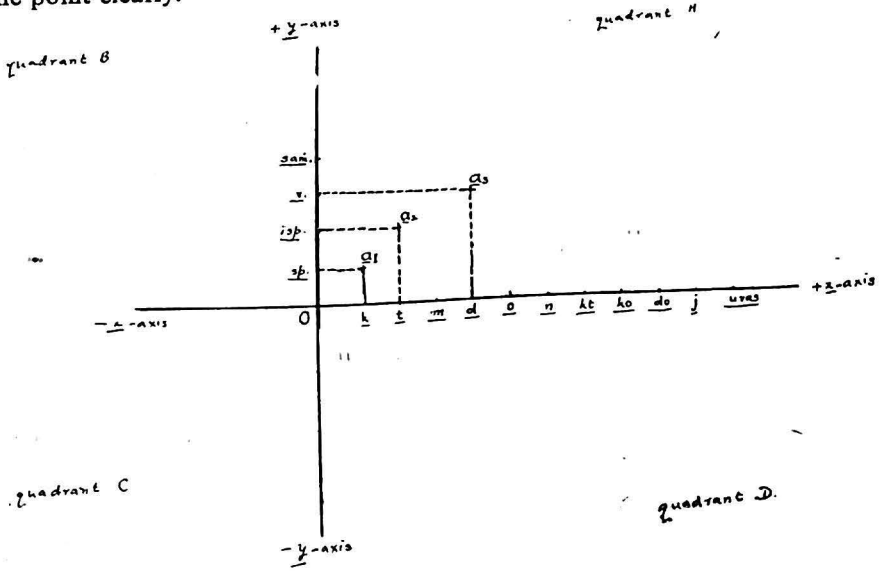


Fig. No. 3

Let x and y be the two axes with the origin O , the axis x representing the places of articulation and y representing the types of internal effort of articulation. The different places and types of internal effort will then be marked on the graph as the points or values of the different sounds. The sound a_1 which has k as the co-ordinate on the x -axis and sp as the co-ordinate on the y -axis will now be clearly marked on the graph. So also the sound a_2 and a_3 . All the possible sounds in the Sanskrit language could thus be pinned down on the graph. The only difference in the present graph and the graph in mathematics is that unlike in the latter, the value of the co-ordinate points on the y -axis in the present graph does not depend upon, or cannot be calculated in terms of, the value of the co-ordinate points on the x -axis;⁸ each of the values is independent of the other since obviously the two axes represent two independent characteristics of the sound. Moreover, unlike in mathematics the values of both x and y do not change; they are constant.

With this background, we can now represent graphically the Pāṇinian *sūtra*, *tulyāsyaprayatnam savarṇam*. 1.1.9, which states the principle of determining the phonetic similarity of two sounds.

8. Take the graph in mathematics, say, $y = 2x$; If $x = 1$, $y = 2$; if $x = 2$, $y = 4$ and so on. The value of y , therefore, depends upon the value of x .

The two points whose similarity is to be found out are both marked on the same graph. The similar phones will be represented on the graph as follows:—

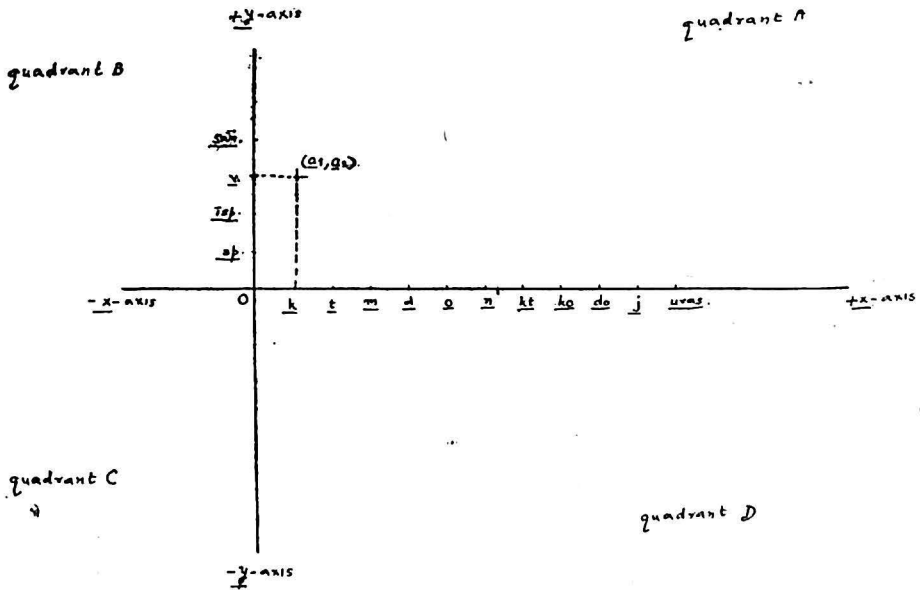


Fig. No. 4

Let the axes x and y represent as before the co-ordinates in terms of place and internal effort of articulation. Let a_1 be the sound with k as the place and v as the internal effect of articulation. It will be marked as a_1 opposite to the respective co-ordinates (see the fig.) Let there be another sound a_2 with the same phonetic description. Now since the phonetic description of a_2 is identical with that of a_1 , the co-ordinate points of both the sounds will co-incide or be identical on the graph. And we actually get that a_2 falls on the same co-ordinates as those of a_1 .

The above graph presumes that both the points a_1 and a_2 are marked in the same quadrant viz. A.

It is also possible to mark the two points separately in two separate, yet adjacent quadrants, say A and B, keeping the y -axis as common. If we draw the two points like that we find that the straight line joining the two points is parallel to the x -axis (see fig. No. 5 below). k_1, t_1 , etc. are the x -co-ordinates of the sound a_1 . and k_2, t_2 , etc. are the x -co-ordinates of the sound a_2 .

9. $k = kaṇṭha$, $t = tālu$, $m = mūrdhā$, $d = danta$, $o = oṣṭha$, $n = nāsikā$, $kt = kaṇṭha-tālu$, $ko = kaṇṭhoṣṭha$, $do = dantoṣṭha$, $j = jihvāmūla$; $sp = sprṣṭa$, $iṣp = iṣat-sprṣṭa$, $v = vivṛta$ and $saṁ = saṁvṛta$, $O = origin$.

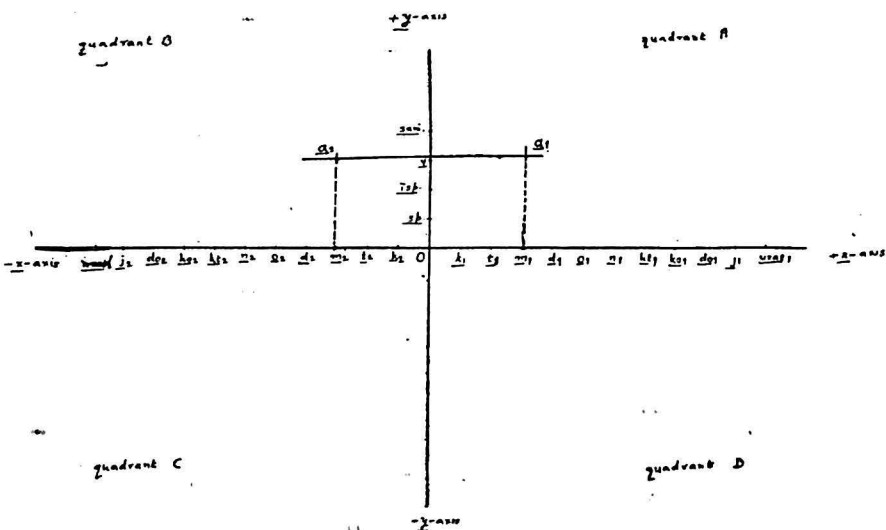


Fig. No. 5

The co-ordinates on both the positive and negative sides of the x -axis are marked in the same order on both the sides of the origin O and have equal distances from it so that $Ok_1 = Ok_2$ and so on. If the graph is in quadrants B and C, the straight line $a_1 a_2$ will be parallel to the y -axis; if it is in quadrants C and D, the straight line will be parallel to x -axis; and if the graph is drawn in quadrants A and D, the straight line $a_1 a_2$ will be parallel to the y -axis again. That the straight line joining the two points is parallel to either of the two axes could be proved mathematically.

The situation is different if we mark the two points separately in separate, non-adjacent, opposite quadrants (see fig. No. 6 below; the quadrants selected are opposite to each other, viz. A and C.).

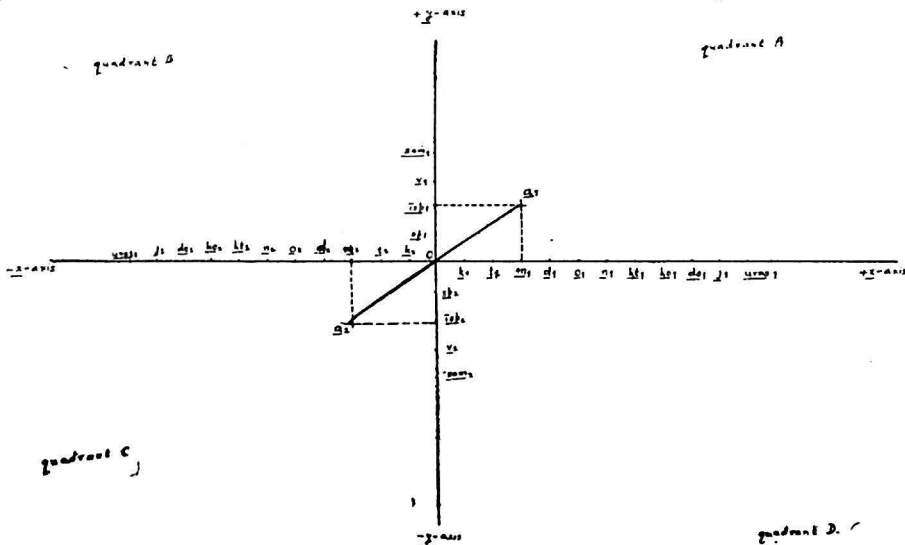


Fig. No. 6

We find that the straight line joining the similar sounds, viz. $a_1 a_2$ passes through the origin. Similar is the case if we draw the graph in the opposite quadrants B and C. Mathematical proofs could be given in this case also to prove that the straight line passes through the origin O .

Graphically represented, the Pāṇinian *sūtra* 1.1.9 gives us three graphical conditions for sounds to be similar to one another. The principle of phonetic similarity can then be graphically enunciated as : Two sounds can be said to be phonetically similar to each other if (1) they co-incide on the graph (as in Fig. No. 4), or if (2) the straight line joining them is parallel to either of the axes (as in Fig. No. 5), or if (3) the straight line joining them passes through the origin (as in Fig. No. 6).

4. The principle of *anuvṛtti* in the Pāṇinian *Sūtra*-system :

It is well-known that *anuvṛtti* or borrowing a word from the previous *sūtras* for the sake of intelligent and consistent interpretation of the following *sūtra* or *sūtras* is one of the wonderful axiomatic devices employed by Pāṇini in the *sūtra*-system. This principle is exploited by Pāṇini to the fullest extent and is conspicuously absent -- with the exception of its scattered employment here and there -- in all other pre-Pāṇinian and contemporaneous *sūtra*-systems. Pāṇini is the first to employ this technique to its perfection and all the post-Pāṇinian *sūtra*-systems owe the debt to Pāṇini.

Before deciphering any principle in the technique of *anuvṛtti* it is necessary to know its working and nature. This will be clear from the following example.

Let us take the *sūtras* 1.3.2-1.3.8, in all seven, laying down the *it*-sounds and the conditions for the sounds to be *it*. The following is the Pāṇinian statement and order of the *sūtras* which are quoted here for easier and clear understanding:—

(a) Pāṇinian statement:

1. *upadeśe ac anunāsika it*, 1.3.2
2. *hal antyam*, 1.3.3.
3. *na vibhaktau tusmāḥ*, 1.3.4
4. *ādir nītuḍavaḥ*, 1.3.5.
5. *ṣaḥ pratyayasya*, 1.3.6
6. *cuṭū*, 1.3.7
7. *laśakvataddhite*, 1.3.8

All of these *sūtras* lay down the different conditions for the various sounds to be *it*; *sūtra* No. 3, viz. 1.3.4, however, prohibits certain sounds to be *it* under certain conditions.

If we read the *sūtras*, we find that except the one, single *sūtra*, viz. 1.3.2. which contains the words *upadeśe* and *it*, all the rest do not have any such words; yet, we have to borrow these words in consistently interpreting the rest of the *sūtras*. While *sūtras* No. 2 and 3, viz. 1.3.3 and 1.3.4 require the *anuvṛtti* of two terms viz. *upadeśe* and *it*, *sūtra* No. 5, viz. 1.3.6 requires the *anuvṛtti* of three terms, viz. *upadeśe*, *it* (both from the

first *sūtra* i.e. 1. 3. 2) and *ādiḥ* (from the forth *sūtra* i.e. 1. 3. 5); *sūtras* No. 6 and 7, viz. 1. 3. 7 and 1. 3. 8 on the other hand are meaningless without the *anuvṛtti* of four terms viz. *upadeśe*, *it*, *ādiḥ* and *pratyayasya* (the last from the fifth *sūtra* i.e. 1. 3. 6). Not only this, but also that the absence of *anuvṛtti* of these terms in places where they are required will create chaos and upset the whole Pāṇinian *sūtra*-system; its absence will hinder any intelligent interpretation of the *sūtras*, will create technical and grammatical difficulties and further lead to interpretational deadlocks, affecting the grammar of the language. The importance of *anuvṛtti* in the Pāṇinian principles of axiomatics can, therefore, hardly be exaggerated.

We know, therefore, that many times and in many places in Pāṇini's *sūtra*-system, we have to read many *sūtras* by assuming the existence of certain words even if they are not there. This assumption of their existence there is based upon their actual inclusion in some preceding *sūtra* or *sūtras*. If, therefore, in the above example, we put the borrowed words in the places in which they are required but are not there, we may perhaps get a clue as to the underlying principle of *anuvṛtti*. The Pāṇinian statements in their solved form will then be:—

(b) Solved Pāṇinian statements :

- 1 *upadeśe ac anuṇāsika it*, 1.3.2.
- 2 *upadeśe hal antyam it*, 1.3.3
- 3 *upadeśe na vibhaktau tusmāḥ it*, 1.3.4
- 4 *upadeśe ādir nītuḍavaḥ it*, 1.3.5
- 5 *upadeśe ādiḥ ṣaḥ pratyayasya it*, 1.3.6
- 6 *upadeśe ādiḥ cutū pratyayasya it*, 1.3.7
- 7 *upadeśe ādiḥ laṣakvataddhite pratyayasya it*, 1.3.8

The dissolved form of the Pāṇinian statements at once now helps us to decipher the principle on which the technique of *anuvṛtti* seems to be employed. We can easily see that the words, which are required to be borrowed are common words in all or some of the Pāṇinian statements. In other words, what Pāṇini did is that he took out all the common words occurring in all the *sūtras* and inserted them in one single *sūtra* which would stand at the head of all its dependent *sūtras*. The concept underlying the principle of *anuvṛtti* is, therefore, purely mathematical, viz. that of taking out of the brackets the highest common factor from amongst the given, numerous algebraic expressions. The Pāṇinian statement of rules, therefore, looks like bracketted algebraic or mathematical expressions with the highest common factor or factors out of the brackets. The point will be clear if we put symbols for all the words and rewrite the *sūtras* symbolically.

Put *a* for the words *ac anuṇāsikaḥ*

<i>b</i>	„	„	<i>hal antyam</i>
<i>c</i>	„	„	<i>na vibhaktau tusmāḥ</i>
<i>d</i>	„	„	<i>nītuḍavaḥ</i>
<i>e</i>	„	„	<i>ṣaḥ</i>
<i>f</i>	„	„	<i>cutū</i>

Put g for the words *laśakvataddhite*

x „ „ *upadeśe*

y „ „ *it*

p „ „ *ādih*

q „ „ *pratyayasya*

and the sign + (plus) for showing the Pāṇinian order of *sūtras*.

We now represent the Pāṇinian statements symbolically as:

$$x.a.y + x.b.y + x.c.y + x.p.d.y + x.p.e.q.y + x.p.f.q.y + x.p.g.q.y.$$

Taking out the highest common factor viz. xy out of the brackets, we get,

$$xy [a + b + c + p.d + p.e.q + p.f.q + p.g.q]$$

We can again take out some more common factors and we get the factorised expression as:

$xy [a + b + c + p \{d + q(e + f + g)\}]$ which is the perfect algebraic representation of the Pāṇinian statement of the seven *sūtras*. While the factors xy are common to all of the *sūtras* (and hence they are taken out of the biggest brackets), the factors p and q are only partially common to some of the *sūtras*.

The principle followed by Pāṇini in employing the technique of *anuvṛtti* in his *sūtra*-system, therefore, seems to be similar to the simple principle of factorisation or finding out the highest common factor from amongst the given expressions in algebra. The highest common factor taken out may be a single word or group of many words.

There are many examples in Pāṇini's grammar in which the technique of *anuvṛtti* is employed. The scope of the *anuvṛtti* varies from two to a number of *sūtras*; cf. the *sūtras* 3.2.95 and 3.2.96 where the *anuvṛtti* of the word *yudhikrñāḥ* is limited only to the two *sūtras*, whereas the *anuvṛtti* of the word *ṭac* extends upto 22 *sūtras* from 5.4.91–5.4.112; while the *anuvṛtti* of the word *dhātoḥ* in the *sūtra*, 3.1.91 covers almost one, entire *adhyāya* (viz. the 3rd) of four *pādas* containing in all 541 *sūtras*, the *anuvṛtti* of the word *prātipadikāt* in the *sūtra* 4.1.1. embraces in all 1190 *sūtras* from two complete *adhyāyas* (viz. 4th and 5th) of four *pādas* each.

The *anuvṛtta* word can either be the subject or what is technically called in Indian grammars as *uddeśya* or predicate or what is called as *vidheya*. The *anuvṛtta* word *it* in the above example belongs to the grammatical category of *vidheya* or predicate while the word *upadeśe* belongs to the category of *uddeśya*.¹⁰

4.1. Types of *anuvṛtti*:

If we examine all the examples wherever *anuvṛtti* is to be resorted to for the interpretation of the *sūtras*, we find three different, broad types:

10. It is to be noted in this connection that according to the Indian Sanskrit grammarians, any category other than *vidheya* is to be included in the category of *uddeśya*; and hence *upadeśe* belongs to the category of *uddeśya* and not to that of *vidheya-vistāra* i. e. extension of predicate.

(1) formal *anuvṛtti* - (2) semantic *anuvṛtti* and (3) semi-formal or semi-semantic *anuvṛtti*.

4.1.1. Formal *anuvṛtti* :

This type is visible in cases in which only the form of the word is to be borrowed; the *anuvṛtta* word there does not refer to any meaning, social or technical. This is the case where the suffixes or *pratyayas* in their form are to be borrowed; cf. the *sūtras* 5.4.91-5.4.92 where the word *ṭac* in the *sūtra* 5.4.91, which is a termination, is to be borrowed in its own form in interpreting all the rest of the *sūtras*; the word *ṭac* does not refer to any linguistic or technical meaning in this context. It is pure *śabdānuvṛtti* i. e. borrowing of the form of the word.

4.1.2. Semantic *anuvṛtti* :

There are circumstances where the linguistic meaning conveyed by the words used in the *sūtra* has to be borrowed in interpreting the following *sūtras*; cf. for example the *sūtra*, *tasyāpatyam*, 4.1.92. Neither the word *tasya* nor the word *apatya* is to be borrowed; what is borrowed in the following *sūtras* is the linguistic meaning that the phrase *tasya apatyam* ('his/her child/progency') conveys. The same is the case with the *sūtras*, *tatra jātāḥ*, 4.3.25, *tatra bhavaḥ*, 4.3.53 etc.

Actually Kāśikā on the *sūtra*, 4.1.92 remarks: *arthanirdeśo'yam* ('this is a reference to the meaning' and not to the form of the words *tasyāpatyam*). This type of *anuvṛtti* is purely on the semantic plane; it is pure *arthānuvṛtti*.

4.1.3. Semi-formal or semi-semantic *anuvṛtti* :

This category is seen where Pāṇini intends a technical term like *prātipadika*, *dhātu* or *aṅga* to be borrowed in the following *sūtra*. The result is that mere physical borrowing of the technical term, say *prātipadika* (in the *sūtra*, 4.1.1), does not work or suffice, since in actual grammatical operation, we have to take into consideration some concrete *prātipadika* like *rāma* or *hari* or *bhānu* which would satisfy the criterion of being a *prātipadika* laid down in the definition (cf. the *sūtras*, 1.2.45; 46). That is to say, a technical term requires that it must be borrowed both physically or formally as well as semantically. This type of *anuvṛtti* is both formal as well as semantic; also, it is neither purely formal nor purely semantic. We may term this type as semi-formal or semi-semantic.

The whole discussion above aims to bring into foreground one important point, viz. that to take out 'only the semantic import' or 'meaning' of the words, detached of their formal counter-parts, as the common factor is certainly a stage further than taking out 'simply the pure form' of the words as the common element. In the case of the pure semantic *anuvṛtti*, Pāṇini seems to have transcended the limits of formal plane. He thus seems to have repudiated the claim that algebraic representation is possible only in the case of formal- and not semantic-categories. And no one

would deny that this is certainly a great advance in analytical techniques if only one takes into account the context of time in which Pāṇini flourished.

We can represent algebraically those *sūtras* also in which semantic *anuvṛtti* is available by ascribing suitable symbols to the semantic categories.

The whole technique of *anuvṛtti* can, therefore, be said to have been based on the same principle on which the technique of factorisation or finding out the highest common factor in algebra is based.

5. Other instances :

The same technique of factorisation or taking out the highest common factor is clearly visible in the definition of *prātipadika* laid down by Pāṇini in the *sūtra*, *arthavad adhātur apratyayaḥ prātipadikam*, 1.2.45.¹¹

The use of the concept of zero indicates the utilisation by Pāṇini of the mathematical principle of 'division';¹² especially the cases where an inserted zero has its non-zero counter-part in other forms lend support to such a supposition. Moreover, the concept of zero itself pre-supposes morphological analysis in terms of position or order of morphemes, bound or free. Pāṇini has actually stated explicitly the position of suffix as *para* i. e. posterior to the *prātipadika*; cf. the *sūtra*, *paraś ca*, 3.1.2. All these points add greater strength to the hypothesis that Pāṇini has utilised certain mathematical techniques and principles in his linguistic description of Sanskrit. The idea of position can only come from mathematical logic.

All this, it must be confessed, presupposes a certain advance in mathematics prior to Pāṇini's times which, in the absence of any pre-Pāṇinian evidence in the field, cannot be proved.

It is at this stage of our study of Pāṇini's grammar very premature to hasten to the conclusion that Pāṇini knew mathematics. The affirmative or negative answers to the questions like whether Pāṇini knew mathematics, or whether he borrowed certain mathematical principles or algebraic techniques to describe the Sanskrit language depend upon stronger internal and external evidences. It is also possible that the ancient Indian mathematicians might also have borrowed certain techniques from Pāṇini.

The question who borrowed from whom, will get its answer only in the context of wider and stronger evidence. The problem could to a certain extent be solved if at least one third, or even one-fourth, of the 4000 *sūtras* of Pāṇini could be suitably represented mathematically. In doing so, it must be borne in mind that linguistic techniques or description can never be represented fully mathematically; that is to say, such mathematical representation of grammatical rules will never satisfy all the rigorous criteria of the mathematical rules. One has, therefore, to make certain allowance for the looseness in such representations. What is important to see in such cases is only the

11. For details, cf. M. D. PANDIT, *op. cit.*, pp. 56-60.

12. Cf. M. D. PANDIT, "Zero in Pāṇini", *Journal of the M. S. University of Baroda*, (Humanities), Vol. XI, No. 1, 1962, pp. 63-65.

basic resemblance of the technical principles employed in mathematics and in Pāṇini's grammar. If the two basically agree, we have sufficient grounds to believe that either Pāṇini has borrowed the principles from mathematics or mathematics owes some debt to Pāṇini.

It is also possible that both might have borrowed from an altogether different yet common source. Can it be logic or mathematical calculus? All this, it must be confessed, would again pre-suppose a certain advance in the field of logic, mathematics and mathematical calculus; unfortunately there is no pre-Pāṇinian evidence in the field. But that certain Pāṇinian and mathematical principles resemble one another cannot be forgotten.



INDIA

OF ADVANCED STUDY

INDIAN INSTITUTE OF ADVANCED STUDY

Author

Acc. No.

Title : Author :

Title :

Borrower

Issued

Returned