Bertrand Russell's Theories of Causation

BY

ERIK GÖTLIND

Almqvist & Wiksells Boktryckeri AB

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To M

Preface

It is scarcely necessary to say that to delve into Russell's philosophical production is an intellectual adventure, even when the object is to consider only a relatively narrow selection from the multitude of ideas and theories which constitute his philosophy, as has been done for this study of causality and causal laws.

A closer study of this particular field requires no justification. Russell's theories of causality are a corner-stone of his philosophy. Many of his other theories are not fully comprehensible without a thorough knowledge of his ideas on causality and causal laws. But his theorising over causal problems is also interesting in itself, apart from its place in the rest of his philosophy, because Russell is one of the comparatively few philosophers who are well acquainted with the literature of physics, and who endeavour to fit their theories to what is really known about the nature of empirical phenomena.

It is perhaps advisable to state that I have made no enquiry into historical facts regarding the origin of Russell's theories. C. D. Broad, in the introduction to his *Five Types of Ethical Theory*, says, "The minute study of the works of great philosophers from the historical and philological point of view is an innocent and even praiseworthy occupation for learned men. But it is not philosophy; and, to me at least, it is not interesting". These are also my sentiments.

It only remains for me to thank all those who have helped me in the writing of this little book. I would first like to express my gratitude to Professor Konrad Marc-Wogau, my teacher in philosophy, for all the trouble he has taken and the good advice he has given me. I wish to thank Fil. Lic. Thorild Dahlquist, Docent Sören Halldén and Docent Per Olov

Löwdin, for the many stimulating discussions I have had with them. I would also like to express my thanks to Mr. Reginald Stedman B. A. for the pains he has taken with the translation of the Swedish manuscript and to Mr. Donald Oman B. A. for many valuable linguistic suggestions.

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Erik Götlind.

Content

A philosopher cannot be an expert physicist, or, at least, we cannot expect him to be. He must therefore accept the physical theories which the experts put before him, after subjecting them to a more or less critical examination. It is therefore quite natural that a philosopher who incorporates in his philosophising about the universe the physical theories which there are good grounds for accepting — and the fact that most expert physicists are for the moment agreed that there is a great deal of evidence for certain theories is undeniably a good ground for accepting them — may change his epistemology to suit the physical theories as they change. There is no arbitrariness or fickleness in this. The philosopher is merely following a maxim formulated by Bertrand Russell, according to which he should not accept anything which there are no good grounds for believing to be true; if experts change their opinion, the philosopher cannot retain those parts of his theory which are undermined by the change without coming into conflict with the maxim in question (unless the experts have made some obvious mistake, or some more complex mistake of a kind which may be exposed by philosophical analysis).

Bertrand Russell, more perhaps than most other philosophers, has had the courage to work in accordance with some such maxim, and this has occasioned many, sometimes revolutionary, changes in his philosophical theories, where they depend on physical theories. This is not least due to the fact that the period during which Russell has written his most important books has seen the introduction of radically new conceptions regarding physical fundamentals. Classical mechanics, with its strict determinism, its absolute time, and action at a distance, has given place to the theory of relativity, according to which the only time which is not just a construction is local time, and action at a distance an impossibility, all processes connected with one another being so connected by processes in the space-time continuum. The revolution implied by quantum mechanics is not less drastic, with its conception of atomic phenomena as discrete, and to some extent unpredictable.

It is therefore only to be expected that there have been changes, following the development of physics and the displacement of one physical theory by another, in Russell's views on the problems we are going to consider, the problems associated with causality, of whether there are relations which we may call causal between events in our universe, the extent to which such relations occur, what is meant by saying that a relation is causal, and so on. These changes are particularly noteworthy where they affect the type of causal law which occur in the more advanced sciences, especially physics, and which Russell calls scientific. The greater part of this book will be concerned with these laws, since they are by far the most important in Russell's philosophy of laws which he calls causal. They constitute the only type of exact causal law which cannot be reduced to some other type he deals with, if we disregard the laws involved in the definition of a thing at an instant, a "momentary thing". Of course, these latter are also very important in Russell's philosophy, but it is only in two books that we may call them causal, and, furthermore, they do not deal with sequences of events.

In his first work treating causality problems in detail, *The Principles of mathematics*,[1] which appeared in 1903, Russell includes only one type of causal relations, those expressed by the laws of physics, with particular reference to dynamics, and including a differential equation as an essential constituent. These laws are the prototypes of the scientific causal laws. The properties Russell assigns to them differ considerably from those which are usually assigned to causal relations. He cannot agree that such relations hold between only two entities, since dynamics provides us with relations enabling us to infer from configurations of a particle

[1] *The Principles of Mathematics*. Cambridge, 1903.

system at two different instants the configuration of the system at any third instant. In other words, dynamics gives us a ternary relation, a relation between three configurations of a system of particles. (The positions and masses of the particles constitute a configuration.) Russell even considers the possibility of general n-ary causal relations, with whose help we may proceed from a knowledge of sets of data regarding events at n-1 different instants to infer a set of data at an nth instant.

These causal laws have a big advantage over the traditional ones of philosophy in that by means of a functional relation they can embrace an infinite number of relations of the type "if A occurs, then B occurs". It is not necessary to formulate a special law for each particle system, nor for each pair of configurations in a system of particles, a law of a type which says that if this particular pair of configurations in this system of particles occurs a certain other configuration will occur at a certain subsequent instant, but we can instead include all these relations in a single formula, which is said to express a causal relation.

It should also be noticed that Russell does not assume that there are any exact causal relations between separate events within a system of particles, such as our world, for instance. It is always a question of complete systems of particles, of configurations of particles within the *entire* universe. The laws which are valid for the parts of a system are always approximate in some degree, even if the margins of error are calculable. The configurations need not have temporal extension. Obviously Russell considers that causal relations may relate entities without temporal extension.

As Russell requires a whole system to be taken into consideration for his causal relations, it is not altogether clear whether he may be said to admit action at a distance or not, because the distance between the causally related events is not only spatial: the configurations occur at different instants. It is true that the instantaneous acceleration of each particle of a system is determined by an instantaneous configuration of the system, and since each particle of the system is involved in the acceleration of the individual particles, we might perhaps say that there is some form of action at a distance between the other particles and something associated with the individual particle. But, according to Russell, "accelerations" are fictive quantities which we introduce for the sake of convenience, and when we speak of three configurations to which the causal relations apply, we do not need to bring in accelerations, neither among the facts we start with, nor among the facts we infer. Accelerations are auxiliary quantities.

In 1912 Russell still stood on the ground of classical mechanics in his essay "On the Notion of Cause"[1], and the changes in his conception of scientific causal laws are insignificant. The criticism of traditional ideas about the relation of cause to effect is extended, and Russell points out, among other things, that the view that a cause cannot exert any influence except in the place where it is situated — that is, that action at a distance is impossible — is a prejudice, which made it difficult to accept Newton's mechanics. This must mean that Russell himself accepts the idea of action at a distance, but he is probably thinking of the abstract form in which the acceleration of a single particle is determined by the configuration of the system at a given instant. It is also stressed that the scientific laws describing the causal relations valid in our universe include a differential equation as their essential mathematical constituent.

The picture begins to alter in *Our Knowledge of the External World* 1914[2] and the alterations are attributed by Russell to the influence of A. N. Whitehead. The causal theories are fitted into a general theory of knowledge in quite a different way, and classical mechanics no longer holds the field unchallenged. The special theory of relativity's conceptions of space and time, and even of things, and the way in which it is established that these entities are constructions from the more primary material of events, are mentioned as the basis of the entire theory of the external world.

[1] "On the Notion of Cause", *Proceedings of the Aristotelian Society*, V. XIII, 1912—13, pp. 1—26. Reprinted in *Mysticism and Logic*, New York, 1918, Ch. IX. The references quoted will be from *Mysticism and Logic*.

[2] *Our Knowledge of the External World*, London and Chicago, 1914.

The primary material, events, is supposed to have extension of some kind. Because of this extension, events may be said to overlap and the concept of overlapping is used to define instants and series of instants. Everything which exists has finite temporal extension, says Russell, and this means that every event appears at two different instants at least. (It is not, however, necessary for the construction of time that all events should have finite temporal extension.) Causal relations now relate events, and they therefore apply to entities which have finite temporal extension, whereas they were previously taken to apply to configurations occurring at given instants, without finite temporal extension. This is the most important change.

The Analysis of Mind,[1] 1921, says nothing notably new about scientific causal laws. The part played by differential equations is again stressed, with the addition of "finite-difference"equations if quantum mechanics is correct. However, in *The Analysis of Matter* 1927,[2] the work in which the analysis of causality problems is carried furthest in many respects, and in the more popular *An Outline of Philosophy* of the same year, the changes are considerable. These two books were written at the time of the great discussion about the fundamentals of physics which was carried on in the twenties, and they are largely a philosophical analysis of the results achieved by physics. Russell is now considering the possibility that the way of regarding physical phenomena that had lead to differential equations in the laws describing them might not be applicable in quantum mechanics. Quantum mechanics gives us chains of events consisting of bounded events for which we can say that causal relations hold, these relations being such that they would not become more exact if events farther off were taken into account. These causal relations are therefore different from those of the approximate type which Russell considered permissible in classical mechanics for individual particles of a system. The theory of relativity also

[1] *The Analysis of Mind*, London and New York, 1921.

[2] *The Analysis of Matter*, London and New York, 1927.

[3] *An Outline of Philosophy*, London and New York, 1927.

provides Russell with reasons for thinking that causal relations apply only to bounded events close together in the space-time continuum. Indeed, it proves to be very difficult to say precisely what is meant by "the state of the universe at a given instant", or "the configuration of all the particles of the universe at a given instant"; and the action at a distance which could be referred to previously now disappears. We may regard the acceleration of a particle (relative to a system of which it is a part) as a function of the surrounding gravitational field. If one spatio-temporal event is to affect another, there must be some process which connects the two events.

Russell's attitude to the question of whether a principle of causality could be correct which stated that all phenomena are predictable from the data we have about the universe has also been influenced by the results of quantum mechanics. Earlier, it seemed that he at least found no decisive reason for rejecting the idea that such a principle could be correct, although there was no sound argument for such a principle either; whereas it now seems that he thinks that there are strong reasons against such a principle, since individual quantum changes appear to be unpredictable, as they occur in a heated gas or at a radioactive breakdown, for instance.

In a paper entitled *Determinism and Physics*,[1] 1936, Russell discusses the question of whether Heisenberg's principle of indeterminacy gives us reason to doubt the validity of determinism, where determinism must be taken to mean that everything which happens is predeterminable; and he answers that the principle does not provide us with any reasons against determinism. This viewpoint is abandoned in *Human Knowledge*, 1948,[2] where it is stated that the principle of indeterminacy seems to indicate an insurmountable difficulty in that it always leaves a certain margin of uncertainty in determinations of such quantities as the future status of an elementary particle.

[1] *Determinism and Physics*, The 18th Earl Grey Memorial Lecture, 1936. The Librarian, Armstrong College, 1936.

[2] *Human Knowledge, Its Scope and Limits*, London, 1948.

The conception of the nature of scientific causal laws which we find in Russell's most recent extensive work on the theory of knowledge, *Human Knowledge*, is not notably different from that expressed in *The Analysis of Matter*. However, there is divergence on one point. Contrary to his view in the earlier work, Russell now says that he cannot accept the view that causal connections imply only regular sequence. Something more than this is required for the relation to be causal, although he does not say what this is.

After *The Principles of Mathematics*, Russell has described relations other than the scientific causal laws which he would be prepared to call causal. In "On the Notion of Cause" and subsequently, the causal terminology has been applied to a type of approximate, pre-scientific relation between bounded events, not too precisely defined, such as the putting of a coin in a slot-machine and the reactions of the machine, or the striking against a box and the ignition of a match. All the relation need express is that when the one event occurs the other will probably follow. It may be that in *Human Knowledge* Russell requires that when an event of one type is really followed by an event of the other, they should be related in a definite way, in a similar manner to events related by scientific causal laws.

In *Our Knowledge of the External World* we meet a new type of law reappearing in the subsequent works: laws which express relations applying, approximately at least, to events that occur in the neighbourhood of what is a thing from a commonsense point of view. Russell uses these relations to define a thing at a given instant. He says in *Our Knowledge of the External World* that it is a question of laws of similarity and continuity. In *The Analysis of Mind* and afterwards, they are called "perspective laws", and they are said to express principles of projective geometry. It seems that they are regarded as causal in *Our Knowledge of the External World* and in *The Analysis of Mind*, but in *The Analysis of Matter* it is evident that they are not regarded as causal, because to some extent they deal with events separated by intervals which may be space-like in the relativistic sense.

A final, fourth type of law appears in *The Analysis of Mind* and *An Outline of Philosophy*, laws which are purely psychological, and to which the causal terminology is applied. In *Our Knowledge of the External World*, Russell has given up his earlier epistemological view, which we might express roughly as the view that most of what common-sense regards as real is real and that there is a distinct difference between mental and physical phenomena, and he has adopted instead a view which may be described as a form of objective phenomenalism. According to this, everything which exists is of a mental nature, or anyway, it could be experienced if there were anyone at the place of its occurrence. This view is most consistently formulated in *The Analysis of Mind*, and a natural feature of it is the prominence of possible laws relating individual events in the same "mental biography". However, Russell is not certain that these "mnemic causal laws", as he calls them, are not reducible to physiological, which in their turn are reducible to physical, in such a manner that they remain a special category merely because of our lack of knowledge. They are no longer given as a special type in *Human Knowledge*.

Thus the number of types of relations which Russell wishes to call causal increases from one in *The Principles of Mathematics* to four in *The Analysis of Mind*, decreasing in *Human Knowledge* to one again, or perhaps to two if the pre-scientific laws are to be regarded as a type of their own.

An important point in Russell's theories regarding causal problems is the Humean question of the ultimate reasons for assuming that causal laws apply to a certain region. This question is not considered in *The Principles of Mathematics*, as a matter of fact, but it is treated thoroughly in *The Problems of Philosophy*.[1] Russell concludes that a principle of induction supports us in the assumption of causation, an *a priori* principle which can neither be verified nor refuted, but which can yet be seen to be correct. But it is doubtful whether we can really derive from this principle the support that it is supposed to give, because of a probability concept

[1] *The Problems of Philosophy*, New York, 1912.

involved which was taken from Keynes. It appears that Russell continued to hold the view that this principle is a sufficient reason for assuming that causal laws occur — or anyway, for assuming that in some of the cases where we think we have grounds for applying causal laws these laws are actually valid — until *The Analysis of Mind* of 1921. In *An Outline of Philosophy*, Russell begins to ask whether a principle involving Keynes's probability does not tell us too little, and even whether we have any grounds to act on statements of the type "there is such and such a probability that something will happen" if we employ the Keynesian concept. He thinks that a concept drawn from some frequency theory would be preferable, if there were some frequency theory which did not involve such considerable difficulties as those which occur in the theories with which he is acquainted.

In *Human Knowledge* the dominating question is that of the ultimate postulates to be adopted to support the assumption of causal laws, and in general, to provide a sound basis for the working methods of science. Russell endeavours to find postulates more fundamental than the principle of induction, postulates which will enable us to make inductive inferences in some cases and inferences by analogy in others. One of them may be regarded as itself a causal law, while two others enable us to infer the existence of causal laws of a certain type. So we do not need a principle of induction in order to justify the assumption that causal laws are valid in some region at least: it is one of the fundamental propositions of all our theorising that there are causal relations.

It is obvious that there are many interesting and important connections between Russells' causal theories and the rest of his philosophy. Indeed, it may be questioned whether there are not many important points which can hardly be understood without a closer knowledge of Russell's ideas about causality; for instance, his theory of the constitution of matter, and his theories about our ability to acquire knowledge of the external world, among others. However, the connection between the causal theories and the rest of his philosophy has not been dealt with in this book.

It is well worth considering, but that would take another volume. The present analysis is confined to Russell's theories of causality and causal laws, and the rest of his philosophy has only been considered where it has been found advisable to do so for an understanding of the theories of causation.

Chapter I

The first proper discussion of problems associated with causation in Russell's works is to be found in *The Principles of Mathematics*, published in 1903. In this chapter we will consider primarily that book.

It is possible to trace two leading principles in what Russell has to say there: the analysis of causal propositions should lead to a result which is expressible in the formal logical notation developed in the book, and the causal propositions should be of such a form that it is possible to say that the laws of physics, especially those of dynamics, express causal connections.

The second of these two principles is put forward by Russell in opposition to the view which takes physic to be merely descriptive, a view held by Mach and Kirchhoff among others. For Russell, the decisive reason against this descriptive outlook seems to be that if it were correct, we would not be able to infer what happens at a certain instant from a knowledge of what happens at certain other instants. All such inferences involve a relation of implication between events at different times, and, according to Russell, this relation is causal.[1] One of his main tasks is therefore to show that physics makes use of causal connections, as this refutes the descriptive conception.

But the logical nature of causal propositions must be established before this can be accomplished. Russell begins by analysing a proposition which is supposed to express causation between particulars. But he says that there is no such causation, and therefore continues his analysis by examining propositions of a more general character.

To begin with, he rejects the possibility that we should be able

[1] *The Principles of Mathematics*, p. 474.

to say that A causes B when the statement "A precedes B" is true, where "A precedes B" does not contain any reference to given instants (A and B representing actual or possible "temporal existents", for instance, events), as well as the possibility that we may formulate a principle of causality with approximately the following content: For every event B we can find a definite series of events which always and necessarily precede B.[1]

The reason given for not accepting such a theory is that there cannot be temporal succession directly between events, but only between instants of time. An event is a quality at a particular instant, says Russell in an article in *Mind* to which he refers.[2] A particular event can only occur at a single instant.[3] Since an event is always referred to a certain instant, the statement "the event A precedes the event B in time" cannot express anything more than a relation between two given events A and B, even if "precede in time" were a relation which could apply to events. This in itself is a sufficient reason that "A precedes B in time" cannot be taken as an analysis of "A causes B".

But the same difficulty recurs with the proposition in which Russell gives the result of the analysis of the logical nature of causal propositions. This reads: "There is an interval t such that A's existence at any time t_1 implies B's existence at a time t_1+t".[4] The interval t between two instants at which A and B occur is finite, according to Russell, because the time series is compact, that is, there is always a third instant between any two instants, however close together. Two instants cannot therefore be consecutive; there must always be a finite interval between them. A and B cannot in this case denote events, since events always occur at particular instants. The expression "A's existence at any time t_1" would be meaningless with such an in-

[1] *Op. cit.*, p. 475.

[2] "Is position in time and space absolute or relative?" *Mind*, N. S. No. 10, 1901, p. 294.

[3] Thus a process lasting a finite time cannot be an event; but it contains events. Many events may occur at the same instant.

[4] *The Principles of Mathematics*, p. 477.

terpretation. Another possibility is that A and B represent "possible existents", and that these "possible existents" are not events. According to Russell everything which can have a causal relation to something else is a "possible existent",[1] where "existent" is an undefined concept.[2] Numbers and other sorts of abstract entities do not stand in a causal relation to anything, says Russell, nor do points of space or moments of time.[3] But we need only observe that events may be "existents" and may be denoted by "A" and "B" in order to see that the difficulties do not disappear because the formulation is considered *even* to cover cases where A and B are events.

If we assume that "A" and "B" denote separate events, which cannot occur more than once according to Russell's theory, and if we wish to say something more in our analysis of "A causes B" than just that these two events occur in connection with one another, the following is a possible interpretation of the proposition previously quoted: There is an interval t and a class of events K, and another class of events K_1 such that if A is an element in K which exists at an instant t_1 the existence of another event B is implied which is an element of K_1 at the instant t_1+t. The events contained in each of the classes K and K_1 have certain properties in common.

This formulation, like that which was criticised above, may be expressed in terms of the logical apparatus developed in *The Principles of Mathematics*, a necessary condition if it is to be accepted

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[1] *Op. cit.*, p. 476.

[2] *Op. cit.*, p. 36.

[3] *Op. cit.*, p. 476. In an article called "Causality, Introduction and Probability", *Mind* 1919, p. 169, Philip Jourdain asserts that causal laws apply to numbers in Russell's theory, since the differential equations which Russell takes as examples of physical laws expressing causal relations do at the same time express a relation between numbers. This criticism may be disregarded if a distinction is made between the mathematical relation R which applies to numbers and the relation $PR\breve{P}$ which applies to empirical phenomena, where P expresses a correlation between phenomena and numbers and $\breve{P}$ the inverse correlation. It is $PR\breve{P}$ which expresses the causal relation, not R.

by Russell. He consistently endeavours to put mathematical and scientific statements in such a form that they may be formalised. In the formal language of *Principia Mathematica*[1] (not yet developed in *The Principles of Mathemattcs*) Russell's formulation may be written as follows:

$$(\exists t)\,(t_1)\,(A(t_1) \supset B(t_1+t)) \qquad (1)$$

$A(t_1)$ denotes A's existence at t_1, $B(t_1+t)$ B's existence at t_1+t. The form modified so that A and B may be events is:

$$(\exists t)(\exists K)(\exists K_1)(t_1)(A)(A\varepsilon K \cdot A(t_1) \supset (\exists B)(B\varepsilon K_1 \cdot B(t_1+t))) \qquad (2)$$

Formal implication is involved in these cases according to the terminology in *The Principles of Mathematics*. That the implication is a formal implication follows from the fact that it takes all material implications of a certain type to be correct — in this case it is t_1 which takes different values giving rise to a set of material implications.[2]

However, as Russell himself points out, there are reasons for believing that causal statements which are actually applicable are more complicated than (1) or (2). One of these is that (1) and (2) refer to isolated events; certain difficulties arise if we assume that such events are causally related when we try to reconcile this assumption with dynamics' description of the way events succeed one another in our world. One such difficulty is "the antinomy of causation" which Russell dealt with in *The Philosophy of Leibniz*,[3] and which he takes up again in *The Principles of Mathematics*. The antinomy arises when a collection of particles has an effect which depends on the separate parts, although the effects of the individual parts are non-existent, that is to say, we cannot point to a particular effect and ascribe it to a particular particle. This means that when one system of particles influences another system of particles, it is not possible to point to certain

[1] *Principia Mathematica*, V. I, Cambridge 1910.

[2] *The Principles of Mathematics*, p. 41.

[3] *A Critical Exposition of thePhilosophy of Leibniz*, Cambridge, 1900, pp. 98 and 136.

isolated events in the one system as being the cause of particular isolated events in the other. Consequently, the causal relations between the systems cannot be described by (1) or (2); causal statements which serve to describe them must be of a more all-embracing character.

Another of Russell's reasons is that he wishes to avoid "the paradox of independent causal series"[1]. If it were possible to isolate a particular event A as the cause of another event B, which in turn were the cause of an event C, we would obtain independent causal series which lead to difficult epistemological problems. For instance, how can we know of more than one such series, namely, the one to which we belong, without finding it necessary to accept some such theory as that we have in some mysterious way an image of the whole universe within us, or that there are within us processes which in some manner mirror the course of events outside us, with which we have no causal contact.

This leads Russell to an analysis of causal statements from other points of view. The final results is the following: "given m events at any moment, m other events at a moment whose distance from the first is specified, and so on till we have n groups of events, then m new events can be inferred at any new moment whose distance from the first is specified, . . .".[2] The values of m and n depend on the kind of events in question.[3] The form of causal connection described by the statement is suitable for dynamics, says Russell, since in a material system of N particles, $m = N$ and $n = 2$ according to dynamics; and the whole of our world may be regarded as a system of particles. This means that two configurations of events, each of which contains a description of the world, the system of particles, at a particular instant, imply a third similar configuration of events. We will return to this idea later.

A peculiarity of Russell's expression for a causal statement is

[1] *The Principles of Mathematics*, p. 476. [2] *Op. cit.*, p. 479.

[3] Presumably, Russell means by this among other things that if every sequence of events were regulated by a differential equation of the rth order, n would be equal to r.

the condition that there must be equally many events at each instant. Russell's explanation of this condition seems to indicate that the type of causal system he has in mind is one in which there are exactly m different events at each instant, and which is therefore determined by n classes of m events at other instants if causation of the type described by Russell applies to the system. According to Russell, the condition is a consequence of the circumstance that causation occurs only when we can make unambiguous inferences about events at any instant whatsoever from other events of a certain class of events.[1] He says that for this to be possible two conditions must be fulfilled: (1) when we have n groups of events from which we infer another group of events, every one of these $n+1$ groups must be implied by the other n groups; (2) the condition already mentioned, that the $n+1$ groups of events contain the same number of events.

However, closer analysis of the question shows that neither (1) nor (2) need be fulfilled for us to be able to make unambiguous inferences about events at any instant. Russell thinks that if we are to be able to make unambiguous inferences about events, the process by which we infer the $n+1$th group of events may be repeated both into the future and into the past. This means that if we have e_1 events at t_1, e_2 at t_2, . ., e_n at t_n, and infer e_{n+1}, (t_i preceding t_j when $i < j$), we may repeat this procedure, taking e_1 of the e_2 events at t_2, e_2 of the e_3 events at t_3, . . ., e_n of the e_{n+1} at t_{n+1}, and infer e_{n+1} new events at a new instant t_{n+2} later than t_{n+1}. We might proceed in the same way to infer events at instants t_{n+3}, t_{n+4}, and so on. In order that this may be possible, we must have $e_r \leq e_{r+1}$ for all r, since if some $e_k > e_{k+1}$ we could not take e_k events of the e_{k+1} at t_{k+1}. A corresponding argument applies to inferences into the past, according to Russell, which gives us the condition $e_r \geq e_{r+1}$. Therefore the only possibility is that $e_r = e_{r+1}$ for all r.[2]

The idea behind this assumption that we must be able to repeat the process either forward or backward in time seems to be that

[1] *Op. cit.*, p. 478.

[2] *Ibid.*

it ensures that we are able to infer events at all instants of the future and past.[1] But we may advance against this that if t_{n+1} may be chosen quite arbitrarily, as is stated in the final formulation, we can reach any instant without having to repeat the process, and if it is not necessary to repeat the process, we do not need to take $e_r = e_{r+1}$ for any r.[2] It seems that Russell adopted implicitly the premiss that causation requires that the process should be repeatable regardless of whether we can infer events at every instant in some other way. Perhaps the conception of causal systems of the type described above, where the number of different events is constant and the configuration of the system at any instant is determined by the configurations of the system at n different instants, lead Russell to adopt this assumption, especially since, according to classical mechanics, our world is of this type.

As regards the first condition, that each group of events is implied by the other n groups of a set of $n+1$ groups when some group is implied by the other n, it is not necessary for unambiguous inferences about events at all instants. But we may regard it as an independent condition which Russell considers must be fulfilled in order that there should be causation.

(1) and (2) bear a certain relation to one another: If (1) is fulfilled, so is (2), if we suppose that none of the $n+1$ groups of events may contain more events than are necessary for the inference, and t_{n+1} may be chosen arbitrarily, since all e_i must then be equal to e_{n+1} on the assumption that e_{n+1} takes the same value for all t_{n+1}. On the other hand (1) follows from (2) if e_r, which is the same for all r, is equal to the number of different events in the system at instant t_r, for all r, assuming that t_{n+1}

[1] *Ibid.*

[2] If t_{n+1} cannot take more than one value in the future, it is not certain that we can reach any instant of the future by repetition of the process, because there might be a case where the interval between t_{n+a} and t_{n+a+1} decreases with increasing a with a rapidity so great that we get a series of intervals which has a convergent sum. There would then be a limiting instant beyond which we could not infer any event by repetition of this process.

may be chosen quite arbitrarily and that each n groups of e_r events permit the inference of a group of e_r new events. Russell has presumably assumed such a system in his final formulation, which only says that $e_r = m$, that is, that (2) is fulfilled, without saying that (1) need be fulfilled.[1] It might be that from m events at t_2, m at $t_3, \ldots$, m at t_{n+1}, we could infer m events at t_1, events quite different from these which were used to infer the m at t_{n+1}, if there are at least $2m$ different events at t_1. In general we may say that if there are more than m events at any instant, (1) need not be fulfilled. It is obvious that the uniqueness of the inference need not be destroyed in this situation.

To sum up: It seems as if Russell considers the following conditions to be necessary if a system is to be causal. (1) That the procedure by which new events are inferred from groups of other events should be repeatable into the future and into the past,[2] as a consequence of which the number of events in the different groups of events involved in the inference is the same for all groups. (2) That each group of events (at the same instant) involved in the inference is implied by the other groups of events involved in the inference. The arguments in support of (1) and (2) themselves contain a condition which must be fulfilled if there is to be causation, although (1) and (2) are not consequences of this condition, which must be regarded as independent. Russell motivates (1) by saying that it ought to be possible to make inferences at all instants of the past or future. This condition, which may be called (3), does not oblige us to assume that it must be possible to carry out the repetition described. The reason for (2) is that it must be possible to draw unambignous inferences to events at any instant, but the property of uniqueness does not vanish if events may be implied, for example, at an instant t_1 by n groups of events after t_1 which are quite different from those events at t_1 which were previously used, together with the n–1 following groups to infer the group

[1] Quoted on p. 25.

[2] The past here means the time preceding the instant at which the first group of events figuring in the inference occur, and the future the time subsequent to the last group.

at the last instant t_{n+1}. The third condition, the one which is contained in the motivation of (1) and (2), is therefore the condition that it must be possible to make inferences about events at all instants. If this further condition (3) were not included then, amongst other things, it is not certain that we would be able to infer events at an instant between instants at which known groups of events occurred. Regarding Russell's final formulation of a causal statement, it follows immediately that (3) and (2) are fulfilled, but it tells us nothing about (1). If we suppose that Russell adopted a system in which the number of events at a given instant which determine the system in certain respects at that instant is one and the same whichever instant we choose and that this number is the same as the m which occurs in the formulation quoted, then (1) is also fulfilled.

If m and n are put equal to 1, we get a proposition which is similar to but not identical with the formulation of a causal statement obtained as the result of the preliminary analysis[1]: if one event is given at a certain instant, we can infer an event at any instant whose distance in time from the first is given. This statement is altogether different from the result of the preliminary analysis, and this latter is therefore not a special case of the final result.

The final formulation may be used to accomplish what Russell set out to do: to show that physics, and particularly dynamics, is not purely descriptive, but that it is possible to formulate causal statements within its framework. If, as in classical mechanics we regard our world as a system of N particles whose relative positions at a particular instant are taken to be events, physics permits us to make causal statements where $m=N$ and $n=2$. This means that the configuration of the world[2] at any instant is determined by the configuration of the world at any other two instants. This proposition may be expressed in the formal mathematical language used in dynamics as the statement that the position of every particle at a particular instant t is a function of the

[1] *The Principles of Mathematics*, p. 477. (p. 22 of this chapter).

[2] Equivalent to the positions of all the separate particles.

coordinates of all particles at two instants t_1 and t_2, together with t, t_1 and t_2.[1]

According to Russell, this form of causation is different in two respects from the one usually accepted by philosophers: (1) the causal relation connects three events or groups of events, not two, and (2) that which remains constant for different cases of causation of a certain type is a certain relation between the events which are said to be causally connected in each case, and not just the occurrence of a particular event which is repeated, and which is followed by another particular event each time it occurs. (1) depends on the form of the equations giving us the coordinates of the particles. They are actually second order differential equations. If we needed to work with differential equations of higher order, it would be necessary to have more than two configurations to determine a new configuration. (2) contains one of the chief results of physics: that there are constant properties associated with a train of events which are independent of the instantaneous situation of the changing system. One consequence of this, according to Russell, is that an infinite class of causes (two configurations may be regarded as a cause) may stand in the same relation to some effect (for example, a configuration) among an infinite class of effects. If this were not so, says Russell, we would require an infinite number of causal laws of the traditional type — same cause, same effect — to describe a continuous movement during a finite interval of time, however brief this might be.[2]

If we accept the sort of causation which Russell has described, we can also solve the problem of causation with regard to separate events. There is no such causation; it is always configurations of the whole system which determine new configurations. The antinomy of causation thereby disappears.[3] There is never any question of effects associated with particular parts of the system.

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[1] *Op. cit.*, p. 486.

[2] *Ibid.*

[3] The antinomy (mentioned on p. 24) was that the whole system did not experience any effect which was not a result of the influence of the individual parts, although these individual influences could not be isolated.

More or less in passing, Russell mentions another solution of the antinomy. He says that the only effect which is traditionally ascribed to particles is an acceleration of the individual particles. But the acceleration of an individual particle with respect to the other particles is not an event or a physical fact, but a mathematical limit.[1] Such a limit cannot be a "possible existent", according to Russell, and cannot be causally related to anything. But an objection which might be advanced against this reasoning is that the mathematical limit is certainly a number and therefore cannot be causally related to anything; but this number is attached to a phenomenon in the same way that the numbers giving the coordinates of a particle are, and this phenomenon may be causally related to something.[2] The fact that the acceleration of a particle in classical mechanics has a number associated with it which is defined in physical theory as a limit does not necessarily make acceleration any less measureable than any other physical quantity. Perhaps it is possible to conceive other descriptions of the physical reality based on other measureable properties, where the phenomena with which in classical mechanics numbers defined as mathematical limits are associated are no longer associated with such numbers. It must be regarded as a property of the system if a measurable phenomenon is associated with numbers defined in the system as a limit. But something which is not a property of the system is a certain relation between the numbers associated with different phenomena. Such relations must occur in all systems intended to describe how the phenomena in question are related to one another.

The theory that no individual event can have effects is accompanied by difficulties which Russell considers in the development of certain ideas often recurring in subsequent works. The important difficulty concerns the possibility of knowing the effect of the whole system on a single part. If we cannot obtain detailed knowledge about the whole of our world, we cannot calculate the

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[1] *Op. cit.*, p. 487.

[2] Some problems connected with this are mentioned on p. 23 note 3.

course of events in particular parts of it.[1] When we make calculations about the movements of individual bodies in the solar system, we regard the solar system as if it were the whole of our universe, says Russell. But how can we be certain that the fixed stars, or the universe beyond them, does not influence our solar system in such a way that the regularities we have observed hitherto may suddenly cease to exist. Russell's answer to this question is that the gravitational laws we assume to apply to the whole universe give such small values for the changes in the solar system due to changes in the distribution of mass outside a sphere with radius R and centre at the sun, that we can assign a very small upper bound to the effects within the solar system, in whatever way the outer universe changes, the bound being partly determined by R. Because of this circumstance, we can regard our solar system as a minor universe where gravitational disturbances from the rest of the universe do not exceed a certain small amount. In "On the Notion of Cause", Russell calls such systems, which, by virtue of the adopted laws, are subject to only a limited disturbance from external factors, *relatively isolated.*

Russell also gives a more exact mathematical formulation of the condition for a system to be relatively isolated gravitationally, or, putting it more precisely, the condition that the disturbing effect from the universe outside a sphere of radius R on the relative acceleration of two particles within this sphere should be less than a certain limiting value. "If ε be any number, however small, there should be some distance R such that, recurring to our previous function Φ[2], if $\frac{d}{ds}$ denotes differentiation in any direction, then

$$\frac{d}{ds}\int_r^\infty \Phi(r)\,dr < \varepsilon \text{ if } r > R\text{"}$$[3]

[1] *The Principles of Mathematics*, p. 487.

[2] The function by means of which the relative acceleration of two bodies is calculated. Defined in *The Principles of Mathematics*, p. 484.

[3] *Op. cit.*, p. 487.

Russell considers this inequality to be fundamental. He states that it would not be possible to test a law in practice if a corresponding inequality did not apply. The example of gravitational attraction varying directly with distance is mentioned, and Russell says that in this case it would be impossible to test the law other than in a finite universe.[1] With such direct variation, changes of a certain order of magnitude would have the same effect on the relative acceleration of two bodies however far away the changes were. If we assume that we cannot know about an infinite number of events, and in any case, not about events which occur at an infinite distance, it appears impossible to calculate the changes of the relative acceleration of two given bodies since all the factors relevant to the calculation are not available.

That the laws of motion allow a part of an independent[2] system to which these laws apply to be disturbed by the rest of the system only within certain limits, and that two configurations of the system determine every other configuration of the system, and that our world constitutes such an independent system are characteristic features of Newtonian and Herzian dynamics, according to Russell. He says that if we do not assume that the laws of motion for our world are of this nature "other principles will give a science having but little resemblance to received Dynamics".[3]

Even if we agree that Russell has been successful in his attempt to find a formulation of causal statements which permits the interpretation of propositions occurring in classical mechanics as causal (and has thereby attained his object of showing that physics contains non-descriptiv elements), the question remains of whether there are propositions in modern physics which are causal in Russell's sense of the term. Both quantum mechanics and relativity theory have pointed to difficulties which make it impossible to use statements about our world which are causal

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[1] *Op. cit.*, p. 488.

[2] A system is independent when it is causally unrelated to anything else, that is, it contains everything which is causally related to anything within the system.

[3] *Op. cit.*, p. 496.

in this sense. That is not to say that physics is purely descriptive, since it still uses statements about relations between observed phenomena and events in the future, although these are somewhat different from those in classical mechanics. Until *The Analysis of Matter* 1927, Russell adhered in the main to the view put forward in *The Principles of Mathematics* that two configurations of our world imply[1] a configuration of the world at any instant whatever. In *The Analysis of Matter*, problems arising from description of our world in terms of quantum mechanics and the theory of relativity are considered in more detail, and because of the discontinuous phenomena described in quantum mechanics, Russell thinks it necessary to abandon the attempt to find a causal description of the world containing causal statements of the sort defined in *The Principles of Mathematics*. But we will consider this point in Chapter VI.

There is little mention in *The Principles of Mathematics* of laws of general causation such as "everything has a cause" or "everything is both a cause and an effect", but the few references provide us with a comparatively clear picture of Russell's views about the possibility of general causation in some form or other holding in our world. If we were to suppose that causal relations of the type described by Russell are valid this general causation would mean that everything which exists at a particular instant is included in a group of simultaneous events, which, together with a series of equally large groups of events, implies another group of events at any chosen instant, and it would mean that every quantity which exists at any instant is implied by some assemblage of equally large non-simultaneous groups of simultaneous events. This signifies that everything which exists at any instant is causally related to something else, which must also exist at some moment, as Russell affirms in the preliminary discussion about the form of causal statements. He says there that if anything is not causally related to something else, it

[1] It is difficult to say what "imply" signifies. It seems that Russell uses "A implies B" and "B can be inferred from A" as synonyms.

cannot exist, from which it follows that everything which exists at any instant is causally related to something.[1]

But this does not mean that everything which exists is both a cause and an effect, for what we might call a cause in Russell's theory of causality is an assemblage of groups of events, and the effect a series of events, which cannot individually be causes of anything; it is only when combined with other events that they become parts of causes. There is one exception to this: when $n=m=1$ for some region, that is, when the number of groups required for the inference of another group is 1 and the number of events in the separate groups is also 1. From the point of view of the reasoning, it is immaterial whether such a region exists; it is sufficient to observe that dynamics describes a region which is not of this type in order to establish the falsity of the statement that everything which exists at some instant is both a cause and an effect.

The fact that this can be established has some interest for Russell, because he says that if we adopt the principle that everything which exists is both a cause and an effect, we are confronted with consequences which are difficult to accept. For instance, (1) we would obtain a reason for the infiniteness or circularity of time[2], and (2) it could be proved that if there

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[1] *Op. cit.*, p. 476.

[2] It is not quite clear what Russell means by the infiniteness of time, but the following interpretation is feasible. That time is infinite means that the compact series of instants of time is infinite. This series has stretches (a stretch is the aggregate of terms between two terms of the series, *The Principles of Mathematics*, p. 181) and distances (a relation between two terms in the series, *op. cit.*, p. 171). We can interpret the statement that the series is infinite in terms of either of these quantities, since both of them have magnitude. If the magnitudes assignable to stretches or distances in a given series may be ordered in a sequence which has infinity as its limit (*op. cit.* p. 189) the series is called infinite. For a stretch the magnitude is expressed by its magnitude of divisibility — *op. cit.*, p. 412. (For the measurement of the magnitude of divisibility, see *Princ.* p. 178; for the measurement of the magnitude of distance, p. 180.) Presumably the circularity of time signifies a certain periodic recurrence of every configuration of the universe. Circular series for which all the terms recur in the

are events at any instant, then there will always be events.[1] However, it is possible to raise some objections to the statement that the principle has these consequences. If we first consider (2): the statement "there have always been and always will be events" may be interpreted in two ways, (a) that there have been and always will be events at all instants, and (b) that there are always instants but that there is no instant before which there were no events and no instant after which there will be no events.

The assumption that there are events at every instant does not necessarily imply (a), since, according to Russell, the events do not last for a finite time, but occur at a single instant, so that two events cannot be consecutive in the sense that they occur in consecutive intervals or at consecutive instants, since the instants form a compact series. Consequently between any two events which are taken to be causally related there is always at least one instant t at which there is no event causally related to the two given events. If we suppose that there is only a finite number of events at an instant t_1, there is consequently always an instant t in the neighbourhood of t_1 at which no effects of the events at t_1 occur; and however many events we suppose to occur at t_1, they do not necessarily have effects at every instant in the neighbourhood of t_1. All this assumes that a cause cannot have more than one effect, for if we suppose, for example, that a cause has effects at every succeeding instant and that an effect has causes at every preceding instant, (a) follows at once from the assumption that something exists which is both cause and effect.

This may be compared with Russell's theory of causality. It

same order have neither measurable stretches nor measurable distances if it is not possible to distinguish the different appearances of the same term, because the rules for the addition of magnitudes do not apply in this case. Whether time is infinite or not is an empirical question. Russell says that one instant does not logically require an earlier instant (p. 147), and we may suppose that neither does he regard one instant as logically implying a later instant. The length of the time series therefore depends on empirical facts.

[1] *Op. cit.*, p. 476.

follows from that theory that if we only have a single set of events as cause (in the Russellian sense) of something, we know that there are events at every instant, since the set implies events at every instant whose distance from the first instant at which some event of the set occurs is specified. This means that (a) is valid whenever general causation in this sense applies to any region.

Neither does the interpretation (b) imply that (2) is a consequence of the principle that everything which exists is both a cause and an effect, unless we assume that there is more than one event which is both a cause and an effect. If we suppose that there is an event at an instant t_1 which is the cause of some other event at t_2, we know that this event is itself the cause of another event at another instant t_3, and so on. A chain of events extending into the past may be obtained in a similar manner. In this way we obtain a series which contains an infinite number of events, extending into the past and into the future, but this does not entitle us to conclude that the length of time covered by this series is infinite, which would mean that (b) is true. It might be that the distance between the instants at which cause and effect occur decreases for each new event which we infer, both forwards and backwards in time. For instance, suppose that we begin with an event A at an instant t, and that the effect A_1 of A occurs after an interval t_1 and the effect A_2 of A_1 after an interval $\frac{1}{2}\,t_1$ (counted from A_1) and the effect A_3 of A_2 after an interval $\frac{1}{2}\cdot\frac{1}{2}\,t_1$ (counted from A_2) and so on. In this case, the infinite series does not extend beyond the instant $t+2\,t_1$, and this would mean that (b) is not true, for possible instants after $t+2\,t_1$.

Another consequence of the above reasoning is that the principle is not a strong reason for (1). It is not necessarily true that time is infinite or that it is circular as a consequence of the principle as long as we do not exclude the possibility of chains of events for which the time intervals between successive events form a convergent series. However, both (2b) and (1) remain valid if we do exclude such series, for instance, if we assume that the interval between cause and effect is always equal to a fixed number k. (2) with the interpretation (b) is also preserved by the assumption that a cause has effects at every succeeding instant and an effect causes at every preceding instant, in the same way as this assumption preserved (2) with the interpretation (a); though not (1), since it is not possible to infer anything about the extent of time from the assumptions that there are events at all instants. It is not excluded on logical grounds that the series of instants forms an open finite interval, that is to say, the series is bounded both forwards and backwards but does not contain instants on the actual bounds. In this case, each instant has an instant before it and one after it (and therefore infinitely many both before and after it as the series of instants is taken to be compact), without the series being infinite or circular.

Russell's assumption that everything which exists is causally related to something (in his sense of the term) does not imply (1), since it only leads to the conclusion that events occur at all instants, and provides us with no conclusion about the extent of time. The series of instants need not even form an open interval, because the causal relation does not require any effect later than the last instant for one of the events in the set of events constituting the cause, and similarly backwards into the past when we consider causes; it only requires that there shall be events at every other instant than those at which events of the set in question occur.

To sum up, we may say that if (1) and (2) are to be consequences[1] of the principle that everything which ever exists is both a cause and an effect, all that is required in the case of interpretation (b) of (2) is an assumption which excludes the possibility that the intervals in a causal chain may form a convergent series, either backwards into the past or forwards into the future. For (1) and (2) with interpretation (a), what is required is that a cause should have effects at all succeeding instants and that an effect

[1] The case where the principle does not imply (1), but, as Russell puts it, constitutes a reason for (1), is not discussed, because the principle does not by itself constitute a reason for (1) in any other sense than that it is in conformity with (1).

should have causes at all preceding instants, and further that there should for every cause be at least one effect whose distance in time from the cause is not less than a certain value t_1, and among the causes for each effect one distant in time from its effect not less than a certain value t_2.[1] This latter is a condition to ensure that there shall be at least one causal chain beginning from a given event for which the intervals do not form a convergent series. Another possibility is to assume that the series of instants is discrete and that the cause and effect always occur at consecutive instants. We could then be certain about (1) and (2 a), and therefore also about (2 b).[2] There is no reason to suppose that Russell did adopt any of the above assumptions for the causality he discusses, so we cannot be certain that (1) and (2) follow from the principle. Russell does not actually use this principle, but instead a similar one, according to which everything which ever exists is causally related to something else. It follows from his theory of causal relations that there are events at all instants if there is any one event, but we cannot deduce anything about the extent of time.

A problem to which Russell devotes a good deal of attention in his later works is the connection between induction and causation, and it is touched upon in *The Principles of Mathematics*. But more precisely, the problem is the following: Must we support causal propositions about events which have not been observed, e.g. events in the future, by means of some principle which is not a principle of causation, that is, not a principle which states explicitly that causality applies to all regions or that everything which exists is causally related to something else, or something of the sort, although the principle does provide us with reasons for accepting such general principles of causation? As early as *The Problems of Philosophy*, Russell answers this question by saying

[1] This condition is sufficient, but there are weaker conditions. They must, however, all give us a divergent sum of timeintervals.

[2] On account of what is said in *The Principles of Mathematics* (p. 193) about the distance between terms of a discrete series, it seems natural to say that time is infinite when the discrete series of instants has no last term and no first.

that we are bound to accept some sort of principle of induction in order to infer unobserved events, but in *The Principles of Mathematics* there is still no theory of induction which would enable such a principle of induction to be developed. He there says that what is usually called induction seems to him to be either deduction in disguise or a "method for making plausible guesses.".[1] In enquiring into what we should understand by an empirical premiss, however, Russell describes a process of which he says that it contains the essence of induction, although it cannot be said to be a form of deduction or a method for making more or less reliable guesses; it might be described as a procedure which there are methodological reasons for using if we wish to build up a comprehensive and unambiguous theory of empirical phenomena.[2]

All that is said about the certainty of causal propositions is that the propositions of a causal nature which are stated in dynamics – e.g. the laws of motion – are not *a priori* truths valid in all worlds. It is not even certain that they apply to our world: "Whether they are ultimately valid, or are mere approximate generalisations, must remain doubtful".[3] This is no contradiction of the postulate that everything which ever exists is causally related to something else; it merely says that our descriptions of the valid causal relations are not necessarily correct. But exactly what is said about our world in the postulate that everything which ever exists is causally related to something else is not discussed in *The Principles of Mathematics*. The difficult question of whether or not "causal relation" is defined in such a way that there *must* be causation in our world, which would mean that the statement "everything which exists is causally related to something" becomes analytical, does not appear before "On the Notion of Cause", where the question of what further conditions should be applied to causal relations in order that they need not necessarily be valid is also taken up. It becomes apparent that the definition of causal relations in *The Principles of Mathematics* is too wide. We will return to this later.

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[1] *Op. cit.*, p. 11.

[2] *Op. cit.*, p. 441.

[3] *Op. cit.*, p. 488.

Chapter II

The first of Russell's publications after *The Principles of Mathematics* to make any important new contribution to the problems associated with causality is *The Problems of Philosophy*, 1912.[1] The analysis of the form of causal relations is not continued in this work, and no principles are formulated of the type "everything which ever exists is causally related to something." But the book gives the first consideration of a problem which occupies a good deal of Russell's attention from then on, gradually assuming a more and more important position in his theory of knowledge, to become the dominant question in *Human Knowledge*, 1948: the question of what reasons we have for assuming that an observed regularity will apply in the future as well.

It is possible to cast doubt on all conclusions about events in the future or the past which are drawn with the help of knowledge about events now, or, more generally, all conclusions about events at a particular instant from knowledge of events at other instants, since there are no logical reasons for accepting such conclusions. The problem may be stated thus: Is there any non-logical principle which enables us to draw conclusions about non-observed events from observed events. Russell's answer is that there is such a principle, the principle of induction, and that if we do not accept such a principle we have no reason to infer from our experience of regularities that causation occurs in any region.[2]

The induction which is thus accorded so important a place is the one merely mentioned in a footnote in *The Principles of*

[1] There are a few remarks about problems of causality in an article "Determinism and Morals" in *The Hibbert Journal* 1908, but they are too few and too unprecise to justify any analysis of them.

[2] *The Problems of Philosophy*, p. 69.

Mathematics, For Russell's ideas about induction in *The Problems of Philosophy*, John Maynard Keynes seems to have had an important influence, Russell having read Keynes's then unpublished work "*A Treatise on Probability*", which first appeared in 1921.[1]

Russell follows Keynes in dividing the principle of induction into two cases, one dealing with induction from an assemblage of events of a particular type and all having a certain property to the statement that the next event of the same type has this property, and the other dealing with induction from an assemblage of events having a certain property to the proposition that all events of this type have this property. The first type of induction (which Keynes calls "inductive correlation") is formulated by Russell in the following way: "(a) When a thing of a certain sort A has been found to be associated with a thing of a certain other sort B, and has never been found dissociated from a thing of the sort B, the greater the number of cases in which A and B have been associated, the greater is the probability that they will be associated in a fresh case in which one of them is known to be present; (b) Under the same circumstances, a sufficient number of cases of association will make the probability of a fresh association nearly a certainty, and will make it approach certainty without limit."[2] The second type ("universal induction" in Keynes's terminology) is formulated in a similar way except that it is concerned with the probability that *A* always occurs together with *B*, instead of with the probability that *A* and *B* will occur together the next time that one of them occurs.[3] It is the latter type of induction which is important in this connection, since it is this principle of induction which, together with empirical

[1] Russell says in the preface to the first edition that he derived a great deal of assistance from unpublished writings of J. M. Keynes on probability and induction. In *Our Knowledge of the External World*, 1922, when dealing with induction, Russell refers to Keynes's work of 1921, without any alteration of his own text from 1914.

[2] *The Problems of Philosophy*, p. 66.

[3] *Op. cit.*, p. 67.

data, gives us a reason for assuming that causality applies to a certain region.

We have *a priori* knowledge of these principles of induction, says Russell, as we have of certain logical and mathematical propositions, and even ethical propositions and principles about the intrinsic desirability of certain things, such as "knowledge is more desirable than ignorance".[1] When our knowledge of these types of propositions is said to be *a priori*, it means that we consider them to be true in the light of our experience of a limited number of events, although the truth of the propositions can never be proved by empirical events.[2] This explanation of *a priori* does not make us any more certain of what is *a priori* knowledge than of what is knowledge in other cases. It appears to be possible for a person to think he has an *a priori* knowledge of a certain proposition and for another person to think he has an *a priori* knowledge of another proposition which is inconsistent with the first, and there even seems to be nothing to prevent a single person from thinking him having an *a priori* knowledge of two inconsistent propositions, as long as he is unaware of their inconsistency. For instance, it is possible to formulate principles which are very little different from the principles of induction put forward by Russell, although they are not reconcilable with these latter, and some people may accept them. Of course, this does not prevent any of these principles from being true, and does not prevent our knowledge of the true principle being merely *a priori*, if we really do have knowledge of it, but its *a priori* character is no guarantee of our being aware of its truth.

We may ask whether the principles of induction formulated by Russell can really be applied as Russell intended them to be — among other things, to provide a reason for assuming that causality applies to a particular region. To be able to answer this question, we must know what is meant by the "probability" which is mentioned in both of them.

We can derive a certain amount of information about the mean-

[1] *Op. cit.*, p. 76.
[2] *Op. cit.*, p. 74.

ing of "probability" from Russell's reasoning showing that the principles are neither verifiable nor refutable. He says that if we have observed a large number of swans which are all white, the second of the two principles allows us to draw the inference that all swans are probably white. This inference is not incorrect if we subsequently see a swan which is black, because the probability is estimated on the basis of the information available at the moment. However many black swans we see after we have made the inference that it is probable that all swans are white, these observations do not affect the correctness of the proposition, but we could not infer such a proposition from the observational material available at this later stage.[1] Thus new experience cannot refute our principle. Neither can it be verified by new experience, since without a principle of induction, we do not know what experience we will undergo in the future.[2] Nor is it possible to obtain verification of a generalisation by a complete check of all cases covered by the generalisation, since without the principle of induction we cannot say anything about the possibility of new cases not included among those we have checked. We cannot conclude that there will be no new cases of a certain sort of event in the future if we are not in a position to apply some principle like Russell's principle of induction.

Entirely similar reasoning is valid for the first principle. We are thus led to the conclusion that the principles of induction are neither verifiable nor refutable by any experience which we may undergo. The same applies to definite statements that the probability of the occurrence of a certain event when another event occurs is of a particular magnitude, statements which we arrive at with the help of our principle of induction and certain empirical data, since, as Russell maintains, the probability is based entirely on these data, and is independent of data derived from later events.

According to Keynes, to whom Russell refers, the probability is an indefinable logical relation between a group of propositions

[1] *Op. cit.*, p. 68.

[2] *Ibid.*

M (about our empirical data) and the proposition *H* which becomes probable on the basis of *M*. Keynes says that with the help of this relation we can accept the proposition "*H* is true" with a certain measure of rational belief when we have *M*, or, at any rate, we may regard "*H* is true" as acceptable with a certain measure of rational belief even though we ourselves may not have this belief.[1]

We might therefore take the principle to mean the following: The larger the number of cases where we observe that a thing of type *A* occurs together with a thing of type *B*, the higher the degree of rational belief which we can assign to the statement "things of type *A* always occur in association with things of type *B*", provided we have not observed any case where *A* was not associated with *B*. Whether or not the degree of rational belief is numerically measurable does not affect the content of the principle; that only tells us whether or not there are certain formal relations between the probabilities.[2]

But we cannot use such a principle to infer anything about future events, or to support the assumption of causality in a certain region, unless we know the relation between the fact that a rational belief of degree *p* is assigned to the proposition "things of types *A* and *B* are always associated" and the actual accurrence of things of types *A* and *B* together. It is conceivable that the principle might be acceptable even though there were no permanent regularity in our world. It is possible to assign a high degree of rational belief to a proposition "there is a permanent regularity *S*" without as a consequence knowing anything more about the actual occurrence of the regularity than we should have known if the degree of rational belief assigned to the proposition had been smaller.

Russell considers this difficulty in *Human Knowledge*, where he states that the probability which occurs in the principles of

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[1] Keynes, *A Treatise on Probability*, London, 1921, p. 6. The probability in this sense is not always measurable, nor is it possible to compare a particular probability with every other probability.

[2] *Op. cit.*, p. 158.

induction must not be an undefinable entity. If it is an undefinable logical relation, it may be that the principle of induction is correct, says Russell, although every inference made with its help proves to be false; nor can we find any empirical reasons for or against the principle, nor can we employ it to draw inferences about future events. If we are to be able to use the principle for generalisations of the type implied by the assumption of causality in some region, "probable" must mean "what in fact usually happens".[1] This indicates that Russell no longer regards the principles of induction he formulated in *The Problems of Philosophy* as serving the purpose he there ascribes to them, if he was there making use of the concept of probability in Keynes's theory, as he evidently was.

It is not necessary, however, to interpret the probability as a frequency in order that the principles may serve their purpose. It is possible to define the relation previously discussed in such a way that when a high degree of rational belief (a large probability) is assigned to a proposition such as "every A' is a B'" on the basis of an experiential material M, it is at the same time usually true that "every A' is a B'" for a material M_1 which is similar to M in all relevant respects. It is obvious that the probability is not a frequency just because of this relation; it may still be an undefinable entity. If Russell adopts such a relation in *The Problems of Philosophy*, though there is no explicit mention of it, it is possible to use the principles in spite of the fact that Keynes's concept of probability is involved.

However, two arguments may be put forward against the adoption of such a relation: (a) the assumption that such a relation continually applies is just such an assumption of a regularity as we wish to make probable with the help of the principles of induction, and (b) in cases to which the principles might be applied, we have assumed that there are certain regularities which are always valid (their frequency is greater than zero), and we therefore have certain reasons for believing that a particular regularity

[1] *Human Knowledge*, p. 420.

observed in a material M is one of these; but this means that, as in case (a), we have assumed something which we wish to make probable with the help of the principles of induction.

The result of this discussion is therefore as follows: either Russell considers that it is not necessary to define any relation of the type described above, and in that case the principles of induction cannot be used for the intended purpose, — amongst other things, as support for the assumption of causality in some region, or Russell has assumed a relation which makes it possible to apply the principles of induction to that which they were intended for; but in this case he has assumed something which was to have been made probable by means of the principle.

Russell does not treat Keynes's discussion of the conditions to be fulfilled in order that we may be able to prove that the principles of induction should be applicable to given cases. According to Keynes, two conditions must be fulfilled: (1) the proposition "every thing of type A is associated with a thing of type B" must to begin with have a certain finite *a priori* probability, which is not obtained by inductive reasoning — as Russell points out in *Human Knowledge*[1] — and which furthermore involves no assumption that there are regularities. (2) the probability that all n observed cases of things of type A will be associated with things of type B when the general proposition does not apply shall differ from 1 (or complete certainty) by a finite amount.

It is clear that even if these two conditions are fulfilled and if it is possible to prove that the principles of induction may be used for a certain specific case, we do not therefore know more about the actual occurrence of things of type A together with things of type B, as long as the previously discussed relation (or some of the same kind) between the fact that a certain degree of rational belief (a certain probability) is applicable to a proposition "in all cases where A' occurs, B' occurs" and the actual occurrence of A' and B' together is not assumed.

Keynes's postulate that there are only a limited number of independent types of property ("the principle of limitation

[1] *Human Knowledge*, p. 453.

of variety"), by which he ensures that the first condition is fulfilled, involves the assumption that there are regularities, something which we wished to establish with the help of the principles of induction. The assumption that the world's "generating properties" are finite in number and less than a certain given number is not sufficient; they must also always remain the same. For, suppose that they were not, and that, for instance, all of them were one at a time replaced by new ones during the course of a certain definite interval of time, and later changed back again (the new properties not being amongst the old), the exchange lasting sufficiently long for the new properties to appear together with many of the others; in this case it might be that no inductive inference could be correct, nor could we have any *a priori* probability that certain properties always occur together, despite the fact that the number of properties in combination is always finite and unchanging. Or suppose all the generating properties were occasionally exchanged for a new set, then we would have no use for our principles of induction, even if they were always correct, since they are of no avail in inferring anything about the state of the world at a subsequent instant. In order to be able to obtain an *a priori* probability for the associated occurrence of certain properties, and even in order to be able to use our inductive inferences, we must assume that the generating properties are always the same. Thus Keynes has not proved that it is possible to use the principles of induction without introducing assumptions of a sort which we should have made acceptable with the help of the principles themselves. If our conclusion here is correct, neither will what Russell says be true when he states that the principles of induction are our only reason for accepting causality in a certain region. This holds even if the relation which has been discussed were defined in a way which gave us some knowledge of what it is that a proposition which is probable expresses (indirectly through the relation) about the actual world, because we have already used certain statements that there are regularities without the support of the principles in question.

As to whether the principles of induction must be *a priori* or not when Keynes's concept of probability is adopted, it must be said that there is no method of deciding empirically whether the undefinable logical relation called probability by Keynes applies to quantities in the manner stated by Russell in his principles of induction; nor is there any empirical method which could show that the undefinable relation does not apply. So, if the principles are correct, our knowledge of them can only be *a priori.*

They are, however, not obviously correct in the sense that whoever were to reflect about the problems in connection with generalisations from a given empirical material would arrive at them or be inclined to postulate them. The view, for instance, that all regu laritiesmay have some exception and that the possibility of exceptions from the regularity would not be affected by the fact that we observe the regularity in an ever-increasing number of cases does not seem to be absurd or even unreasonable.

Let us then sum up the discussion: The problem which Russell has formulated in *The Problems of Philosophy* is: what principles must we adopt in order to be able to infer from a certain experiential material propositions about events which are not included in this material, in a scientific description of the world which assumes that deviations from observed regularities are purely a matter of chance, and one of whose main principles is that if the world is a causal system, in the sense that everything in the world is causally related to something else, it is a causally isolated system, that is to say, everything causally related to something in the world also belongs to the world. But the principles which Russell formulates do not seem to accomplish this aim.

Russell first gives a special treatment of the problems associated with causation in a paper entitled "On the Notion of Cause", which was read before the Aristotelian Society in 1912 and published in the society's journal. The question about which most of the essay revolves concerns which principle occupies the same place in the sciences as philosophers, according to Russell, suppose to be occupied by traditional causal laws or general principles of causation like 'same cause, same effect'. Sometimes Russell is inclined to the view that it would be better to abandon causal terminology altogether, and not to call any new relations causal, not to speak of "cause" and "effect" longer, but he still maintains that there is a statement about certain relations occurring in the world which has more right to be called a "law of causality" than any law "to be found in the books of philosophers".[1]

In order to establish this, Russell begins with a criticism of what he regards as the most usual theories of causality and causal relations. His object is to show that science cannot apply any of these theories, and he accomplishes this by showing that certain statements about causal relations such as "same cause, same effect" say too little to be of any use, and that certain other statements, e.g. that the connection between cause and effect must always be of the same sort as the connection between an act of the will and the action which we think is linked with this act, are false. Also to be included in the critical part of the essay is the analysis of some common ideas about determinism and free will and about determinism and teleological theories, where Russell makes use of the positive results he has arrived at in his theory of causation. As in *The Principles of Mathematics*, Russell does not recognise

[1] *Mysticism and Logic*, p. 195.

as causal any expressions used in science for causal connections which do not contain a mathematical function, or, more precisely, relations not including differential equations.

However, there are some new points. What is to be understood by a "deterministic system" is defined by means of a general functional relation; and he considers the difficult problem which arises in this connection, of limiting the allowable functions so that "determinism applies to all worlds" is not an analytical statement, although there is no proposal which would give a complete solution of the problem.

Perhaps the most important extension of the theory of causation is that Russell now recognises an entirely new type of statement as causal, besides those which express a functional relationship. These statements are of a more approximate character, and do not give us such certain knowledge of actual events as do the others, and they are therefore not used in science, but only on a pre-scientific level. They do, however, fulfil the requirement for causal statements that they may express something about events or phenomena which are not part of our experiential material, but which are nevertheless determined by that material.

If we have observed that a certain regularity has prevailed up to now (without exception), we may infer that it will probably continue to prevail in the future, according Russell. In order to make this inference we must employ a principle of induction, which is admittedly open to doubt, but which we assume without discussion for the sake of the reasoning.[1] Russell does not say anything more precise about the form of this principle, but it seems likely that he refers to the second of the two given in *The Problems of Philosophy,* both because the book appeared the same year that "On the Notion of Cause" was delivered, and because it is *the* inductive principle in the definite form which is referred to, as if there were only one and it is only the second which is relevant in this connection. If what was said in the preceding chapter about the possibility of using the inductive principle described in *The Problems of Philosophy* to infer the actual occurrence of some-

[1] *Op. cit.*, p. 192.

thing from a certain empirical material is correct, the only propositions we are entitled to make about new events with the aid of the principle state that a certain degree of rational belief may be assigned to them, or, more exactly, to propositions about the occurrence of new events. But we cannot say anything about the actual occurrence of these events. This criticism would also apply to "On the Notion of Cause". The probability statements which we can make on a basis of the principle of induction do not fulfil the condition that they should deal with the actual occurrence of events which are not included in the initial material. It is clear that probability statements of the type Russell works with may say something about events not included in the initial material, although not about their actual occurrence.

The fact that it is the actual occurrence which interests Russell appears when he says that the assumption that there is a causal regularity may be refuted.[1] If such an assumption merely implied that we assign a certain degree of rational belief to the proposition "there is a regularity S" under certain conditions, this could not be refuted, as Russell himself points out in *The Problems of Philosophy.*[2]

For the sake of the discussion which follows, let us suppose that there is a principle of induction which can be used as Russell wishes to use it, and let us now consider the theory of causal regularity which he develops. Suppose we often observe a particular combination of two events in this case, of which we call the first "the cause" and the second "the effect". "Event" is here used in a broader sense than in *The Principles of Mathematics*, where an event was a unique happening confined to an instant of time. But it must be possible for our event to recur, if a series of events is to recur. So we now mean by "event" a universal with sufficient breadth of definition for it to recur if need be.[3] Striking a match against a box, insertion of a particular kind of coin in an automatic machine, etc., are events in this sense.

[1] *Op. cit.*, p. 194.

[2] *The Problems of Philosophy*, p. 68.

[3] *Mysticism and Logic*, p. 187.

Russell says that the relation between cause and effect which we have described differs from the "traditional" causal relation in four respects.

(i) The relation is not a necessary one. All it says is that it is probable that when A (the first event in the series) occurs, B (the second event) also occurs near to A, and we support this statement with a number of observations of A and B together. Russell says that A may even be the cause of B in the sense which we are discussing, even when there may be cases where B does not follow A. The actual form of Russell's statement is not entirely unambiguous. Two conceivable possibilities are the following; (a) A may be the cause of B even though we observe cases where B does not follow A. Russell says that striking a match against a box is the cause of the match igniting, even though some matches may be damp and therefore do not ignite,[1] — an experience which most people have probably had. This example lends credibility to interpretation (a), but it also leads to certain difficulties. We cannot arrive at the conclusion that B will probably occur when A has occurred without employing Russell's principle of induction. But when it is formulated in *The Problems of Philosophy*, it is explicitly stated that we must not have observed any case in which B does not follow A. If we assume that a principle of induction such as Russell would consider applicable here contains such a clause, our first interpretation cannot be correct. Let us then try another interpretation (b), whereby we are not allowed to have observed cases where B does not follow A, but where the possibility is not excluded that such cases have actually occurred or may occur in the future. Then we are able to use the principle of induction. If we have a series of observations at a certain time which allow us to say that A and B form a causal sequence, and if we afterwards observe a case where A occurs but not B, we can no longer say that A and B form a causal sequence because we cannot now employ the principle of induction. So a sequence may be called causal until we find an example to the contrary.

[1] *Op. cit.*, p. 193.

(ii) All that is necessary in order that we may call a sequence causal is that it has occurred sufficiently often, says Russell, and he regards his causal concept as different from the "traditional" in this respect also.[1] In this sense, night is the cause of day, in spite of the fact that we can easily imagine happenings which would prevent day from following a night. Russell criticises Mill for saying that we cannot accept the sequence of day and night as causal because it is so easy for us to imagine a state of the world where there would be no such sequence, and because a causal sequence must be thought of as unexceptional and unconditioned and he says that any causal sequence may have an exception. So the causal concepts of Mill and Russell are different.

(iii) As regards the other two respects in which the causal concept under discussion is said to differ from the "traditional", one of them might be described as a difference in the assumptions as to whether general causation applies or not: "it will not be assumed that *every* event has some antecedent which is its cause in this sense".[2]

(iv) In the other case it is rather a question of similarity: Russell says that laws of this causal type tend to be replaced by laws of quite another type as soon as a science has firm ground under its feet. They are only used in a pre-scientific stage. But this is just what Russell says about causal laws of the "traditional" type, that they are of no use in the advanced sciences, though philosophers have sometimes supposed that it is just there that they would be used.[3] It is simplest if we take it that in this last respect Russell just means that his causal concept differs from what some philosophers *thought* with regard to more traditional causal concepts.

Russell says that the laws used in science are quite different from the approximate sort described above. For instance, in the laws of gravitation there is nothing which we can call the cause and nothing we can call the effect; all that we have is a mathematical formula. It is not just a question of the same cause always

[1] *Ibid.* [2] *Ibid.*

[3] *Op. cit.*, p. 186. Russell states that Bergson belongs to this category.

having the same effect, but instead a question of one and the same relation between two assemblages of events. It should be noticed that the mere circumstance that there is a formula which describes the movement of mutually gravitating bodies is not sufficient to make it impossible that something in the relation could be called a 'cause' and something an 'effect'. Not, of course, in the "traditional" sense of these terms if that is taken to mean that the effect is brought about by the cause in some way or is a logical consequence of it, but in the sense that they are well-defined and it is likely that they will recur, and that it is possible to build causal chains, that is to say, that it is possible to regard the effect in its turn as the cause of a new effect or that the cause is itself an effect of another cause.[1]

However, Russell considers that for our world the appropriate formula are such, — for phenomena described by physics,[2] at least — that we cannot even use cause and effect in the above restricted sense. The relevent formulae are specified as differential equations, which are, moreover, of not more than the second order: "Certain differential equations can be found, which hold at every instant for every particle of the system, and which, given the configuration and velocites at one instant, or the configuration at two instants, render the configuration at any other earlier or later instant theoretically calculable."[3] This is a concenttrated expression of the ideal conception of causality in classical mechanics, which Russell put forward in *The Principles of Mathematics*.

It has already been mentioned that this formulation is not in accord with the quantum mechanical picture of the world, and it also contains difficulties which arise for reasons connected with relativity.[4] However, the fact that "certain differential equations"

[1] The difficulties which arise if we allow every conceivable mathematical relation to figure in causal relations is discussed later on in connection with deterministic system, p. 66.

[2] *Mysticism and Logic*, p. 194.

[3] *Ibid.* The calculable configurations include those occurring between the two initial configurations.

[4] There is further discussion of this point in Chapter VI, pp. 114—118.

have been mentioned may be disregarded in the analysis of the proposition quoted above, and we may just take it that "certain equations" are referred to. If only a few of the more important propositions of quantum mechanics are true, it means that differential equations are not sufficient in themselves, even though they constitute part of the relations between our observables. We do not lose anything essential when we replace "certain differential equations" by "certain equations", since, according to Russell, the important thing is that we can use these equations to express relations between an infinite number of cases without having to provide a new causal law for each new case (a law saying that the relation applies to that case also). It is similarity between the relations that scientific causal laws must express, not merely that the same cause has the same effect.[1]

The principle "same cause, same effect" need not conflict with the fact that there is a similarity between relations which we wish to find, since if A is one of the phenomena described in the first part of the relation, and if the relation in question is many—one or one-one, the same quantity is obtained in the second part of the relation every time A occurs. This is not affected by the fact that it may be extremely improbable that two identical cases should occur. According to the "traditional" view of causation, the assumption that a certain event A is always followed by a certain event B is based on the observation of a series of cases where A and B have been encountered together,[2] while in Russell's version, if A has been observed in association with B, the assumption that this association will persist in the future is not based on other observations of A and B together, but on observations of other pairs A′ and B′, which are related in the same way as A and B.

Russell formulates the proposition that causal laws (of the type

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[1] *Op. cit.*, p. 195. Also *The Principles of Mathematics*, p. 486.

[2] There is clearly also a difference if the causal relation is taken to be logical, since the relation would then be found from an analysis of A, whereas for Russell the assumption of causation in a certain region is ultimately based on the inductive principle.

used in science) apply to our world, regarded as a system of particles, in a manner which apparently leads to difficulty. He says: "There is a constant relation between the state of the universe at any instant and the rate of change in the rate at which any part of the universe is changing at that instant, and this relation is many-one, i.e. such that the rate of change in the rate of change is determinate when the state of the universe is given."[1] This "law of causality " is incorrect, as has already been pointed out. It expresses the belief that every course of events in the universe regarded as a material system may be described by differential equations of at most the second order, which is approximately the same as saying that the acceleration of every particle of the universe is determined by the configuration of the universe at any instant. It is difficult to see how it is possible to calculate coming events from a knowledge of the state of the universe at any one instant, since this law of causation does not deal with relations between entities at other instants than the given one and entities at the given instant, but instead talks about entities which exist at the same instant. The difficulty disappears if we remember that the acceleration requires the movements to be continuous for a small interval of time around the instant to which the relation refers. When the law says that the relation is constant, this means that the relation applies to every instant, even to instants in the future. This ensures an unbroken continuity in the movement of bodies from the present moment to an instant in the future at which events occur that we wish to calculate. This is a vague description of a fact which has a very exact mathematical description, but it suffices for our present purpose.

Russell appends some explanatory remarks to this law which is the one he says has a better right to be called a law of causality than those laws which are so called in the books of philosophers.

(1) It is not *a priori* or a necessity of thought or even self-evident.

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[1] *Op. cit.*, p. 195.

It is merely an empirical generalisation. By making observations in a number of fields described by physics and discovering the laws for this description, it has been found that, in classical mechanics anyway, all processes which occur in our universe may be described by differential equations of the second order at most.

Russell does not say what we are to understand by "*a priori*", but we may suppose that the expression has the same significance as in *The Problems of Philosophy*. He says in that book: "some knowledge is *a priori*, in the sense that the experience which makes us think of it does not suffice to prove it, but merely so directs our attention that we see its truth without requiring any proof from experience".[1] This provides us with a definite mening for "*a priori*". The fact that the law is not *a priori* in this sense, that we cannot have *a priori* knowledge of the law, follows from the fact that it is refutable as it stands. The fact that it is refutable must mean that it cannot be *a priori*, since as soon as we can think of some logically possible experiment to refute a law, or some state of the universe to which it would not apply, we can hardly say that we can see that the result of the experiment would be thus or thus.

(2) Russell also stresses that according to the law the past is determined by what happens now in exactly the same sense as the future is. "Determine" has a special meaning here: a variable is determined by certain other variables if it is a function of them. When it is said that one term of the relation includes the rate of change in the rate of change at a given instant t, nothing in this point to a difference between past and future and presumably Russell means that when we make calculations to unobserved events these may be brought out in the same way whether the value of the interval between t and the instant at which the unobserved events occur is positive or negative.

(3) As in *The Principles of Mathematics*, Russell states that the law is not empirically verifiable if we cannot find an isolated system

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[1] *The Problems of Philosophy*, p. 74.

for which the state of the rest of the universe does not influence events to any appreciable extent. There is no mathematical formulation of this idea such as there was in *The Principles of Mathematics*.[1] But something new is a distinction between two types of causally isolated system — we will consider this later.[2]

(4) The other comment which Russell appends to the law is not so directly concerned with it as are the preceding, but it is of fundamental significance for the whole theory of causality. He says that although the traditional law of causality (by which he presumably refers to that expressed by "same cause, same effect"[3]) is not assumed by science, scientists work with another hypothesis, "the uniformity of nature", for which there are supposed to be inductive reasons. This hypothesis implies an assumption of a principle about the permanence of laws which Russell describes in the following way. If we have found that a law has applied in a certain region for all relevant observed cases, we expect it to apply to all future cases, or, if this proves not to be so, that there is another similar law which applies both to all the old cases covered by the old law and to the new exceptional case. It is stressed that we have only inductive reasons for this principle, that is, it is only probable, to a greater or less degree.[4]

However, it seems possible to put forward some objections to this reasoning. (i) The principle is analytical in one sense. Later on, Russell himself points out that we can find a law which covers all cases in the past and all unforeseen irregularities occurring subsequently. We do not require inductive reasons to support this principle if we are not restricted in our interpretation of "law".[5] (ii) As science progresses, laws which were taken to be applicable are found to be refutable and are replaced by new. As

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[1] *The Principles of Mathematics*, p. 487.

[2] p. 64.

[3] *Mysticism and Logic*, p. 194, the idea that the same cause brings about the same effect is considered in connection with "the old 'law of causality' ".

[4] *Op. cit.*, p. 196.

[5] Considered on pp. 66 ff.

far as empirical laws are concerned, there seems to be no law which must be irrefutable. This disregards interpreted analytical laws such as that two stones plus two stones make four stones. This means that we have inductive grounds for the proposition "no laws are eternally valid". This means that for the case at hand inductive grounds are of doubtful value. (iii) It is even questionable whether such reasons tell us anything at all. The "probability" ascribed to a proposition on such grounds is not properly defined, but it seems likely that, as in *The Problems of Philosophy,* the probability concept is that used by Keynes in *A Treatise on Probability.* From this point of view to say that the principle of the permanence of laws is probable tells us nothing about the actual occurrence of permanently valid laws, only something about the degree of rational belief ascribable to the proposition "the principle of the permanence of laws is valid" on the basis of the empirical material at our disposal.[1]

Russell contends that our knowledge of nature's obedience to law is not of an *a priori* nature but an empirical generalisation. He emphasises that the principle of the permanence of laws is therefore less probable than the individual laws of which it is a generalisation. Thus, according to Russell, there is no reason to regard the principle as a major premiss in scientific reasoning, since in any particular case we wish to obtain a conclusion as certain as possible and therefor ought not to assume more than is absolutely necessary for the case in question.[2]

Let us return now to the original principle about the similarity of relations or the similarity of differential equations, to give it its more precise formulation. Perhaps we ought first to point out that the idea that all causal relations — assuming we can talk about such — may be expressed with the aid of differential equations is abandoned in Russell's later works, partly for reasons taken from physics. In *The Analysis of Matter* it is done so explicitly.[3]

[1] Problems associated with this probability concept were discussed in Chapter II, p. 45.

[2] *Mysticism and Logic,* p. 197.

[3] See Chapter VI, p. 112.

The way in which the principle is formulated shows that the constant relation mentioned is one between numbers and not between events. "The rate of change in the rate of change" of something is an abstract entity, in physical connections a number, and the state of the universe is also described with the help of numbers — the state of the universe as it is considered here, at least. We cannot therefore say that one part of the relation is the cause of the other part, since we cannot say that one number is the cause of another number. The difficulty pointed out by Philip Jourdain, which also appears in *The Principles of Mathematics* may be avoided in the manner indicated in Chapter I.[1] The mathematical relation between the numbers is a part of the causal relation, which is of the form $RT\bar{R}$, where R is a relation of correlation between an empirical phenomenon and a certain number, $\bar{R}$ the inverse transformation, and T a relation between the numbers. When Russell says that the relations between configurations of the universe are expressed by differential equations, this means that the differential equations constitute the part T of the relation $RT\bar{R}$ which applies between the configurations considered as empirical entities, and when it is said that the past is determined in the same way as the future, "determine" denotes concept applicable to referents and relata of T not of $RT\bar{R}$.

Interpretation of the principle is made more difficult by the fact that Russell uses the term "configuration" in two different senses. Sometimes he uses it to mean an aggregate of measurements pertaining to the universe at a given instant, as when he says "... an acceleration as a function of the configuration ...".[2] On other occasions he uses it to refer to the empirical phenomena to which these measurements appertain, as when, in speaking about the laws applying to particles in a system of particles, he says "... or the configurations at two instants, render the configuration at any other earlier or later instant theoretically calculable".[3] According to Russell, the laws in question are to be of a type of causal law also found in the advanced sciences. It

[1] Chapter I p. 23, note 3.

[2] *Mysticism and Logic*, p. 196. [3] *Op. cit.*, p. 194.

is therefore simplest to assume that "configuration" in the latter quotation denotes an empirical entity, since the laws otherwise apply to numbers and not to empirical phenomena, and cannot in consequence be regarded as causal.

There is also a degree of ambiguity about the expression "the state of the universe", but this need not lead to any difficulty, since the sense is always apparent. But we may ask what "the state of the universe" signifies in the case of empirical phenomena. Russell uses it to denote the phenomena in the universe to which measureable magnitudes may be assigned, but that does not mean to say that the universe is completely determined. It does not follow that the universe could not be different in some respects without the state of the universe in this special sense necessarily being different. In other words, it is neither stated that everything which can be measured is included in "the state of the universe", nor that everything which exists can be measured.

There are further difficulties of another kind. (a) In quantum mechanics it has been shown that it is impossible to give all the data for a given system which are required for calculation of all the measurable data for the system at a subsequent instant. (b) For reasons appearing in relativity theory, it is in practice impossible to know whether events are happening at other places which will afterwards affect events in our surroundings. For instance, we can never know anything about processes which are taking place "at the same time" in a region moving towards us with a velocity equal to the velocity of light. We can never be certain that a description of the state of the universe at a given instant includes everything relevant to the calculation of future events.

Points (a) and (b) make it impossible to accept the law of causality put forward by Russell. (a) makes it incorrect or unusable, depending on how we interpret the expression "the state of the universe" mentioned in the law. If we require the state of the universe to be given empirically, that is, by means of observations, then we find that there is no such state which determines uniquely "the rate of change in the rate of change" in accordance with the law, but that those states of the universe which have been observed only allow us to calculate coming changes within certain limits. The law of causality is invalid in this case. If we do not require that the state of the universe be given empirically but also permit the inclusion of hypothetical date to fill in the gaps made by quantum mechanical considerations when assembling data for the calculation of future events, the law is in a certain sense arbitrary. With one group of supplementary data we may obtain one law, and with another group of supplementary data another, although the body of empirical data used in both cases is the same. Even if the different possibilities for the supplementary data only give us different functions for calculating future events, the law tells us nothing as long as we have no criteria regarding the data to supplement the empirical given data in the particular cases. Without such criteria, the law tells us nothing more about the universe than it did without these supplementary data.

(b) makes the law unverifiable in individual cases, even if it is correct, since we can never be sure that we have considered all the data relevant to a state of the universe at a given instant, because all processes which bring us knowledge of this state proceed at a finite rate. This does not mean that the state of the universe at a given instant cannot be a properly determined magnitude; it means that we can never be sure whether we know about the whole of the state or only a part of it by direct perception or conclusion from empirical observations.[1] But (b) does not provide us with any important argument against the law of causality put forward by Russell.

We must regard as satisfactory Russell's answer to the question he posed in "On the Notion of Cause": what law or principle occupies the place on the advanced sciences which we usually suppose the traditional type of causality principle to occupy, from the viewpoint of classical mechanics, but as has been said in (a) unsatisfactory from the viewpoint of quantum mechanics.

[1] Problems associated with the theory of relativity are considered in Chapter VI, pp. 114 ff. The difficulties arising from quantum mechanics are also considered in that chapter, p. 111 and forward.

Quantum mechanics was not so far developed at the time, but there had been an every increasing amount of literature on the subject since Planck's work of 1901. Bohr's fundamental work was published in 1913, but it was not until 1927 that Russell dealt with the consequences which quantum mechanics and the theory of relativity have for theories of causality in his important work *The Analysis of Matter*.

But let us return to the problem of details. Russell develops one of his preoccupying ideas, that of the isolated system, in connection with some explanatory remarks. If we are to be able to detect regularities which occur in the universe, there must be parts of it which are in some way or other isolated from the rest in so far as the phenomena of these regularities are concerned. Russell distinguishes between two forms of isolated system, relatively isolated and practically isolated. The first type of system is defined in the following way: "A system relatively isolated during a given period is one which, within some assignable margin of error, will behave in the same way throughout that period, however the rest of the universe may be constituted".[1] When conditions for a system to be isolated were discussed in *The Principles of Mathematics*, the isolated system in question must have been relatively isolated, since the discussion bears upon whether the rest of the universe outside a certain sphere may be arbitrarily constituted.

The other type of system is defined as follows: "A system may be called 'practically isolated' during a given period if, although there *might* be states of the rest of the universe which would produce more than the assigned margin of error, there is reason to believe that such states do not in fact occur".[2] Russell's formulation here might lead us to believe that a system is practically isolated if we *believe*, or have reason to believe, that the state of the rest of the universe does not influence the system more than a certain limited amount during a given period, but it is clear that the only thing which can be meant is that the rest of

[1] *Mysticism and Logic*, p. 197.
[2] *Op. cit.*, p. 198.

the universe *actually* does not influence the system more than a certain amount.

Russell employs this last concept to analyse the traditional idea of what is meant by the statement that an event A causes another event B. Russell says, "It may happen that, as a result of general scientific laws, whenever A occurs throughout a certain period, it is followed by B".[1] In this case we have an example of a system, namely, A and B, which is practically isolated during a certain period. What makes it impossible to accept such associations as that between A and B as scientific is that their occurrence always depends on special circumstances which do not need to be present in order to render the assumed scientific laws correct.[2] It may be pointed out that if, in considering all relevant factors for the determination of B's cause, A becomes A' and the system A', B is relatively isolated. It is conceivable that A' would refer to the whole universe, but this would not conflict with the definition of a relatively isolated system.

However, the question of the properties of an isolated system is not very important for the main problems of causality, since the isolated systems defined by Russell are important with regard to the possibility of discovering scientific laws, but not important for the form of the final scientific theory.

But questions intimately associated with these main problems do appear in Russell's discussion of what is meant by a deterministic system. The definition given for such a system is: "A system is said to be 'deterministic' when, given certain data, $e_1, e_2, \ldots, e_n$ at times $t_1, t_2, \ldots, t_n$ respectively, concerning this system, if E_t is the state of the system at any time t, there is a functional relation of the form:

$$E_t = f(e_1, t_1, e_2, t_2, \ldots, e_n, t_n, t). \qquad \text{(A)}$$"[3]

Russell points out that it is possible to imagine systems which do not satisfy the condition that (A) shall be valid for every

[1] *Ibid.*

[2] *Op. cit.*, pp. 198–199.

[3] *Op. cit.*, p. 199.

instant t, systems for which (A) only applies during a certain period. The system is then deterministic during this period.

It is obvious that the e_i and t_i in formula (A) are constants, so that the only variable argument of f is t. Therefore, E_t, the state of the system at the instant t, is a function of t only. f should not be regarded as the symbol for a single mathematical function, but as the symbol for a group of mathematical functions, all with t as the only variable, with the help of which the properties characterising E_t may be calculated. Russell calls $e_1, e_2, \ldots, e_n$ the *determinants* of the system.

One point is not clear in the definition of systems deterministic during a certain period: it is not stated whether the determinants are at times within the period or whether they may be at times outside the period. If we consider the extreme possibility that none of the determinants are within the period, but all at instants after it, we have a situation in which there were no past or present events by means of which any configuration of the system belonging to the period could be calculated during the period. From the point of view of an observer living in the system, it would not be deterministic in the sense that it could be determined from available data.

The system would be deterministic in this sense if, at any instant t there were a set of determinants at instants before t but not in the period of which t is an instant. Or if the interval has a first instant T, the system would be deterministic in this sense if there were a set of determinants at T (or, naturally, before T).

But another difficulty connected with this definition of a deterministic system is much more fundamental than that above. (A) may be read in the following way: there are mathematical relations $f_1, f_2, \ldots, f_h$, which we include under the name f, which relate the numbers constituting the data of the system at an instant t to a certain set of numbers $e_1, e_2, \ldots, e_n, t_1, t_2, \ldots, t_n, t$. Since Russell also calls $e_1, e_2, \ldots, e_n$ events,[1] and not merely data, and since he obviously would not recognise numbers as events, we must suppose that the events e_i are described by numbers r_1,

[1] *Ibid.*

$r_2, \ldots, r_m$ which are assigned to them in some way or other. In the same way E_t, which is the state of the system at the instant t, has to be described by numbers $s_{1t}, s_{2t}, \ldots, s_{kt}$. The meaning of (A) is then as follows: There is a group of functions f which, for every t, relates the numbers $s_{1t}, s_{2t}, \ldots, s_{kt}$ to the numbers $r_1, r_2, \ldots, r_m, t_1, t_2, \ldots, t_n, t$.[1] If, as is usual in mathematics, we regard each correlation between numbers in one aggregate and numbers in another aggregate as a function, the statement that (A) applies to a certain system becomes an analytical statement: for, it is always possible to find functions f_1 which fulfil our conditions, since whatever s_{jt} we have for a certain t, we can always relate these s_{jt} to the numbers $r_1, r_2, \ldots t_n, t$, and when t runs through all conceivable values, we obtain a set of correlations which we can incorporate in one function. In order to avoid this difficulty we may put a restriction on the functions permitted so that f_i may only be of certain types of correlations, and Russell discusses some possible limitations.

Before we proceed to a consideration of this discussion, it is worth noting that the system seems to be necessarily deterministic, even if the conditions imposed on it are very broad. From the purely theoretical point of view we can imagine a system in which k (the number of s_{jt} determining E_t) is not constant but varies with t. This would mean that the number of phenomena in the system changes with t, for instance, by the inclusion of new phenomena which may be measured in the same way as the old. If k increases from k_1 at the instant T_1 to k_2 at the instant T_2, we can imagine an (A) which gives an uninterpretable value, for example, a complex value, to those s_{jt} which do not correspond to any phenomenon at T_1, but which do correspond to phenomena at T_2 or at any later instant, in accordance with the rules of interpretation, or rather, rules of correlation, which we have. It is also conceivable that the functions f_i could be functions of t in such a way that they become more many-valued as t increases, so that we have k_1 values at T_1 and k_2 at T_2, and if the f_i are regarded as

[1] Thus it is already assumed here that the instants have numbers assigned to them, and that these numbers are the t_i.

merely denoting sets of relations, the increase or decrease of k gives rise to no problems since it is always possible for a correlation to apply to the classes of numbers in question, regardless of the variation in t or k.

Russell says it may happen that some f_i is not describable by a finite number of symbols, and if this is so we cannot calculate the system completely at a future instant from (A), but that does not mean that there are not functions f_i for which (A) would apply to the system.[1] Thus, the universe must be a deterministic system, regardless of whether the laws applying to it can be written down or not.

If we do not restrict the permissible functions, another difficulty arises, according to Russell. The universe is necessarily deterministic and bound by laws up to the instant t at which we find ourselves, but there are an infinite number of laws which are similar in the past and in agreement with the empirical material, though different for instants after t and therefore unusable in (A). Russell points out that in every instant an infinite number of laws are refuted which applied up to that instant. If we had no formal criterions whatever for deciding which of the infinite number of laws hitherto applying to a certain phenomenon will also apply in the future, it would be impossible for science to work, and no hypothesis assuming something not experienced would be acceptable. Science must therefore have a method for selecting from the infinity of conceivable laws, and Russell says that this consists in choosing the simplest law which is in accordance with the facts.[2] The fact that simple formulae or laws are chosen does not depend on some law of Nature which says that we should do so; it is based on purely methodological considerations. When new facts come to light and we are obliged to abandon the simplest formula hitherto for one more complicated, but the simplest taking the new facts into account, science has "no sense that an axiom has been falsified".[3]

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[1] *Op. cit.*, p. 204.

[2] *Ibid.*

[3] *Ibid.*

This reasoning involves a concept of simplicity which is very vague.[1] It should be noticed that it is formulae and functions which are more simple or less, not the relation which these express. A one-one relation between numbers belonging to two classes is not simpler than any other one-one relation, but the descriptions of these relations may differ in complexity. There is some justification for saying that a one-one relation is simpler than a one-many, and that it is possible to draw up a hierarchy of relations with the aid of a "simplicity" concept which deals with thr topological complexity of the relations[2], but it is not thus that Russell wishes to use the concept. The description of a particular one—many relation may be simpler than the description of a particular one—one relation, and it is such simplicity which concerns us here.

Let us consider some interpretations of "simplicity" applied to a function or formula.

(i) One function is simpler than another if it contains fewer signs in all or signs of a certain kind than the other function (or if it has some other formal advantage over the other function). We must refer to a certain formal language in this case, since a function A may be simpler than a function B when both are expressed in one formal language, while the contrary is true if another formal language is used to express them, owing to different notations and abbreviations in the two languages. There is obviously a degree of arbitrariness involved in the choice of a particular formal language in this case.[3]

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[1] Russell himself mentions this vagueness, e.g. in *Human Knowledge*, p. 497.

[2] Russell's relation-numbers measure this topological complexity. *Introduction to Mathematical Philosophy*, London and New York, 1919, Chapter VI, p. 52.

[3] A simplicity concept of a similar kind is discussed by Adolphe Lindenbaum in an article "Sur la simplicité formelle des notions", *Actualités Scientifiques et Industrielles*, vol. 394, 1936, p. 29. He uses a concept of logical type in the determination of the simplicity of an expression. A recent discussion of a simplicity concept, also of a similar kind to those mentioned in (i), may be found in Nelson Goodman, *The Structure of Appearance*, Cambridge, Massachussetts, 1951, pp. 59—85.

(ii) One function is simpler than another if the number of operations needed to arrive at the value or values of the first function for a given value of the arguments is less than the number of operations for the other function for the same value. This definition does not ensure that different numbers of operations are not required for the same function with regard to different values of the argument (still supposing that 'operation' is a well-defined concept); which difference might lead to a situation where the number giving the degree of simplicity is greater for one function for one value of the argument than it is for the same value for another function and less for the same function for another value of the argument than the degree of simplicity for the other function for this value. This would mean either that the functions must be considered as not comparable or as equally simple. If we do not wish to say that two such functions are equally simple[1], we may modify (ii) to (ii′): One function is simpler than another if the largest least number of operations required to evaluate the first function from values of its argument ('largest' referring to different values of the argument) is less than the corresponding number for the other function. If neither function is simpler than the other, we say that they are equally simple. However, it is essential for these definitions that "operation" is a well-defined concept, because we cannot be sure that one function is simpler than another for all definitions of "operation".

(iii) One function is simpler than another if the least number of steps required to define it in a certain formalised system — for example, that of *Principia Mathematica* — is less than the corresponding number for the other function. As before, we say that two functions are equally simple if neither of them is simpler than the other. As in the preceding definitions, the hierarchy of functions which may be drawn up by this method is arbitrary, depending in this case on the fact that "step" may be interpreted in different ways and because quite another order of simplicity

[1] It cannot be that they are not comparable if we are to use their relative simplicity in making a choice between them.

may be obtained by using another formal system. It may even happen that if another axiom system is chosen for *Principia Mathematica* the hierarchy may be changed in such a way that a function formerly simpler than another becomes less simple than the other.

(iv) Jeffreys has another suggestion for what is to be understood when we say that one function or law is simpler than another. His formulation is: "Of two laws, the simpler is the one that contains fewer parameters left free to be adjusted to fit the observations".[1]

These four definitions of simplicity possess certain common weaknesses. For instance, we can never be certain in a particular situation where we wish to choose the simplest function (or law) conforming with the facts at our disposal that we will not have to choose between a large number of equally simple functions, or even an infinite number. This means that we cannot be certain of a unique choice merely with the aid of a simplicity concept, so that in order to avoid an arbitrary choice we need some other criterion to assist us in our choice. It is however clear that even if we have a large number of equally simple functions to choose between, the application of a simplicity test to them reduces the number of acceptable functions considerably. It may be possible to achieve a successive reduction of the field of choice by first choosing those functions which are equally simple$_1$ (simple in a first sense), subsequently selecting from this group those which are equally simple$_2$, and so on.

In choosing between functions which apply to the experiential material at hand but differ with regard to unobserved cases, we cannot derive any help from a principle of induction, since all

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1 Harold Jeffreys, "Bertrand Russell on Probability" *Mind*, Vol. LIX no. 235, p. 316. Jeffreys criticises Russell's statement in *Human Knowledge* that 'simplicity' is a vague concept. The definition he puts forward talks about laws and not about functions, but if we have (A), $E_t = f(e_1, \ldots, t)$, we can transfer Jeffreys's definition to functions f_i or F_i when (A) symbolises laws of the type $r_{it} = f_i(e_1, \ldots, t)$, or $F_i(r_{it}, e_1, \ldots, t) = 0$ if there is no explicit expression for r_{it}.

the functions we have to choose between are equally probable in the sense that they are correct for all observed cases. At least, this is so if we do not assign greater probability to certain classes of functions on formal grounds.

Jeffreys has attempted to solve the problem by dividing the functions into classes with different probabilities. His formulon of the problem reads: "if there is no limitation on the choice of laws no prediction from observation is possible".[1] Jeffreys proposes that the problem may be solved by halving the probability assigned to a law when it is necessary to formulate a new law involving more parameters in the light of new facts not accounted for by the old law.[2] But this gives us no solution of the problem if we have not already assumed that the most probable of two laws which both accounted for the old facts is that which is simplest in the sense suggested by Jeffreys. Without this assumption, we have no reason to start with any one function among those which conform with the facts, and therefore will not arrive at a situation where we have to choose between a function which has hitherto been valid but has now encountered an exceptional case and a function which remains valid for all cases. But to point to one class of functions or laws as more probable than certain other classes in spite of the fact that all of them account for the observed facts seems to imply a purely arbitrary procedure, or else it means that the probability used is irrelevant to the question of what functions will conform with future observations. It is one thing to choose between laws according to their formal simplicity for methodological reasons, and another to choose between laws because one believes that the world is so constituted that it is best described by laws as formally simple as possible, that these laws are in some respect probably in agreement with the empirical facts and that more complicated laws are "less probable".

Let us return to the problem of restricting the permissible functions. For some reason, Russell has refrained from considering the

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[1] *Ibid.* [2] *Ibid.*

restriction of the permissible functions for f in (A) to prevent the statement "the universe is a deterministic system" being merely analytical, as, for example, by permitting only functions of a certain simplicity.

Russell says something which makes it seem that he thought the problem could be solved by not using fucntions in which time occurs explicitly. The time may appear in integrated form, since we are concerned with time intervals, but absolute time must not appear.[1] Perhaps such a principle makes the statement "the universe is a deterministic system" a synthetic proposition. Let us examine such formulae in which the time does not occur explicitly. Examples given are the laws of mechanics, since the acceleration in them is a function of the system's configuration, not a function of the configuration and the time as Russel points out.[2] In this case two configurations of the system (e.g. the universe) constitute a set of determinants according to classical mechanics, whereas one configuration is sufficient to determine the acceleration of the different particles of the system.

If we required of the laws that they should determine certain data $A_1, A_2, \ldots, A_n$ for the system at a particular instant from certain other data for the system at the same instant, we would not even need to admit laws in which the time occurred in integrated form. In this example, where the acceleration of different particles is determined by one configuration, we cannot quite avoid referring to other instants, but the reference is in a way indefinite since the laws which Russell has in mind do not explicitly refer to events at other instants, nor even to the existence of other instants. But the concept of the acceleration at an instant t involves the assumption that the body in question exists in a neighbourhood of t, however small this neigbourhood is. The acceleration of the bodies of a system at a certain instant tells us something about changes in this system in the neighbourhood of this instant. We cannot,

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[1] "The difficulty we have been considering seems to be met partly, if not wholly, by the principle that the time must not explicitly enter into our formulae". *Mysticism and Logic*, p. 205.

[2] *Op. cit.*, p. 205.

however, calculate the configuration of a system of bodies at a subsequent instant merely from a knowledge of their accelerations at the instant t; we also need to know the velocities of the bodies at this instant. So this example does not provide us with an illustration of a law which only contains the time in integrated form and which may be used for predictions (without being combined with other laws).

As regards A_i, at every instant the system contains a set of determinants for determining A_i at that instant, but we need to know much more in order to be able to make predictions.

Since we are concerned with restrictions on functions relating one set of data with earlier or later data, it is natural to assume that the laws may be used for predictions. We might suppose that a law which accords with the principle and which may be used for predictions is of the form $D_\tau = f(e_1, \tau_1, e_2, \tau_2, \ldots, \tau_{n-1}, e_n, \tau)$, (B), where τ_i is the interval between e_i and e_{i+1}, and D_τ is part of the state of the system after an interval τ from an instant T which is calculated with the help of the instants at which e_i occurs, the e_i being numbers assigned certain definite properties of the system. The law says that if we have a set of e_i we can calculate D_τ. For instance, any two configurations provide us with a set of e_i from which the state of the system D_τ (considered as a system of particles) may be calculated at an instant within an interval of time τ from one of the configurations if we know the interval of time τ_k between the configurations. In this case, τ_i can only take the values 0 or τ_k.

Whether we take it that any set of data which fulfils certain conditions may be used as a set of e_i for the determination of D_τ or that only certain of these sets are acceptable, or perhaps that there is only one set of e_i for (B), we have still not restricted the number of possible functions by not allowing absolute time to occur explicitly in our formulae. All we have done is to select other sets of numbers to be related by the functions. If we take it that Russell wishes to restrict the permissible functions or formulae to those in which the lapse of time only occurs in integrated form, we have a real restriction on the possible functions for (A). Our formulae must be describable in such a way that they contain τ as one limit of a definite integral. In this case the statement "the universe is a deterministic system" is not analytical, but probably false. There are laws of atomic phenomena which are not of this type. We need not assume that Russell intended to restrict the functions in this way, but instead, that the essential thing is that absolute time does not occur explicitly in them. We may add in this connection that no important alterations result if our laws are of type (B) and there is only one set of e_i for determining D_τ. This would mean that the T from which we reckon τ may be regarded as the zero point in a system of absolute time and τ as the absolute time, or, if we wish, we can regard τ as constant and the absolute time as variable without taking τ from the origin, in the same way that we had for (A). In order to obtain a difference between the cases (A) and (B), we must suppose that the instants at which the e_i occur are irrelevant. Only the distances between them are relevant in this connection, and these are variables in (B), each denoting the value assigned to a certain property of the system.

A deterministic system of this sort is on the whole equivalent to a system where there is causality[1], of the general type described in *The Principles of Mathematics*. The only difference is that the "data" e_i which enter into the causal relations must be divided up into groups of simultaneous data with equal numbers of elements in each group, whereas no similar condition is imposed in *On the Notion of Cause* on the data e_i which enter into the relations symbolised by (A) in order that the system may be deterministic.

But, as already mentioned, this does not signify that a restriction has been imposed upon the permissible functions; we have merely restricted the classes of numbers to which the relations apply. The statement "the universe is a deterministic system" is no longer analytical, because it is possible that exactly the same determinants could occur twice and be followed by different series

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[1] When we say causality applies to a system we mean that every event belonging to the system is causally related to something else in the system.

of events each time, so that an event a time τ after the determinants the first time need not recur the second time at the corresponding instant, if it recurs at all. If the universe behaved in this way, it would not be a deterministic system with Russell's proposed restrictions of the permissible functions.

The result of this discussion is therefore that the statement "the universe is a deterministic system" does not remain analytical when we introduce Russell's restrictions of the permissible functions, in spite of the fact that the proposed restrictions are actually not restrictions on the functions but mean that the sets of numbers related by the functions are determined in a particular way.

It is not difficult to find a restriction on the permissible functions so that not all systems are deterministic, since any prohibition of a group of possible functions serves this purpose, even if the group contains only one function, which we may chose quite arbitrarily. What is difficult is to find a restriction which is not merely arbitrary, but for which we can give some good reason. For practical reasons, we may stipulate that all functions considered must be expressible by a finite number of symbols, but all that such functions have in common is this property, and we do not seem to have said anything worth-while about the properties assignable to processes in a system which is said to be deterministic in the sense that (A) applies to it and that the functions denoted by f are expressible by a finite number of symbols. What would tell us something about the properties of events or changes in a deterministic system is a restriction of the permissible functions to a certain structural type, comparable with exponential functions, differential functions, analytically continuable functions, etc., regardless of whether the functions are finite or not. The problem is important, but further discussion of possible restrictions on permissible functions for the description of deterministic systems would be beyond the scope of this work.

The criticisms which Russell levels against some theories of causality different from his own give us a good deal of information about his own theorising, but they also have their interest when regarded as contributions to the great slow-moving philosophical debate, where there may be centuries between the replies.

Three main aspects of Russell's criticism will be considered. (1) The first concerns laws of causality such as the following: "Given any event e_1, there is an event e_2 and a time interval τ such that, whenever e_1 occurs, e_2 follows after an interval τ"[1], which is rejected by Russell on the grounds that the advanced sciences, especially physics, do not use laws of this type. (2) The second concerns certain assumptions about cause and effect or about causal relations, such as that there shall be spatial contiguity or temporal contiguity between cause and effect. These are rejected for similar reasons to those given for the laws in (1), because the scientific laws accepted by Russell do not satisfy these assumptions. (3) The third concerns Russell's opinions on the problem of the freedom of the will, and his solution in terms of his theory of causality.

(1) Russell derived the object of his criticism in this case from a passage on causality in Baldwin's *Dictionary of Philosophy*[2], which achieved the form cited above after certain improvements. Perhaps there would be little interest in criticism of this law if it were not that some philosophers have adopted it or a variant of it, Russell mentions J. S. Mill[3] and Bergson.[4]

There are two main line of argument in the criticism of this

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[1] *Op. cit.*, p. 183.

[2] James Mark Baldwin, *Dictionary of Philosophy*. Vol. I–III, New York and London 1901–1905.

[3] The following quotation from Mill is given: "The law of causation, the recognition of which is the main pillar of inductive science, is but the familiar truth, that invariability of succesion is found by observation to obtain between every fact in nature and some other fact which has preceded it". — *Logic*, Bk. III, Chap. V, § 2. One difference between Russell's and Mill's laws of causality is that the former implies that every event has an effect—the latter implies that every event has a cause.

[4] The following is part of what is qouted from Bergson: "Now, it is argued, this law means that every phenomenon is determined by its conditions, or, in other words, that the same cause produce the same effects." *Time and Free Will*, p. 199.

law. (a) It is not possible to define the events mentioned in the law with any great precision, since it is necessary that an event may recur as was the case even with the events which the prescientific laws connected. Russell says that an event must be a universal, something of which there may be many instances. The striking of a match, for instance, is an event in this sense.[1] Otherwise, if it is improbable that an event should ever recur, we cannot make any predictions with the help of such causal laws as this; actually, the law would be useless for anything. But the events which the advanced sciences — physics, anyway — deal with are described very exactly, and the form of the laws used by these sciences is such that it is immaterial whether the events recur or not. In some cases the quantities to which these laws apply cannot be regarded as events with definite limits like the striking of a match, for instance, a configuration of the entire universe considered as a particle system.[2]

(b) The other argument depends on the fact that there is an interval of time between a cause and its effect. Something may happen during this interval, which is always of finite length, that prevents the expected effect. We can lessen this uncertainty by extending the meaning of 'cause' to include the immediate neighbourhood of the instantaneous causal event. But the more we extend our cause the less it is a definite event, and the less probable it becomes that it will be repeated.[3] We can never, according to Russell, be really certain until we have included the whole universe in the cause, and the law then approaches those of the advanced sciences in certainty; but at the same time it has diverged so much from the original law dealing with approximately determined events of precisely defined extent that we can no longer say that it is of the same type as that quoted in (1).

Russell proves that τ in (1) must be finite in the following way. If τ is not finite, it is either infinitesimal or zero. The first possibility

[1] *Mysticism and Logic*, p. 186.

[2] This is also why Russell is doubtful about using causal terminology at all in connection with advanced scientific laws.

[3] *Op. cit.*, p. 187.

is dealt with quite shortly by saying that there are no infinitesimal intervals of time. This presumably means that infinitesimal quantities are mathematical concepts which lack an empirical interpretation. The possibility that τ is zero, so that there is temporal contiguity between cause and effect is rejected by consideration of a dilemma. As in *The Principles of Mathematics*[1], Russell says that two instants cannot be contiguous because the time series is compact[2], a compact series being one for which there is at least one element of the series between any other two. This means that if the cause and effect are to be contiguous in time, either or both must be finite in extent. So Russell assumes that they both last for a finite time; it does not affect the reasoning whether we make this assumption or an assumption where only the cause or the effect lasts for a finite time.

Suppose, says Russell, that the process constituting the cause includes a change. Then we may divide up the cause into successive parts between which there are causal connections (if general causality applies). We must choose one of these subdivisions as the cause (which would be temporally contiguous with the effect), since we have not supposed it possible that there may be several causes of one event. It is natural to choose the last of the subdivisions because it is the one contiguous in time with the effect. If this includes a change, we can repeat the procedure, and obtain a still smaller cause. And so on, until we arrive at a cause which does not include a change. There may not be such a cause not including a change, in which case there is nothing which we can specify as the cause we are looking for. If, on the other hand, we do arrive at a static subdivision which proceeds our effect, it is incomprehensible why this cause suddenly brings about the effect at the moment when it actually does so instead of at some earlier or later instant.

Russell says that we are forced to abandon the idea of the temporal contiguity of cause and effect in view of this dilemma, that

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[1] *The Principles of Mathematics*, p. 477.

[2] *Mysticism and Logic*, p. 184.

we either can never single out anything as the cause, or must assume that a static state suddenly changes to another state.

This reasoning may also be looked upon as an immanent criticism of a theory of causality which says, among other things, that there is only one cause for every event, and that the cause and effect are contiguous in time, and furthermore, that there are events of unlimitedly short duration. These suppositions lead to the above dilemma, and we must therefore reject the theory. It is perhaps worth noticing that the criticism cannot be applied to a theory which does not assume that every part of an event is an event, however short a time it lasts, for instance, theories which assume that an event must contain at least one sense-datum, and that there are no sense-data of shorter duration than a certain specifiable time.

It may be pointed out, however, that the formulation of the law does not imply that it is only possible to have a single cause, since the law merely says that for every event e_1 there is an event e_2 which always follows e_1 after a time τ, and this is not irreconcilable with the statement that there are many events e_i which always precede e_2 and which all occur at a time τ before e_2.[1] We could then say that there are cases of temporal contiguity where $\tau = 0$, without falling into the above dilemma.

The same uncertainty which may occur because something might happen during the interim time τ to prevent the occurrence of the expected event also applies to the case $\tau = 0$, however, since the events e_1 and e_2 have been defined by Russell in such a way that they must be of finite extent; it may happen that some interference occurs when only a part of the expected effect has occurred or just when it begins, such that the whole expected effect does not occur.

(c) A third reason against causal laws of this type is that the whole of the terminology of causality is of little use for advanced scientific laws, according to Russell, since there is nothing which

[1] The time interval may be measured from the end of e_i to the beginning of e_2.

we might justifiably call a cause and nothing we might justifiably call an effect in the formulae used in physics, for example. Presumably Russell means that the criticised causal laws are unusable in descriptions of how processes proceed and that they must be usable to this end if they are to be accepted.

(2) The other main point of Russell's criticism of causal theories concerns five maxims about causes, effects, and causal relations. The criticism may be divided under two headings, one that the laws of the advanced sciences do not always satisfy the maxims in question, and the other a criticism on a psychological basis, whereby an attempt is made to show that the maxims are analogies built on the relation between volition and willed events.

The five maxims are: (a) cause and effect must be more or less similar[1], (b) the cause is analogous to volition, since there must be an intelligible nexus between cause and effect[2], (c) the cause compels the effect in some sense in which the effect does not compel the cause[3], (d) a cause cannot operate when it has ceased to exist, because that which has ceased to exist is nothing[4], (e) a cause cannot operate elsewhere than where it occurs.[5]

It is obvious that (d) and (e) are not fulfilled by a law which connects two configurations of a system of particles with a third, where the configurations exist at different instants and are different in appearance. With special reference to (d), Russell mentions the criticism described in (1), of temporal contiguity of cause and effect. The example with the configurations also makes it clear that (b) need not apply, since physics does not assume any special connection between the configurations of a particle system other than the mathematical functions which describe a certain configuration at a distance from two other configurations, if these are of a particular form. Perhaps it ought to be pointed out that "nexus" in (b) cannot be interpreted as a continuous chain of events joining the cause to the effect, because the analogy

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[1] *Op. cit.*, p. 189.
[2] *Ibid.*
[3] *Op. cit.*, p. 190.
[4] *Op. cit.*, p. 191.
[5] *Op. cit.*, p. 192.

with volition would then disappear. We can hardly maintain that we are aware of a continuous process between the application of our volition and the event which we say is the result of this volition — except in some psychological sense of experienced continuity, something which cannot have an analogue in the case of three configurations of a system of particles.

Russell rejects (a) on the ground that we can hardly say that there is any similarity between a part of a configuration and two whole configurations of the same system determining that part. But Russell himself says elsewhere[1] that if the configuration and velocities of the particles of a system are given at a particular instant, the velocities and configuration of all the particles of the system are theoretically calculable for every other instant. Cause and effect in this case seem to resemble one another, even with very high requirements for resemblance. Thus, if this last example is accepted, it seems as if there *may* be resemblance, although there *need* not be, so it does not constitute any essential relation between cause and effect.

Russell divides his consideration of (c) into two. (c′) The cause compels the effect in some sense in which the effect does not compel the cause, compulsion being regarded as a psychological concept. Only someone with a desire can be compelled in this way, according to Russell, and a configuration of a system of particles has no desire — at least, we have no reasonable grounds for assuming that the whole universe, taken as a system of particles, has any desire — so this suffices to refute (c′). (c″) the cause determines the effect in some sense in which the effect does not determine the cause, "determine" being interpreted as signifying that the effect is a function of the cause or a one-many relation relates effect and cause. If the cause and effect are described with equal precision, this lack of symmetry does not occur, Russell points out. In his own theory of causality, according to which two configurations of a system of particles determine every other configuration and every configuration is determined by every pair of

[1] *Op. cit.*, p. 194.

other configurations, the relation is many-many, and there is no asymmetry in the respect under discussion.

He gives a psychological derivation of these five maxims which he considers untenable, and he contends that all, apart from (a) and (c″), are formulated after the pattern of mental phenomena. (b) expresses this directly, (c′) contains "compels" which means that desire is involved, (d) and (e) contain "operate", but only volition can operate, according to Russell. No psychological derivation is given for (a) and (c″).

(3) In the discussion of problems associated with free will, which is intended to illustrate the section on deterministic systems, two ideas are particularly criticised: (a) the idea that our subjective feeling of freedom is an argument for free will, and (b) the idea that determinism (the doctrine that our volitions belong to some deterministic system) implies that our actions are necessitated in some sense relevant to this discussion.

(a) According to Russell, the idea that a subjective feeling of freedom is an argument for free will is associated with the picture of the causal relation given in (2c′), which describes the cause as compelling the effect. But (2c′) need not apply to a deterministic system, and to say that we have a feeling that we are under no compulsion of circumstances to act in a certain way therefore tells us nothing about the freedom of the will.

(b) Russell treats three meanings of "necessity" which often occur in connection with the problem of free will. The first is associated with a fatalistic belief that all events and actions occur in the way they do however much we may wish them not to. Determinism does not imply that such fatalism is correct, because determinism considers desires to be determined to the same extent as events. The other two meanings concern logical necessity, necessity which may be an attribute of a propositional function and necessity with respect to a particular constituent of a proposition.[1] The former is irrelevant in this connection, and the latter,

[1] "A proposition is necessary with respect to a given constituent . . . when it remains true however that constituent may be varied." *Op. cit.*, p. 207.

though relevant, has no emotional significance, says Russell. In this latter sense, he goes on to say, the connection between the will and its determinants is necessary (if our volitions belong to a deterministic system) if the instant at which the determinants occur is the constituent which varies and the lapse of time between the volition and the determinants is kept constant. We may remark here that if determinism is to imply this, a "deterministic system" should not be defined in such broad terms as it was with the help of (A)[1], since with that definition what is determined by the determinants may change according to the instants at which these occur, whether the time interval between the determinants and what they determine is constant or not. On the other hand, determinism will imply this if we follow Russell's suggestion on the restriction of the permissible functions for (A) and assume that absolute time may not occur explicitly in them.

It is not easy to see why this form of necessity cannot have emotional significance. We need only assume that the determinants are very few and often occur. It may be that as soon as they occur, I know that I shall have a certain volition, though knowing this need not diminish my feeling of freedom of choice in individual cases. It does not affect this argument that, if determinism is correct, it is improbable that the determinants are so few that they may all be comprehended, and improbable that they will ever occur more than once.

Russell also points out that if there are a set of non-mental determinants in a system including our volitions, this does not mean to say that it is not possible for the same system to include a set of mental determinants, and Russell thinks that this ought to diminish our reluctance to accept determinism as correct.[2]

[1] *Op. cit.*, p. 199.
[2] *Op. cit.*, p. 207.

Chapter IV

There is practically no mention of the problem of induction in "On the Notion of Cause", and practically no discussion of causal laws or laws of causality in *The Problems of Philosophy.* But both induction and causality are discussed in *Our Knowledge of the External World* 1914. The theories of this latter work are on the whole in agreement with those in the two from 1912. There are, however, certain new points depending on changes in Russell's theory of knowledge which need to be considered.

Two main points will be discussed. (I) What Russell means by "data". He now says that causal relations are relations between sense-data, sometimes just called "data", or entities which are of the same logical type as sense-data, and that data cannot be instantaneous, whereas he previously said that causal relations related, for instance, three instantaneous configurations of a particle system. (II) What he means by "causal laws". Certain statements indicate that it is possible for a causal law to be valid only during a limited period, which is contrary to his previous use of the term. Moreover, it seems that Russell has extended "causal laws" to embrace laws about certain relations between sense-data, either actual or merely possible, besides the two types of law which Russell called causal in "On the Notion of Cause", (i) the pre-scientific approximate laws where all that was required in order that we might say there was a causal connection was that a sequence often occurred, and (ii) the type which occurs in the advanced sciences, which, according to Russell, had a differential equation as their essential element.

The chapter which deals primarily with problems associated with causality exemplifies, like the rest of the book, a method of philosophical analysis which Russell sketches at the beginning of the chapter. There are two essential parts to this method: (a) an analysis of the logical connection between propositions which are part of our knowledge, which involves the establishment of premisses from which the rest of our knowledge may be derived, and a reduction of the propositions to more simple and precise ones and (b) an enquiry into the degree of certainty to be assigned to our premisses and the statements derived from them. This programme is also carried out. Of the four questions about problems associated with causation which Russell sets out to answer[1], the first, "what is meant by a causal law?" is an example of a reduction of the type described i.e. of (a), and the next two, "what is the evidence that causal laws have held hitherto?" and "what is the evidence that they will continue to hold in the future?" examples of (b), while the fourth, about how the causation accepted by the advanced sciences is related to that accepted by common-sense and traditional philosophy, partly falls under (a) and partly falls outside both (a) and (b), that is, constitutes a problem which must be regarded from the point of view of the history of ideas, and which would be expressed in Russell's terminology as: How does the body of knowledge which we possess compare with the body of knowledge which certain circles, e.g. certain philosophers, possessed at some earlier time?

The division of problems associated with causality into these four questions and a fifth dealing with the consequences of his theories for the question of free will, implies really only a redistribution of material which was presented in "On the Notion of Cause", and the incorporation of theories of induction from *The Problems of Philosophy* in the answer to the third question. We will endeavour to bring out what is new in our analysis of the first and second of the above-mentioned questions.

(I) The problem of what is meant by "data" makes its appearance in Russell's description of the general form of a causal law: "Whenever things occur in certain relations to each other (among which their time-relations must be included), then a thing having a fixed

[1] *Our Knowledge of the External World*, p. 212.

relation to these things will occur at a date fixed relatively to their dates."[1] We may first notice that this description is wider than that in *The Principles of Mathematics*, and probably even wider than the widest interpretation of "causal law" derivable from the part of "On the Notion of Cause" which deals with deterministic systems. But we will discuss this latter comparison later. Russell says that "the things given" cannot be such entities as only exist for an instant, since such entities, if they exist, cannot be data. This seems to indicate that entities which last for a finite time and which are called data may be elements of causal relations. He also says in another place that causal relations relate terms made of sense-data or entities of the same logical type as sense-data. Furthermore, he says that if a causal law is to be directly verifiable, both referent and relatum must be data.[2] This brings us at once to the question of the sense in which Russell uses the term "data".

One meaning of data is that assigned to the term when he says that philosophic analysis begins from a stock of common knowledge, which constitutes our data.[3] The knowledge in question may even have been acquired through description, and it may therefore refer to things which can never be experienced by us directly. But this cannot be the meaning of "data" which is relevant in this connection. Russell says in another place that a causal law must deal with sense-data if it is to be verifiable.[4] If we compare this with the statement just mentioned, which says that a verifiable causal law must deal with data, it appears that "datum" and "sense-datum" are used as synonyms. This conclusion is supported by the fact that Russell says that if we wish to extend our knowledge of what exists with the help of a causal law, we must use the causal law to deduce something which is not at the moment a datum.[5] This must mean that in

[1] *Op. cit.*, p. 216.
[2] *Op. cit.*, p. 213.
[3] *Op. cit.*, p. 211.
[4] *Op. cit.*, p. 81.
[5] *Op. cit.*, p. 213.

such a case what we infer cannot be anything of which we have knowledge other than through the causal inference, that is to say, we must not be immediately aware of it; hence "datum" is here used for "sense-datum", or perhaps even for "sensation". Anyway, sense-data seem to belong to the quantities which even in this case are covered by the term "datum". But if "datum" and "sense-datum" are synonymous — or, at least, if a sense-datum is a datum, it is obvious that we have two different meanings for "datum", because a sense-datum can never be knowledge, according to the theories in *Our Knowledge of the External World*, and the meaning of "datum" which we first discussed therefore cannot be relevant, here, where the term is used in connection with causal laws.

If we now take it that causal relations do hold for sense-data, we still have to consider the question of the other entities to which causal relations may apply, entities which are not sense-data, but which are of the same logical type as sense-data. It seems that these entities include possible sense-data, or "ideal aspects", as Russell calls them. An aspect is a sense-datum which is part of a perspective at an instant, and a perspective is, for instance, the field of vision at a given instant, our auditive surroundings at a given instant, or some eqvivalent in the case of our other senses. If we stand in a room and look at a table at a particular instant, we see at that instant an aspect of the table, an aspect of the wall behind the table, an aspect of part of the floor under the table, and so on, and these aspects together make up our visual perspective at that instant. At the same instant, various ideal aspects of the table exist, which are part of ideal visual perspectives from points in the room at which there is no observer— to one side of me, in front of me, and from most of the other points in the room.[1] It is a matter for discussion whether these ideal aspects exist in exactly the same sense that sense-data exist, but however that may be, Russell assigns some sort of existence to them. Since the ideal aspects are not constructions of perceived aspects, and since Russell

[1] *Op. cit.*, pp. 88, 111.

thinks that all aspects, both ideal and perceived, may be grouped together in "the system of 'perspectives'"[1], it is clear that both types of aspects are considered to be of the same logical type.

Besides these ideal and actual aspects, there is another important category of entities which might be elements in causal relations, and which are of the same logical type as aspects, namely, all entities which might be called "mental events". In *Our Knowledge of the External World,* Russell adheres to the viewpoint that sense-data are experienced by special mental acts, which he calls "sensations"[2], a viewpoint which is abandoned in *The Analysis of Mind,* where mental acts are not taken to be special entities. Mental phenomena include sensations, introspective facts of various kinds, thoughts, and facts of memory.[3] All these phenomena exist in the same sense that sense-data do, and, together with sense-data and possible sense-data, they constitute the sum total of existence, according to Russell's point of view at this time, a point of view which is best described as a sort of phenomenalism.

But causal laws also apply to events, and we need to know how events are related to sense-data and entities of similar logical type. It is stated in the discussion of the general scheme of causal laws that the things which are given, and for which causal laws are valid may be processes, especially motions, and we naturally assume that Russell regards movements as events. It is stressed, moreover, that there is a sense in which movements may be data[4], that we can have immediate experience of movements.[5] This must mean that there are events which may be data. Furthermore, if a causal law applying to events is to be verifiable, it must be a

[1] *Op. cit.,* p. 88.

[2] *Op. cit.,* p. 76.

[3] We read on p. 76 that it is natural to look for the external cause of a sensation in the sensible object (=sense-datum) which is perceived through the sensation, and on p. 215 the relation between a man's thoughts and his words is stated to be causal. The conception of logical laws and of entities of any logical type may be termed mental events, but not logical laws regarded as propositions, nor other entities of higher logical type than those which occur in sense-data.

[4] *Op. cit.,* p. 216. [5] *Op. cit.,* p. 139.

statement about sense-data or constructions of sense-data, and since no abstract objects can be relata or referents in a causal relation, but only objects "having the kind of reality that belongs to objects of sense"[1], there can hardly be any question of other "constructions" from data than series of data. Events involving ideal aspects may also figure in causal relations, as well as separate ideal aspects. We infer that there has been a flash of lightning from the fact that we hear a particular sort of rumbling[2], and the event we infer in this case consists partly of ideal aspects in any case, or, if no one has seen the flash, entirely of ideal aspects. Even the laws of physics — those of dynamics, for instance — deal with series of aspects, although these series are called "things". Russell says the very fact that we can verify physical laws depends upon the existence of such series of aspects, which allow of physical description. "Things are those series of aspects which obey the laws of physics".[3] This must mean that "thing" in this context means something else than a class of aspects or some other logical construction of aspects[4], since physical laws are causal[5], according to Russell, and verifiable, and verifiable causal laws must deal with sense-data; moreover, causal laws do not deal with abstract entities, and therefore certainly not with classes of sense-data.

The discussion has thus provided us with the result that causal relations relate sense-data (which are not ideal aspects), ideal aspects, series of aspects and ideal aspects, and mental events.

Before we proceed to the problem of whether the view that causal relations relate only sense-data and things of the same type as sense-data can be reconciled with Russell's earlier theories about causal relations, we shall first briefly consider a remark which he makes about relata in causal relations. Unlike the entity which we say is causally related to something, the entity which we infer

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[1] *Op. cit.*, p. 214. [2] *Ibid.*

[3] *Op. cit.*, p. 110.

[4] *Op. cit.*, p. 89. "All the aspects of a thing are real, whereas the thing is a mere logical construction».

[5] *Op. cit.*, p. 109.

with the aid of the relation and the given facts can only be described. The given entity may also be given directly, "by acquaintance", while the quantity we infer can only be known to us by description.[1] It is perhaps a matter for discussion whether it is always true that an inferred quantity can *only* be known to us by description in those cases when we infer from facts which we know by description that something exists, at the same moment as we experience, for instance, a visual aspect of what we have inferred. But quite apart from this circumstance, we cannot use causal laws to infer something which cannot *also* be described, since laws which can be expressed verbally do in fact say that something which may be described in a certain way is related to something else which may be described in a certain way.

The consequence of this is that causal laws, whatever their import, cannot be used to infer the existence of something which does not permit of description. It may well be that some elements of sense-data may be described by setting different sense-data in relation to one another, using different of their properties to build hierarchies of them, and correlating sense-data with numbers; and so on but it is disputable whether all the elements in a sense-datum can be described. If we suppose that there is at least a remainder in an individual sense-datum which does not permit of description, we arrive at the consequence that there is something connected with sense-data which could not be inferred with the help of causal laws.

Let us now consider whether Russell requires that everything which may be causally related to something else and everything to which something may be causally related must be of finite temporal extension. One of Russell's statements makes it appear that he regards at least mental events and sense-data (aspects) as finite in extent. He says that events of which we are conscious are finite in extent, and in this connection mentions explicitly sensations and objects of sense.[2] The question remains whether ideal aspects also must have finite temporal extent.

[1] *Op. cit.*, p. 214.
[2] *Op. cit.*, p. 116.

If we assume, as seems only natural, that an ideal aspect cannot include an element which would render it impossible for someone to observe it if he were in a position from which the ideal aspect could be observed, and furthermore assume, with Russell, that, an aspect must be temporally finite in order that we may experience it, we must also assume that ideal aspects are finite in temporal extent.

Although we have decided that the referents and relata of causal relations must be of finite temporal extension, we have of course said nothing about how short their duration may be. Disregarding Russell's attitude to this last question, we have here an essential difference between his present theory of causation and the earlier theories of causation according to which a causal relation related three instantaneous configurations of the universe (considered as a system of particles). There, the referent consisted of two instantaneous configurations and the relatum of one. (This is presumably associated with the fact that what Russell previously meant by "configuration" would now be interperted by him as an entity of higher logical type than sense-data.) The law of universal causation which Russell now puts forward, and which may be said to correspond to the law in "On the Notion of Cause"[1] about universal causation in our world and to the law given in *The Principles of Mathematics*[2], is formulated in a way reflecting the difference mentioned: "There are such invariable relations between different events at the same or different times that, given the state of the whole universe throughout any finite time, however short, every previous and subsequent event can theoretically be determined as a function of the given events during that time."[3] He says here that the state of the universe must be given for a finite time in order that it shall be possible to infer previous or subsequent events, and, moreover, the events we infer also have finite temporal extension. Here we have a new point of view.[4]

[1] *Mysticism and Logic*, p. 195.

[2] *The Principles of Mathematics*, p. 479.

[3] *Our Knowledge of the External World*, p. 221.

[4] According to the law of universal causation, the events inferred are

It might be supposed that it is possible to unify the two points of view by saying that the previous theories were not concerned with laws applying to sense-data and entities of a similar logical type to sense-data, but with logical constructions from such entities and that these constructions might occur instantaneously while the laws were verified with the help of the entities themselves. But this does not take us away from the fact that the laws which, in the earlier theories, applied to entities which lasted only an instant were regarded as causal, whereas Russell now requires entities obeying causal laws to be temporally finite.

II. It has been possible to discern this difference without considering which relations or laws are accepted hy Russell as causal. According to one of the most general pronouncements on causal laws in *Our Knowledge of the External World*, Russell means by

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determined as functions of the given events. If there is not some restriction of the permissible functions in this law, the same state of affairs arises as with the deterministic systems described in "On the Notion of Cause", namely, that there *must* be universal causation in all conceivable worlds, so that the statement "there is general causation in the universe" becomes analytical. Philipp Jourdain has put forward the opinion that the formulation of the law implicitly involves a restriction of the possible functions. But his argument appears to be unconvincing. He says that it is assumed that the functions possess some general characteristic: they must be analytically continuable or "monogenic", as Jourdain expresses it. For, if the functions did not possess this characteristic, we could not infer states of the universe at every instant from a knowledge of the states of the universe during a finite interval of time. (*Philipp Jourdain, Causality, Induction and Probability*, Mind, Ser. 2, 28, 1919, p. 173).

It looks as if Jourdain has confused two different things here. It is, of course, impossible to infer the form of a function in regions where he have no values for it, unless it is analytically continuable but this is a difficulty which merely concerns our ability to know the complete form of the function. But the problem is not that of determining from a knowledge of the states of the universe during a given finite interval of time the form of the function or functions with which we may calculate future events, or, more generally, events not among those given; the problem is primarily concerned with whether there actually are functions, analytically continuable or not, by means of which we could infer — at least, in principle — from a knowledge of the universe during a certain period of time any preceding or succeeding event, if we only knew them.

such a law any proposition by whose aid we can infer the existence of a sense-datum, or some entity of the same logical type as sense-data, or an event, from some one or more other entities included in these categories.[1] Or, to take a later definition, a causal law expresses a relation for which the referent and the relatum each belong to their own particular class of phenomena, which are defined by means of some general characteristic, — general because the law must be applicable to many cases.[2] As in previous works, it is pointed out that it is not a matter of a principle like "same cause, same effect", but of whether the same relation applies to different phenomena. That it is a question of expressing how the same relation applies in different cases is obvious with scientific causal laws, and the condition that referent and relatum must be assigned some general characteristic applies both to scientific and to pre-scientific laws. These two types of causal laws seems to be the same as before in all respects, except that they do not relate instantaneous entities. But beside these laws Russell introduces a new type which also is regarded as causal.

In defining an instantaneous thing, a thing at a given instant, Russell proceeds from the perspectives we have of our surroundings — for instance, the visual perspectives — and the separate parts of these perspectives which we can say are aspects of different things. The visual perspectives of a person nearby resemble ours to a certain extent, and his perspectives include aspects of most things which we also have aspects of. The different aspects of one and the same thing at the same instant are put together, and are said to be the thing, and the laws we use to put together the different aspects which we say constitute a thing state that there is similarity[3] between actual aspects, and that some form of continuity is to be fulfilled by the assemblage of aspects — a continuity with whose help we can infer ideal aspects.[4]

It seems that Russell calls these laws "causal". He says in a very general statement, "In this very general sense," (that causal

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[1] *Op. cit.*, p. 213.
[2] *Op. cit.*, p. 214.
[3] *Op. cit.*, p. 89.
[4] *Op. cit.*, p. 111.

laws are non-logical laws applying to events) "the laws of dynamics are causal laws, and so are the laws correlating the simultaneous appearances of one 'thing' to different senses."[1] Presumably this latter reference is not merely to the laws applying to simultaneous experiences of the same thing through the different senses of one person, but to the laws applying to aspects of the same thing for different persons, because in the following passage he asks how these laws can help us to define things, and he later shows how they can help us with this definition; but he does not say anything about the laws applying at a particular instant to different experiences of the same thing by one person. For these laws, relatum and referent may pertain to any one of the senses, and there are no special rules regulating their structure. If we wish to say that relatum and referent are to be defined through some general characteristic (except, of course, that they are existents and not abstract entities), it can only be in some transferred or attenuated sense.

Since these laws are to be regarded as causal, it is evident that we may not formulate them in such a way that they may be used to infer from the existence of certain events in the neighbourhood of a thing the existence of other events in the neighbourhood of the same thing, as this would give us an analytical statement, "thing" denoting an assemblage of aspects (or events) grouped in a way described by the laws.[2] It is conceivable that there is no thing, in the sense of "thing" used here. For there to be any meaning in calling them causal, Russell must mean that it is often the case that events are related to one another in the way described by the laws. If we experience a particular event, we can infer with the help of the laws that there are probably similar events, at least, withing a small neighbourhood of the place where we are. But we cannot be certain that there are similar events in the neigh-

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[1] *Op. cit.*, p. 109.

[2] The requirement that the aspects of the same thing are to be arranged about a centre is not emphasised so much as in later works. But he does say, "The aspects of a thing in different perspectives are to be conceived as spreading outwards from the place where the thing is, . . .". *Op. cit.*, p. 93.

bourhood; we may be experiencing an hallucination or a dream or the universe may consist of punctually distributed aspects where no aspect has a similar aspect in its neighbourhood. This uncertainty may be increased by the influence on the degree of similarity of neighbouring aspects due to the intervening medium between the place occupied by the person who perceives the aspect of a thing and the place where the thing is (in the perspective space which we may construct with the help of the different existing perspectives).[1]

To summarise the differences between the theory of causation as developed earlier and that presented in *Our Knowledge of the External World*, we may say that(1) the entities related by causal relations are now taken to be temporally finite, whereas previously they could be instantaneous and that there were in fact causal relations between such entities and (2) the entities have now been defined as mental events, or sense-data, or possible sense-data, or series of these entities, and (3) a new type of relation is regarded as causal, — a relation of similarity, or a "principle of continuity", which is used to group together separate aspects at one instant to a "thing".

One might perhaps say that there is also a difference in the fact that it is not until now that Russell has discussed with any thoroughness the connection between causal propositions and the principle of induction, a theme which assumes increasing prominence in his later works. The question at issue is: how can we justify the assumption of a causal connection, is there any principle we can employ to infer from a certain empirical body of facts by a purely deductive method that an event will probably occur, or that a certain relation which is called causal probably always holds? The principle which provides us with good grounds for adopting causal laws is, according to Russell, the principle of induction which was discussed in "The Problems of Philosophy", and against which the same criticism can be advanced as was put forward in Chapter II.

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[1] *Our Knowledge of the External World*, p. 93.

Chapter V

In *The Analysis of Mind*, 1921, the use of the term "causal law" is extended to include yet another category of relations besides the three already mentioned in *Our Knowledge of the External World*. "Causal law" now covers four different kinds of laws or relations. (1) Scientific laws of the type encountered in physics, especially in dynamics, the only type of law which Russell treats as causal in *The Principles of Mathematics*. The presentation in *The Analysis of Mind* may be described as a deepened conception of what laws like these express, but the divergences from earlier treatments are unimportant. (2) The type of relation which is taken to be causal in "On the Notion of Cause" and afterwards, namely, the pre-scientific and approximate type which is said to apply when there is an often repeated sequence although the quantities appearing in the sequence need not be connected by a relation of necessity, and, indeed, need not even always appear together. (3) The type of law which is first termed causal in *Our Knowledge of the External World*, the laws by means of which separate sense-data and possible sense-data may be grouped together to a "thing" at a given instant; they are now called "perspective laws", or "intrinsic laws".[1] (4) Besides the three foregoing categories of relations, which are obviously different from one another, a fourth is discussed, that of purely psychological laws, or "mnemic causal laws". Russell prefers not to commit himself over the question of whether these laws are only approximate, and replacable with laws included under cate-

[1] "Intrinsic laws" are mentioned already in the essay "The Ultimate Constituents of Matter" (*Monist*, July 1915, pp. 399—417. Reprinted in *Mysticism and Logic*, 1918, Chapter VII), but the term there seems to denote only relations between events at different times. (*Mysticism and Logic*, p. 129).

gory (1) when we know more about physiological processes and the connection between physiological and mental processes, but he thinks there is much to support this view. His opinion seems to be that in their present form, the mnemic causal laws are not reducible to any of the preceding types, but it may be that this is due merely to our lack of knowledge.

When we have considered these types, we will conclude with a supposition which Russell makes with regard to the scientific laws, that it may be possible to reduce them to still simpler laws applying to separate sense-data, actual or possible, instead of to the whole groups of sense-data or possible sense-data which constitute material bodies, so that, for instance, physical laws — which, according to Russell, are concerned with relations between material bodies — will be derivable from these new laws.

1) Russell employs similar reasoning to that which in "On the Notion of Cause" led him to the conclusion that there must be a finite interval of time between cause and effect, and he now arrives at the conclusion that the only certain laws acceptable to science are laws containing differential equations (or finite-difference equations if quantum mechanics is correct[1]) as essential elements. He says that all events turn out to be processes when examined closely. If one event is to be the cause of another, they must in the first place be contiguous in time, because otherwise something may happen in the interval between them which prevents the occurrence of the effect. But, he continues, since we are dealing with processes, it is difficult to see how the first part of the cause is relevant for the effect. We can therefore contract the cause more and more, and likewise with the effect, until we finally arrive at a mere description of how the change occurs at the instant when we say that the original cause is transformed into the effect. If we wish to describe the change, and the direction in which the change takes place at every instant of a process (cause and effect may be regarded as one process), we are led to differential equations, as they are especially designed to express this. These laws may be called causal despite the fact that they

[1] *The Analysis of Mind*, p. 306, note 1.

only deal with instantaneous changes because we can use them to infer unobserved phenomena from the data of a given situation. The laws themselves do not actually express the relation between the given situation and a future situation which is inferred, but they are intimately associated with this relation. We can find the relation between two particular situations by using our laws and certain processes of calculation — integration, and so on; or, as Russell puts it for such a law holding in a system of particles: "it tells us how the particles's motion is changing at each moment, not where the particle will be at some future moment"[1] This is no new feature in Russell's theory; in "On the Notion of Cause"[2] it occurs, for instance, in the description of the meaning of the differential equations which express causal relations, but it appears that it has not been expressed so explicitly before.

It might be asked whether a reason analogous to that which Russell gives for taking cause and effect to be contiguous in time cannot also be applied to the consideration of causal laws which are expressed with the aid of differential equations. As already mentioned, it is stated that cause and effect must be contiguous in time, because otherwise something may happen in the interval between them which prevents the occurrence of the effect, or, to put it another way, if the cause is not immediately followed by the effect, a disturbance may intrude between them. We do have to assume something of the sort even when we express the laws with the help of differential equations: we must assume that the processes described by our laws go on undisturbed by other processes. As soon as a disturbance occurs during an interval of time, our differential equations are incapable of describing the change involved in the processes in question during this interval. And when we use these laws to calculate events or states of processes at a later instant t by integration over a certain interval of time or in some other way, we must assume that the processes are undisturbed during the entire period up to t. It would be equally reasonable to assume that cause and effect do not need to be

[1] *Op. cit.*, p. 95.

[2] *Mysticism and Logic*, p. 195.

contiguous in time, but, that the interval between them must be free from disturbance. It might be objected that we run the risk of defining freedom from disturbance in such a way that the proposition "the effect B always follows the cause A after a time interval τ, if the interval is free from disturbance" becomes analytical. But this risk does not seem greater than the risk that, when we say of a process " the direction of change in this process is adequately described by the following differential equation, assuming that the process procedes free of disturbance", this proposition will be analytical.

The fact that, in spite of this, we prefer the laws which describe the course of change in a process, instead of the traditional kind of causal laws which postulate a certain ever-present relation between two events, depends on factors which Russell himself has to some extent stressed in previous works: they embrace in one expression an infinite number of cases, they describe more immediately that which exists — namely, certain characteristics of process change — and they may even be used to deal with events that are never repeated. Moreover, there is reason to suppose that for certain of these laws the universe is an isolated system in the sense that no disturbances can affect the universe in respect of the subject matter of these laws.

2) The pre-scientific type of relation which Russell wishes to call causal is of the form "A is usually followed by B"[1], where A and B denote events which are not too narrowly defined, and it is described in the same way as in "On the Notion of Cause", except that it is now emphasised that we can have many causes for one event B, if the 'cause' is taken in the special sense that it is an event which stands in the actual pre-scientific causal relation to B. If workmen leave a factory F at midday when the hooter sounds, we can say that the sounding of the hooter is the cause of the men's leaving the factory. But other hooters at other factories all over the country are sounding at the same time, and we are just as much justified in saying that the sounding

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[1] *The Analysis of Mind*, p. 96.

of any one of these hooters is (in the present sense) the cause of the workmen's leaving factory F.[1]

The usage of language suggested here by Russell has been criticised by C. D. Broad.[2] Broad considers that the unreasonableness of the idea that causation may not imply more than a regular sequence is manifest in this very example of the workmen and the hooter. There are two main points in his criticism: (a) there is no continuity of space and time in the example given[3], and (b) it may be that a regular sequence is not at all a part of what we mean by a causal relation, but only a not-infallible indication that we have to deal with a causal relation.[4]

If Broad requires that there shall always be spatio-temporal continuity for quantities related to one another in a certain way in order that the relation between them may be called causal, he cannot accept Russell's use of words on any point, because in three cases for certain and probably in the fourth, when Russell talks about causal relations, spatio-temporal continuity in Broad's sense is not an essential or even a necessary condition. In the first case, that of laws for the advanced sciences, Russell even approves relations containing "finite-difference equations", provided quantum mechanics proves to be correct, and these equations do not involve continuity, only nearness in time and space of the phenomena in the relations, or, to put it more exactly, between the phenomena in certain relations derivable from these laws. This must mean that Russell does not regard spatio-temporal continuity as an essential condition to be fulfilled if the relations with which the advanced sciences deal are to be called causal. However, it is not certain that the discontinuity here is not of a kind which even Broad could accept. The second case concerns the pre-scientific laws to which Broad specifically objected, on the ground that they permit discontinuity. In the third case, that of the "perspec-

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[1] *Op. cit.*, p. 97.

[2] Susan Stebbing agrees with this criticism in *A Modern Introduction to Logic*, London, 1930, pp. 282—285.

[3] *The Mind and Its Place in Nature*, London, 1925, p. 456.

[4] *Op. cit.*, p. 454 and 456.

tive laws", laws applying to aspects occurring at one and the same instant, it is obvious that there is no spatio-temporal continuity in any sense intended by Broad. Nevertheless, it is possible to speak of a certain kind of continuity. Between any two aspects A and B which are part of a "thing" at a given instant, there is normally a series of other aspects which gradually change from those resembling A more than B to those resembling B more than A, if we move from the A position to the B along any line which does not change direction.[1] But all these aspects from points on the line exist at the same instant, and it is therefore inadequate to speak about continuity in time. As Broad himself observes, there is no continuity of the kind in question when we come to consider the fourth case, the mnemic laws. If we can assume that these laws are irreducible, we have no reason to suppose that there is continuity of space and time between, for instance, a past experience and a present memory of it.

It thus seems that Broad would find himself unable to accept Russell's causal terminology at any point, with the possible exception of scientific laws, if he were to require spatiotemporal continuity between all phenomena entering into a relation called causal. The only point on which Broad's criticism may be important is in that Russell's usage of language does not conform with that generally accepted, that Russell has committed the error of believing his analysis to have yielded four purified types of the general use of causal terms; and the error of this should be most evident in the second case, where it is a question of regular sequence, because Broad says that many people attribute more to the causal relation than merely regular sequence. Broad himself thinks, as was previously mentioned, that a regular sequence is merely an indication that a causal relation *may* be involved. However, Russell has never claimed that his use of language on this point is in accordance with any common usage.

[1] What is meant by the same direction is that if the line is described in polar coordinates with the pole at the centre of the "thing", the angular coordinate of a point moving along the line always increases or always decreases.

The only general condition Russell has imposed on a causal relation is that it must be possible to infer with its help the existence of something from the existence of something else, and at least the first, third and fourth cases fulfil this. It is more doubtful whether the second also does[1], but if Russell is prepared to use the causal terminology in this case as well, the mistake to which he is thus liable is quite different from that which Broad wishes to ascribe to him.

3) As regards the perspective laws, the laws by means of which the various aspects of a thing (in the ordinary sense) at one instant are assembled to an entity which is called a "thing", *The Analysis of Mind* contains no pronouncement where they are explicitly called causal. Thus there are only two types of causal laws included in the concluding remarks about the laws of future sciences, which we will consider later. They are (i) physical, which might be described in terms of groups of aspects called "things", and (ii) mental, comprising the laws which are not physical, and which describe relations between "particulars", a particular being a sensation or an image.[2] But the perspective laws can neither be transcribed in terms of groups of aspects — and are therefore not physical laws, — and neither does any of the terms need to be sensation or image — so they are not mental causal laws either. Perspective laws might apply to aspects which are all ideal, not experienced by anybody. But this does not signify that Russell has said that perspective laws are not causal. Presumably, all that he wishes to express in the place quoted is the relations between the future scientific laws and the laws which we have just referred to as physical or mental.

Let us consider these perspective laws a little more closely. The most general definition given is the following: "In a generalised sense, all the intrinsic laws of change of appearance may be called 'laws of perspective' ".[3] Simultaneous aspects or "appearances" of a thing may be different at different distances from the thing,

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[1] For a criticism of this case, see Chapter III, pp. 51—52.

[2] *The Analysis of Mind*, p. 306.

[3] *Op. cit.*, p. 106.

depending on the medium surrounding the thing, but the laws used to determine the differences between these aspects are not intrinsic in Russell's terminology. The "laws of perspective" which thus deal with simultaneous aspects include as special cases the ordinary laws of perspective that apply to visual phenomena, but there are also laws dealing with the way in which non-visual aspects of a thing change with the distance from the thing, and in general, with position.[1] Russell says that if we knew sufficient about these laws, we could express them with the help of differential equations.[2] This (or that they can be expressed with the help of "finite-difference equations") is for Russell an important condition if they are to be developable to causal laws in a perfected science.

The fact that they are "laws of change" fits them for Russells' causal terminology in other respects as well. The laws which in science replace the traditional causal laws dealing with well-bounded cause and well-bounded effect are described in just this way, as "laws of change".[3] But the situation here is not exactly the same: the changes are not now functions of time.

If we know what an aspect "looks like" — to use a vague term — at a certain place for a certain thing, we can employ the perspective laws and the laws applying to the medium surrounding the thing to infer what other aspects of the same thing look like from other places, and if we are very near to the thing, the disturbing effect of the intervening medium on the hypothetical appearance of the aspects if only these perspective laws applied would be so small that it might be neglected.[4] The perspective laws therefore fulfil the general condition to be satisfied by causal laws, given in *Our Knowledge of the External World*, according to which they

[1] Two positions are associated with an aspect: (i) the place where the thing is said to be situated, called the active place, and (ii) another place, called the passive, in which position any person present would experience a sense-datum. Here we are concerned with (ii).

[2] *Op. cit.*, p. 125.

[3] *Op. cit.*, p. 93.

[4] Russell states more precisely that "the laws of perspective account for the differences to the first order of small quantities, and other laws are only required to account for second-order differences". *Op. cit.*, p. 125.

may be used to infer from the existence of something the existence of something else. It is true that what we infer will not be so precise if we assume that certain disturbances may arise from the surrounding medium, but we can still infer something with the sole help of perspective laws, contrary to what is the case if only laws for the medium were available, since then we could not infer from the existence of something the existence of something else. It is obvious that the inferences made with the help of the laws are also uncertain in the way described in the preceding chapter for the corresponding laws mentioned in *Our Knowledge of the External World*, — we cannot be certain that every event is an element in a set which makes up a thing.[1]

In brief, we may say that it is quite in accord with Russell's causal terminology to call perspective laws causal and that there is reason to suppose that Russell really regards them as causal, in spite of the fact that he nowhere provides us with a statement from which it would immediately follow that they are causal in his system. Yet another reason for saying that it is appropriate to call the perspective laws causal in Russell's system appears when we analyse what Russell means by causal dependence.

4) The fourth form of relation which Russell calls causal, the purely mental relation, enters into the discussion by way of the general theory of knowledge which is developed. When the sensations and images[2] (particular acts by which sense-data are experienced are no longer assumed) which belong to a certain place — in Russell's terminology "the passive place" — are composed into a mental biography, the question arises of whether there are regularities, laws linking different entities in the same biography, which cannot be included among the physical causal laws. Russell answers that, as far as our knowledge extends at present, there

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[1] Chapter IV, p. 95.

[2] "I believe that the stuff of our mental life, as opposed to its relation and structure, consists wholly of sensations and images". *Op. cit.*, p. 109.
Russell describes the difference between sensations and images thus: "Images, as opposed to sensations, can only be defined by their different condition: they are caused by association with a sensation, not by a stimulas external to the nervous system — . . ." *Ibid.*

is nothing which directly contradicts the occurrence of irreducible mental laws, but it is likely that these laws will be reducible when we know more about physiological and mental phenomena.[1] It is therefore likely, though not certain, that mental causal laws are derivable from physical laws.[2]

According to Russell, the mental causal laws which are exemplified by laws of association, laws to which the memory is subject, and others, are of the form: "A, B, C, . . . in the past, together with X now, cause Y now"[3], where "A", "B", "C", "X" and "Y" denote either parts of perspectives — for example, separate aspects of objects — or complete perspectives of, for example, the whole of the visual field at an instant, or the whole of a person's mental biography at a given instant. A, B, C, . . . , Y are thus sensations or parts of sensations, or images; experiences in dreams, hallucinations, or something of the sort. In contrast to what is possible with every part of a sensation, it seems to be impossible, according to Russell, to assemble purely mental phenomena together with other entities to a "thing", therefore only one place can be assigned to them, namely, the place at which the mental biography to which they are said to belong is situated.[4]

One of the characteristics which distinguishes purely mental laws from physical, and which makes it impossible to transcribe them from their present form to that of physical laws is that they (as follows from the quotation) do not state that one present situation causes another present situation but that a series of events in the past, together with an event (stimulus) in the present, causes something in the present (a memory image, for instance). If psychological laws were irreducible, we could not regard the state of the universe — which includes stimuli, everything which may be denoted by "X" above — during a finite interval of time as the cause of all mental events during the interval, since it is

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1 *The Analysis of Mind*, p. 304.

2 *Op. cit.*, p. 307.

3 *Op. cit.*, p. 87.

4 "Some particulars, such as images, have no 'active' place, and therefore belong exclusively to psychology." *Op cit.*, p. 301.

not only events within this interval which constitute the cause in this case, but also a series of events prior to the interval. It is, of course, always possible to make the interval so short that some of the past events which figure in the cause fall outside the interval. It may be that the events in the past which must be taken into account in psychological laws can be determined by means of non-psychological laws from the state of the universe during the given interval, but we cannot definitely take it that it is always so without first assuming that the psychological laws are reducible. Suppose the past events are dreams, which therefore contain images, and that, in accordance with Russell's view[1], the problem of the reducibility of mental laws may be translated into the question of whether images are causally dependent on physical phenomena or even subject to other than purely physical laws. Then we cannot suppose that the past events may be determined with the help of non-mental laws from the state of the universe during the given interval, without assuming what we wish to establish as probable.

If these images could not be related with any physical object, that is to say, they could not be incorporated in the constitution of a physical thing, and if, as Russell maintains, physical causal laws apply to sets of entities which are called things, not all psychological laws could be reduced to physical, since at least some of them deal with images. But if the images are related to physical objects and the relation mentioned implies that "the images" are causally dependent on physical events, events involving the physical things to which the images are related and with which physical laws may deal, then the psychological laws are reducible.

Under these circumstances what Russell would accept as causal dependence is of central importance.

He says that A is causally dependent on B if (i) there is a many-one relation between A and B such that we can infer B when A is given but not A when B is given, and (ii) the relation is causal.[2]

[1] *Op. cit.*, p. 303.

[2] *Op. cit.*, p. 302.

Sensations are causally dependent on physical phenomena because, says Russell, a perception of a thing from a place near to it indicates the appearance of the thing from places farther off, in a manner in which a perception of the thing farther off does not determine its appearance when closer. And the thing is defined as the limiting set of appearances of the thing as we approach it closer and closer.[1] This limiting set is the closest and most exact conception we have of the thing; therefore, all other appearances of the thing depend on the thing (defined as the limiting set of appearances) in the manner described. In order to be able to say that the dependence is causal, we need to know that the relations between the appearance of the thing and the thing are causal. The laws we use in determining the appearance of a thing far off from the appearance of the thing close at hand are perspective laws and laws for the intervening medium. Therefore, when Russell says that our sensations are causally dependent on physical phenomena, we have in this a statement which indicates indirectly that he regards the laws of perspective as causal.

So the question of whether or not there are irreducible psychological laws is bound up with that of whether or not there is a many-one causal relation between physical phenomena and images. If there are such relations, then we should be able to calculate with the help of non-psychological causal laws how we would react mentally and bodily during a finite interval if we know enough about the physiological state of the body and the condition of the physical environment during the interval. There would be no room for specifically psychological laws. Russell does not wish to express a definite opinion on whether images are causally dependent on physical phenomena, but he does say, "I think the bulk of the evidence points to the materialistic answer as the more probable".[2]

We may regard it as a natural consequence of Russell's conception of material things as "fictitious systems of regular appear-

[1] "This limiting set of appearances may be defined, for purposes of physics, as the piece of matter concerned." *Op. cit.*, p. 107.

[2] *Op. cit.*, p. 303.

ances"[1] which can never be experienced at one in their entirety, and which we must look upon as constructions, that he suggests that the only primary causal laws science would contain if it were more complete would be relations between individual particulars, quantities to which existence may be ascribed, and which may be immediately perceived under favourable conditions. The laws of physics, he says, would then be derivable from these fundamental causal laws, but it is not certain that all psychological laws need necessarily be derived since sensations and images are particulars immediately perceptible, not constructions. In any case, the primary causal laws would resemble the psychological laws more than the physical.

It appears that Russell is correct in this suggestion, because modern quantum mechanics works with principles which quite closely resemble the ones expressed here. We will say something more about this in the next chapter.

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[1] *Op. cit.*, p. 306.

Chapter VI

In the two works which appeared in 1927, *The Analysis of Matter* and *An Outline of Philosophy*, the mnemic causal laws of *The Analysis of Mind* are again discussed, as well as perspective laws, causal laws employed in the advanced sciences, and pre-scientific laws (these latter to a very limited extent, however). The categories are the same, but the interpretation of the laws within them has undergone fundamental changes, evidently on account of the influence on Russell of the great debate about the basic problems of physics which went on during the twenties. With regard to quantum mechanics, we must remember that it achieved its more systematic form about the middle of that decade, through the work of Heisenberg, Born, Jordan and Dirac. So it was hardly possible for Russell to publish an analysis of the problems presented by the new theory before 1927. Furthermore, Eddington's broad presentation of the theory of relativity, which Russell often has recourse to in *The Analysis of Matter*, was published in 1922, one year after the publication of *The Analysis of Mind*.

The analysis of causal problems is carried considerably further than in any of his other works, and it is more precise than in any of the works we have previously considered, with the possible exception of *The Principles of Mathematics*. This is only to be expected, as the book is intended to give an analysis of the technical and abstract modern physics, which employs, at least in some domains, carefully defined relations and a terminology of extraordinary exactness.

The biggest differences, compared with earlier works, are to be found in the interpretation of scientific laws. Differential equations are no longer of such obvious importance; Russell introduces the idea of well-defined events as the cause of certain other well-defined events which may be called the effect, and he doubts whether he can talk about the state of the universe at a given instant as he had done previously. Even the terminology is changed in many respects. Thus, for instance, "intrinsic laws" has a different meaning from what it had in *The Analysis of Mind.*

We will mainly discuss the analysis of scientific causal laws in this chapter, but to some extent perspective laws as well. The final part of the chapter will deal with a problem which receives little attention in *The Analysis of Mind,* though it is discussed in *Our Knowledge of the External World* and *The Problems of Philosophy,* and which now emerges again — the problem of the certainty of causal laws.

Pre-scientific causal laws, those involving only a certain "regularity of sequence", which were treated at length in "On the Notion of Cause", are again mentioned, but hardly distinguished as a special category; mnemic causal laws and mnemic causation are discussed in *An Outline of Philosophy*[1], and the treatment does not seem to have undergone any important changes since *The Analysis of Mind.* We will therefore not consider either of these types.

1. Let us first consider causal laws used in the advanced sciences — notably physics. Study of quantum mechanics seems to have lead Russell to make two important changes in his interpretation of causal relations valid in our world. He now thinks (a) that the causal relations are not always expressible solely by means of differential equations, and (b) that we are entitled to speak about bounded causal systems where isolated events are causally related to each other.

(a) In "On the Notion of Cause", it is stated that scientific causal laws express a "sameness of relation", or, more precisely, "sameness of differential equations".[2] Russell's belief in the unique role of differential equations is still part of the foundations. It is not until *The Analysis of Mind* that he mentions that ordinary differential equations are perhaps not usable in quantum mechanics, though "finite-difference equations"[3] may be employed there; this develop-

[1] *An Outline of Philosophy,* pp. 298 ff.
[2] *Mysticism and Logic,* p. 195.
[3] *The Analysis of Mind,* p. 306.

ment is insignificant, and cannot be said to imply anything new in principle. Differential equations retain their importance in *The Analysis of Matter*, especially for the laws used to relate events occurring at different instants as apertaining to the same "thing". The whole of the theory of relativity is constructed with the help of differential equations. But Russell makes the reservation, "It may well be" that there is another field, "that dealt with by quantum theory — in which this whole apparatus is inapplicable".[1] This supposition has turned out to be correct. In present-day quantum mechanics, it is not only differential equations that constitute the mathematical part of the relations connecting observable quantities, but the relation between these quantities also involve other expressions, such as integrals expressing the probability that the entities are to be found within a certain region, and such as expressing the "range" of a calculated value. Even when the range is zero, when we are dealing with a quantity which can only take values of a discrete scale, the relation involves expressions which are not differential equations.

(b) The laws describing quantum changes, which occur in quantum mechanics but not in the theory of relativity and not in classical mechanics, are, according to Russell, examples of "extrinsic causal laws", or, in other words, laws where "one piece of matter is mentioned as concerned in the behaviour of another".[2] Russell thinks that in this respect the laws are comparable with those of classical mechanics, where, for instance, the acceleration of the Earth in its motion round the Sun is a function involving data about the Sun. But in Einstein's gravitational theory, the motion of the Earth about the Sun need not be regarded as something involving Sun and Earth (as a causally related couple); in-

[1] *The Analysis of Matter*, p. 108. Elsewhere Russell utters in a quite general statement about the process of replacing the unscientific approximate laws by scientific, more exact laws. "We arrive in the end (possibly not at the very end) at differential equations." The statement in *An Outline of Philosophy* that "Scientific laws can only be expressed in differential equations" (p. 122) is presumably to be regarded as a rhetorical exaggeration.

[2] *The Analysis of Matter*, p. 324.

stead, the motion of the Earth at any instant is determined by the metrical characteristics of the field in the immediate neighbourhood of the Earth's position at that instant.

In a quantum mechanical process where one atom emits energy in the form of radiation of a definite frequency, which is absorbed by another atom, it is difficult to avoid a description of a type where "one piece of matter is mentioned as concerned in the behaviour of another".[1] The notable point is that in contrast to the classical laws of mechanics, which are also extrinsic — the new mechanics admits a bounded process, where we may speak of one particular event as the cause of another. However, we need not, and indeed, we cannot, say what part atoms near to the one which emits the quantum of energy play in that emitting, and despite the eventuel influence of the surrounding atoms we may regard an atom as "the same" before and after the emission of energy; and, what is perhaps even more illuminating, we may associate the light quanta which radiate from an atom during a quantum change with the atom until they are absorbed, and may regard the phenomena as a bounded causal chain, whereby the emitted light is causally connected with a certain atom in a manner in which it is not connected with any other.[2] As Russell has so often pointed out, in classical mechanics a knowledge of two configurations of the material of the whole universe, or a knowledge of the state of the universe during a finite interval of time, is required if we are to be able to calculate what is going to happen in a subsidiary region. Every phenomenon in a particle system is there as much — and as little — connected with a certain other phenomenon as it is with any phenomenon.

Not less important are the changes in Russell's conception of

[1] *Op. cit.*, p. 331. Russell even considers the possibility that laws of quantum change are the only ones which cannot be interpreted so that their extrinsic character disappears.

[2] *Op. cit.*, pp. 368 and 372. From the standpoint of a theory, however, where the photons do not have any coordinates for their positions but belong to a field of their own, it seems to be difficult to maintain the view about the photons as connected with a certain atom in the way Russell thinks that they are.

causal relations which he is obliged to make from relativistic considerations. The more important of these changes will be treated in the following sections (c) — (e).

(c) He is now prepared to abandon "action at a distance", a concept so intimately associated with Newtonian mechanics.[1] As has already been pointed out, in Einstein's theory of gravitation it is not necessary to distinguish one body as the cause of another's behaviour; instead, each body behaves in a manner dependent on the metrical properties of the surroundings. These metrical properties might be different if the distribution of mass in the universe were different, or, to put it more precisely, if the metrical properties of other parts of the universe were different; but this does not prevent us from regarding the behaviour of a body as determined by the metrical properties of the body's immediate surroundings. It is the form of the gravitational field at the place where the body is that is decisive. To obtain a clearer idea of what it was that encouraged Russell to alter his theory at this point, consider what would happen if the sun were suddenly to disappear without being replaced. According to relativity theory, the gravitational field in the neighbourhood of the Earth would not change for some minutes, and during this time the Earth would continue to move in the same path with respect to the fixed stars (to express it in classical mechanical terms) that it would have followed if nothing had happened to the Sun. The concept of action at a distance seems to be as much out of place here as it is in the case of a light ray from the Sun striking the Earth. Russell says that we may, however, talk about "action according to distance", and he presumably means the same with "according to distance" as when we say that the solar radiation striking a unit surface varies according to the distance of the surface from the Sun. To develop the comparison a step further, the light within any small volume of the universe is derived from a number of dif-

[1] In "On the Notion of Cause", p. 192, Russell says of the maxim "A cause cannot operate except where it is", "This maxim is very widesperad: it was urged against Newton, and has remained a source of prejudice against 'action at a distance'".

erent bodies, or, we may say, from a number of different places, and the form of the "light field" within the small volume depends partly on the distances to the places from which we say the radiation was emitted. In this case we may say, however, that what happens within the small space is causally connected with what happens in other places, actually, with the quantum changes at the places from which the radiation is said to come. It is related to the events at the other places by extrinsic causal laws. But, for the gravitational field, there are no events at other places which are connected to events in the small space we are considering, of a kind which might be said to depend on the gravitational field. Consequently, Russell is inclined to regard the Einsteinian laws of gravity as "intrinsic", which means in *The Analysis of Matter* that they refer to the relations applying to the same piece of matter at different instants.[1] The intrinsic laws are there used to determine which events are to be assigned to the same thing at different instants, not as in *The Analysis of Mind*, where intrinsic causal laws determined which events should be associated with the same thing (or, should constitute the thing) at an instant.[2]

The laws of gravitation relate events belonging to the same thing with one another, without bringing other material bodies into the relationship. No extrinsic laws relate what happens at other places to the happening that a body moves in the surrounding field in a certain way at a particular place. The field may be described as a superposition of several parts (without relative order, and combined in a whole in a way which makes it impossible to identify the "parts"), the magnitude of each part depending on the distance to a particular place. "This is not action *at* a distance", says Russell, "but action *according to* distance". From this point of view, we can hardly say that something has an effect on something else at a distance from it.[3]

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[1] *The Analysis of Matter*, p. 246 and pp. 322—323.

[2] The closer characterisation of intrinsic causal laws is discussed later, on pp. 124 ff.

[3] *Op. cit.*, p. 328. The same problem is discussed in *An Outline of Philosophy*, p. 123.

The rejection of action at a distance means that if we wish to assume that some A affects some B, A and B must overlap in some sense in the space-time continuum, which in its turn means that at least one of them must have finite spatio-temporal extension, and for two entities of which neither is in its entirety a part of the other, that both must have such finite extension. According to Russell, the most fundamental laws must deal either with events of finite extension which are "compresent", or with how chains of compresent events are built up.[1] We will return to Russell's description of these events in (f), when we consider the general definition of causal laws. Meanwhile, we will consider two further changes in Russell's conception of the causal relation which relativistic considerations obliged him to make.

(d) Earlier, he never says that causal relations must relate events apart in time; on the contrary, it seems possible for causal laws to relate "simultaneous" events. For instance, in "On the Notion of Cause", a system is said to be deterministic if the state of the system at every instant is determined by a series of data referring to the system and the instants at which they occur. It is obvious that the laws used to determine the state of the ystem at any given instant are regarded as causal. This means therefore that there is nothing to prevent the use of these laws to determine events which are simultaneous with certain of the events involved in the determination. However, this need not mean that causal laws may relate events which are all simultaneous. But in *Our Knowledge of the External World*, it is stated explicitly that causal relations may apply to events which *all* occur at the same instant.[2] Now, on the other hand, causal laws in our universe connect only events separated by a time-like interval in the relativistic sense.[3] There can be no causal connection between events of which no one occurs at an instant simultaneous with or later than the instant at which a light-ray would arrive at it which started from any one of the other. This depends on the fact that the greatest speed of any

[1] *The Analysis of Matter*, p. 356.

[2] *Our Knowledge of the External World*, p. 216.

[3] *The Analysis of Matter*, p. 313. See also pp. 212 and 368.

process is the speed of light.[1] It takes about eight minutes for light to travel from the Sun to the Earth, so what happens on the Sun after a ray of light has left the Sun cannot be directly causally related with anything occurring before the time of the ray's arrival at the Earth about eight minutes later. Events on Sun and Earth which are related to each other in the manner described are not separated by a time-like interval, but by a space-like one. However, there are indirect causal relations between such events, events which, to express it roughly, enable us to infer what happens on the Sun even when we cannot be affected causally by it in a direct way. These relations are not of their nature different from the directly causal; it is merely that they describe events which proceed in another direction of space-time than that in which we are situated.

This condition on causal laws that they should express relations between events separated by a time-like interval might be supposed to be one which need be fulfilled only in our world, not, for instance, in a world where there was no highest velocity, or, at least, where there was a velocity higher than any finite velocity such as a causal process might have. But, if "interval" is defined by means of the most rapid process, as it is in the theory of relativity, and if there were processes of infinite rapidity, there would be no space-like intervals. Every interval might be causally related to any other, but all intervals between them would be time-like.

(e) A difficulty associated with the occurrence of space-like intervals between events is that of avoiding ambiguity in such expression as "the state of the universe at a given instant" or "the state of the universe during a given finite interval of time", expressions which occur in Russell's earlier formulations of laws of causality.[2]

Russell states in *An Outline of Philosophy* that two events separated by a space-like interval are not definitely simultaneous in relativity theory, though it is impossible to distinguish one of

[1] A phasevelocity may be greater but this velocity is not relevant in a discussion of causal processes. A phasevelocity in a given case does not describe a velocity with which energy is transmitted in that case.

[2] For example, *Our Knowledge of the External World*, p. 221.

them as the first. If A and B are two such events, there is always a place with a local time such that A precedes B in time, another place where B precedes A, and yet another where they are simultaneous. "Place" here should not be taken as a region fixed to something else in some sense; it is only a question of local bodies to which we can assign a local time. There is no cosmic time, only a lot of local times, and it is therefore impossible to talk meaningfully about "the state of the universe at a given instant", for, which of all the events separated from an event at a given instant (in the local time of the latter event) by a space-like interval are to be included in the "state of the universe"?[1]

(f) Russell's conception of causality exhibits innovations in several other respects as well. He rejects the classical mechanical idea that configurations of a particle system at two given instants determine causally the configuration of the system at any other instant, and the idea expressed in *Our Knowledge of the External World* that the state of the universe during a finite interval, however short, determines the state of the universe at any other instant, and he gives as his reason that what is meant by "the configuration of a system at a given instant" is not unambiguously defined, nor is "a configuration during a short interval of time."

Russell's causal relations do not now apply to configurations or other entities with so great extension in space-time, but to events in the immediate neighbourhood of each other. Nowhere is the analysis of the entities which may figure in a causal relation carried

[1] *An Outline of Philosophy*, p. 115. This difficulty is possible to overcome by placing an arbitraty three-dimensional surface on which all intervals are space-like through the light-cone, as is pointed out in an article "The Radiation Theories of Tomonaga, Schwinger, and Feynman" by F. J. Dyson in *Physical Review* 75, Feb. 1. 1949, p. 486. The arbitrariness will not disappear by use of this method but we get something analogous to the state of the universe in a given instant (in the prerelativistic sense) in the sense that all events on such a surface are together enough for the determination of everything happening in the universe not on the surface with the accuracy admitted by quantum mechanics. The only difference is that we have many (perhaps an infinite number of) different surfaces to choose between at every instant whereas classical mechanics assumed only one definitely determined state of the universe at a given instant.

so far as in *The Analysis of Matter*. A new feature is the division of such entities into different categories, and the exact specification of them.

The following sentence in which Russell gives a general description of the relations he wishes to call causal appears to have a central importance: "We shall say, then, that all causal relations consist of a series of rythms or steady events separated by 'transactions'".[1] This is a more precise form for our world of a yet more general definition of a causal relation, which reads, "There is a causal relation whenever two events or two groups of events of which one at least is co-punctual, are related by a law which allows something to be inferred about the one from the other".[2] Russell does not discuss this more general form, but proceeds at once to the more special definition. It can be seen, however, that the more general form is in conformity with both "action at a distance" and with states of a whole system at given instants as causal determinants for the whole system at other instants, if it is assumed that there are no events in the system separated by a space-like interval.

The more special definition of causal laws mentions "transaction", "rhythms" and "steady events", three terms with well-defined meanings in Russell's terminology.

A "transaction" is a quantum change, which involves a change in the energy of a system.[3] When an atom emits light in passing from one energy level to another, a transaction occurs, as also when light is absorbed by an atom. The problem of whether transactions have temporal extension is discussed, and that of their possible duration, but Russell does not take any definite standpoint. He prefers to leave the question to physicists. Nevertheless, he does advance certain arguments in support of the idea that they must last at least as long as a complete wave, in the case of a periodic process transporting energy.[4]

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[1] *The Analysis of Matter*, p. 368.

[2] *Op. cit.*, p. 367. [3] *Op. cit.*, p. 355.

[4] *Op. cit.*, p. 365. On p. 360, he writes, "We do not know whether quantum changes are really sudden or not; . . ."

A "rhythm" means a periodic process, or, more precisely, a "qualitative cycle of events". Russell thinks that only changes of position cannot form a series of events constituting a rhythm; there must be qualitative changes, and the series of events involved (with certain periodically recurring qualities) must advance even if we measure time from the place at which the events occur. That is, the process must be periodic, and the cycles must succeed one another even in the local time of the region where the events occur. The period of a cycle could be measured, according to Russell's suggestion, by counting the number of separate events occurring during the period, that is to say, the number of qualities in the least series of qualities which is repeated.[1] Russell further says that if it were not a question of a qualitative cycle, but instead, only of positional changes, a rhythm could not be periodic, regarded from the region of space-time in the immediate neighbourhood of the place where the rhythm is situated, since, there, the observer is at rest. Whether or not these qualitites can be correlated with similarly periodic metrical changes in the surrounding region, or with other periodic processes, is a matter by itself, which does not render it any the less necessary that the rhythm involves a qualitative cycle.

One example of a rhythm given by Russell is the Bohr-Rutherford atom, but this is not simply a matter of a periodic change of position of an electron with respect to a nucleus. We must postulate that there is a qualitative cycle at the place where the electron is situated and a qualitative cycle at the place where the nucleus is situated, in spite of the fact that we cannot observe the atom when it is in a definite rhythm, or, more correctly, when it contains rhythms which are going on without change, but can only detect it when the rhythms undergo changes and energy is emitted or absorbed. As Russell himself points out this view is abandoned in the Heisenberg quantum mechanics, where the atom is represented by a matrix in which its different energy states are included, so that it is not at all necessary to suppose that something happens

[1] *Op. cit.*, p. 357 (*m* occurrences of the "same" quality are counted as *m* qualities).

to the atom when it is not undergoing a transaction. This theory has the great advantage that it reduces the non-observables involved in physical theories.[1]

The above definition of rhythmic processes, as Russell points out, does not make light a rhythm, although it may at first sight appear to do so. That light is not a rhythm in this sense depends on the fact that an observer moving at the same speed as the light cannot experience any part of the wave other than that where he is. For such an observer, its periodicity can only be space-like, comparable in a way with that of corrugated iron.[2] But light appears to be a rhythmic process for all other observers than those moving with it at its own speed. Seen from the light wave itself, every part of it according to Russell is a "steady event" that is to say, an event lacking physical structure, and compresent with at least two events which are not themselves compresent.[3]

However, it is not altogether clear that every part of a light wave is a steady event in Russell's sense of the term, since the whole light wave may be said to have some sort of structure, even if only a space-like one seen from the wave itself. If we consider just a small part of a light wave, it may be impossible to say that it is in any sense periodic, that it contains any recurring space-like structure, but, nevertheless, it may contain *some* space-like structure. Russell assumes that each part of a light wave is without structure, and it therefore seems likely that he either regards the parts he speaks about as without space-like structure, or regards the question of whether something has space-like structure or not as irrelevant for the question of whether or not it is a rhythm or a steady event, and when he says that a steady event has no physical structure, merely space-like structure is not taken into consideration. We will suppose that the latter alternative is the valid one, because without any close definition of what is meant by "part" it is stated that "each part of a light-wave is a steady event in the sense described above" and because it seems as if

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[1] *Op. cit.*, p. 358.
[2] *Op. cit.*, p. 357.
[3] *Op. cit.*, p. 356.

Russell thinks that there really are *some* part of the light-wave which has a space-like structure as the comparision with the corrugated iron makes probable.

Even perceptions are mentioned as an example of events which contain steady events. Russell does not wish to exclude the possibility that even perceptions may be rhythmic processes, like the physical phenomena to which they are correlated, as, for instance, a light ray which is periodic with respect to the body which absorbs it, and which is correlated with a visual perception. If perceptions were rhythmic processes and contained steady events we should have an example of steady events being parts of rhythms. Indeed, he mentions the possibility that there is perhaps no steady event which is not a part of a rhythm.[1] If we assume that all perceptions are periodic processes, and that parts of perceptions do not involve a qualitative cycle recurring within the parts, if only these are sufficiently small, a consequence is that not every part of a perception is a perception. However, all that Russell definitely says in this connection is that "perceptions ... are systems of steady events".[2]

Let us now consider the question of whether a causal relation in Russell's system may be said to be ternary or quaternary. If we suppose that a transaction is not a special event incorporating more than the rhythms and steady events which surround it (which is possible whether a transaction occupies a finite interval of time or not), it is simplest to regard causal relations as ternary. One rhythm is succeeded at an instant *t* or after a certain interval by another rhythm, and these rhythms plus a steady event (the light which is emitted or absorbed or some other process transmitting energy) are the only events we have, although we say that a transaction occurs at *t* or during the interval as there were a special category of events called transactions; but we only mean by this that one rhythm ends and another begins.

The situation is quite different if we assume that the transactions cannot be regarded as either rhythms or steady events,

[1] *Ibid.*

[2] *Op. cit.*, p. 363.

but must instead be regarded as special events not reducible to rhythms or steady events only. In this case, the causal relations are quaternary.

As was previously mentioned[1] Russell leaves the question about the duration of a transaction to the physicists but this question is not the same as the one just mentioned: they are however connected with each other. It is reasonable to think that Russell would regard a transaction as a special event if it lasted for a finite time. If it took no time, it could not be regarded as a causal process with finite extension in space-time, because Russell accepts that causal processes proceed with a finite velocity not greater than that of light.

It is obvious that the laws for quantum changes are reconcilable with the formulation of causal laws which is given. With them, according to Russell, it is always a question of rhythms and steady events which are separated by transactions. An atomic change may then be described by the following sequence: a certain rhythm in the atom, a transaction, and finally a new rhythm (the atom in its new state) plus a steady event (a light wave moving away from the atom). It may be remarked that modern quantum-mechanics does not include in the description of the world an interpretation of "transactions" as peculier occurrences going on at the time they are said to occur.

Since every physical event would be either a rhythm, a transaction, or a steady event, physical laws — and, therefore, intrinsic laws as well — must deal with series of such entities. If the formulation of a causal law must be interpreted as meaning that every causal law states the manner of the connection between rhythms or steady events separated by transactions, not all of the laws that Russell regards as causal in other contexts could be accommodated in this general formulation. For instance, the Einsteinian gravitational laws, which are counted as intrinsic, would not be causal, since they can tell us something about the gravitational reactions of electrons to the field surrounding them even

[1] This chapter, p. 119.

when no transactions occur in the neighbourhood. Moreover, what happens near the electron is unimportant except when it involves a change in the field about the electron; all that is important is that the electron itself, regarded as a free particle, does not undergo quantum changes. The formulation must therefore be taken to mean that laws relating series of rhythms or steady events may be causal even when not separated by transactions. In this case it does not make sense to talk of the causal relations as being n-ary, with a definite n.

Even though, according to Russell, it is perhaps necessary that quantum changes must figure in the series of events which provide us with knowledge about the electron, or, indeed, about anything, we cannot assume that all laws state something about series of events which involve transactions. It is not necessary to say anything about this other series of events which connect what happens outside us with certain happenings in our brain and which may involve a transaction, in order to state something about the electron. We may therefore take it that Russell did not mean by his formulation of a causal law that a law must include a description of transactions in order that it may be called causal. It is clear that intrinsic causal laws then satisfy the formulation, since they state something about physical occurrences, and Russell does not admit physical occurrences which are not rhythms, transactions, or steady events.

(g) There are, however, certain difficulties in interpreting intrinsic laws, since — in some cases, at least — the events which constitute a piece of matter cannot be experienced; such is the case, for instance, with the events constituting an electron in the Bohr-Rutherford theory, when it is revolving undisturbed in an orbit about the nucleus.

But before we treat these difficulties, it is necessary to look more closely into what is meant by intrinsic causal laws — these laws which occupy a position of such essential importance in Russell's theory of the external world, since it is with their help that material things are constituted. We should also remember that, without these intrinsic laws, the extrinsic causal laws remain indefinite, as these external laws are formulated in terms of matter.

There is already a discussion of the laws which apply to events that are said to pertain to one thing in *Our Knowledge of the External World,* but it is not carried so far as in *The Analysis of Matter*, nor is it so technical. As in *Our Knowledge of the External World,* we still have the proposition that matter is groups of events related by certain causal relations[1], but these relations are now described more precisely than they were previously.

As in earlier works, Russell begins by defining a thing at a particular instant with the help of "perspective laws". These have the same significance here as in *The Analysis of Mind.*[2] Events, he says, are arranged round centres in accordance with these perspective laws, and such a centre is said to be the location of a physical object. Where such a group of events occurs, there are as a rule similar groups of events at nearby points in space-time between which the intervals are time-like. It is not essential for these groups that there should be a continuous transition from one to the other via a group of intervening events (the possibility of discontinuities due to quantum changes must be taken into account). Suppose, Russell says, that in such a series of events, the difference between two groups a and b separated by a very small interval t is very nearly the same as the difference between groups of events b and c, also separated by an interval t; and furthermore, suppose this is true for any three groups of events in the series where the interval between the first and second is very small and equal to the interval between the second and third; the difference between the groups becoming smaller as t decreases. Then we say that we have a piece of matter. The description may be put more briefly by saying that if the series' groups of events are related by differential equations of the first order, the series constitutes a piece of matter. If there are discontinuities in the series, they must be of a kind which is described in quantum mechanics.[3]

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[1] *Our Knowledge of the External World,* p. 110; *The Analysis of Matter,* p. 246; and *An Outline of Philosophy,* p. 124.

[2] *The Analysis of Matter,* pp. 208 and 211

[3] *Op. cit.,* pp. 245 and 322.

According to Russell, this is still only an approximation, since we must also take into account physical laws of the second order relating quantities that may be described approximately in the above manner. Or, in Russell's formulation: "if there is an event x at time t, there will be, at any neighbouring time $t + dt$, an event $x + f_1(x)\, dt + f_2(x)\, dt^2$, where $f_1(x)$ is a continuous function of time, while $f_2(x)$ is determined by the second order differential equations of physics. The string of events so connected is called one piece of matter".[1]

These laws, by means of which we differentiate certain groups of events among the total complex of events, to form a series which we call a material thing (the events in each group related by perspective laws as described above), are called intrinsic causal laws by Russell, and the thing so defined a causal line.

It should be noticed that an intrinsic causal law may at the same time be an extrinsic law, since $f_2(x)$ may include a reference to some event which is not among the events constituting the material thing to which x pertains. Russell's formulation does not exclude this possibility.

If we take an electron as an example of a piece of matter fulfilling this condition that an intrinsic law relates all the events which are said to pertain to the electron, the question arises of what type of events constitute the electron. If x is an event at the instant t, is there really a group of events at different instants $t+dt$ which may be described by $x + f_1(x)\, dt + f_2(x)\, dt^2$?

Let us first consider an electron which is not bound to a particular atom, but is a free particle moving in a gravitational field. We know quite certainly that the laws describing the movement of the electron do not assume the occurrence of transactions along the path. So rhythm or steady event supply the only remaining possibilities. It may be that all the events in question are steady events but that they together form a rhythm.

However, if the free electron is a rhythm, the rhythm must continue unchanged during the whole of the time that the electron is

[1] *Op. cit.*, p. 245.

free; otherwise, the electron would include a transaction, which would not accord with the theories Russell has adopted. Such a rhythm would consist of a cycle of qualities occurring at the place where the electron is said to be, but it is difficult to see what the qualities would be in this case. From the quantum mechanical point of view, waves of some kind may be brought together to form an electron, but what it is that these waves represent is not immediately clear. According to one interpretation, the waves are merely a description of how probable it is that the electron will appear in the different places. But these "probability waves" cannot be events in Russell's sense of the term. It is only possible to call them cycles of qualities in a transferred and tenuous sense.

Perhaps it is possible to give the name, a cycle of qualities, to a rhythmic change in the electromagnetic field or the gravitational field about the place where we say an electron is situated. This would conform with Russell's conception of a set of events grouped about a centre, these sets of events at different instants being related by certain laws. It would not be unreasonable to have the qualities or properties applying to the field in a particular region constitute a piece of matter, in this case an electron which we say is at a certain place in the region (the place being determined by the way in which the qualities or properties are grouped), while at the same time the form of the field around the place where the electron is said to be must be taken into consideration in calculating the motion of the electron.

However, it is not easy to see what these qualities would be which would occur cyclically, grouping themselves about a centre in accordance with perspective laws, and which might be included in the constitution of a free electron. Whether or not there are such qualities is a question for physics, not for philosophy. And the same holds when the free electron is a series of steady events not forming a rhythm.

Russell points out that in Heisenberg's theory, we cannot identify an electron at one instant with an electron at another instant.[1]

[1] *Op. cit.*, p. 322.

This does not necessarily mean that there are not events grouped about centres with whose help electrons may be constituted; nor does it mean that these centres cannot be arranged in series by means of certain laws, assuming that certain other events do not occur in the neighbourhood of the series. But Heisenberg's quantum mechanics does imply that the events which we may observe and associate with some electron do not suffice for a complete calculation of its path, and that a later event which is associated with an electron cannot be said to pertain to a particular electron observed previously, since it is not known exactly where the later electron has come from. Only in very special cases is an identification possible. This is obviously not consistent with Russell's view of the electron as constituted of events which are related to each other according to the special intrinsic laws Russell adopts.

For electrons which are bound to a particular atom, it may be assumed, according to Russell, that a cyclic series of qualities occurs at the place where the electron is. In the Bohr theory of atomic structure, it was natural to assume that a rhythmic process went on in the atom, a process which changed to another to the accompaniment of a transaction, and which could be described most simply as a rotation of an electron around a nucleus. But no rhythmic process had been observed, and it was impossible to determine the position of the electron in its "orbit" around the nucleus. This was one of the factors which lead Heisenberg to a quantum mechanics where a definite position or path for an electron in the neighbourhood of the nucleus is meaningless, just as is the idea of continuous processes in the neighbourhood of the nucleus when no transactions occur. This does not mean that there cannot be such processes as Russell mentions near to the atomic nucleus, processes which might be said to constitute the bound electron; it merely means that we have no reason to assume that there are such processes, and, in any case, the observations we can make are not sufficient to construct a physical space in the neighbourhood of the atom, with electrons moving in definite paths at definite distances from the nucleus. Thus, in Heisenberg's mechanics, there are fewer assumptions about quantities which our empirical experience does not oblige us to accept. It might be said that, in some respects, modern quantum mechanics approaches the ideal of fundamental causal laws which Russell put forward in *The Analysis of Mind*, and which he still holds to[1]; according to this, the laws which deal with groups of events called pieces of matter are derivable, the fundamental laws being laws relating individual physical occurrences.

But Russell bases his ideas on the Bohr theory of the atom, and he is well aware of the difficulties attendant on his way of constituting the electron when regarded from the viewpoint of the Heisenberg theory.[2]

Our original question, whether there are events that may be grouped together to entities called electrons by means of intrinsic causal laws of the type formulated by Russell, receives from modern quantum mechanics the answer, that no such events can be observed. Of course, single events may be connected with an electron, but not sufficiently many and not distributed in the way Russell describes. The conditions under which we make observations are such, that we can say with certainty that we can never define intrinsic causal laws which defined an electron in such a way that we could definitely assign a certain event to a certain electron. The same thing applies to all elementary particles. It seems therefore that intrinsic causal laws cannot be used for the constitution of the smallest parts of matter. They were formulated with larger bodies in mind, such as emit light and thus make it possible for us to think of visual pictures of the bodies in question from various viewpoints, a mass phenomenon which is only possible because of the large accumulations of atoms involved, and which, by virtue of its constancy, enables us to construct a space with defined positions and distances in the neighbourhood of the body. A series of events which we associate with a certain body at a certain instant are thought of as related to a series of events

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[1] *An Outline of Philosophy*, p. 292.

[2] *Op. cit.*, p. 46.

at another instant by means of definite causal laws, and the events so associated are said to pertain to the same body, the same material thing.

This method breaks down when applied to the elementary particles. There are not sufficient observable events connected with a given elementary particle for the construction of a space around it of the type we are accustomed to think about in macroscopic considerations, or even of the more abstract type formulated by Russell, where a body is said to be at the centre of the events which are grouped together by means of perspective laws, and which constitute the body at that instant. The fault with Russell's intrinsic causal laws is perhaps that they assume a physical space of relatively conventional form which is unsuited to the world of the elementary particles, or, more exactly perhaps, the elementary events. We therefore have reason to suppose that the intrinsic causal laws formulated by Russell are not fundamental, but that they are derivable from more fundamental laws, valid for the events associated with single elementary particles; these particles cannot be constituted by means of intrinsic laws from the observable events with which they may be associated.

This does not mean to say that intrinsic laws of the type in question might not be taken to apply to non-observable events, as, for instance, when a free "electron" moves in a gravitational field without being disturbed by observations. But this is not sufficient for the laws to fulfil the conditions which Russell imposes on them, namely, that we may use them to identify a body at a certain instant as the same body as one we observe at an earlier instant.

(h) Associated with this difficulty in identifying elementary particles is the fact that we cannot calculate everything about later events from available data. The actual observations disturbs the phenomenon observed, and this finds expression in Heisenberg's principle of indeterminacy, which we will not attempt to deal with. Russell's reason for thinking that we must probably abandon the idea that subsequent events are completely calculable is that we only have statistical laws for radioactive disintegration, so that we cannot say when a certain atom of a radioactive substance disintegrates. Something similar applies to transactions, for instance, in a glowing gas, for which we can only say how much radiation there is per unit time and unit volume, but nothing about which atoms will emit light at a particular instant.[1]

This means that it is no longer possible to suppose that laws of causality of the type found in *Our Knowledge of the External World*[2] are universally correct, not only because we cannot speak meaningfully about the state of the universe at a given instant or during a given interval[3], but also because the data we can obtain about certain phenomena do not suffice for the complete calculation of later events, even for events within a very small region of space-time.

2. Some change may be detected in Russell's attitude to perspective laws, the laws by which events are grouped together to form a "thing" at a particular instant, in *The Analysis of Matter*. If events separated by a space-like interval cannot be causally related, the perspective laws cannot be described as causal, since they relate groups of events which have the common property that they are supposed to have started from a certain region simultaneously (travelling with approximately the same velocity). The interval between these events may consequently be space-like, and therefore there may be cases when they are not causally related. Hence it would not accord with Russell's other theories in *The Analysis of Matter* to hold that the perspective laws are causal.

We may question whether the perspective laws are really fundamental in the sense that they cannot be deduced from other laws which Russell has already assumed. It is stressed that the perspective laws are not special psychological or physiological laws; the differences and similarities between different perspectives is a question for physics, and the laws are simple geometrical laws.[4]

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[1] *Op. cit.*, p. 368. *An Outline of Philosophy*, p. 152.

[2] *Our Knowledge of the External World*, p. 221.

[3] Discussed under e. on p. 117.

[4] *An Outline of Philosophy*, p. 158.

This must mean that they state something about structure and structural similarity, not about qualities, since, according to Russell, physics does not deal with qualities. It may seem as if two different types of perspective laws were needed, one for physical occurences and one for sensations — but this is not the case. If we know the distribution of physical occurrences in a certain place we may, according to Russell, infer the sensations which are connected with them in the same place through causal laws which relate for example light-waves with visual sensations and sound-waves with auditory sensations.[1] These are not a new category of causal laws but a special case of the scientific laws. "A sensation", states Russell, "is merely one link in a chain of physical causation; . . ." and consequently the laws relating sensations with events in such a chain are physical causal laws.[2] It is not mentioned of what sort they are but there is no reason to think that Russell does not regard the causal chain in question as describable ultimately in terms of extrinsic and intrinsic causal laws of the previously mentioned types.

Physics provides us with certain laws for the transmission of light from an emitting body, and with the help of these laws we can calculate what happens at every point near to the body, that is, we can calculate all conceivable visual stimuli associated with the body. This calculation does not require the use of perspective laws. Actually, we might make many errors if we did make use of them, as they give only a very approximate description of the radiation of light from a source. This is so if we have an opaque screen near to the emitting body, with small holes, let us say two, in it of a size comparable with the wavelength of the light emitted and with a comparable distance between them. The light which passes through the small holes spreads out radially, and gives rise to interference phenomena, which means that comparatively bright light would be seen in some places, and practically no light at all in other. Perspective laws tell us nothig about this phenomenon,

[1] *An Outline of Philosophy*, p. 155. *The Analysis of Matter*, p 227.
[2] *An Outline of Philosophy*, p. 156.

but there are laws which may be used to calculate the light distribution behind the screen, although they belong to physics and not to projective geometry. In order to construct around an object a space filled with something which may include stimuli, we do not require physical laws plus perspective laws, but only physical.

It is quite a different matter if we use rough perspective laws as a preliminary basis for physical analysis, in order to arrive at the more exact laws of physics; we do not need them once we have the physical laws, no more than we need less exact laws of the type "most objects fall when released" once we have the laws of mechanics.

It is also another matter that when we build up a conception of a room in which we find ourselves such laws as the perspective certainly play an important part in our conception of distance, our ability to infer how the room would look from another position, and in our ideas of what should be called the same object when we look at the room from different positions. But that does not mean to say that we should use these rough laws when we come to define what we mean by a certain thing at a certain instant, when this thing is to be subjected to the more exact laws of physics. When we wish to define a thing at a given instant as the group of all perspectives of the "thing", or, more correctly, as a group of events occurring at certain instants, which we can associate with one another in a particular way, our definition will probably correspond better to what Russell means if we base it on physical laws instead of perspective laws. One of the important properties of the "thing" Russell wishes to define is that when some of the events pertaining to a thing change (if the thing moves, not if something in its environment moves), all the other events will change as well, in accordance with certain definite laws. It is therefore important to include in the constitution of a thing all the phenomena which regularly change together with the others in this way, and it is not possible to do this with only perspective laws at our disposal. For instance, we should pass over some of the optical phenomena behind the screen with the small holes in it if we only use perspective laws, in spite of the fact that these phenomena change at the same time as the more usual visual aspects of the object, when the object moves.

It might be supposed that the laws for the intervening medium which Russell assumes, and which are also applied in the constitution of a thing, might suffice to explain deviations in the case we are discussing. But we cannot attribute our difficulties to an intervening medium if we take "medium" in its normal sense. Of course, there is nothing to prevent us from calling the physical laws regulating the phenomena occurring in this experiment laws of the effect of the medium between the light source and the place of observation on the intensity of the observed light. But such a use of the term "laws for the intervening medium" finds no support in Russell's use of the term. If we were prepared to admit such a significance for the term, we could also say that the Einsteinian laws by means of which the bending of a light ray in the neighbourhood of a heavy body may be calculated are such laws of the intervening medium, but Russell appears not inclined to do this.

It therefore seems that we cannot accept the perspective laws as anything more than auxiliary laws by means of which we arrive at the more exact laws, and which we can abandon in the more exact system. They are not fundamental in any other sense than that we are aware of them before the exact laws, and we should fall into hopeless difficulties if we actually used them for constituting "matter" to which physical laws were to be applied.

3. The problem of the certainty of causal laws received little consideration in *The Analysis of Mind*, but now returns to prominence once again. As previously, it is pointed out that causal laws are not laws of necessity. They do not say that laws must express necessary relations in order that they may be called causal. All they express is that there are certain persistent correlations between certain events.[1]

That these correlations referred to in causal laws persist, if they really do so, must according to Russell be accepted as "brute facts"[2];

[1] *The Analysis of Matter*, p. 327.

[2] *An Outline of Philosophy*, pp. 121 and 156.

we cannot infer it, and it is not unreasonable to think that there are exceptions to every regularity.

We cannot be certain that any causal law is valid, says Russell, but inductive arguments may render individual laws about correlations probable.[1] We obtain these inductive arguments by the assumption of a suitable inductive principle. In *The Analysis of Matter*, it is stated that such a principle must include the provision, "that a correlation which has been found true in a number of cases, and has never been found false, has at least a certain assignable degree of probability of being always true".[2] We can assign a certain degree of probability to certain regularities according to such a principle, but we cannot use it to determine anything about the actual occurrence of persistent regularities or the actual exceptions to a certain regularity, as long as the term "probability" is not defined more exactly. Two possible interpretations of "probability" are discussed in *An Outline of Philosophy*: according to Keynes's[3] definition of the term — which raises the question of an undefined logical relation between propositions, or as it is defined in some frequency theory.[4]

Russell has changed his mind about Keynes's theory, and now regards it as unsatisfactory. He says, "Mr. Keynes leaves an uncomfortable gap between probability and facts, so that it is far from clear why a rational man will act upon a probability".[5]

He would prefer a frequency theory, if there were one not open to objections, because probability would then no longer need to be an undefined quantity, and because such a theory would establish a much closer contact between probability and what actually

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[1] *Op. cit.*, p. 156.

[2] *The Analysis of Matter*, p. 167. No difficulty arises from the fact that the principle does not explicitly state a relative standard for the probability of the correlations. The probability must be interpreted as a probability relative to all our relevant knowledge.

[3] J. M. Keynes, *A Treatise on Probability*, 1921.

[4] Russell refers to R. H. Nisbet, "The Foundation of Probability", *Mind* 1926, Vol. 35.

[5] *An Outline of Philosophy*, p. 286.

happens.[1] However, he does not wish to accept any existing frequency theory, since the associated difficulties are considerable. The implication of the term "probability" in the principle of induction is thus left open, as no other possibility is referred to, than Keynes's theory and frequency theories.

However, it is questionable whether a frequency theory could provide us with a probability that a regularity would always apply without assuming that there are regularities which always apply. We will consider this question briefly in the next chapter, when we mention Russell's analysis of different frequency theories in *Human Knowledge*.

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[1] *Ibid.*

The differences between Russell's opinions about the problems associated with causality, as he presented them in the two publications which appeared in 1927 and in those which appeared after that year, most important being *Determinism and Physics*, 1936, and *Human Knowledge, Its Scope and Limit*, 1948, are considerable in some respects, but not of decisive significance apart from one point. Russell considers, at least in the latter of these works, that it is no longer possible to accept the view that causation implies merely a strictly invariable sequence.

The new items consist of (1) a discussion of the implications of the principle of indeterminacy and the statistical laws of modern physics for the question of whether everything that happens has its place in a causal connection — a discussion which is confined to *Determinism and Physics*, — and (2) an examination of the relevant facts — in Human Knowledge — in an endeavour to establish a certain justification in terms of probability for the adoption of causal laws in certain connections. With this end in view, five postulates are put forward to justify the way in which the empirical sciences proceed.[1] The term"probability" occurs in two of these postulates, and it is evident that Russell now definitely favours a probability concept of the type employed in frequency theories.

The categories into which causal laws are divided are not appreciably different from those given in *The Analysis of Matter*, although the terminology is somewhat changed. The distinction between intrinsic and extrinsic causal laws in the sense in which the terms "intrinsic" and "extrinsic" are used in *The Analysis of Matter* is still maintained, but the intrinsic laws are now called laws of "quasi-permanence" and the extrinsic laws laws of "inter-

[1] *Human Knowledge*, p. 506.

action". The term "intrinsic causal law" is reserved for a limited group of the laws which were previously called "intrinsic". The Einsteinian laws of gravitation, for instance, are no longer called "intrinsic", but the first law of motion is, since it is taken to be an example of a law which says something about a thing at a series of instants without it being necessary to take into account anything apart from the thing.[1] The Einsteinian laws of gravitation require a consideration of the metrical properties of the immediate environment of the body concerned.

But we know of no case of a body moving in complete isolation, entirely unaffected by anything around it; it is even questionable whether such a case could find a place in our causally bounded world, whether it would not fall outside the range of things which science can deal with. The things which we can observe therefore obey intrinsic laws only approximately.[2]

As before, perspective laws are mentioned, laws which express

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[1] There are many kinds of process to which intrinsic laws are applicable besides those which deal with the position and velocity of a particle, according to Russell, and he gives as examples the cooling of a red-hot object, and the rapid diminution of the sound from a large bell (p. 495). It is not clear why in the first case the cooling cannot be accounted for by a series of quantum changes in the individual atoms and a series of collisions between the molecules of the body and those of the body and its immediate surroundings. The example of the bell can probably by analysed in the same way. If such an explanation is possible, we can no longer regard the cooling of the body as a process which proceeds according to intrinsic laws only.

However, it is possible that Russell does not assign the same meaning to "intrinsic law" on this occasion as he assigns to "intrinsic causal law" on other occassion (for example, as the expression is used on p. 508), that is, the former expression may by taken to cover a wider concept than the latter. This is possible but not probable, in view of the fact that "intrinsic laws" in this connection are said to express "intrinsic causation", so it would be unnatural not to regard "intrinsic causal law" and "intrinsic law" as synonyms. If we may assume that these two expressions have the same meaning wherever they are used, perhaps we must interpret the statement about cooling as meaning that intrinsic laws are applicable to part of what happens in this process.

[2] *Op. cit.*, p. 509.

principles in projective geometry, and the intention seems to be the same as in the previous books.[1] They have the same significance as in *The Analysis of Matter*, and cannot be regarded as a type of causal laws. Pre-scientific laws are still regarded as causal[2], but they are no longer different in any important respects from scientific causal laws, as these are described in Human Knowledge. It may be that Russell requires his scientific laws to relate events between which there is some other connection in addition to that of an invariable sequence in order that they may be called causal, whereas he does not require such a relation between events which we associate with one another in our ordinary generalisations in order that these generalisations may be said to express causal laws. But it is most probable that Russell considers that the generalisations which are to express approximate causal laws should be concerned with events, which, when they occur together, are related to one another in a special way. The generalisation that if a match is struck on a matchbox it will ignite might thus be called a causal law if the striking of the match on the box stands in a particular relation to the appearance of the flame (in most cases, at least), in the cases when an event of the former type is actually followed by an event of the latter type.

There is no longer mention of special psychological laws like the mnemic laws which were discussed in *The Analysis of Mind.* Applying a postulate about the continuity in space and time of the various elements which enter into a causal relation, we find that such laws are either not causal or (and this is the alternative preferred by Russell) that there is a certain sort of continuity about the different elements related by mnemic laws, in which case they are not different in any essential point from the other laws which Russell calls causal.[3]

1. (a) But before we consider the problems in *Human Knowledge*, let us consider some reasoning in *Determinism and Physics*. Russell there discusses more thoroughly than in any other of his writings

[1] *Op. cit.*, p. 482.
[2] *Op. cit.*, p. 327.
[3] *Op. cit.*, p. 510.

the question of whether Heisenberg's principle of indeterminacy definitely implies that the world we live in does not constitute a deterministic system. It seems possible to advance some objections to the conclusions he arrives at. The reasoning is as follows: The principle of indeterminacy (one of many principles which each give rise to the same problems) states that we cannot determine the momentum and position of an electron at a given instant to an arbitrarily high degree of exactitude, if we wish to determine both these quantities simultaneously. The degree of precision which is possible for a simultaneous determination is such that the product of the margin of indeterminacy for one and the margin of indeterminacy for the other is equal to h, h being Planck's constant. The more accurate our determination of one factor, — the position, for instance — the more indeterminate the other, the momentum, becomes. Russell says that this has been interpreted, by Eddington, among others, as meaning that the electron does not really have a definite position and a definite momentum at the same time, contrary to what we usually imagine to be the case with material bodies. From this point of view, the principle of indeterminacy does not imply that something exists which we cannot determine. Everything which exists can be determined, says Russell[1], or, at any rate, the principle of indeterminacy gives us no reason to think otherwise.

The word "determinate" is used in this reasoning as an expression of the fact that we can obtain values without ambiguity by observation and experiment, and that we can give descriptions as exhaustive as physics may require them to be of phenomena which we say are determinate. Such determinacy does not imply predeterminacy, however. It could quite well happen that no phenomenon is predeterminate, but that they are all determinate in detail in the sense just mentioned, when they do actually occur.

If physicists had kept to the view that the principle of indeterminacy did not mean that there is something which is not determinate in the first sense, says Russell, they would not have

[1] *Determinism and Physics*, pp. 12 and 18.

made the mistake of believing that the principle of indeterminacy shows us that we cannot find causal laws which allow us to determine (in the sense of predetermine) with complete exactitude what is going to happen to particular elementary particles.[1]

This objection to physicists' ways of reasoning does not seem to be justified. In actual fact, after the determination of the position of an electron with great precision, and of the momentum with a corresponding small degree of precision, we have only a statistical forecast of the results of a subsequent determination. We may say that a certain electron will be somewhere within a certain sector, the limits of which are determined by our values and our margins of error for the momentum and position. The probability that an electron will be at a certain place within a sector at a certain time is furthermore the same as the probability that it will be at some other place in the sector.[2] When we come to observation of an electron within the sector, we can determine every property that it would be meaningful to ascribe to the electron (if the interpretation given by Russell and Eddington and others is correct), but we cannot say certainly beforehand what the result of this determination will be.

Bohr interprets the principle of indeterminacy as a description of the fact that everything we determine must be determined with the help of something else, and the least intermediary thing with which we can determine anything about an elementary particle is a light quantum with a certain action (dimension energy $\times$ time). If we accept this interpretation, it seems that we have to accept the idea that not all the phenomena in the universe which we can determine (in a sense corresponding to the first one assigned to "determinate") are predeterminate.[3] But it does not follow

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[1] *Op. cit.*, p. 18.

[2] For a more detailed consideration of this consequence of the principle of indeterminacy, se Moritz Schlick, "Die Kausalität in der gegenwärtigen Physik". *Die Naturwissenschaften* 19, 1931, p. 152.

[3] It seems that Eddington wished to say something of this sort when, in discussion with Russell, he said, "I will grant if you like, that our knowledge of the universe at the present moment can be regarded as perfectly

from this interpretation that we cannot conceive of other worlds where the least intermediary in a determination is smaller than a light quantum, or even of worlds where the phenomena have such relationships to one another that they are predeterminate in spite of the fact that observations disturb what is observed, if the result of the disturbance is exactly calculable. What quantum mechanics says is that if certain fundamental assumptions about our world are correct (and the reasons for believing so are overwhelming) there cannot be any exact predetermination of certain phenomena, although they can be determined within certain limits, using statistical methods.

(b) This brings us to the second question, which Russell discusses in connection with the problem of whether physics has shown that the world is not deterministic in the sense that everything in it cannot be predetermined with whatever degree of precision we wish: the question of whether the statistical laws occurring in physics must be taken as implying that determinism is not valid.

Russell's answer to this seems to be that we are not justified in concluding that, because a statistical law is valid in a certain field, there cannot be causation in that field. On the contrary, it seems likely that there cannot be statistical regularity, he says, if there are not regularities of a causal type among the phenomena concerned, although we may not know what they are. If we do not assume that there are some laws applying to certain phenomena, then we have no reason to suppose that a random distribution exists in a given field, or that such a distribution would be any more likely than another sort of distribution. A random distribution would in this case have no precedence over any other distribution. If we accept the frequency theory, says Russell, we are not entitled to

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determinate (the supposed indeterminacy being introduced in translating it into an inappropriate frame of conception). But that does not remove the 'indeterminism' (which is distinct from 'indeterminacy'), namely, that this knowledge, however expressed, is inadequate to predict quantities which, independently of our frame of conception, can be directly observed when the time comes." *The Philosophy of Physical Science*, p. 91.

state that non-random distributions are less likely (in the sense associated with this theory) than other distributions, unless we know of a law which applies to the phenomena concerned, and which justifies us in saying that "non-random" distributions seldom occur.[1] So if certain phenomena are supposed to have a property of statistical regularity, this must be taken as a sign that causal laws apply to these phenomena.[2]

This idea is precisely the opposite of one which is put forward in *The Analysis of Matter*, according to which, if the number of electrons is finite, and if the time interval between two events involving the same electron must always be a multiple of a certain number, the regularity which we think occurs in the world about us may be due to the absence of laws.[3]

We will not embark on a closer analysis of this problem, but we can at least say that if statistical regularity is taken to indicate that causal laws exist which account for the statistical regularity in some way, these causal laws should not be regarded as merely statistical at bottom, regardless of whether they say that there is a certain correspondence between certain phenomena in a hundred per cent of all cases occurring or in some smaller percentage of the cases, because we then need to look for the causal laws which account for these regularities, and find ourselves in an

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[1] *Op. cit.*, p. 16.

[2] This is directed against Eddington, amongst others, and he replies to it in *The Philosophy of Physical Science* by adopting a more emphatic point of view, maintaining that "in the new physics the so-called probabilities are actually the real entities — the elemental stuff of the physical universe. We have precise knowledge of *them*; and it would seem retrogressive to postulate other entities behind them of which our knowledge must always be uncertain" (p. 90). (It should be remembered that this is an extreme interpretation of quantum mechanics.) It is obvious that if there are no other entities than statistical ones, there cannot be causal laws applying to them which help to maintain the statistical distribution. It appears, however, as if Eddington sometimes confuses what physics describes (according to him) with what exists. If physics only describes statistical distributions, presumably this is not to be taken as implying that nothing apart from statistical distributions exists.

[3] *The Analysis of Matter*, p. 235.

infinite regression. That would mean that there is no law which is final, and might even mean that a continued search for laws is meaningless, since it is not really important at which law of the series the search is abandoned.

This difficulty does not arise if we say that it is only when the statistical regularities do not refer to one hundred percent correlations between phenomena that other statistical regularities underlying them are indicated, these other regularities being such that they denote a connection between certain phenomena in one hundred per cent of all possible cases. Of course, the difficulty also disappears if we regard the causal laws as expressing more than just a statistical connection. We may therefore suppose that Russell considered both these possibilities. Which of them he accepted is not clear. But the first possibility involves a distinction between statistical regularities without any significant differences being given, and implies that when it seems possible to assign the probability 1 (in the sense used in frequency theory) to this regularity, we may give up our search for other regularities to support this one. But in all other cases we have reason to look for an underlying regularity to account for the observed statistical relation, which seems to imply pure arbitrariness if there are no intrinsic properties to distinguish one regularity from another. When there is a correlation in 99.99 per cent of all conceivable cases, this indicates that there are one or more underlying 100 per cent relations, but when there is a correlation in 100 per cent of all conceivable cases, no underlying relation is indicated which accounts for just this particular distribution. Such a theory appears to be quite arbitrary. So it seems more reasonable to suppose that Russell thought of the causal connections as expressing something more than a mere statistical connection. This would conflict with something which Russell had often said previously, that we have no reason to regard causal laws as anything more than expressions of a correlation[1], but, on the other hand, it would be

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[1] A point of view put forward, for example, in *The Analysis of Matter*, p. 327.

in accordance with the ideas which he puts forward in *Human Knowledge*.[1]

One objection which might be advanced against this opinion that a random distribution indicates underlying causal laws just as much as other distributions do, because we have no reason to suppose that such a non-random distribution is less probable than any other distribution from the viewpoint of frequency theory, is that the number of possible random distributions is much larger than the number of non-random ones, for reasons connected with possible combinations, and that the ratio of the former to the latter rapidly approaches zero as the number of possible cases included in the distributions increases. It is not clear why these combinatory circumstances are not the only ones we need to know about in order to be able to say that if there are no laws governing a certain group of phenomena to which these combinatory conditions apply, then a random distribution is probable (in the sense of frequency theory), and more probable than a non-random, and that this probability becomes very large if the number of possible cases of the phenomena is very large.[2]

It follows from Russell's conclusions that, although we cannot regard the fact that certain phenomena show a statistical regularity as a proof that there are causal laws governing these phenomena (and the least these causal laws express is that a correlation of some sort exists in a hundred per cent of all possible cases), neither can we regard this fact as an indication that no causal laws apply to the phenomena in question. "I conclude that, while there is at present no decisive reason in favour of complete determinism in physics, there is no reason against it, . . ."[3], says Russell, and we can agree with him that the statistical laws of physics provide us with no reasons either for or against determinism as it is defined at the beginning of the article, a doctrine which

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[1] *Human Knowledge*, p. 508.

[2] This should not be confused with the fact that a given distribution is just as probable as any other distribution.

[3] *Determinism and Physics*, p. 18.

states that there are laws with the aid of which we may conclude that certain events will occur at one instant of time when certain other events have occurred at other instants.[1] But, on the other hand, it does seem that the principle of indeterminacy proves that it is impossible in our world to predetermine *everything* which is determinable at a particular instant from what may be experienced at earlier instants; and "complete determinism" must be taken to imply that it is possible to calculate everything which occurs at an instant of a kind dealt with by physics, from previous experience with the help of causal laws.

It seems in *Human Knowledge* that Russell has abandoned this point of view and returned to one which he had held previously in *The Analysis of Matter*. We read in *Human Knowledge*, "There are reasons for believing that this absence of complete determinism is not due to any incompleteness in the theory, but is a genuine characteristic of small-scale occurrences."[2]

2. The theories put forward in *Determinism and Physics* do not exhibit any other important departures from the theories of causation and causal laws which were argued in the works immediately preceding that one. But the shift of view point apparent in *Human Knowledge* is much more far-reaching. Perhaps the most fundamental change is that Russell altogether abandons the view which he held previously, that we do not have any reason to attribute anything more to causation than that there is a certain correlation, which need not imply more than a regular sequence. He now says: "I cannot accept the view that causation is merely invariable sequence."[3] This does not mean only that Russell's usage of words is different. If that were the case, it might be that he had not changed his views about any empirical phenomena, but had decided to reserve expressions such as "causal law", "causation", and others, for other phenomena than they were hitherto reserved for. However, he continues to describe much the same phenomena as causal as he did before, so the change

[1] *Op. cit.*, p. 3.
[2] *Human Knowledge*, p. 37.
[3] *Op. cit.*, p. 508.

of view must be due to a change in his conception of the relationships between the different elements of these phenomena, that is to say, to a change in his conception of our world.[1]

The other changes in Russell's theoretical outlook on causal problems are best illustrated by his discussion of the essential postulates we must adopt in order to construct a theory of the type which is already to be found in physics, in order that this theorising, together with material derived from experience, may tell us something about what has not yet been observed. In what follows, we will give our main attention to this discussion.

It is not immediately apparent what status these postulates have in Russell's system. However, it does seem to be clear that three of them — the first, the second, and the fourth — are of the same type as ordinary empirical generalisations, and arrived at in the same way. But they are generalisations about the most simple and fundamental facts, and they are confirmed by such an overwhelming mass of experience that they seem to be quite self-evident in one sense. Russell thinks that we actually assume them in most of our statements about empirical data.[2] This also applies to the other two of the five postulates, the third and the fifth, although they cannot be regarded as empirical generalisations in quite the same sense as the others. The purpose of these postulates, according to Russell, is to give us the "antecedent probabilities" we need in order to justify the inductive

[1] It is not immediately apparent from the text whether Russell thinks of the pre-scientific laws as dealing with events which have a special causal relation to one another. When Russell discusses these laws, he does not speak about "invariable sequence". It would, however, be in accordance with the other views expressed by Russell in *Human Knowledge* if we suppose that he considers that there must be a special causal relation, direct or indirect, sometimes at least, between events associated with one another by these laws, because if there were never such a relation, it would not be possible to state that the probability is of such and such a magnitude, the occurrence of the events near to one another in this case being only a matter of chance. The condition stated in the general formulation of causal laws, that the probability of the applicability of the generalisation must be greater than $\frac{1}{2}$, would not then be fulfilled.

[2] *Human Knowledge*, p. 526.

conclusions which actually occur in the construction of scientific theories about our world.[1] It is not explicitly stated that the postulates only justify induction with respect to the facts of our world, but it is obviously this which is intended, and not induction in "all conceivable worlds". It is easy to imagine worlds where any one, or several, of the postulates might lead to incorrect conclusions.

With regard to the problem we are treating, three types may be distinguished among the postulates.

(i) To the first type belongs only the first postulate ("the postulate of quasi-permanence"), which, although it is in the form of a postulate, may be regarded as an approximate causal law, or, if preferred, a pre-scientific law: "Given any event *A*, it happens very frequently that, at any neighbouring time, there is at some neighbouring place an event very similar to *A*."[2] It seems that the postulate might be covered by Russell's general definition of a causal law, according to which a causal law is a general principle with the aid of which we can conclude from sufficiently many data about one region of space-time something about another region of space-time, and the conclusion need not be more than probable (the probability of the correctness of the conclusion must exceed $\frac{1}{2}$).[3] If we assume that "very frequently" implies that the probability exceeds $\frac{1}{2}$ in any case, the postulate may be taken to be entitled to the name of causal law. This means that a causal law figures among the essential primary assumptions which we make when we begin to construct a scientific theory, and which are to make the conclusions we draw from this theory probable, that is, which give us reason to rely on these conclusions. It is clear that there is one difference between other causal laws and the first postulate, in so far as the other laws are based on one or more of the postulates. But this is a system property of the laws, a property depending on the position of the laws in the system.

[1] *Op. cit.*, p. 506. H. J. McLendon discusses the status of the postulates in an article, "Has Russell answered Hume?". *The Journal of Philosophy*, vol. XLIX, no 5, Febr. 1952, p. 145.
[2] *Op. cit.*, p. 506. [3] *Op. cit.*, p. 326.

There is no fundamental difference between these laws and the first postulate as far as their application to our experience is concerned. They serve us in the same way when making inferences about unobserved phenomena. The status of these five postulates may be compared to that of the axioms in a deductive logical calculus. The fact that particular logical laws are taken as the axioms of such a calculus, is a system property of these laws, and the fact that they are taken to be the axioms does not differentiate them in any respect, considered as logical laws, from the other logical laws which are not axioms but deduced theorems.

Moreover, it is only to be expected that a causal law should occur in the form of a postulate, because as soon as we adopt the frequency theory, as Russell does without reservation in *Human Knowledge*, we are obliged to assume that there are invariable sequences in our world, if we wish to conclude from observed regularities in a finite material at our disposal that these regularities, or some of them at least, probably recur in cases which do not enter into our material or probably have a permanent existence. The bare structure of the conclusion is roughly as follows. We have an assumption according to which there are invariable sequences. We have encountered a regularity in a limited experience. Since there are actual invariant sequences, the probability that the regularity we have observed is one of these invariant sequences is not zero. If we wish to assign a high degree of certainty to the assumption of invariant sequences of a particular kind, we must assume that there are actually invariant sequences of this kind or a category of invariant sequences, not altogether too broad, which includes that one to which we wish to assign a high degree of probability. This is presumably what Russell aimed to do with his first postulate. According to him, one of the properties distinguishing those quantities which we call things is that they are made up of a series of events, each of which is related to some other event in the series in the way described in the postulate. If we wish to assign a certain likelihood to the assumption that future events are also often connected with one another in a similar way to that we have already observed, which impelled us to call an assemblage a "thing", we have to adopt the first postulate, or something similar.

(ii) The second and fourth postulates belong to the second type. They are characterised by the fact that with their help (together with some observations) we may conclude that there are causal laws in our world, laws of a more exact nature than the first postulate.

The second postulate ("the postulate of separable causal lines") is introduced to support the assumption of separate "things", something the first postulate did not suffice for, and it reads as follows: "It is frequently possible to form a series of events such that, from one or two members of the series, something can be inferred as to all the other members."[1] We make these inferences about all the other members of the series by means of a causal law, according to Russell. The postulate therefore says that it is often the case that events are members of a series to which special causal laws apply. There are at least some series of events which obey causal laws. We know this from the postulate, without any knowledge of empirical data.

The fourth postulate ("the structural postulate") may be said to express more or less the same thing as Russell otherwise usually expresses by saying that phenomena are often arranged about a centre in accordance with perspective laws. In contrast to the second postulate, the fourth assumes that we have observed certain empirical facts before we may use it to infer that there is something which obeys causal laws. It reads: "When a number of structurally similar complex events are ranged about a centre in regions not widely separated, it is usually the case that all belong to causal lines having their origin in an event of the same structure at the centre."[2] It will be seen at once that by formulating the postulate without resort to the perspective laws, and without specifying how the causal lines mentioned are related to the centre, Russell avoids the difficulties which were discussed in the previous chapter, and which were connected with the idea that events

[1] *Op. cit.*, p. 508.
[2] *Op. cit.*, p. 511.

around what is ordinarily called a body may be connected with the help of perspective laws expressing certain principles in projective geometry. It was found that such laws probably give us an only approximately correct picture of the facts.[1] "Causal line" in Russell's terminology signifies a series of events which are connected by some causal law.[2] The postulate does not say that it is often the case that events are ranged about a centre, and it does not even say that there is a single case where this is so; it merely says that if we find structurally similar events grouped about a centre, our postulate makes it likely that each of these events belongs to a causal line.[3] When we have observed that there are many cases where events are grouped in this manner, we may conclude that there are causal laws regulating at least something in our world. It would be contrary to the postulate if there were no case of events arranged in this way belonging to causal lines while there were a number of cases of events arranged in the way mentioned, since the postulate says "it is *usually* the case that all belong to causal lines" (my italics).

(iii) The postulates of the third type, the third ("the postulate of spatiotemporal continuity") and the fifth ("the postulate of analogy"), presume the existence of associations to which causal laws apply. We cannot employ them to conclude that there are causal laws for certain phenomena if we do not already know that causal laws apply in some connection. The third postulate reads: "when there is a causal connection between two events which are not contiguous, there must be intermediate links in the causal chain such that each is contiguous to the next, or (alternatively)

[1] This was discussed in Chap. VI, pp. 131 ff.

[2] *The Analysis of Matter*, p. 477.

[3] This does not contradict the general statement about the postulates, which says that "Each of these postulates asserts that something happens often, . . .", since Russell puts this more precisely on p. 506; where he says that each postulate "asserts that something happens in most cases of a certain sort; . ." So the postulate need not say anything about the frequency of phenomena of a certain sort, and does in fact only express something about the distribution of different kinds of event in connection with phenomena of a certain sort.

such that there is a process which is continuous in the mathematical sense".[1] This postulate defines for us what is meant by "causal connection", and thereby what is meant by "causal law".[2] A correlation between phenomena not joined by a continuous process or series of processes which are only separated by quantum changes, cannot therefore be regarded as causal. But presumably the postulate is to be interpreted as not merely expressing something about the usage of language which Russell finds acceptable, but also as codifying certain general facts about the world. When the postulate is described as expressing a denial of action at a distance[3], this does not mean just that if there were something called "action at a distance" the laws which regulate it would not be causal, but it also means that a correlation between events which are not related in the manner described in the postulate is

[1] *Op. cit.*, p. 509.

[2] The definition of causal connection which is given in this postulate must agree with the general definition of a causal law which Russell gives in another chapter — a definition which is so wide that it embraces even pre-scientific laws. This latter definition says that a causal law is a general principle with the help of which we can conclude something about a certain region of space-time if we have sufficient data about another region of space-time. This conclusion need only be probable, however, (p. 326); so a causal law may be of the type: if we have data a, b, c about a region F, it is likely that phenomena k, l, m of a certain kind will occur in another region G. If a causal law is actually intended to express a causal connection, it follows that in those cases where phenomena of the type k, l, m occur after other phenomena have occurred in the region F (they may be described by a, b, c), there is a causal connection between the two sets of phenomena, in the cases the law refers to, at least. The law might be rewritten as: If data a, b, c of a certain type apply to phenomena in a region F, there is a certain probability that phenomena k, l, m of a certain kind will occur in another region G, where these phenomena are causally connected with the phenomena which data a, b, c refer to. This does not say that the phenomena k, l, m might not occur after the other phenomena occur in F without there necessarily being any causal connection between them. The law does not need to say anything about such cases. Perhaps Russell just means that there is such and such a probability that a phenomenon is causally connected with another phenomenon of a certain type in another region, when the first phenomenon occurs.

[3] *The Analysis of Matter*, p. 509.

always accidental, and no inductive conclusion may be based on such a correlation. (Of course, this should not be confused with the situation where we have a correlation between two phenomena but do not know whether there is a chain of events between them.)

We should not press the statement that no inductive conclusion may be based on a correlation between events which we do not know to be connected by a chain of events. If two processes of the same periodicity are going on parallel to one another, and if the processes are of the same kind, we may conclude that they will probably continue to go on parallel to one another in the future also, or that they have done so in the unobserved past. But this correlation does not entitle us to conclude anything about the two processes than we could have concluded by observing them separately, and nothing more about what happens in the universe; and we are only able to arrive at the conclusion that the processes will probably continue parallel to one another in the future if we know the laws governing the individual processes.[1]

To incorporate the idea that all causal relations occurring in our world have a definite property among the postulates of a system, before constructing any of the theory, is something new in Russell. But he has found this to be necessary to justify the assumption of the existence of physical objects even when we

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[1] It is not certain that the third postulate is as appropriate as the others. If it is really true, as Bohr says, that insurmountable paradoxes arise in certain experiments if we assume that there is a continuous chain of events ranged one after the other, like Russell's causal chain, between two appearances of the same elementary particle, for example, a free electron, — in other words, a group of events determining a definite "path" for the particle between its two appearances — the consequence would be that the laws giving us the approximate values for the second appearance of the particle from the most exact observations of it possible at its first appearance could not be called causal in Russell's terminology; and we are not entitled to say that there is a causal connection between the two events, although we are able to connect them by means of quantum mechanics. It is difficult to see why these particular laws, out of the many occurring in physics, should be marked out as non-causal.

cannot perceive them. Thus the third postulate ensures that the theories we construct with the aid of the five fundamental assumptions are not a kind of phenomenalism or at least, not attributable to the sort of phenomalism which says that all events which occur are of a mental nature.

The first postulate, according to which it is often the case that in the neighbourhood of any event other similar events occur, already allows us to infer that there are probably events which we do not observe, that is to say, events which are not of a mental nature. But the postulate does not allow us to say with certainty that there are such non-mental events. Nor do the second or third postulates provide us with any justification for saying that there are certainly non-mental events; they do not even seem to give any likelihood to the existence of such events. It may be that every series of events which obeys a causal law in the manner described in the second postulate consists of events which are mental. It is also conceivable that every time we think we have some reason to suppose that there is a causal line between two mental events— for instance, the sight of a dog running into some bushes and the sight of him coming out again — there is really no such causal line. We could not use the third postulate to infer that non-mental events do occur between the two appearances of the dog, because a necessary condition for the application of this postulate is that a causal line does exist between the two experiences. It is possible to accept all the postulates as true, and to experience what we do experience, without being obliged to accept as true in any single case that two apparently separate causal lines containing similar events are parts of the same causal line.

It appears that the fourth postulate, together with the third, constitute our best justification for the assumption that non-mental events exist. If there are events ranged about a centre as described in the fourth postulate, all of them being elements of a causal line going towards the centre, it is unlikely that all the events which go to make up all these causal lines (for which the third postulate is valid) are mental. If one of the events were mental, a causal line would connect this event with the centre, and, according to phenomenalism, all the events of this line must also be mental. If we take it that a mental event is one which must be experienced by someone, this implies that there is someone to experience each event which occurs at a point of the line. This places us in a dilemma. Either the mental events pertain to more than one experiencing being or they appertain to just one. Let us suppose that there are two beings; then we arrive at a difficulty if we assume that every thing which experiences events must be finite and that the experience is located somewhere within the thing, since the events which the one being experiences somewhere on the causal line must join up with the events experienced by the other, and these events must be as close together as the third postulate requires them to be (if they do not merge continuously into one another). If there is only one experiencing the events on the line, this observer must move out from the centre along the causal line at the same rate as the events succeed one another along the line. This must apply to all the causal lines proceeding from the centre in question, if all the events grouped about this centre are to be mental. Such a possibility cannot be excluded on logical grounds, even if we accept all the postulates, however incredible it may seem to be. If may be that, every time structurally-similar events are grouped about a centre and causal lines proceed from the centre to these events, all the events of these causal lines are mental.

But since the fourth postulate states that it is usually the case that all events ranged about a centre in the way described are connected by causal lines to the centre, and since it often happens that we experience events which are associated with other events (for example, those experienced by others) about a centre, and since in many cases we do not move with respect to these centres and we do not see any other mowing in this way[1],

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[1] We must have a space in order to talk about moving, arrangement about a centre, and causal lines joining an event which happens at a centre with an event which happens outside this centre. When Russell says that the causal lines proceed outwards from a centre, this is presumably to be understood in the light of what has been said about perspective laws in

it seems likely, or we think it reasonable to suppose, that there must be at least some cases where causal lines do not contain mental events. Perhaps we do not need anything further to be able to say that phenomenalism is untenable. Therefore, if we accept the first four postulates, we cannot accept phenomenalism, because it is then extremely unlikely that it is a tenable point of view. As a matter of fact, Russell considers phenomenalism to be a system especially liable to arbitrary reasoning. In order to be able to suppose, as the phenomenalists do, that other mental events and experiences than their own exist, it is necessary to use conclusions drawn from one's own experience, says Russell, and if we allow such a conclusion, there is no reason why we should not allow conclusions drawn from events experienced about events which no one ever experiences.[1]

So, if we accept the third and fourth postulates, we must regard the form of solipsism which says that there are no other events than my own mental events as probably untenable, since we can use these postulates to infer that non-mental events probably occur. But we cannot use them to infer that there are other mental events than our own. In order to support such inferences about the mental experience of others, Russell introduces the fifth postulate, which reads: "Given two classes of events A and B, and given that, whenever both A and B can be observed, there is reason to believe that A causes B, then if, in a given case, A is observed but there is noway of observing whether B occurs or

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earlier works as meaning that the causal lines include a series of structurally similar events (or, more exactly, that a series of sections through a group of causal lines running out from a centre gives us a series of structurally similar phenomena), and that these events may be arranged in some manner according to their size. If I sit and look at an object (which other people are also looking at), and if I cannot arrange my experience of the object in a series with the help of a gradation according to sizes so that the later experience is larger (or smaller if apparent sizes are regarded) than the earlier, it is probable that my experience belongs to causal lines containing other elements than my mental events, which, together with my mental events, may be arranged in the way described.

[1] *Op. cit.*, p. 196.

not, it is probable that B occurs; and similarly if B is observed, but the presence or absence of A cannot be observed."[1]

The postulate says only that it is "probable" that B occurs if A is observed but B cannot be observed, but this is quite sufficient for us to be able to conclude after some observations that there do exist at least some other mental events than our own. If we experience some mental phenomena of our own belonging to a class B, and these are experienced at the same time as other phenomena belonging to a class A, which may be connected with something which is not our own mental phenomena — our behaviour, for instance — and if there is a causal relation between events of the former category and events of the latter, and if we also know that A[2] often occurs without it being possible to observe whether or not B occurs, it follows from the postulate (since 'probable' there refers to a frequency probability) that B do actually sometimes occur unobserved by us. However, this occurrence need not be in connection with any of the occasions when we have observed A, but it must be associated with someone's observation of A at some time, in the past or in the future, under circumstances which preclude the observation of whether B occurs or not. For, if B does not occur on some occasion under the conditions described the probability mentioned in the postulate is zero; but 'probable' signifies a probability greater than zero, because otherwise there would be no point in formulating the postulate. Therefore, if the requirements mentioned are fulfilled, we may infer by means of the postulate that mental phenomena exist which we cannot observe, and which are associated with other organisms than ourselves.

The only difficulty is that A and B must be causally related when they are both observable. It is true that the postulate says that there only need be reason to believe that A and B are causally connected, but presumably this must be interpreted as meaning that there is actually a causal relation between A and B, or,

[1] *Op. cit.*, p. 511.

[2] For the sake of brevity, we will not repeat that the phenomena concerned belong to a definite class of phenomena.

that there is at least sometimes a causal relation between them, when there is reason to believe that there is a causal connection. If, every time there is reason to believe that A and B are causally connected, they are not actually so related, it is difficult to see how it can be probable that B occurs unobserved on an occasion when A occurs. But, of course, we can always assign an arbitrary probability to the occurrence of B in such a case, although a postulate involving this would not be very acceptable.

If the essential condition to be fulfilled in order that we may use the postulate is not that A and B must be causally related, but that there are reasons for believing that they are so related, and if we may really employ the postulate when there is never any causal relation between the two categories of events concerned, although we have reason to believe that there is (at least sometimes) then the question of reasons for believing that there is a causal relation is important. However, such a question is not discussed. Nor is it made clear what are the criteria for the existence of a causal relation. Whichever alternative we choose, it is difficult to decide in an actual situation whether the case we are concerned with is one for which the postulate says anything more than we already know.

But this difficulty has no significance for the question of how much we must assume about the existence of causal laws in order that we may accept the conclusion, arrived at with the aid of the postulate, that other mental phenomena exist than our own. Let us consider this problem. It is at once clear as we have already seen, that it is sufficient to assume that I do occasionally have reason to believe that there are causal connections between my own mental phenomena and surrounding phenomena or my behaviour, and that it is actually the case that sometimes, though not always, this belief corresponds to a causal connection which actually occurs. Suppose we have three classes of mental phenomena, A_1, A_2 and A_3, and that phenomena belonging to a class B_1 sometimes occur in association with phenomena belonging to A_1, and that there are corresponding classes of phenomena B_2 and B_3 for A_2 and A_3; and let us further suppose that we have reason to believe that A_1 is causally related to B_1[1], A_2 to B_2, and A_3 to B_3. Then it is sufficient if there is actually a causal relation in one of these cases — for instance, between A_1 and B_1. It even seems to suffice if we assume that there is a connection between A_1 and B_1 which is so weak that it cannot be described by any rule covered by Russell's general definition of a causal law; namely, a connection which allows certain inferences to be drawn from phenomena belonging to B_1, to phenomena belonging to A_1, the probability associated with this connection being non-zero but not necessarily "considerably more than a half".[2]

It seems to be necessary that the third postulate be fulfilled, for there to be a chain of events between the phenomena we are taking to be connected, since this is the only known way of ensuring that the correlation between the phenomena is not merely accidental.

If these extremely weak conditions are fulfilled, it must at least sometimes be the case that some one of my mental phenomena constitutes a link in a chain of events in which some of the other phenomena concerned occur. This appears to be sufficient for the fifth postulate to be acceptable.

Corresponding conditions ought also to be fulfilled in all other regions where the postulate is applied. It is quite obvious that the postulate itself cannot be a causal law, since it does not suffice to know of the existence of certain phenomena to be able to infer the existence of others with its help; it is also necessary to know that certain further conditions are fulfilled, which are concerned with the occurrence of certain relations between particular categories of phenomena.

It should be noticed that we cannot even employ this postulate to infer the existence of causal connections in any region, or, in general, in the universe, since the postulate only says it is probable that B occurs (or A) when the given conditions are fulfilled. It is not

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[1] A shorter expression for "a causal law exists relating phenomena belonging to A_1, and phenomena belonging to B_1".

[2] *Op. cit.*, p. 326.

stated that the probability is greater than a half. Neither do we know whether the unobservable B (or A, as the case may be), which we infer must sometimes occur when A (B) is observed, is related to A (B) in the way described in the third postulate.

But if we do not require more of a causal law than that it may be used in inferring the probable existence of one thing from the existence of another, we must accept the fifth postulate as a principle which helps us to extend the field of applicability of certain causal laws.

In conclusion, let us briefly consider whether the five postulates are sufficient to accomplish Russell's intention with them. Let us consider the postulate which is perhaps the most important for the determination of what a "thing" is, namely, the second, "the postulate of separable causal lines". If Russell thinks it is very important to know what intrinsic laws govern the events in the individual causal lines, the question arises as to how much guidance the second postulate gives us in this matter (taking into account the other postulates as well). It seems that the system of postulates is not sufficient in this respect. Anyway, it is not at all obvious how we may overcome the following three difficulties:

(i) If we find ourselves in the position of having to choose between different intrinsic laws for a given series of events, all of them conforming equally well with the observations made, the second postulate does not provide us with a basis for making a choice (and neither do the other postulates). The only condition to be fulfilled by the laws is that they shall enable us to infer at least something about every other event belonging to a series if we know one or two elements of the series. But if we have several such laws, which are in conformity with experience but which differ with regard to future events which may belong to the series, it is important that we should have some rule for selecting one or more of the laws as more likely than the others.[1]

(ii) The postulate does provide us with the possibility of inferring

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[1] This difficulty resembles the one discussed by Russell in *Mysticism and Logic*, p. 204.

from the occurrence of certain events that we are probably dealing with a material body having a certain unit and distinguishable from other bodies, but we have not gained very much by this, since we do not know what laws will apply to the series of events we are to encounter in the future. This postulate does not make it probable that series which we have not yet observed will obey the same intrinsic causal laws as govern the series with which we are already acquainted. We cannot even say it is probable that the laws for the unknown series will in any way resemble those we are acquainted with in any other respect than that they may be used in inferring something about the other elements of a series from a few of the elements. But it is essential that there should be laws which tell us that the bodies we will encounter in the future will probably obey exactly the same laws as the bodies we have already considered; or to put it more exactly, that the intrinsic laws which enable us to single out various series of events from the mass of events are very limited in number, or at least, that some of the laws for future series are of the same kind as the laws appling to series now. It is impossible, with nothing more than Russell's postulates, to make certain inferences which have actually been made on other bases, for instance, the inference from the knowledge of the elementary particles we have at present that the number of elementary particles is probably constant, or that the number is probably very small.

(iii) Neither does the second postulate say anything about the number of elements in a series to which an intrinsic causal law applies or about the extent of the series in time. It only tells us that observed events probably belong to such a series, and that there is perhaps some likelihood that they belong to series which have already been observed; but it does not tell us that it is probable that events will occur in a region where a body has been observed shortly beforehand. The first postulate does tell us this, however, because it states that, in the neighbourhood of a phenomenon, another phenomenon will probably occur which is similar to the first. So we know that after the latest observations in a series of events, it is probable that we can observe other events which are similar to the latest ones observed. But this is not the same as that the events which we should be able to observe could also be calculated from the observed events of the series with the aid of the intrinsic laws valid for the series. For this to be so, we must have some postulate which says that the valid intrinsic laws probably connect such events as the first postulate deals with, at least some-times when there is a time-like interval between them.