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R. GATTO: Problems in the Theory of Weak Interaction* 137



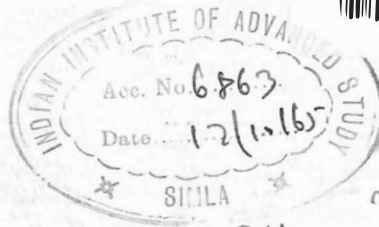
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Problems in the Theory of Weak Interactions¹⁾

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1. Introduction

The theory of weak interactions has now come to a very definite point after the brilliant success of the universal $A-V$ theory proposed by FEYNMAN, GELL-MANN, MARSHAK, and SUDARSHAN [1] (which we shall call the FGMS theory). We shall base this discussion on the hypotheses of that theory. We shall not review here the development of the FGMS theory, but we shall try to reproduce all those steps that will be necessary to make our discussion essentially selfcontained, so that no preliminary reading of papers on the subject will be required. We shall mainly be concerned with those aspects of the theory of weak interactions which still appear to be essentially unsettled or not yet at a definite stage. Thus in most cases we shall present speculations which may very well turn out to be wrong when more or better experimental data will be available.

2. General Description

2.1 Weak interacting currents and their classification

The older viewpoint for describing weak interactions consisted in grouping the particles into different pairs, $(\mu\bar{\nu})$, $(e\bar{\nu})$, $(p\bar{n})$ etc., and assuming that each pair interacts with the other pairs. A different possibility is now suggested by the AV theory. For couplings of such kind there is a possibility that the weak interaction Lagrangian takes the form of the coupling of a current with itself

$$J_\mu J_\mu^\dagger + \text{h.c.} \quad (1)$$

The currents J_μ , of course, are not to be understood as currents in the usual sense. In J_μ both vector and axial contributions are present. One difference with respect to the older description is, for instance, that now there are couplings also of a pair to itself, such as $(p\bar{n})(p\bar{n})$. Moreover the new description is certainly much more specific. According to which of the two descriptions we want to assume we can either classify the different pairs into a few classes (lepton class, $\mathcal{L}$, and various strongion²⁾), classes, S_0 , S_{-1} , S_1), or, correspondingly, decompose the current J_μ into different components. For brevity we shall denote with

¹⁾ Based on a series of lectures given by the author at the "Seminar über Hochenergie-Physik" in Oberwolfach, West Germany. September 1958. Part of the present paper overlaps the content of lectures given by the author in Varenna, August 1958.

²⁾ We call strongion a particle which is either a baryon or a meson (with the exception of the μ -meson which is a lepton).

the same symbol of the class the corresponding current, so that

$$J_\mu = \mathcal{L}_\mu + S_{0\mu} + S_{-1\mu} + S_{1\mu}. \quad (2)$$

Let us now explain in detail the classification:

$\mathcal{L}$: lepton class (the corresponding current $\mathcal{L}_\mu$ is the lepton current).

It consists of the pairs

$$(\mu^+ \bar{\nu}) \quad (e^+, \bar{\nu}) \quad (3)$$

For a pair $(A\bar{B})$ the transfer of charge is $\Delta Q = Q_B - Q_A$. Note that the other possible pairs such as

$$(\mu\bar{e}), \quad (\nu\bar{\nu}), \quad (\mu\bar{\mu}), \quad (e\bar{e}) \quad (4)$$

are excluded from $\mathcal{L}$. Their inclusion would lead to processes such as

$$\mu^+ \rightarrow e^+ + e^- + e^+, \quad \mu^- + p \rightarrow e^- + p \quad (5)$$

not observed. We note that the excluded pairs have $\Delta Q = 2$ or $\Delta Q = 0$. Therefore it seems that one has to assume at this point that only pairs with $\Delta Q = \pm 1$ are coupled. The reason for this choice is a mystery so far. We shall come back later in more detail to this point. To be definite, without limitation to our discussion, let us consider here only the pairs with $\Delta Q = -1$. Thus the lepton current is given by $\mathcal{L}_\mu = (\mu^+ \bar{\nu})_\mu + (e^+ \bar{\nu})_\mu$.

S_0 : strongion class with $\Delta S = 0$ (the corresponding current is the strangeness-conserving current). One finds a total of six possible baryon pairs which satisfy the limitations $\Delta Q = -1$, $\Delta S = 0$, and of course also $\Delta N = 0$ because they must couple to the lepton pairs:

$$(p\bar{n}), \quad (\Sigma^+ \bar{\Lambda}), \quad (\Sigma^+ \bar{\Sigma}^0), \quad (\Lambda \bar{\Sigma}^-), \quad (\Sigma^0 \bar{\Sigma}^-), \quad (\Xi^0 \bar{\Xi}^-). \quad (6)$$

All such pairs can go into one another by strong interaction. (We shall call such pairs "equivalent".) They are also equivalent to the π^+ . The π^+ will dissociate through them. It will be convenient to add to these pairs possible boson terms corresponding to vector interactions of the boson currents

$i \left(\varphi \frac{\partial \varphi^*}{\partial x_\mu} - \varphi^* \frac{\partial \varphi}{\partial x_\mu} \right)$ with the lepton fields. One finds two such terms

$$(\pi^+ \pi^0), \quad (K^+ \bar{K}^0). \quad (7)$$

The presence of such terms is necessary if one accepts the hypothesis of non-renormalizability for the vector part, as we shall see later.

S_{-1} : strongion class with $\Delta S = -1$ (strangeness non-conserving current of the first type). We find again a total of six pairs

$$(p\bar{\Lambda}), \quad (p\bar{\Sigma}^0), \quad (n\bar{\Sigma}^-), \quad (\Lambda \bar{\Xi}^-), \quad (\Sigma^+ \bar{\Xi}^0), \quad (\Sigma^0 \bar{\Xi}^-) \quad (8)$$

They are equivalent to the K^+ , which will decay through virtual dissociation into them. Two meson terms are possible, namely

$$(K^+ \pi^0), \quad (K^0 \pi^+). \quad (9)$$

Differently from the corresponding situation with S_0 , there is nothing here forcing the assumption that (9) are coupled. The only observation that can be made is that the simultaneous existence of both $K \rightarrow \mu + \pi + \nu$ and $K \rightarrow \mu + \nu$

excludes the possibility that in $\mathcal{S}_{-1}$ only the boson terms (9) are coupled — some fermion pairs also have to be coupled. This can be shown readily by parity considerations.

One can verify that by coupling to each other $\mathcal{L}$, $\mathcal{S}_0$ and $\mathcal{S}_{-1}$ one has accounted for all the observed weak processes. The requirement that the pairs carry a unit charge (or equivalently, that the current J_μ is singly charged) has eliminated unwanted processes such as

$$\mu^+ \rightarrow e^+ + e^- + e^+, \quad \mu^- + p \rightarrow e^- + p, \quad K \rightarrow \mu^\pm + e^\mp + \pi, \quad K \rightarrow \pi + \nu + \bar{\nu}. \quad (10)$$

The advantage of classifying the pairs into classes is mainly that of avoiding the embarrassing question of which pairs in each class are originally coupled: for instance only one pair in each class may be coupled, the other pairs will then still be indirectly coupled through the strong interactions, or all pairs may be coupled according to some particular unknown symmetry. It is quite relevant to investigate the possibilities for such symmetries, and we shall touch upon this question when speaking of the possible non-renormalization of the vector part.

The two strongion classes $\mathcal{S}_0$ and $\mathcal{S}_{-1}$ introduced so far can be characterized by saying that $\mathcal{S}_0$ contains all pairs equivalent to the π^+ , and $\mathcal{S}_{-1}$ all pairs equivalent to the K^+ . Moreover for $\mathcal{S}_{-1}$, $\Delta S = \Delta Q$.

2.2 Limitations from the selection rule $\Delta S = \pm 1$ for strange particle decays

Let us now consider the class $\mathcal{S}_{+1}$.

$\mathcal{S}_{+1}$, strongion class with $\Delta S = +1$ (strangeness non-conserving current of the second kind). It contains

$$(\Sigma^+ \bar{n}), \quad (\Xi^0 \bar{\Sigma}^-) \quad (11)$$

and a possible meson term

$$(\bar{K}^0 \pi^+). \quad (12)$$

Pairs of this class would be equivalent to a positive meson with $S = -1$. No such meson exists in the Gell-Mann scheme. Pairs of this class are equivalent to the system $(\bar{K}^0 \pi^+)$ which behaves like a positive meson with $S = -1$. Now the point is that if we couple $\mathcal{S}_{+1}$ to $\mathcal{S}_{-1}$ we find a possible reaction such as $\Xi^0 \rightarrow n + \bar{\Sigma}^- + \Sigma^-$ which would lead to the sequence

$$\Xi^- \rightarrow \Xi^0 + \pi^- \rightarrow n + \pi^0 + \pi^- \rightarrow n + \pi^-. \quad (13)$$

Now $\Xi^- \rightarrow n + \pi^-$ has not been seen and instead $\Xi^- \rightarrow \Lambda^0 + \pi^-$ has been seen in all the cases reported (not so many however) thus suggesting the selection rule $\Delta S = \pm 1$ for strange particle decays [2]. This might be a motivation for excluding class $\mathcal{S}_{+1}$ ¹⁾. What is important is that one can then obtain a number of predictions which can be compared with experiment. Let us first see what these predictions are. Then we shall look into the question of how to exclude

¹⁾ Note that all strong virtual steps in (13) involve the stronger pion interactions and no K interactions.

this class by theoretical arguments. By coupling S_{+1} to $\mathcal{L}$ one would have the decay modes

$$\Sigma^+ \rightarrow n + l^+ + \nu \quad \Xi^0 \rightarrow \Sigma^- + l^+ + \nu \quad (14)$$

(where l means either e or μ), and

$$\bar{K}^0 \rightarrow \pi^- + l^+ + \nu \quad (15)$$

and its charge conjugate

$$K^0 \rightarrow \pi^+ + l^- + \bar{\nu}. \quad (15')$$

Such decay modes are forbidden if S_{+1} is not coupled. What does this mean in terms of the physical particles K_1^0 and K_2^0 ? Recalling that [3]

$$K_1^0 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0) \quad K_2^0 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0) \quad (16)$$

one finds immediately the following relations between the decay amplitudes

$$(K_1^0 \rightarrow l^+) = (K_2^0 \rightarrow l^+) = \frac{1}{\sqrt{2}}(K^0 \rightarrow l^+) \quad (17)$$

$$(K_1^0 \rightarrow l^-) = - (K_2^0 \rightarrow l^-) = -\frac{1}{\sqrt{2}}(\bar{K}^0 \rightarrow l^-) \quad (17')$$

In (17), (17'), $(K_1^0 \rightarrow l^+)$, for instance, is the matrix element for $K_1^0 \rightarrow l^+ + \pi^- + \nu$.

OKUN and PONTECORVO have pointed out that the presence or absence of $\Delta S = \pm 2$ interactions is of great relevance in the problem of the $K_1^0 K_2^0$ mass difference [4]. If S_{+1} is coupled, a term contributing to the mass difference would already be present at the first order in the weak coupling constant, for instance, through the virtual scheme

$$\bar{K}^0 \rightarrow \Sigma^+ + \bar{n} + \pi^- \rightarrow \Xi^0 + \Sigma^+ + \pi^- \rightarrow K^0 \quad (18)$$

The Pais-Piccioni oscillations [5] would then be very frequent ($\sim 10^{17}$ per second) as compared to the inverse lifetime ($\sim 10^{10} \text{ sec}^{-1}$). If S_{+1} is not coupled, a typical mass difference contribution would come from

$$\bar{K}^0 \rightarrow p + \bar{n} + \pi^- \rightarrow K^0, \quad (19)$$

with two weak steps, leading to a mass difference presumably of the same order of the inverse lifetime. In this situation one may observe the Pais-Piccioni oscillations.

The requirement $\Delta S = \Delta Q$ excludes class S_{+1} . A stronger requirement which also leads to exclusion of class S_{+1} is that the strangeness-non-conserving current transforms in isotopic spin space like the component of a spinor. One sees immediately that there is no way of satisfying this requirement in the class S_{+1} since there are no suitable linear combinations available, for instance of a Σ and a nucleon. We shall later discuss again such questions.

Finally, if one still wants to complicate the situation, one can think of currents $S_{\pm 2}$ for which $\Delta S = 2$. Again one meets the trouble of $\Xi^- \rightarrow n + \pi^-$ by coupling for instance a term $(n\bar{\Xi}^-)$ to S_0 . Also decay modes such as $\Xi^- \rightarrow n + e^- + \bar{\nu}$ would be expected in that case.

2.3 Possible neutral currents

Let us now discuss the possibility of neutral currents ($\Delta Q = 0$). We consider two neutral lepton currents, $\mathcal{L}^{(0)}$, containing $(\mu^+\mu^-)$, (e^+e^-) and $(\nu\bar{\nu})$, and $\mathcal{L}^{(0)'} containing (μ^+e^-) . $\mathcal{L}^{(0)'}$ is apparently not coupled at all. One finds out that it is still consistent with present experiments to couple $\mathcal{L}^{(0)}$ to a strangeness-conserving neutral current $S_0^{(0)}$.$

$S_0^{(0)}$: neutral strongion class with $\Delta S = 0$ (neutral strangeness-conserving current). It may contain the baryon terms

$$(p\bar{p}), (n\bar{n}), (\Lambda^0\bar{\Lambda}^0), (\Sigma^+\bar{\Sigma}^+), (\Sigma^0\bar{\Sigma}^0), (\Sigma^0\bar{\Lambda}^0), (\Sigma^-\bar{\Sigma}^-), (\Xi^0\bar{\Xi}^0), (\Xi^-\bar{\Xi}^-) \quad (20)$$

and the meson terms

$$(\pi^+\pi^-), (\pi^0\pi^0), (K^+K^-), (K^0\bar{K}^0). \quad (21)$$

The coupling $\mathcal{L}^{(0)} S_0^{(0)}$ does not lead to any inconsistency with present data. There would be weak interactions such as $\mu + n \rightarrow \mu + n$ which however will be hard to detect because of the stronger electromagnetic effects, and processes such as $\nu + p \rightarrow \nu + p$ also hard to detect. One also will have interactions $(e^+e^-)(\nu\bar{\nu})$ and $(\mu^+\mu^-)(\nu\bar{\nu})$ which can be rearranged to look as $(e\bar{\nu})(e\bar{\nu})$ and $(\mu\bar{\nu})(\mu\bar{\nu})$ again in the AV combination, and they would therefore interfere with the corresponding terms arising directly from the charged currents. A similar conclusion also holds for terms $(p\bar{p})(n\bar{n})$ which can be rearranged in the form $(p\bar{n})(p\bar{n})$.

The reason which suggested that we look into the possibility of a neutral current it connected with an extension of the hypothesis of non-renormalization of the vector part of the strangeness-conserving S_0 , which we shall discuss later. From such a point of view, in trying to build up the neutral current $S_0^{(0)}$ one is strongly tempted to require that its vector part be the 3 current, $t_\mu^{(3)}$, of the isotopic spin, the axial part being then fixed by the requirement that the fermions occur in the $\frac{1}{2}(1 + \gamma_5)$ projection. This would lead automatically to the non-renormalization for the vector coupling constant in $S_0^{(0)}$, and would produce a highly symmetric scheme. In such a case $S_0^{(0)}$ would consist of

$$(p\bar{p}), (n\bar{n}), (\Sigma^+\bar{\Sigma}^+), (\Sigma^-\bar{\Sigma}^-), (\Xi^0\bar{\Xi}^0), (\Xi^-\bar{\Xi}^-), (\pi^+\pi^-), (K^+K^-), (K^0\bar{K}^0). \quad (22)$$

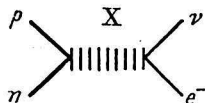
One has further to look into the possibility of neutral strangeness-non-conserving currents. One is here however immediately faced by a difficulty. In fact, by coupling a term $(n\bar{\Lambda})$ with $(\nu\bar{\nu})$, (e^+e^-) , and $(\mu^+\mu^-)$, one would find processes such as $\bar{K}^0 \rightarrow e^+ + e^-$ and $\bar{K}^0 \rightarrow \mu^+ + \mu^-$ of which the first one would proceed at a very low rate, but the second one could very well be observed. Even more, one would find processes such as $K^- \rightarrow \pi^- + \nu + \bar{\nu}$ and $K^- \rightarrow \pi^- + e^+ + e^-$, which would favorably compete with $K^- \rightarrow \pi^0 + e^- + \bar{\nu}$, which is a known reaction.

2.4 The requirement $\Delta Q = \pm 1$ and the intermediate meson hypothesis

If one wants to keep only the charged currents, a rather suggestive hypothesis is that weak interactions are mediated by an agent X (for instance a massive boson) which carries unit charge. Each pair would interact with X, and indirectly with all the other pairs coupled to X. The resulting interaction between two pairs would then be slightly non-local. If X is a boson with mass M , it must have spin 1 to obtain in the local limit the AV theory, and its propagator would be in momentum space

$$\frac{1}{K^2 + M^2} \left(\delta_{\mu\nu} + \frac{K_\mu K_\nu}{M^2} \right). \quad (23)$$

The effective weak interaction would occur through the X meson, for instance β^- decay would be given by a graph:

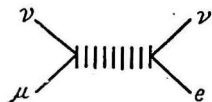


The non-locality contained in the $\delta_{\mu\nu}$ term is difficult to observe because of the smallness of the momentum transfers K^2 for most of the known processes

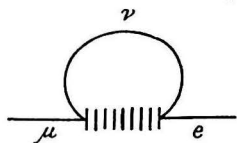
$$\frac{1}{K^2 + M^2} \approx \frac{1}{M^2}. \quad (24)$$

Even more difficult would be the observation of the $K_\mu K_\nu$ term. This term would contribute effective scalar and pseudoscalar contributions to the resulting direct interaction between the currents. Since X is supposed to have only electromagnetic interactions (because it is charged) and "semiweak" interactions with the currents, its production cross-section would be very small (for instance it could be produced by the reaction $\gamma \rightarrow X^+ + X$) so that no difficulty concerning its apparent absence may occur. Its decay would occur at first order in the semiweak interactions of X with the currents. Let us however see what difficulties one encounters by postulating the existence of X. First of all, there is evidence for some non-local effects in μ^- decay which were suggested [6] to explain the deviations of the ρ -value from $3/4$ (the best ρ -value determination so far is $\rho = 0,68 \pm 0,02$, due to CROWE). A deviation of this order of magnitude could be accounted for by a value of the mass around 300–400 Mev. However, as we shall discuss later in more detail, the sign of the deviation cannot apparently be accounted for by a real meson. For a real meson the deviations would always be such as to increase the ρ -value above $3/4$. Of course, before reaching a definitive conclusion one should try to estimate the radiative corrections to the ρ -value for this more complicated model (in which a virtual photon can also be emitted from the non-local vertex). The radiative corrections which were subtracted to get the experimental figure of $0,68 \pm 0,02$ are those calculated for a point direct interaction between the four fermions. A second difficulty for the hypothesis of the intermediate X occurs in connection with the

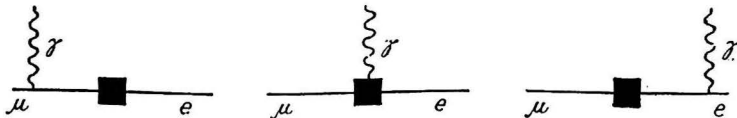
apparent non-existence of the decay mode $\mu^+ \rightarrow e^+ + \gamma$. If an intermediate X exists, the diagram for μ -decay, $\mu \rightarrow e + \nu + \bar{\nu}$, would look as



From this diagram we can deduce the existence of an indirect term looking as $\bar{\mu}_e(x) K_{e\sigma}(x - x') e_\sigma(x')$ in the interaction Hamiltonian, where K is some form factor. It comes from the graph



If the electromagnetic interactions are now turned on, the existence of the above term would imply the existence of diagrams of the kind

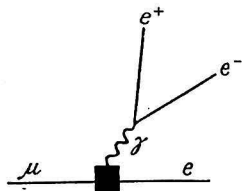


which lead to $\mu \rightarrow e + \gamma$. If the kernel K is expanded in powers of the momentum transfer and only the linear term is kept, then one can easily show that gauge invariance requires a perfect compensation of all the three graphs above so that no $\mu \rightarrow e + \gamma$ occurs at this order. This approximation is however not recommended because of the smallness of the wavelength of the emitted γ . FEYNMAN and GELL-MANN [7] and FEINBERG [8] have calculated the probability for $\mu \rightarrow e + \gamma$ with the conventional highly divergent quantum electrodynamics of a charged spin one meson X . The matrix element for $\mu \rightarrow e + \gamma$ is found to be

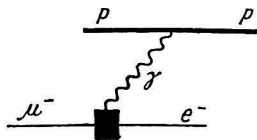
$$\frac{e}{96\pi^2} \frac{f_X^2}{M^2} m_\mu N (\bar{u}_e \sigma_{\alpha\beta} F_{\alpha\beta} (1 - \gamma_5) u_\mu) \quad (26)$$

where f_X is the semiweak coupling constant and N is a number depending on the cut-off. The expression (26) is valid only for large values of M . With a cut-off around the value M , N turns out to be essentially one, depending logarithmically on the cut-off. The probability for $\mu \rightarrow e + \nu + \bar{\nu}$ also contains the factor f_X^2/M^2 , which thus drops out in the expression for the branching ratio of $\mu \rightarrow e + \gamma$ to $\mu \rightarrow e + \nu + \bar{\nu}$. One finds thus a value 10^{-4} for such a branching ratio. The experimental value is due to LOKANATHAN and STEINBERGER who report a figure of $2 \cdot 10^{-5}$ as an upper limit for the branching ratio. It is also evident that since one has an effective vertex operator in (25) one then has

also the possibility of obtaining processes such as $\mu \rightarrow e + e^+ + e^-$, from the graph



and $\mu^- + p \rightarrow e^- + p$ from



Such processes would also occur at the same order in the weak coupling constant as the ordinary decay $\mu \rightarrow e + \nu + \bar{\nu}$ or the ordinary absorption $\mu^- + p \rightarrow \nu + n$. They occur however at the second order in e^2 , whereas $\mu \rightarrow e + \gamma$ occurred at first order in e^2 . It is funny how, although all this business of the intermediate charged X was invented to avoid processes such as $\mu \rightarrow e + e^- + e^+$ or $\mu^- + p \rightarrow e^- + p$, such processes still occur at higher order in e^2 . However practically the probabilities come out to be so small that no inconsistency with the present data arises for the moment. What seems instead to give rise to a real inconsistency is the absence of $\mu \rightarrow e + \gamma$. It should however be emphasized that a cut-off calculation like this may very well be wrong. No other theoretical scheme is known however to calculate such things.

Now, why do not such difficulties occur in the local case, with a direct four fermion interaction? One can easily convince himself that the neutrino loop (25) is zero in the local case, because of symmetry considerations. Its expression is however far from being unambiguous.

3. Possible Properties in Isotopic Spin Space

3.1 Isotopic spin changes in the decay of strange particles

It may at first sight seem rather useless to start a discussion about the possible properties of weak interactions in isotopic spin space, since isotopic spin is certainly not-conserved in such interactions. To such a point of view one may object, first, that in a similar situation where isotopic spin conservation is violated, namely with electromagnetic interactions, it is indeed very useful to analyse the amplitudes in terms of eigenstates of isotopic spin; second, that there is in fact some experimental evidence for an apparently simple behaviour of the decay amplitudes in isotopic spin space, and at least the rule $\Delta S = \pm 1$ for strange particle decay directly implies $\Delta I_3 = \pm 1/2$; third, some general requirements to the strangeness-conserving terms, such as that of a definite behaviour of the current $[6]$ under charge conjugation times charge symmetry, or that of nonrenormalization for the vector part, lead us to assume a behaviour

like that of the component of a vector in isotopic spin space. An important point to be noticed first is that, by requiring $\Delta I = 1/2$ to the strangeness non-conserving current, one obtains immediately $\Delta I_3 = +1/2$ for the decay of strange particles, *though in general both $\Delta I = 1/2$ and $\Delta I = 3/2$ will be present in the interaction.* That is one has a scheme conceptually simpler than the $\Delta I = 1/2$ rule for the interaction which still gives $\Delta I_3 = \pm 1/2$. This result can be verified very easily. As we said already, it may very well be that S_0 , the strangeness-conserving current, behaves, like a component of a vector in isotopic spin space. This follows if one postulates a definite behaviour of the current [6] with respect to the product operation of charge conjugation and charge symmetry, or also if one assumes the hypothesis of nonrenormalization of the vector part. We shall discuss both these questions later. If one assumes that the strangeness non-conserving current behaves like the component of an isotopic spinor, then, by coupling it to a strangeness-conserving current behaving like a component of an isotopic vector, one would have selection rules $\Delta I = 1/2$ and $\Delta I = 3/2$ for experimental situation is concerned everybody knows that $\Delta I = 1/2$ [9] can be at most an approximate selection rule. The main argument for it being only approximate is that one cannot explain simultaneously the ratios of the lifetime of K^0 to the lifetime of K^+ and the ratio between the probability for $K^0 \rightarrow \pi^+ + \pi^-$ and the probability for $K^0 \rightarrow \pi^0 + \pi^0$ with only $\Delta I = 1/2$. One finds instead that one has to introduce both $\Delta I = 3/2$ and $\Delta I = 5/2$ contributions to account simultaneously for the above ratios. The estimated magnitude of such contributions, of the order of 10% [10], is such as to make not very plausible the suggestion that they come from electromagnetic corrections. However $\Delta I = 1/2$ has the great merit of explaining, though not completely of course, for the above mentioned difficulty, why the K^+ has so much longer a lifetime than the K^0 . In fact, if $\Delta I = 1/2$ rigorously holds, K^+ cannot decay into two pions because, starting from $I = 1/2$, $I_3 = 1/2$ one can only obtain $I = 1$ for the final charged 2-meson state which should therefore be antisymmetric in space coordinates, in contrast with the value for the orbital angular momentum which is $l=0$ if the K has spin zero. Moreover $\Delta I = 1/2$ also explains the ratio of 2 of $\Lambda^0 \rightarrow p + \pi^-$ to $\Lambda^0 \rightarrow n + \pi^0$. Observation of such a value for this ratio does not however imply absence of $\Delta I = 3/2$ contributions. It must also be noticed that $\Delta I = 1/2$ also explains the experimental ratio $\tau'/\tau \approx 1/4$. However such a value would also be obtained if a $\Delta I = 3/2$ component is present because if the τ has spin zero the final state is expected to be predominantly symmetric in the space coordinates and from group theory one then can show that the possible values of the total isotopic spin of the three pions can only be $I = 1$ and $I = 3$. The latter, however, would not still be accessible even if $\Delta I = 3/2$ were present. We have seen from our preceding discussion about a general description of weak interactions that it is rather difficult theoretically to present a reasonable scheme leading to $\Delta I = 1/2$ always. The fact that $\Delta I = 1/2$ seems to predominate in the decay of strange particles should then be explained in each case by means of a particular mechanism suppressing $\Delta I = 3/2$ or by showing that the same ratios are also obtained by more physically plausible combinations of $\Delta I = 3/2$ and $\Delta I = 1/2$. An example where such a situation may occur is Λ decay. Here one finds that by calculating directly from the coupling $(p\Lambda)(np)$, keeping only the simplest diagram, one obtains, the branching ratio of 2 between the decay modes $\Lambda \rightarrow p + \pi^-$ and $\Lambda \rightarrow n + \pi^0$, as with the $\Delta I = 1/2$ rule, though the interaction now contains also a $\Delta I = 3/2$ term.

3.2 Behaviour of the strangeness-conserving current under the product of charge conjugation and charge symmetry

In looking for more general symmetry requirements that one would ask from the strongion currents, it may be convenient to recall that the strangeness-conserving current S_0 contains all those pairs which are equivalent to a π^+ . Now a π^+ is transformed into itself by performing a charge conjugation and then a charge symmetry. Therefore we can try to formulate the requirement that S_0 has a definite behaviour under the product operation of charge conjugation and charge symmetry, and investigate to what restriction it leads for S_0 . It is unfortunate that a similar possibility does not exist for the strangeness-non conserving currents. The current S_{-1} is equivalent to a K^+ and this would be transformed into an $\bar{K}^0$ under the above product operation, so that it is impossible to formulate the problem of a definite behaviour of S_{-1} under this operation. A similar conclusion is also evident for S_{+1} .

The convenience of classifying the different terms in S_0 according to their properties under the product operation of charge conjugation and charge symmetry have been pointed out recently by WEINBERG [11]. The relevant operator is $G = Ce^{i\pi I}$, where C is the charge conjugation operator and $e^{i\pi I}$ performs a rotation of 180° around the second axis in isotopic spin space, which is one way of realizing charge symmetry as a proper orthogonal transformation. For the nucleon field $G\psi G^{-1} = i\tau_2\psi^c$ and for the pions $G\phi G^{-1} = -\phi$. This last relation is responsible for the known rule which forbids an even number of pions to go into an odd number of pions if G is conserved. Moreover G satisfies the relations $[G, N]_+ = 0$, $[G, S]_+ = 0$, $G^2 = (-)^{N+S}$, $[GI] = 0$, where N is the nucleon number, S the strangeness and I the total isotopic spin operator. We next consider the bilinear expression $(\bar{\psi} \Gamma_i \tau_+ \psi)$ and we find $G(\bar{\psi} \Gamma_i \tau_+ \psi) G^{-1} = -\xi_i (\bar{\psi} \Gamma_i \tau_+ \psi)$; where $\xi_i = +1$ for $i = S, A, P$ and $\xi_i = -1$ for $i = V, T$. Now we say that all covariants Q_i which transform according to $GQ_i G^{-1} = -\xi_i Q_i$, namely in the same way as the simple $(\bar{\psi} \Gamma_i \tau_+ \psi)$ of the same tensor character i , are of the first class; those which transform according to $GQ_i G^{-1} = \xi_i Q_i$ are of the second class. Let us now go back to the strangeness non-conserving current, and consider for instance its vector part. The first term $(\bar{\psi} \gamma_\mu \tau_+ \psi)$ is of the first class, because of definition. When however, we want to add a similar term for the Σ -particles we find that we can choose among all the possible linear combinations of the form $a\bar{\Sigma}^+ \gamma_\mu \Sigma^0 + b\bar{\Sigma}^0 \gamma_\mu \Sigma^-$. If, however, we require that it behaves as a first class covariant vector, then we find that $a = -b$. This means that we can write our term for the Σ -particles as $\bar{\Sigma} \gamma_\mu T_+ \Sigma$, where T_+ is the appropriate isotopic spin matrix. One finds in general that one is led to the conclusion that the selection rules for a vector strangeness conserving current consisting of the pairs (6) are those of the component of a vector in isotopic spin space. This holds also for the total current including the axial which can then be constructed from the requirement that fermions appear in the projection $(1/2)(1 + \gamma_5)$.

If one now assumes as a principle that in the strangeness-conserving current there are only first class quantities, how can one verify such a principle experimentally? In β -decay there is generally not much hope, because a term such as $\langle p | \bar{\psi}_p \gamma_\mu \psi_n | n \rangle$ where $\langle p |$ and $| n \rangle$ are the physical states is always well approximated by a term of the form $\bar{u}_p \gamma_\mu u_n$ between free spinors, because of the low

momentum transfer — and similarly for $\langle p | \bar{\psi}_p \gamma_\mu \gamma_5 \psi_n | n \rangle$. One has to study energetical β -decay to discover such effects. In $\mu^- + p \rightarrow n + \nu$ the momentum transfer is larger. A matrix element $\langle p | \bar{\psi}_p \gamma_\mu \psi_n | n \rangle$ will be expressible in general as

$$\langle p | \bar{\psi}_p \gamma_\mu \psi_n | n \rangle = a_\nu (K^2) \gamma_\mu + b_\nu (K^2) K_\nu \sigma_{\mu\nu} + i c_\nu (K^2) K_\mu \quad (27)$$

where K_μ is the momentum transfer $K_\mu = K_{p,\mu} - K_{n,\mu}$ and a, b, c are invariant form factors. Similarly

$$\langle p | \bar{\psi}_p \gamma_\mu \gamma_5 \psi_n | n \rangle = a_A (K^2) i \gamma_\mu \gamma_5 + b_A (K^2) K_\mu \gamma_5 + i c_A (K^2) K_\nu \sigma_{\nu\mu} \gamma_5. \quad (28)$$

One can immediately convince himself that in each of the above expressions the first two terms behave as first class quantities. In fact, as we said before, V and T take on the same ξ_i and also A and P take on the same ξ_i . The last terms in each of the two expressions are instead of the second class. In $\mu^- + p \rightarrow n + \nu$ the term $i c_\nu K_\mu$ contributes an effective scalar interaction of magnitude $\sim m_\mu c_\nu (K^2)$. A guess about $c_\nu (K^2)$ would suggest that it is $\sim (m_\mu/m_N) a_\nu$. Observation of this scalar term would indicate the presence of second class quantities. Furthermore since the nucleon contributions to the current are always of the first class [they must always be of the form $(\bar{\psi} \Gamma_i \tau_\pm \psi)$] observation of such a scalar term would inform us about the hyperon contributions to the nuclear structure. The term $i c_A K_\nu \sigma_{\nu\mu} \gamma_5$ contributes in $\mu^- + p \rightarrow n + \nu$ an effective derivative coupling $i c_A (K^2) (K_\lambda^{(\mu)} + K_\lambda^{(\nu)}) \sigma_{\mu\lambda} \gamma_5$. The observation of the first two terms in each of the above expressions does not tell anything about second class quantities, but it tells us of nucleon structure. The term $b_\nu K_\nu \sigma_{\nu\mu}$ introduces an effective derivative coupling in $\mu^- + p \rightarrow n + \nu$, while the term $b_A K_\mu \gamma_5$ introduces an effective pseudoscalar. GOLDBERGER and TREIMAN have used dispersion relation techniques to estimate the effective pseudoscalar and they find that it is eight times larger than the original axial part. We shall discuss later this result when speaking of the nuclear interaction of μ particles.

In concluding this discussion about the behaviour in isotopic spin space we want again to state the result that both $\Delta I = 1/2$ and $\Delta I = 3/2$ contributions are expected to be present in the decay of strange particles according to this theory. There seems to be no way of avoiding this conclusion in the present formulation. If one wants an overall $\Delta I = 1/2$ rule for the interaction, then one has necessarily to introduce neutral currents into the picture. We have seen in 2.3 that a strangeness conserving neutral current may certainly be introduced, but its coupling with leptons seems to give rise to some inconsistencies.

4. The Hypothesis of Universality and the Hypothesis of Non-renormalization for the Vector Strangeness-conserving Current

4.1 Form of the weak interaction Hamiltonian and calculation of the weak coupling constant

Let us go back to the strangeness-conserving current. To discuss the question of the non-renormalization of the vector part we have to review first the main steps leading to the weak interaction Hamiltonian of the FGMS theory. The main assumption is that the fields always appear in the interaction with the projection $(1/2)(1 + i\gamma_5)$. (We are using here Feynman's notations.) Let us call

such a projection operator $\mathbf{a} = (1/2) (1 + i\gamma_5)$ and let us also define the Hermitian conjugate $\bar{\mathbf{a}} = (1/2) (1 - i\gamma_5)$. One easily verifies that $\mathbf{a}^2 = \mathbf{a}$, $\bar{\mathbf{a}}\mathbf{a} = 0$. The most general local non-derivative interaction Hamiltonian for β -decay will then be of the form

$$\sum_i g_i (\bar{\mathbf{a}} \bar{\psi}_n \Gamma_i \mathbf{a} \psi_p) (\bar{\mathbf{a}} \bar{\psi}_e \Gamma_i \mathbf{a} \psi_e). \quad (29)$$

Using $\bar{\mathbf{a}}\bar{\psi} = \bar{\psi}\mathbf{a}$ one can write (29) as

$$\sum_i g_i (\bar{\psi}_n \bar{\mathbf{a}} \Gamma_i \mathbf{a} \psi_p) (\bar{\psi}_e \bar{\mathbf{a}} \Gamma_i \mathbf{a} \psi_e). \quad (30)$$

Now $\bar{\mathbf{a}} \Gamma_i \mathbf{a} = 0$ for $i = S, P, T$ and $= \Gamma_i \mathbf{a}$ for $i = A, V$ so that (30) can be written as

$$\sum_{i=A,V} g_i (\bar{\psi}_n \Gamma_i \mathbf{a} \psi_p) (\bar{\psi}_e \Gamma_i \mathbf{a} \psi_e). \quad (31)$$

Furthermore one verifies easily that $\Gamma_i \mathbf{a} = \gamma_\mu \mathbf{a}$ is also equal to $\Gamma_A \mathbf{a} = i\gamma_\mu \gamma_5 \mathbf{a}$ so that (31) can be written as

$$\sqrt{8} G (\bar{\psi}_n \gamma_\mu \mathbf{a} \psi_p) (\bar{\psi}_e \gamma_\mu \mathbf{a} \psi_e). \quad (32)$$

This coupling is nothing else than $A-V$ in the usual notations. It has been supposed here that the nucleons rather than the antinucleons have the projection $\mathbf{a}$ in front. Such an assignment is quite arbitrary of course and no theoretical justification can be given for it. It can however be shown to be the right one experimentally by the following argument. If antinucleons instead of nucleons have the projection $\mathbf{a}$ in front, the β -decay Hamiltonian would be

$$\sqrt{8} G (\bar{\psi}_p \gamma_\mu \mathbf{a} \bar{\psi}_n) (\bar{\psi}_e \gamma_\mu \mathbf{a} \psi_e). \quad (33)$$

By using the properties of the charge conjugation matrix C and the relations $\psi^C = C\bar{\psi}^T$, $\bar{\psi}^C = C^{-1}\bar{\psi}^T$, one can rewrite the above Hamiltonian as

$$\sqrt{8} G (\bar{\psi}_n \gamma_\mu \mathbf{a} \psi_p) (\bar{\psi}_e \gamma_\mu \mathbf{a} \psi_e). \quad (34)$$

This is the $A + V$ combination. Now, in the decay of polarized neutrons $A - V$ would give no asymmetry (neglecting the small deviations from the equality of the axial and of the vector coupling constants which are due to renormalisation effects), while $A + V$ would give maximum asymmetry (also in the same approximation). The observed asymmetry is consistent with $A - V$, thus suggesting the form (32) for β decay Hamiltonian and the assumption that nucleons have the factor $\mathbf{a}$ in front and antinucleons the factor $\bar{\mathbf{a}}$. To build up the current J_μ one adds the contributions from the pairs (pn), (ve), ($\nu\mu$), et cetera

$$J_\mu = (p \gamma_\mu \mathbf{a} n) + (\bar{\nu} \gamma_\mu \mathbf{a} \mu) + (\bar{\nu} \gamma_\mu \mathbf{a} e) + \dots \quad (35)$$

It has been assumed here that μ^- is a particle. The other choice would have led to a μ -decay Hamiltonian given by $\sqrt{8} G (\bar{\nu} \gamma_\mu \mathbf{a} \bar{\mu}) (\bar{\nu} \gamma_\mu \mathbf{a} e)$ which would lead to a ρ_+ -value of zero in contradiction to experiment. That the electron spectrum should go down to zero independent of the statistical weight at the maximum electron energy if two neutrino or two antineutrons are emitted follows from the Pauli principle. In fact at the maximum electron energy the two neutrinos would travel in the same direction, although in general with different momenta.

The wavefunction of a two component neutrino does not however contain any indication of the magnitude of its momentum except in the exponential factor which contributes to the δ -function expressing the total momentum conservation, and since the decay amplitude has to vanish when the two momenta are equal it will also vanish in the general case. So at maximum electron energy the decay amplitude has to go to zero by virtue of the Pauli principle. From the μ -meson decay Hamiltonian $\sqrt{8} G (\bar{\mu} \gamma_\mu \alpha \nu) (\bar{\nu} \gamma_\mu \alpha e)$, one finds for the μ -meson lifetime the expression $\tau = 192 \pi^3 / G^2 m_\mu^3$ and noting that no appreciable effects are expected here to renormalize the value of the coupling constants (the largest radiative corrections arise from virtual photons) one can use this expression to calculate G , the bare coupling constant of the weak interaction Hamiltonian. Surprisingly one finds that the coupling constant so determined is identical, in the limits of the small experimental errors, to the effective coupling constant for the Fermi part of β -decay, that one can measure from the lifetime of O^{14} . Namely from the lifetime of O^{14} one finds $G = (1.01 \pm 0.01) \cdot 10^{-5} (1/m_N^3) = (1.41 \pm 0.01) \cdot 10^{-49} \text{ erg/cm}^3$, and inserting such a value into the μ -lifetime formula one obtains $\tau = (2.26 \pm 0.04) \cdot 10^{-6} \text{ sec}$, to be compared to the experimental lifetime $\tau = (2.22 \pm 0.02) \cdot 10^{-6} \text{ sec}$. This agreement seems to suggest strongly that the hypothesis of universality holds, namely that the formulation with a current J_μ of the form (35) is perhaps right, and furthermore, it indicates that, surprisingly, the renormalization effects due to strong interactions are absent for the V part of β -decay. As far as this latter circumstance is concerned, one would think that, unless some unknown accident takes place, a symmetry principle is operating at this place in a way similar to gauge invariance in quantum electrodynamics which leads to the non-renormalization of the electric charge e . Gauge invariance implies that in the zero energy limit the charge of the proton is rigorously e and furthermore it insures that the result holds independent of the structure of the nucleon. We may say that physically this happens because when a proton emits a virtual π^+ to become a neutron the virtual π^+ must, because of gauge invariance, interact with the photon field and again exhibit the charge e . It is only for smaller wavelengths, that are able to explore the structure, that an effective charge different from e may be exhibited. From this analogy it will not appear surprising that, in order to have non-renormalization for the V coupling in β -decay, one has to introduce a direct interaction of the pion field bilinearly with the lepton pairs. We shall later discuss fully the formal problem that is involved here and the physical implications of the scheme. We would like however to show first how the argument given above for the non-renormalization of the vector part, namely from the equality of the coupling constant from O^{14} to that from μ -decay, would be invalidated if the present evidence for a g -value in μ -decay definitively different from $3/4$ would be confirmed. It is therefore very important at this point to have a very accurate measurement of the g -value in μ -decay.

4.2 Incompatibility of the hypothesis of non-renormalization of the V -part with the present experimental g -value

Let us write the μ -decay Hamiltonian in the form

$$\sqrt{8} G (\bar{\mu} \gamma_\mu \nu_L) (\nu_L \gamma_\mu e) \quad (36)$$

where $\nu_L = \alpha \nu$ is the lefthanded neutrino of the FGMS theory. The ϱ -value from such an interaction is exactly $3/4$. To explain its deviations from the measured value $\varrho = 0,68 \pm 0,02$ (radiative corrections included, the value is due to Crowe) we follow the LEE-YANG proposal [12] that the interaction has a more complicated structure and does not occur at a single space-time point. The above Hamiltonian will therefore be modified to

$$\sqrt{8} G (\bar{e} \gamma_\mu \nu_L(x)) K(x - x') (\bar{\nu}_L \gamma_\mu \mu(x')) \quad (37)$$

where a form factor $K(x - x')$ has been introduced. Of course (37) will then be generalized to the expression for the interaction of J_μ with itself, which will look something like $J_\mu(x) K(x - x') J_\mu^+(x') + \text{h. c.}$ This form is not the most general one, since one could still have something looking as $J_\mu(f(K^2 + K_\mu K_\nu g(K^2)) J_\nu^+ + \text{h. c.}$; however we expect (37) to be at least a good approximation because of the smallness of the electron mass. We do not assume here that the form factor is due to the meson X. It may originate from any complicated cause, or even reflect a true elementary non-locality. If K is expanded in a series of derivatives of the δ -function [12]

$$K(x - x') = \delta^4(x - x') + \frac{\eta}{m_\mu^2} \frac{\partial}{\partial x_\mu} \frac{\partial}{\partial x_\mu} \delta^4(x - x') + \dots \quad (38)$$

we find for the electron spectrum from polarized muons

$$d^2 N \propto x^2 dx d\xi \left\{ 6 - 4x - \frac{2}{5} \Re(\eta) (9 - 16x + 5x^2) + \right. \\ \left. + [(1 + \frac{6}{5} \Re(\eta)) / (1 + \frac{3}{5} \Re(\eta))] \xi [2 - 4x - \frac{2}{5} \Re(\eta) (6 - 12x + 5x^2)] \right\}$$

where $\xi = \cos \theta$ is the angle between the muon spin and the electron momentum and x is the ratio of the momentum of the electron to its maximum energy. The energy spectrum turns out to be

$$dN \propto x^2 dx \left\{ 3 - 2x - \frac{1}{5} \Re(\eta) (9 - 16x + 5x^2) \right\} \quad (39)$$

A ϱ -value from this spectrum is obtained by a least square fit to a Michel's spectrum. The relation is [12]

$$\frac{3}{5} \Re(\eta) = -1,8 \left(\frac{3}{4} - \varrho_{\text{exp}} \right). \quad (40)$$

One sees from (40) that $\Re(\eta)$ turns out to be negative thus excluding an interpretation of the non-locality as due to a virtual boson. The lifetime can also be calculated and it comes out to be

$$\tau^{-1} = \tau_0^{-1} \left(1 + \frac{3}{5} \Re(\eta) \right) \quad (41)$$

and by substitution of (40) into (41)

$$\tau^{-1} = \tau_0^{-1} [1 - 1,8 \left(\frac{3}{4} - \varrho_{\text{exp}} \right)] \quad (42)$$

Here τ_0 is the lifetime for the local Hamiltonian ($\eta = 0$). To calculate the lifetime (42) one still needs the value of the coupling constant, contained in τ_0 . This value however can be taken from the β -decay of O^{14} if, per absurdum, we assume that the non-renormalization principle holds. The coupling constant so determined is again the same that one would obtain neglecting the non-local effects that were introduced in our universal weak interaction. This because the momentum transfer in β -decay is very low and the non-locality is not yet felt. Taking

the coupling constant from O^{14} and $\varrho = 0,68 \pm 0,02$, and computing errors, we find the predicted lifetime for μ -decay to be $\tau = (2,60 \pm 0,13) \cdot 10^{-6}$ sec. The previous surprising agreement with the experimental value has now changed into a disagreement of at least 10%.

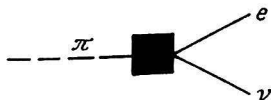
There is one point in which the above argument may be slightly incorrect. Namely the experimental ϱ -value used here has been obtained after subtracting radiative corrections as calculated for point interaction. It is impossible to estimate what the radiative corrections would become in the non-local theory, unless one makes a very definite model of the non-locality (for instance intermediate boson). In general, however, one does not expect great changes from these effects and therefore we would conclude that it is certainly very important at this point to have a very accurate determination of the ϱ -value.

4.3 Experimental tests of Universal Fermi Interaction

The equality of the effective coupling constant for β -interaction as derived from the lifetime of O^{14} to the true coupling constant derived from the μ -meson lifetime suggests first that there may be strict universality in the primary interactions, meaning that the coupling constants may be all equal, and, second, it suggests the absence of renormalization effects and a possible symmetry explaining such absence. It appears therefore to be important to have direct separate tests of the hypothesis of universality, possibly independent of the question of the non-renormalization of the constant by the strong interactions. The most direct test for universal interaction is provided by the ratio of $\pi \rightarrow e + \nu$ to $\pi \rightarrow \mu + \nu$ [13]. The ratio can be calculated uniquely independent of assumptions on the strong interactions for any forms of the weak coupling, provided it is assumed that the two leptons emerge from the same point. For S , V , T both decays are forbidden, for PS the ratio r

$$r = \frac{w(\pi \rightarrow e + \nu)}{w(\pi \rightarrow \mu + \nu)} \quad (43)$$

is 5.4, for A it is $13,6 \cdot 10^{-5}$. To illustrate the argument let us consider the case of A interaction. The decay is represented by the black box



The momenta of the two leptons are assumed to appear only in the combination $P_L^\mu = p_e^\mu + p_\nu^\mu$, corresponding to the assumption of locality. Moreover energy momentum conservation implies $p_\pi^\mu - P_L^\mu = 0$, so that we can choose p_π^μ as the only independent four-vector allowed by kinematics. The matrix element will therefore be of the form

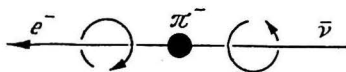
$$M \propto f(p_\pi^2) (\bar{e} \gamma_5 \gamma_\mu p_\pi^\mu \nu) \propto m_e f(p_\pi^2) (\bar{\mu} \gamma_5 \nu) \quad (44)$$

and the ratio r_A is easily calculated

$$r_A = \left(\frac{m_e}{m_\mu} \right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2} \right)^2 = 13,6 \cdot 10^{-5}. \quad (45)$$

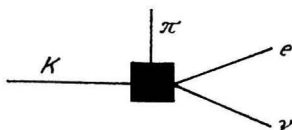
The first factor $(m_e/m_\mu)^2$ is the ratio of the two squared matrix elements, the second factor is the ratio of the two phase spaces. For PS interaction the first factor has to be replaced by 1, and the ratio r_{PS} is identical to the ratio between the two phase space volumes.

It may perhaps be instructive to understand a little more physically the appearance of the factor m_e in the matrix element for the case relevant here of A interaction (44). Consider π^- decay. What is emitted is an l^- and an $\bar{\nu}$. The l^- is coupled in the FGMS theory as a lefthanded particle, while the $\bar{\nu}$ is coupled as righthanded. Of course, it is equivalent here to talk of the FGMS theory or of a pure A interaction, as far as the ratio (43) is concerned, but it is really more instructive to discuss directly in the FGMS theory. Nothing destroys the handedness of the $\bar{\nu}$ which has zero mass, and only the mass term will destroy the handedness of the l^- , since it is known that the electromagnetic interaction preserves the handedness. So in the limit of $m_l = 0$ a lefthanded and a righthanded particle would have to be emitted from a zero spin system, which is impossible because of angular momentum conservation



Therefore the matrix element (44) has to go to zero for $m_l = 0$, as it in fact does according to (44).

What is important here is that radiative corrections (which a priori could not be regarded as negligible as compared to such a small decay amplitude in the case in which l^- is e^-) do not change this conclusion, as long as quantum electrodynamics holds in its conventional formulation. If, however, quantum electrodynamics contained for instance a $\sigma_{\mu\nu}$ coupling, which would destroy the handedness, then the radiative corrections would not have any more to be proportional to the lepton mass, and for the case of the electron, they could produce essential changes in the calculated ratio (45). The delicate nature of the dependence of the calculated value for r_A on the form of quantum electrodynamics was in fact pointed out by different authors [14]. The main interest in the question, at the time in which such discussion was made, came from the apparent strong disagreement between the experimental value for r and its predicted value r_A in the FGMS theory. Recent experiments have however reduced this discrepancy and are quite consistent with the theoretical value [15]. Apparently this case of the ratio of $\pi \rightarrow e + \nu$ to $\pi \rightarrow \mu + \nu$ (together with the corresponding case of $K \rightarrow e + \nu$ and $K \rightarrow \mu + \nu$) is the only case in which a test of the hypothesis of universality can be made directly without going into the various complications due to the presence of strong couplings. There is still another case, which although a little more complicated offers a similar possibility, namely the comparison of the $K \rightarrow e + \nu + \pi$ and $K \rightarrow \mu + \nu + \pi$ modes [16]. A decay mode $K \rightarrow l + \nu + \pi$ can also be represented by a black box



The structure of the black box is here a little more complicated because there is one more prong coming out from the box. One finds that in the K rest system one now needs two functions of the pion energy to describe the box. In fact 4-momentum conservation requires

$$p_K^\mu = p_\pi^\mu + P_L^\mu \quad (46)$$

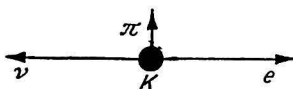
and, taking p_K^μ and P_L^μ as independent, the most general matrix element will be of the form

$$(\bar{l} \gamma_\mu P^\mu a \nu) \quad (47)$$

where P^μ is a linear unknown combination of p_K^μ and P_L^μ . Using the Dirac equation one can write (47) as

$$m_l X(\bar{l} a \nu) + i Y(\bar{l} \gamma_\mu p_K^\mu a \nu) \quad (48)$$

where X and Y are unknown functions of $(p_K^\mu p_\pi^\mu)$, a quantity which is essentially the pion energy in the K rest system. The hypothesis of universality implies that X and Y are the same functions of the pion energy independent of l being a μ or an e . To see in a particular case how one does effectively find limitations on the decay probabilities from the hypothesis of universality, consider the particular configurations where the pion is emitted at rest



The situation is exactly the same as for $\pi \rightarrow e + \nu$ and $\pi \rightarrow \mu + \nu$ and in fact for such configurations the ratio

$$r'_A = \frac{K \rightarrow e + \nu + \pi}{K \rightarrow \mu + \nu + \pi}$$

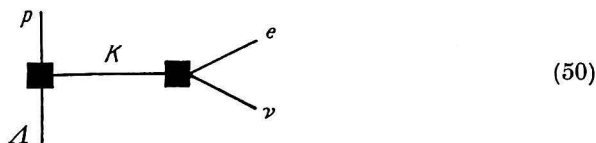
is uniquely determined

$$r'_A = \frac{m_e^2}{m_\mu^2} \left(\frac{\Delta_0^2 - m_e^2}{\Delta_0^2 - m_\mu^2} \right)^2 = 2,6 \cdot 10^{-6} \quad (49)$$

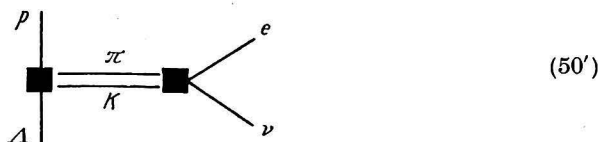
where $\Delta_0 = m_K - m_\pi$. The only change with respect to the pion case has been the appropriate modification of the phase space factor. In the general case one finds the following consequences from the hypothesis of universality: if the pion energy is measured the electron energy spectrum is uniquely determined and only two possible muon spectra are allowed. For each of such two spectra the complete muon polarization (longitudinal and along the pion direction) is uniquely predicted [16]. More stringent conditions follow if it is supposed that as a first approximation the energy dependence in X and Y can be neglected.

Finally, a few words about the β -decay of hyperons. If such decay modes were much more frequent than they really are, one could think of proposing tests of the universality hypothesis by comparing for instance $\Lambda^0 \rightarrow p + e^- + \bar{\nu}$ or $\Lambda^0 \rightarrow p + \mu^- + \bar{\nu}$. The leptonic decay modes of hyperons are however surprising-

ly absent or very rare. If one assumes that the coupling constants are not renormalized (or at least only little renormalized as it appears in β -decay), then one expects from a direct $(\bar{p}\Lambda^0)(\bar{e}\nu)$ interaction, $\Lambda \rightarrow p + e^- + \bar{\nu}$ at a rate of $5.3 \cdot 10^7 \text{ sec}^{-1}$, corresponding to 1.6% of the decays. Similarly from $(\bar{\Sigma} n)(\bar{e}\nu)$, $\Sigma^- \rightarrow n + e^- + \bar{\nu}$ would occur at a rate of $3.5 \cdot 10^8 \text{ sec}^{-1}$, corresponding to 5.6% of the cases. No evidence has been reported so far for such decay modes. We want to make a few remarks here in this connection. First, we know that leptonic decay modes of hyperons cannot be completely absent. In fact even if there were no direct interaction of the sort $(\bar{p}\Lambda^0)(\bar{e}\nu)$ one still would have graphs such as



and



and we know that the four vertices in the above graphs do exist, corresponding to the production experiments and to the $K \rightarrow e + \nu$, $K \rightarrow \pi + e + \nu$ decays ($K \rightarrow e + \nu$ must be inferred from $K \rightarrow \mu + \nu$). Graphs of this sort will occur in a dispersion theory treatment of the process. The intermediate states are taken in the Heisenberg representation. It can be shown that (50) gives a negligible pseudoscalar contribution (the K is assumed to be pseudoscalar), whereas the contribution of (50') is quite appreciable, but its evaluation is a little uncertain. The second remark, that we want to make is that if, as we also considered, in the strangeness-non conserving current also terms $(K\pi)$ are present, there would be also contributions arising from $\Lambda \rightarrow p + \bar{K} \rightarrow p + e^- + \bar{\nu} + \pi^0 \rightarrow p + e^- + \bar{\nu}$ and similar processes. However the axial contribution (no interference $A - V$ in the total rate if the electron mass can be neglected!) is left unchanged. The restriction only to the $(\bar{p}\Lambda)(\bar{e}\nu)$ term is much more stringent than the hypothesis of universality. Finally, numerical coefficients may very well be present in front of the term $(\bar{p}\Lambda)$ in the current, depending on the way in which universality has to be formulated. Such coefficients may have to do with the question whether one really has to put the Λ -field operator into the original formulation, which has to do with universality, or whether one has instead to use particular linear combination of the different hyperon fields. It is apparent that no answer to these questions will be possible unless the available experimental information is greatly increased. Of course one can also always invoke renormalization effects for the coupling constants to explain deviations of the data from the theoretical predictions, but it would seem to us that such justification would be a little dubious if one has to use it for all the cases in which no leptonic decay modes are observed.

4.4. The hypothesis of non-renormalization for the vector part of the strangeness-conserving current

We can now formulate explicitly the hypothesis of non-renormalization for the vector strangeness-conserving current, according to the Feynman — Gell-Mann proposal. We shall also reproduce a recent suggestion by Gell-Mann for a possible experimental test of the hypothesis [17]. Experimentally the β -decay Hamiltonian is found to be given by

$$\sqrt{8}G (\bar{p} \gamma_\mu \frac{1}{2} (1 + \alpha \gamma_5) n) (\bar{e} \gamma_\mu \nu) + \text{h. c.} \quad (51)$$

where G is the same (inside experimental errors of 2%) as the μ -decay coupling constants appearing in the interaction term

$$\sqrt{8}G (\bar{\nu} \gamma_\mu \nu) (\bar{e} \gamma_\mu \nu) + \text{h. c.} \quad (52)$$

The constant α is found to be $\infty + 1.20$ (18). The sign $+$ is determined from the experiment on the decay of polarized neutrons, which gives very small asymmetry, thus suggesting the $A - V$ form instead of the $A + V$ form. We have already discussed in 4.1 how the ambiguity between the $A - V$ and $A + V$ combinations could not be solved from the theory alone but had to be decided by experiment. As to the magnitude of α it can also be derived from the polarized neutron experiment, but with a rather large error. Better determinations are obtained from the $B - X$ -diagram, and from the free neutron lifetime. In this section we shall mainly discuss the problems arising from the result 1), namely from the equality of the renormalized vector constant in β -decay to the value of the bare coupling constant as obtained from μ -decay. The problems arising from 2), namely, why is the A renormalization so small and why is it such as to increase the value of the coupling constant, will be discussed in the next sections.

The Feynman — Gell-Mann suggestion to explain the absence of renormalization effects in the V part of the β -decay interactions consists in postulating that the strangeness-conserving vector current is divergenceless. The absence of renormalization effects on the charge e in quantum electrodynamics is a consequence of the divergenceless of the charge current, which for instance for a system containing nucleons and pions is given by

$$j_\mu = j_\mu^{(N)} + j_\mu^{(3)} = \frac{1}{2} (\bar{\psi} \gamma_\mu \psi) + (\bar{\psi} \gamma_\mu \tau_3 \psi) + i(\varphi^* T_3 \partial_\mu \varphi - (\partial_\mu \varphi)^* T_3 \varphi). \quad (53)$$

Here the first term is divergenceless because it is the local nucleon number current, while the last two terms are divergenceless because they add up to the local current of the 3-component of isotopic spin. The decomposition corresponds strictly to the decomposition of Q according to

$$Q = \frac{1}{2} N + I_3 \quad (54)$$

for a system with zero strangeness. The interaction with the electromagnetic field is given by

$$-e j_\mu A_\mu = -e j_\mu^{(N)} A_\mu - e j_\mu^{(3)} A_\mu \quad (55)$$

and the first term contributes to the Hamiltonian a term behaving like a scalar in isotopic spin space, while the second term contributes a term behaving like

the 3 component of a vector. Now one can by analogy postulate that in the vector part of the weak strengeess-conserving interaction what is coupled is the $j_\mu^{(\pm)} = j_\mu^{(1)} \pm i j_\mu^{(2)}$ component of the isotopic spin current

$$j_\mu^{(i)} = \frac{1}{2} i N \tau_i \gamma_\mu N + \varepsilon_{ikl} \pi_k \frac{\partial \pi_l}{\partial x_\mu} + \dots \quad (56)$$

More explicitly the V part of the β -decay interaction is given by

$$- G j_\mu^{(+)} l_\mu^{(+)} + \text{h. c.} \quad (57)$$

where

$$l_\mu^{(+)} = i \sqrt{2} (\bar{e} \gamma_\mu a \nu). \quad (58)$$

The lepton covariant $l_\mu^{(+)}$ plays here a role entirely similar to the field A_μ . The term (56) is directly comparable with the term $-e j_\mu^{(3)} A_\mu$ in (55). The matrix elements are strictly proportional. For any γ -transition due to the second term in (55) there is a corresponding transition due to (57). The Hamiltonian densities that contribute are, for the two transitions,

$$- G j_\mu^{(+)} l_\mu^{(+)} + \text{h. c.} \quad (59)$$

and

$$- j_\mu^{(3)} A_\mu. \quad (59')$$

Let us study in detail the analogy by making a multipole expansion for both the isotopic vector part of the electromagnetic interaction and for our vector weak coupling. The monopole terms are (limiting the discussion to e^- emission)

$$- G I_+ \sqrt{2} (\bar{e} \gamma_4 a \nu) \quad (60)$$

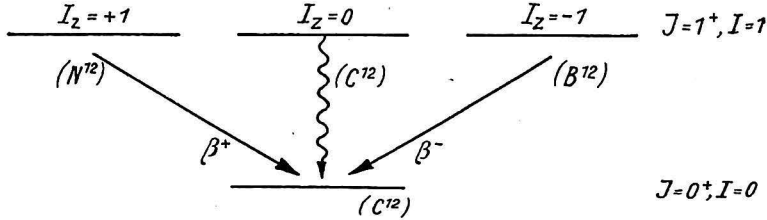
$$- e I_3 A_0 \quad (60')$$

where I is the total isotopic vector and the fields are taken at the nucleus. Equations (60) and (60') may be used if one can neglect the energy release to the lepton and to the photon fields. For instance one can compare

$$\pi^\pm \rightarrow \pi^0 + e^\pm + \nu \quad (61)$$

with the β -decay mode of O^{14} . They both are $O \rightarrow O$ -transitions, so only V is involved. One has to evaluate the matrix element of I_+ between π^- and π^0 , and between O^{14} and its daughter nucleus. This can be done by only isotopic spin considerations and one actually finds that $\pi^\pm \rightarrow \pi^0 + e^\pm + \nu$ and O^{14} have the same ft -value. The phase space available in (61) is however so small to make any experiment to determine its rate very difficult. In general in any $O^+ \rightarrow O^+$ β -transition, which is allowed and only involves the V interaction, the matrix element will be that of I_+ in the approximation in which (60) holds. This is however not a strong prediction of the theory. It would automatically be satisfied in any nuclear model in which the nucleus is described as a system of dressed

nucleons. For a possible verification of the theory Gell Mann examines a situation like the following. Consider a scheme like this:



The nuclei indicated in parentheses correspond to a particular example of the situation. In such a particular example the excited state of C^{12} is at 15.11 Mev. The effective Hamiltonian for the γ -decay, for instance from the $J_z = 0$ level, is

$$-\frac{\mu e}{2M}(\text{rot} \mathbf{A})_z. \quad (62)$$

In fact the transition occurs through magnetic dipole, since $\Delta I = 1$ and there is no parity change, so the matrix element will essentially be that of the pseudo-vector $\mathbf{H}$. In isotopic space $\Delta I = 1$ for the transition so that only the isotopic vector part of the electromagnetic interaction Hamiltonian contributes to the decay. Consider now the β^- transition. Here $\Delta J = 1$ and there is no parity change, so one has a Gamow-Teller matrix element, due to the axial, which is given by

$$-(\sqrt{2}G\alpha) M_{GT}(\bar{e} \sigma_z \mathbf{a} \nu) \quad (63)$$

always considering $J_z = 0$. This matrix element will of course give the main contribution. One may however be able to measure higher order corrections. There will be a matrix element contributing, which is completely analogous to the electromagnetic matrix element (62), in the sense of the correspondence given by (59) and (59'). This matrix element can be obtained directly from (62) by the suitable substitutions, and it is

$$-\frac{\mu G}{m_N}(\text{rot}(\bar{e} \beta \boldsymbol{\alpha} \mathbf{a} \nu))_z. \quad (64)$$

In (64) a factor $\sqrt{2}$ has been introduced to account for the fact that the matrix element of $j_\mu^{(+)}$ is $\sqrt{2}$ times larger than that of $j_\mu^{(3)}$, for the particular transitions. Now it turns out that the term corresponding to (64) in the absence of the non-renormalization hypothesis would be much smaller than (64). The reason is that the transition magnetic moment μ here contains also the contribution to the magnetic moments from the meson clouds, since the pions are now also coupled to the leptons. Thus, for the transition considered one has so far the matrix element (63) and the matrix element (64). However since one is introducing with (63) "forbidden" contributions for the vector, one should for consistency also look for "forbidden" contributions from the axial at the same order. It comes out that two more contributions have to be added. For the total matrix element Gell-Mann gives the expression

$$-(\sqrt{2}G\alpha) M_{GT}[\bar{e} \beta \{\sigma_z - i a(\mathbf{K}, \boldsymbol{\alpha})_z - b K_z \boldsymbol{\alpha} \gamma_5\} \mathbf{a} \nu] \quad (65)$$

where $a = \mu(\sqrt{2}m_N\alpha M_{GT})^{-1}$, and b is another unknown constant. For high energy β -transition the term proportional to b can be neglected and one finds that the spectrum will have the usual allowed shape apart from a correction factor

$$1 + \frac{8}{3} Ea \quad (66)$$

where E is the electron energy. To calculate a one needs μ and M_{GT} . The latter can approximately be calculated from the lifetime for the transition

$$(ft) \propto (\alpha M_{GT})^{-2}. \quad (67)$$

The transition magnetic moment μ can be determined from the magnitude of the γ -decay rate, as can be calculated from (62)

$$\Gamma_\gamma = \alpha \frac{\mu^2}{3m_N^2} \omega^3 \quad (68)$$

where ω is the γ ray energy. For the mentioned example of B^{12} , C^{12} and N^{12} Gell-Mann finds $|a| \approx 2.3m_N^{-1}$. One can estimate in this way that by comparing the spectra from B^{12} and N^{12} one finds an effect of the order of 20%. The correction factor will be $1 + (8/3)aE$ for $\beta^-(B^{12})$ emission and $1 - (8/3)aE$ for β^+ emission (N^{12}). The ratio of the two factors will be $1 + (16/3)aE$, and $1 + (16/3)aE$ at the maximum energy just amounts to about 20%. Many similar possible tests of the theory can be proposed for forbidden transitions, but we think that the test here reported will be sufficient to illustrate the kind of argument.

We shall next discuss the problem of the renormalization of the axial coupling constant.

4.5 *Impossibility of a non-renormalization hypothesis for the axial part of the strangeness conserving current. Quantitative estimate of the renormalization effects*

As we said before the Gamow-Teller coupling constant in β -decay is by now definitely different from the Fermi coupling constant and thus one should not insist further in trying to make the axial coupling constant also non-renormalizable. Also, it might be difficult, mathematically, to make the axial current divergenceless. — although this question is perhaps unrelated to the previous one. One has an argument due to GOLDBERGER and TREIMAN which shows that absence of divergences of the axial current would anyway be in conflict with established experimental evidence in β -decay [19]. Let us sketch here the argument. Write the axial β -decay matrix element in the form

$$(\bar{u}_e \gamma_\mu a u_\nu) \langle p | j_\mu^{(A)} | n \rangle. \quad (69)$$

Now $\langle p | j_\mu^{(A)} | n \rangle$ can be written as

$$\langle p | j_\mu^{(A)} | n \rangle = (u_p [a(K^2) \gamma_\mu \gamma_5 + i b(K^2) K_\mu \gamma_5] u_n) \quad (70)$$

where $K_\mu = K_\mu^{(p)} - K_\mu^{(n)}$ is the momentum transfer, $a(K^2)$ and $b(K^2)$ are nucleon form factors. The form (70) follows from Lorentz invariance, time reversal, and charge independence. In particular a possible term of the form $u_p \sigma_{\mu\nu} \gamma_5 K_\nu u_n$ does not appear because of the circumstance that n and p belong

to the same charge multiplet. Using the Dirac equation for the leptons and momentum conservation, one finds for the A -matrix element of β -decay

$$a(K^2)(\bar{u}_e \gamma_\mu \alpha u_n)(\bar{u}_p \gamma_\mu \gamma_5 u_n) + m_e b(K^2)(\bar{u}_e \alpha u_n)(\bar{u}_p \gamma_5 u_n). \quad (71)$$

The effective A coupling constant is $g_A = a(0)$, because of the smallness of the momentum transfer, and moreover there is an effective PS coupling constant $g_P = m_e b(0)$. If now we postulate

$$\frac{\partial j_\mu^{(A)}}{\partial x_\mu} = 0 \quad (72)$$

we find in momentum space

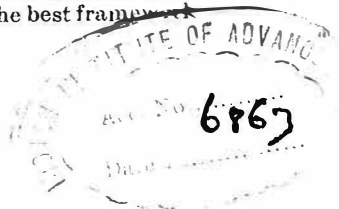
$$K_\mu \langle p | j_\mu^{(A)} | n \rangle = 0. \quad (73)$$

Substituting (70) in (73) we find $(\bar{u}_p [a(K^2)\gamma_\mu K_\mu \gamma_5 + i b(K^2)K^2 \gamma_5] u_n) = 0$ and using the Dirac equation for the nucleons, we finally find the relation

$$g_P = - \frac{2m_N m_e}{K^2} g_A \quad (74)$$

between the PS and the A coupling constants. The ratio g_P/g_A turns out then to be energy dependent, because of the K^2 dependence, and very large, ~ 1000 at least. This is inconsistent with experiment. No such difficulties arise for the V part. Here the matrix element analogous to the (70) contains only a γ_μ term and a term $\sigma_{\mu\nu} K_\nu$. The term with K_μ alone disappears by invariance arguments. It is then evident that the condition (73) can indeed be satisfied. There is perhaps a more direct argument to show that the axial strangeness conserving current is not divergenceless. In fact the decay modes $\pi \rightarrow \mu + \nu$, $\pi \rightarrow e + \nu$, could not occur through such a divergenceless current. Since they both exist, the A current cannot be divergenceless.

Finally let us speculate about the quantitative aspect of the renormalization problem for the axial coupling constant. If one accepts the Feynman-Gell-Mann suggestion that the vector part is non-renormalizable, one is then left with the conclusion that the renormalized axial coupling constant is α times larger than the bare coupling constant (with $\alpha \pm 1.20$). The first question concerns the sign of the renormalization effect. Why is the effective coupling constant larger than the bare coupling constant? We first should point out that the situation here has not very much to do with simple cases where essentially only one field is involved. One likes to imagine that an original bare charge polarizes the vacuum and is then surrounded by charges of the opposite sign, so that the total effective charge is diminished. Here, however, the situation is certainly quite different. More appropriately one can discuss the effect in the static source theory. One can easily convince himself that the renormalization factor which multiplies the bare coupling constant to give the renormalized coupling constant is something like $(1 + (1/3)x^2)(1 + 3x^2)$, where x^2 is the probability of finding in the cloud a pion of given charge. This factor is certainly less than unity. So one is led to the conclusion that, if the renormalization effects are such as to increase the value of the coupling constant, a more sophisticated description of the nucleon is necessary to evaluate them — in particular nucleon-anti-nucleon pairs will presumably play an important role. Perhaps the best framework



for a discussion of the problem is offered by the dispersion relation approach. In particular we refer to the GOLDBERGER TREIMAN work on the form factors for weak interactions [19]. We shall discuss this approach later in more detail, and make here only a few comments concerning the question of the renormalization of the axial coupling constant.

It is clear that if one wants to calculate with dispersion relations the renormalization of the axial coupling constant, one is not allowed to make any subtraction in the relations. Such a subtraction would essentially mean the introduction of the renormalized coupling constants itself in the expressions. One has then to assume that a dispersion relation

$$a(\xi) - c(\xi) = \frac{1}{\pi} \int d\xi' \frac{\text{Im } a(-\xi')}{\xi' + \xi - i\varepsilon} - \frac{1}{\pi} \int d\xi' \frac{\text{Im } c(-\xi')}{\xi' + \xi - i\varepsilon}$$

with "no subtraction" holds for the difference between the form factor $a(K^2)$ of equation (70) and the analogous form factor $c(K^2)$ for the vector coupling. There is certainly no definite theoretical reason why (75) should be valid. Unfortunately the question of knowing when and where one has to make subtractions in dispersion relations to have convergent expressions is still quite unsettled. GOLDBERGER has expressed the "feeling" that there is a close connection between subtraction and bare couplings in the original Lagrangian [20]. So for instance in the GOLDBERGER-TREIMAN work on the form factors in β -decay [19] the dispersion relation for $a(\xi)$ is used in a subtracted form corresponding to the existence of the bare coupling for axial β -decay $n \rightarrow p + e^- + \bar{\nu}$. Another subtraction is made for the vector form factor $c(\xi)$. However, in calculating $\pi \rightarrow e + \nu$, which goes through a loop and for which there is no primary interaction postulated, no subtraction has to be made. So in principle with two numbers, namely two subtraction constants g_A and g_V , and all the required informations about strong interaction amplitudes, one should have a complete description. This possibility does not appear in the usual perturbation theory approach to renormalization. Renormalization here is intended for a theory in which weak interactions are taken only up to first order [22]. Thus in a theory where pion and electromagnetic interactions are taken at any order, and one then adds a bare A , V coupling between n , p , e and ν , one finds that apart from the renormalized A and V coupling constants one still has two possible renormalization counterterms so that a total of four constants, apart from all the necessary knowledge about strong interaction amplitudes, is now required to have a complete description. If one adopts the non-renormalization hypothesis, one finds that apart from the renormalized A and V coupling constants (the latter coincident, of course, with the bare coupling constant) one still has to add a renormalization counter term for the axial. More specifically, in the first case (no hypothesis of non-renormalization for the vector part) if the original weak couplings are

$$(\bar{\psi}_p \gamma_\mu \psi_n) (\bar{\psi}_e \gamma_\mu \alpha \psi_\nu) + \text{h. c.} \quad (76)$$

and

$$(\bar{\psi}_p \gamma_5 \gamma_\mu \psi_n) (\bar{\psi}_e \gamma_5 \gamma_\mu \alpha \psi_\nu) + \text{h. c.} \quad (76')$$

one finds three more terms that may occur as counter terms, namely

$$\varphi_0 (\partial_\mu \varphi) (\bar{e} \gamma_\mu \alpha \nu) + \text{h. c.} \quad (77)$$

$$(\partial_\mu \varphi_0) \varphi (\bar{e} \gamma_\mu \alpha \nu) + \text{h. c.} \quad (77')$$

and

$$(\partial_\mu \varphi)(\bar{e} \gamma_5 \gamma_\mu \mathbf{a} \nu) + \text{h. c.} \quad (77'')$$

Here ∂_μ is $\partial/\partial x_\mu - ieA_\mu$ in the presence of electromagnetic interactions. From charge independence considerations it follows however that (77) and (77') have to occur in a definite combination, so that only one renormalization constant has to be introduced for the two counter-terms (77) and (77'). The counter terms (77) and (77') would correspond to processes

$$\pi \rightarrow \pi + e + \nu (+\gamma)$$

while the counter term (77'') corresponds to

$$\pi \rightarrow e + \nu (+\gamma).$$

If the non-renormalization hypothesis for the vector part is assured, then the coupling constants of (76), (77) and (77') are related in a definite way from the requirement that the total interaction is of the form $j_\mu^{(\nu)}(\bar{e} \gamma_\mu \mathbf{a} \nu) + \text{h. c.}$ $j_\mu^{(\nu)}$ being the current for the "plus" component of isotopic spin. In this case one needs three constants only, for a complete description. In both cases, however, $\pi \rightarrow e + \nu$ corresponds in lowest order to a primitive divergent diagram and a specific counter term has to be introduced for it, in contrast to the dispersion relation approach where one does not make any subtraction in the relation for the relevant form factor. From this discussion one sees that one is here confronted with two quite different possibilities. The dispersion relation techniques make it possible to give unambiguous answers for questions which could not be consistently formulated in perturbation theory, unless by recourse to arbitrary "cut-off" procedures. Their correct formulation in perturbation theory would imply the introduction of renormalization counter terms. The mechanism by which such difficulties are avoided in the dispersion relation technique is through the automatic appearance of relevant cutoffs in the expressions. The convergence of the dispersion relations with the minimum number of subtractions has however to be postulated. An example which illustrates very sharply these differences is provided by a recent formulation by CINI, FERRARI and GATTO [23] of the problem of the neutron-proton mass difference. In perturbation theory the physical masses would already be introduced through suitable counter terms. With the dispersion relation technique the problem can be formulated unambiguously, provided the convergence of the dispersion relation is assumed. In particular in the lowest, and certainly inadequate, approximation in which only the intermediate state of a physical nucleon is considered, the formula that one obtains is similar to that obtained by FEYNMAN and SPEIS-MANN with a cut-off perturbation theory [24]. In the dispersion relation approach the natural cut-off is provided by the Stanford form factors for the nucleon. Let us thus point out that there are at least two possible attitudes as far as the question of subtractions in dispersion relations is concerned. There is an optimistic attitude: one has to make a subtraction every time there is a corresponding bare coupling or a renormalization counter term in the Lagrangian. The attitude expressed by (23) is more optimistic than what we called the optimistic attitude. There is however a chance that such a super-optimistic attitude may

be correct. At zero momentum transfer (75) becomes $a(0) - c(0) =$ (renormalized axial coupling constant) — (vector coupling constant) =

$$= \frac{1}{\pi} \int d\xi' \frac{\text{Im } a(-\xi')}{\xi' - i\varepsilon} - \frac{1}{\pi} \int d\xi' \frac{\text{Im } c(-\xi')}{\xi' - i\varepsilon}. \quad (78)$$

Equation (75) has been suggested to us by Symanzik, who thinks that it may be conjectured to be correct by an argument similar to one developed by KÄLLEN [25] in quantum electrodynamics. Unfortunately the evaluation of the integrals in (78) is certainly a big task. The first contribution comes from the 2π state, but it does not seem to be able to give the correct sign and magnitude of the renormalization effect.

4.6 About possible extensions to other currents

Let us summarize what we have seen about the strangeness-conserving current S_0^μ . The vector part may very well be non-renormalizable. There is a difficulty with the ρ -value, but here one has to wait for better experiments. The axial part has almost no chance of being non-renormalizable. Now what about the divergences of the strangeness non-conserving currents? If we say that the K is pseudoscalar then the A part of the strangeness non-conserving current S_{-1}^μ is certainly not free of divergences because of the existence of $K \rightarrow \mu + \nu$, which occurs through the A part and implies a divergence. For the V part one can hope to have experimental information from the $K \rightarrow e + \nu + \pi$ spectra. Such decay modes in fact occur entirely through V and the condition on the current to be divergenceless imposes restrictions on the form factors for such decay. This has been discussed by GOLDBERGER and TREIMAN [19] and also by WEINBERG et al [26]. The last authors however do not use explicitly the hypothesis of universality and thus they obtain less stringent requirements. The matrix element for $K \rightarrow e + \nu + \pi$ equation (4) is written in the form

$$(\bar{l} \gamma_\mu a \nu) \langle \pi | V_\mu | K \rangle \quad (79)$$

where $\langle \pi | V_\mu | K \rangle = c(K^2) P_\mu + d(K^2) K_\mu$, $P_\mu = p_\mu^{(K)} + p_\mu^{(\pi)}$, and $K_\mu = p_\mu^{(K)} - p_\mu^{(\pi)}$. The hypothesis of non-renormalization would imply $K_\mu \langle \pi | V_\mu | K \rangle = 0$, thus giving a relation between d and c , namely $K^2 d = (m_K^2 - m_\pi^2) c$. Therefore in (79) only one form factor is unknown. It can be shown that the decay rate for $K \rightarrow e + \nu + \pi$ has then necessarily to be larger than that of $K \rightarrow \mu + \nu + \pi$. This conclusion is not inconsistent with experiment. More stringent tests along the lines of reference (16) are still possible however but one will have to wait for better data. We should like, however, to point out that it may in fact be difficult to formulate theoretically a conservation law for the vector part of the strangeness non-conserving current. For the strangeness conserving current we employed conservation of isotopic spin. However the strangeness non conserving current will behave in general like a superposition of $T = 1/2$ and $T = 3/2$, and no similar possibility can be thought of in this case. A final remark concerns the possibility of including in the scheme for the strangeness conserving terms beside the T_+ and T_- currents of isotopic spin, also the neutral T_3 current, which could be coupled to neutral lepton class $\mathcal{L}^{(0)}$ according to the scheme proposed in 2.3. The only motivation for such a scheme so far would be its greater symmetry. Furthermore one would perhaps have

some new possibility open for the relevant question of defining in a local way rotations in isotopic spin space — a question that was explored some time ago by YANG and MILLS [27].

4.7 Further discussion on the role of strong interactions in weak processes

Comparison between absorption and decay

We shall here discuss a little further the problem of the role of strong interactions, and in particular the question of the comparison of μ -absorption, $\mu^- + p \rightarrow n + \nu$, with β -decay $n \rightarrow p + e^- + \bar{\nu}$.

The reason why a problem like this is perhaps easier to reduce to a quantitative analysis than for instance the problem of the leptonic decay of hyperons, can be readily understood. In comparing μ -absorption to β -decay, one is only studying structure functions of the kind

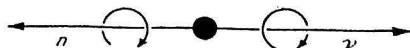
$$\langle n | J_\mu | p \rangle$$

where J_μ can be either $\bar{\psi}_n i\gamma_\mu \gamma_5 \psi_p$ or $\bar{\psi}_n \gamma_\mu \psi_p$ at different momentum transfers. Instead, to compare for instance $\Lambda^0 \rightarrow p + e^- + \bar{\nu}$ to $n \rightarrow p + e^- + \bar{\nu}$ one would have to compare two different structures, namely, $\langle \Lambda^0 | J'_\mu | p \rangle$ to $\langle n | J_\mu | p \rangle$. One also notices how such a comparison cannot be done if one has to make subtractions in the dispersion relations. In fact the subtracted effective coupling constants for the two structures would be unrelated, so that no definite comparison, for instance between the decay rates, would be possible. When comparing instead μ -absorption and β -decay, one is studying the same structure functions for different momentum transfers, and the subtraction constants can be identified with the effective coupling constants in β -decay. The process of μ absorption can then be compared explicitly with β -decay, provided one is able to evaluate the absorptive parts of the structure functions with sufficient accuracy. A similar situation occurs in the comparison of $K \rightarrow \mu + \pi + \nu$ to $K \rightarrow e + \pi + \nu$, studied by CABIBBO and GATTO [28]. Here again the same structure, namely

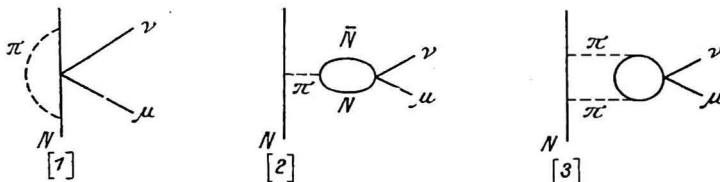
$$\langle K | J_\mu^\nu | \pi \rangle$$

is studied at different momentum transfers. Nobody knows whether a bare coupling has to be postulated for $K \rightarrow \pi + l + \nu$ (see our remarks in 2.1), as it was postulated for $\pi \rightarrow \pi + l + \nu$.

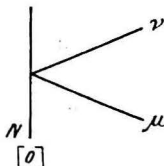
In order to get a first qualitative insight into the problem of μ -absorption let us first make a few simple-minded remarks. It is easy to see that if the interaction is $A - V$ and if one completely neglects any strong interaction effect, the absorption only occurs from the singlet state of the $\mu^- - p$ system. In fact the coefficient of the non spin-flip term in such a case is opposite to that for the spin-flip term, in a non-relativistic approximation, so that no absorption can occur from the triplet state. Now for singlet absorption from the lowest s-state there is apparently no asymmetry in the distribution of neutrons with respect to the μ -spin, and the neutron longitudinal polarization will be maximum (the neutron will be left-handed)



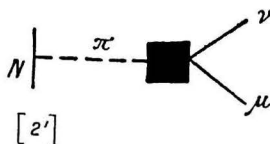
Of course one expects modifications from this oversimplified picture, when pion effects and relativistic effects are taken into account. Virtual pion effects in μ -capture have been considered by WOLFENSTEIN [29] and by GOLDBERGER and TREIMAN [21]. WOLFENSTEIN uses ordinary perturbation theory, while GOLDBERGER and TREIMAN apply the dispersion relation techniques. The results are essentially the same. WOLFENSTEIN considers the diagrams



which express pion radiative corrections to the original diagram



The diagram [1] contributes a renormalization of the coupling constants of [0]. Diagram [2] can be studied in more detail. Instead of [2] it is convenient to consider all the diagrams of the kind

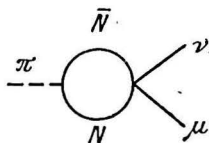


where the black box represents not only the pair state of diagram [2] but also includes all possible contributions with hyperons, et cetera. If the pion in [2'] were on the energy shell the calculation of [2'] would only require the renormalized $\pi - N$ coupling constant and the $\pi \rightarrow \mu + \nu$ decay matrix. Since the pion is off the energy shell for about $2m_\pi$, such an approximate evaluation may introduce errors $\sim m_\pi/M$. The contribution of [2'] will consist in the addition of a PS coupling to [0] and the magnitude of the coupling constant can be calculated in the above approximation as

$$|g_P| = 9 \cdot 10^{-5} \frac{m_\pi^2 + q_0^2}{M^2(m_\pi^2 + q^2)} \quad (80)$$

where q is the four-momentum transfer and $q_0^2 \simeq (1/2)m_\pi^2$. The sign here is left undetermined because the pion lifetime does not determine the phase of the transition matrix. Comparing with the known value of the vector coupling constant one finds $g_P \approx \pm 9$ (vector coupling constant). Moreover g_P can be

related to the axial coupling constant if one introduces into [2'] the contribution of the simplest diagram



for π decay, instead of the black box. In this way one finds that g_P has the same sign of the axial coupling constant. The GOLDBERGER-TREIMAN treatment leads to the same conclusion. Finally let us consider diagram [3]. This diagram contributes a vector and a scalar term as can be seen from the following argument. Take $P^\mu = p_1^\mu + p_2^\mu$ and $Q^\mu = p_1^\mu - p_2^\mu$ as independent momenta to describe the two pions with momenta p_1^μ and p_2^μ . The intermediate two pion state must have isotopic spin one, as is evident from charge independence. Therefore it must be odd in Q^μ in order to be totally even. One can construct three matrix elements for the $\pi + \pi \rightarrow \mu + \nu$ process, namely

$$(\bar{\mu} Q \cdot P a \nu), \quad (\bar{\mu} \gamma_\mu Q^\mu a \nu), \quad (\bar{\mu} \gamma_\mu Q^\mu \gamma_{\mu'} P^{\mu'} a \nu). \quad (81)$$

By use of the Dirac equation one finds however that the last term in (81) reduces to the second (P^μ is also $p_{(\mu)}^\mu + p_{(\nu)}^\mu$). So one has a scalar contribution ($\bar{\mu} Q \cdot P a \nu$) and a vector contribution ($\bar{\mu} \gamma_\mu Q^\mu a \nu$). The scalar contribution is however expected to be small since it is proportional to P_μ which is small for this process. The vector contribution can be considered as a further renormalization of the original vector part of [0]. Keeping only [0] with the renormalized constants and [2'] WOLFENSTEIN calculates, assuming that the two renormalized constants satisfy $g_V = -g_A$, that the asymmetry parameter from polarized muons changes from 0 to -0.22 by the inclusion of [0'], while the longitudinal polarization of the neutron is reduced from 1 to 0.93, and the hyperfine splitting effect $(1/4)(\lambda_+ - \bar{\lambda})/\bar{\lambda}$, of which we shall talk later in more detail, changes from -1 to -0.93 . Such values are furthermore essentially not changed by the inclusion of relativistic effects.

A confirmation of the validity of the above picture comes from the work of GOLDBERGER and TREIMAN on form factors in β decay and μ -capture [21]. Their calculations are based on dispersion relation techniques of the same kind as used for the study of the electromagnetic structure of the nucleon. The S matrix is expressed in terms of nucleon form factors, which are assumed to satisfy dispersion relations. They can thus be expressed, roughly speaking, as integrals over their imaginary parts. The imaginary parts are then estimated by introducing a set of intermediate HEISENBERG states and keeping only those states that are supposed to contribute mostly (this in theory; in practice one keeps only those states for which one is able to say something reliable). The weak interaction Lagrangian is supposed to be of the form

$$(\bar{\psi}_p \gamma_\lambda (c_V + c_A \gamma_5) \psi_n) (\bar{\psi}_e \gamma_\lambda a \psi_\nu). \quad (82)$$

It is irrelevant to make assumptions about c_V and c_A since they are bare coupling constants and the subtracted dispersion relations will only contain the renormalized coupling constants. The only assumption is that c_V, c_A are the same, independent of l being a μ or an e . No direct $\pi \rightarrow \pi + e + \nu$ interaction is assumed

to exist in the Lagrangian. The S matrix element for e^- (or μ^-) + $p \rightarrow n + \nu$ (K capture or μ capture) can be written as

$$S = i(2\pi)^4 \delta^4(n + p_\nu - p - p_l) [(\bar{a}\bar{\nu} i \gamma_e \gamma_5 l) \langle n | A_e(0) | p \rangle + (\bar{a}\bar{\nu} \gamma_e l) \langle n | V_e(0) | p \rangle] \quad (83)$$

where, n , p_ν , p and p_l are the four momenta of n , ν , p and l respectively. $A_e(0) = c_A (\bar{\psi}_n i \gamma_e \gamma_5 \psi_p)$ and $V_e(0) = c_V (\bar{\psi}_n \gamma_e \psi_p)$. One then uses invariance arguments to express the nucleon matrix element in terms of suitable form factors

$$\langle n | A_e(0) | p \rangle = \left(\frac{m_N^2}{p_0 n_0} \right)^{1/2} (\bar{n} [a(K^2) i \gamma_e - b(K^2) K_e] \gamma_5 p) \quad (84)$$

$$\langle n | V_e(0) | p \rangle = \left(\frac{m_N^2}{p_0 n_0} \right)^{1/2} (\bar{n} [c(K^2) \gamma_e - i d(K^2) \sigma_{e\tau} K_\tau] p) \quad (85)$$

where a and b are functions of the invariant momentum transfer $K^2 = (n - p)^2$. It is important here to note that in deriving (84) and (85) one has to make explicit use of charge symmetry, otherwise additional terms would appear. What limits the structure functions to the simple form (82) and (85) is the fact that (82) behaves as a first class quantity under the operation G , and the strong interactions are invariant under G . This has already been discussed in section 3.2. Substituting (84) and (85) into (83) one finds

$$S = i(2\pi)^4 \delta(n + p_\nu - p - p_l) \left(\frac{m_N^2}{p_0 n_0} \right)^{1/2} [a(K^2) (\bar{a}\bar{\nu} i \gamma_e \gamma_5 l) (\bar{n} i \gamma_e \gamma_5 p) + m_l b(K^2) (\bar{a}\bar{\nu} \gamma_5 l) (\bar{n} \gamma_5 p) + c(K^2) (\bar{a}\bar{\nu} \gamma_e l) (\bar{n} \gamma_e p) + i d(K^2) (\bar{a}\bar{\nu} \gamma_e (p_e - p_\nu)_\tau l) (\bar{n} \sigma_{e\tau} p)] \quad (86)$$

Looking at the structure of this S matrix element one finds that one has again an A (with coupling constant $a(K^2)$), a V (with coupling constant $c(K^2)$), and moreover a P_S (with coupling constant $m_l b(K^2) = g_A$) and a derivative term (with coupling constant $d(K^2)$). Calling $(n - p)^2 = \xi$ one then writes down dispersion relations for the form factors a , b , c , d :

$$a(\xi) = g_A - \frac{\xi}{\pi} \int_0^\infty d\xi' \frac{\text{Im } a(-\xi')}{\xi'(\xi' + \xi - i\varepsilon)} \quad (87)$$

$$b(\xi) = \frac{1}{\pi} \int_0^\infty d\xi' \frac{\text{Im } b(-\xi')}{\xi' + \xi - i\varepsilon} \quad (87')$$

$$c(\xi) = g_V - \frac{\xi}{\pi} \int_0^\infty d\xi' \frac{\text{Im } c(-\xi')}{\xi'(\xi' + \xi - i\varepsilon)} \quad (87'')$$

$$d(\xi) = \frac{1}{\pi} \int_0^\infty d\xi' \frac{\text{Im } d(-\xi')}{\xi' + \xi - i\varepsilon}. \quad (87''')$$

One has made subtractions for a and c , and called $g_A = a(0)$ and $g_V = c(0)$. These are the form factors at zero momentum transfer: they are the renormalized coupling constants. From $n - p = p_1 - p_\nu$ we find $\xi^2 = -l^2 - 2[(\vec{p}_1 \vec{p}_\nu) - (p_1^0 p_\nu^0)] = -l^2 + 2lE_\nu$ and from $E_\nu \approx l$; $\xi^2 \approx l^2$. Therefore for K capture the momentum transfer in $\xi^2 \approx m_e^2$ and one can approximately identify g_A and g_V with the coupling constants for A and V in β -decay, namely with $a(m_e^2)$ and $c(m_e^2)$ respectively. For μ -capture one has to deal with $a(m_\mu^2)$, $b(m_\mu^2)$ et cetera, which may be appreciably different from their values at zero momentum transfer. One has next to evaluate the imaginary parts of the form factors in the equations (87). By use of the reduction formula one can show that the absorptive parts of (84) and (85) are given respectively by:

$$A_e = \pi \left(\frac{m}{n_0} \right)^{1/2} \sum_s \bar{n} \langle 0 | A_e | s \rangle \langle s | F(0) | p \rangle G(p_s + n - p) \quad (88)$$

$$V_e = \pi \left(\frac{m}{n_0} \right)^{1/2} \sum_s \bar{n} \langle 0 | V_e | s \rangle \langle s | F(0) | p \rangle \delta(p_s + n - p) \quad (89)$$

where

$$F(x) = \left[\gamma_\mu \frac{\partial}{\partial x_\mu} + m \right] \psi(x) \quad (90)$$

and one has inserted a complete set of physical states δ . Graphically one can represent expressions (88) and (89) by diagrams of the kinds



where the dotted line represents the intermediate states. The states that can occur in (88) are $1\pi, 3\pi, 5\pi, N\bar{N}$ et cetera. So one has the decomposition

$$\begin{aligned} \text{Diagram} &= \text{Diagram} + \text{Diagram} + \text{Diagram} + \dots \\ &= \text{Diagram} + \text{Diagram} + \text{Diagram} + \dots \end{aligned} \quad (91)$$

The states that can occur in (89) are $2\pi, 4\pi, NN$ et cetera, graphically

$$\begin{aligned} \text{Diagram} &= \text{Diagram} + \text{Diagram} + \text{Diagram} + \dots \\ &= \text{Diagram} + \text{Diagram} + \text{Diagram} + \dots \end{aligned} \quad (92)$$

In (91) and (92) the first terms correspond to the subtracted renormalized coupling constants. The second term in (91) gives only a PS contribution which can be calculated by inserting the $\pi \rightarrow e + \nu$ decay matrix at the second vertex.

According to GOLDBERGER and TREIMAN this term should be in μ -capture the most important correction due to strong interactions, apart from the renormalization of the coupling constants. In β -decay the only essential correction is the renormalization of the coupling constants. The actual calculation of the various terms in (91) and (92) is performed by inserting suitable analytical continuations of the various matrix elements at the different vertices which in the physical regions would correspond to observable physical processes. Of course this procedure can only be done for the simplest terms of the expansions (91) and (92) and even in an approximate way. It is however a conclusion of both the perturbation calculations about which we have already reported, and of this dispersion relation approach, that the effective Hamiltonian for μ -absorption should be well approximated by the effective $A V$ β -decay Hamiltonian, with the addition of a pseudoscalar term with a coupling constant about eight times larger than the axial coupling constant. This conclusion is valid if one starts from a bare $A V$ Hamiltonian. If, according to the non-renormalization hypothesis, also a direct pion lepton interaction exists, one may use the methods discussed in 2.4 for a phenomenological derivation for the terms contributed from the vector part.

It is unfortunate that experiments of μ -absorption by protons are difficult because of the very low rate as compared to the μ -decay rate, and also because of formations of molecular ions and effects due to small deuterium impurities. BERNSTEIN, LEE, YANG and PRIMAKOFF have recently pointed out an interesting possibility to get information on the effective Hamiltonian for μ -absorption [31]. They have pointed out that there is a difference in the absorption rate of captured muons from the two hyperfine states, and this effect can be measured and give informations on the absorption interaction. When a μ^- is captured in the K-orbit around a nucleus of spin I , the spin of the total system can be $F = I + 1/2$ or $F = I - 1/2$. Absorption from the states with $F = I + 1/2$ occurs at a rate different than for absorption from the states with $F = I - 1/2$. In fact when the μ^- is absorbed by a proton the ratio is different according as to whether the system is in a triplet or singlet state. However, the spin orientation of the proton is correlated to the direction of the spin I of the nucleus, so the rate will be different for the different spin correlations of the μ -spin with I . The above authors also give an intuitive explanation for the difference between the absorption rate λ_+ from the state with $F = I + 1/2$ and the rate λ_- from the states with $F = I - 1/2$. If the emitted neutrino has a long wavelength it will carry out only $1/2$ unity of angular momentum. Therefore the final nucleons have a total angular momentum $I' = F - 1/2$ or $F + 1/2$ and, if $F = I + 1/2$ then $I' = I$ or $I + 1$; if $F = I - 1/2$, $I' = I - 1$ or I . So there are different final states that contribute in the two cases. The quantity of interest is $(\lambda_+ - \lambda_-)/\bar{\lambda}_{\text{capt}}$ where $\bar{\lambda}_{\text{capt}}$ is the average capture rate. Its estimated value is $\leq 20\%$ for nuclei around $Z \approx 10$, where absorption and decay are about equally probable. How to observe such an effect? Since the total decay rates of the μ^-p system are different according as to $F = I + 1/2$ or $F = I - 1/2$, one has two lifetimes and one may be able to observe a two-exponential decay curve into the mode $\mu^- \rightarrow e^- + \nu + \bar{\nu}$. Such a possibility is discussed in detail by BERNSTEIN, LEE, YANG and PRIMAKOFF.

As already discussed the asymmetry in the neutron distribution with respect to the muon spin direction is exactly zero for an $A - V$ interaction, neglecting pion effects. Furthermore the muon is appreciably depolarized in some cases

when cascading through the various atomic orbits to reach the lowest S-state. It is noteworthy that, for the same $A - V$ interaction, always neglecting pion effects, the neutron longitudinal polarization in $\mu^- + p \rightarrow n + \nu$ is maximum. For the practical case of absorption from a complex nucleus the resulting polarization has been recently calculated using a Fermi model to describe the primary absorption process in the nucleus and an optical model to estimate the rescattering corrections [32]. For neutrons in the upper end of the spectrum the above estimates show that a polarization of at least 50% is to be expected. Measurements of such polarizations will however be rather difficult. Many other important suggestions have been made to obtain information on the μ -absorption process, this rather inaccessible corner of the universal Fermi interaction. Unfortunately the interpretation of the experiments necessarily depends on particular nuclear models.

5. Conclusions

It appears from our discussions that, although an enormous advance has been made in the last two years in our understanding of weak interactions, there still remain many points as yet obscure and many more experiments are needed at present. Even if the formulation of weak interactions in terms of a current interacting with itself will turn out to be correct, the explicit definition of the charged current J_μ is still to be formulated. If the non-renormalization principle for the vector strangeness-conserving part holds, the strangeness-conserving current is uniquely determined from the requirement that its vector part is the plus component of the isotopic spin current and that its axial part is such that fermions always appear in the projection $(1/2)(1 + \gamma_5)$. No similar rules are, unfortunately, available for the strangeness non-conserving current, but we hope that some general symmetry will sooner or later manifest itself to provide us with an unambiguous theoretical definition of the strangeness non-conserving current. For the moment one should also consider the possibility of the contribution of $(K\pi)$ terms to such a current. A considerable simplification is due to the $\Delta S = \pm 1$ rule. Its most evident empirical basis is the apparent absence of $\Xi^- \rightarrow n + \pi^-$, but we lack so far a precise experimental upper limit for this reaction. It is theoretically important to know whether a neutral strangeness-conserving current is also present. There is a quite definite theoretical possibility that such a current exists, if the non-renormalization hypothesis for the vector part is valid. In such a case the suggestion is that such a neutral current may be the 3 component of the isotopic spin current and may also be coupled to leptons, with the exclusion of the (μe) pair. Such a scheme would perhaps offer some new possibilities for the question of defining in a local way isotopic spin rotations. The important question of knowing whether a charged boson can be taken as responsible for the charge transfer between the weak interacting pairs rests now experimentally on a better determination of the upper limit of $\mu \rightarrow e + \gamma$, though the present upper limit already leaves rather little hope. The most important experimental question is here connected with a very precise determination of the electron spectrum in μ -decay. The question of the old $\Delta I = 1/2$ rule has not made great advances so far. If such a rule (for the interactions and not for the strangeness non-conserving current) has to be encompassed in the scheme then one needs a more elaborate formulation. However, the coupling of a neutral strangeness non-conserving current to leptons may produce incon-

sistencies with experiment. The necessity of independent and accurate experimental tests of the non-renormalization hypothesis for the vector strangeness conserving current is hardly to be emphasized. Also the comparison between the μ and the O^{14} lifetime has to be performed with greater theoretical accuracy. It is important here to note the incompatibility at present between such a hypothesis and the present experimental ρ value of μ -decay. The question of the apparent absence of β -decay of hyperons is certainly relevant to the problem of defining the strangeness non-conserving current. The possibilities of a large renormalization of the constants and of contributions from a possible direct $(K\pi)$ -lepton interaction and K -lepton interaction may of course be invoked. The sign and the smallness of the renormalization of axial β -decay couplings opens many theoretical problems. They are unfortunately connected with the dubious validity of forms of non-subtracted dispersion relations, and also, again unfortunately, connected to our very poor knowledge of the deepest parts of the nucleon structure. The great merit of the dispersion relation approach is that of offering a definite possibility (not yet proved to be correct) of surpassing the limitations of the renormalization theory which required with the specification of the counter terms more theoretical input to obtain a complete description of the weak processes. A less ambitious program to understand in part the role of strong interactions in the weak processes consists in comparing reactions in which the same structure appears, such as $\mu^- + p \rightarrow n + \nu$ to $e^- + p \rightarrow n + \nu$ or $K \rightarrow \pi + e + \nu$ and $K \rightarrow \pi + \mu + \nu$. For such cases a subtraction in the dispersion relations (either to remove theoretical criticism or only to obtain a better convergence) is still tolerable. However the μ^- absorption process is still a dark corner of the universal Fermi interactions and much more experimental light is certainly necessary to contemplate its details.

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