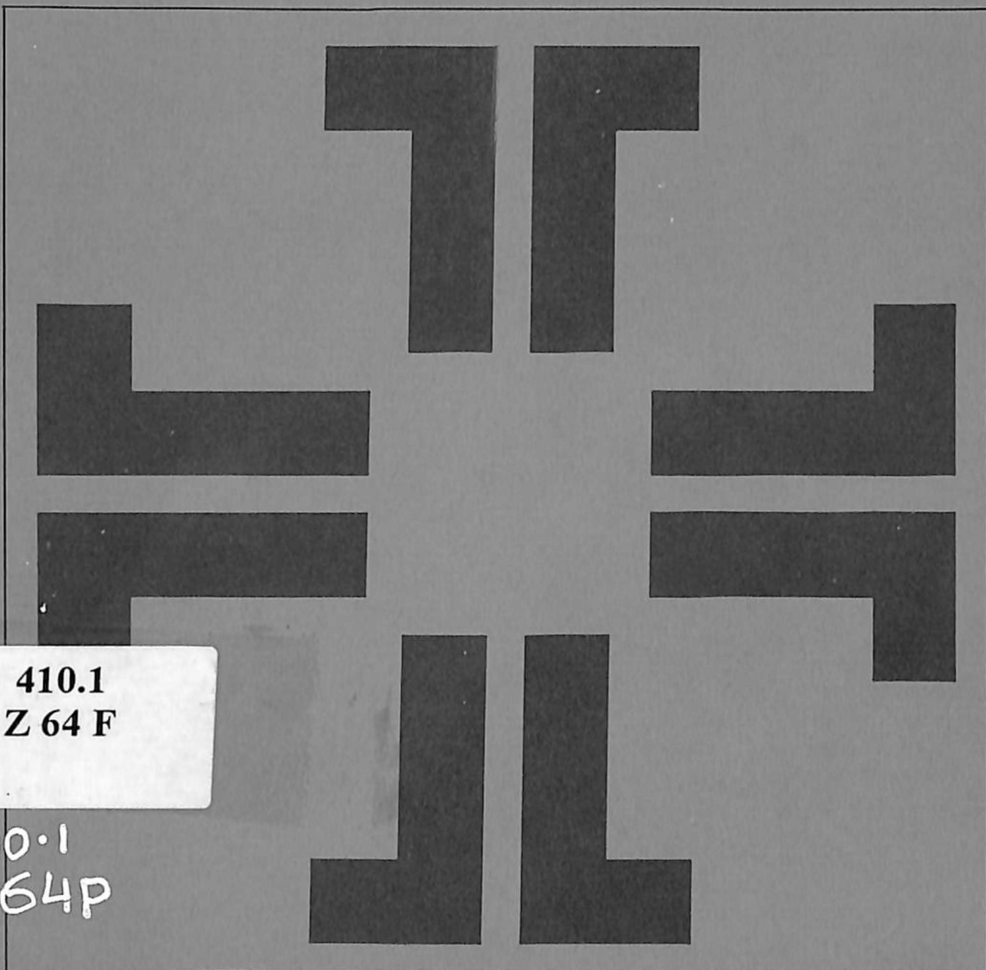
Ernesto Zierer

Formal Logic and Linguistics



JANUA LINGUARUM

STUDIA MEMORIAE
NICOLAI VAN WIJK DEDICATA

edenda curat

C. H. VAN SCHOONEVELD

Indiana University

Series Minor, 102

FORMAL LOGIC AND LINGUISTICS

by

ERNESTO ZIERER
NATIONAL UNIVERSITY OF TRUJILLO, PERU

1972
MOUTON
THE HAGUE · PARIS

Original title:
La lógica formal en la lingüística
Translated from Spanish by Alun L. W. Rees

LIBRARY OF CONGRESS CATALOG CARD NUMBER: 78-134547

Printed in The Netherlands by Mouton & Co., Printers, The Hague.

FOREWORD

This is the third publication in our series *Mathematical Methods in Linguistics* based on an interdisciplinary seminar held at the National University of Trujillo and organized by the Department of Foreign Languages and Linguistics.

In this number it is hoped to lead the reader towards an understanding of the important part that formal logic may play in the study of some aspects of linguistics. With the help of numerous examples, not all of which bear fruitful results, the reader should eventually be able to judge for himself the extent to which modern logic can help linguistics form its statements with greater accuracy.

Pertinent though lengthy discussions arising during the course of the meetings have not been included in the text to avoid excessive deviation from the main theme. However, the present paper will have fully achieved its aim if it succeeds in providing the reader with an appreciation of the definite advantages offered by the application of mathematical methods to certain areas of linguistics.

An important contribution to the success of the most recent seminar has been made by Prof. Alejandro Ortiz of the Mathematics Department, whose valuable co-operation is gratefully acknowledged.

Trujillo, Peru, December 1968
National University of Trujillo

CONTENTS

I. INTRODUCTION

During the course of the seminar, three areas were considered in which formal or symbolic logic may be applied to linguistics:

(1) To FORMULATE MORE STRICTLY AND CONCISELY THE RULES defining particular phenomena at various levels of linguistic structure:

Example: In the phrase, *all my first attempts*, the words *all*, *my*, and *first* belong to three distinct classes — here designated *A*, *B*, and *C* — and the order in which they occur before the nucleus of this nominal syntagm is fixed, so that sequences such as the following are unacceptable

**my all first attempts* or **my first all attempts*, etc.

In terms of formal logic, the rule corresponding to this can be written as:

$$\forall a \in A \wedge \forall b \in B \wedge \forall c \in C \rightarrow a \prec b \prec c$$

where $\forall$ is the UNIVERSAL QUANTIFIER and stands for *all*, $\in$ signifies *belonging to*, $\wedge$ is the connective of conjunction *and*, and $\prec$ indicates that the element occurring before it precedes the one coming after. The arrow denotes an implication.

Such an application, which merely borrows the symbols of formal logic to express a rule, achieves nothing new from the standpoint of linguistics, for it is based on a prior linguistic analysis.

(2) The DISCOVERY OF PARTICULAR LINGUISTIC PHENOMENA THAT ARE REGULATED BY CERTAIN LAWS OF FORMAL LOGIC:

Example: If two nouns are combined in Spanish, the ending of the predicative adjective referring to both of them, agrees with the logical conjunction.[1] If we arbitrarily denote feminine gender as 0, and masculine as 1, then the possible combinations are shown in the following truth table:

	S_1	S_2	$S_1 \wedge S_2$
Juana y María son buenas.	0	0	0
Juana y Juan son buenos.	0	1	1
Juan y Juana son buenos.	1	0	1
Juan y Carlos son buenos.	1	1	1

(3) The SEMANTIC EXPLANATION OF CERTAIN LINGUISTIC PHENOMENA:

In the following examples the same copula conveys a different meaning in each case:

(1) *John is a man*:	belonging to a class
(2) *John is happy*:	predication
(3) *John is the author of these verses*:	identity
(4) *Catholics are Christians*:	the inclusion of one class in another

In symbols: (1) $x \in X$ (3) $x = y$
(2) $H(x)$ (4) $A \subset B$

where x stands for 'John'
H stands for 'happy'
X stands for the class of 'male'
y stands for 'author of these verses'
A stands for the class of 'Catholics'
B stands for the class of 'Christians'
$\subset$ stands for inclusion
$=$ stands for identity
$\in$ stands for 'is an element of'

[1] This particular application was made by Bernard Pottier, "Vers une sémantique moderne", *Travaux de Linguistique et de Littérature*, II (Strasbourg, 1964).

II. SENTENTIAL LOGIC

1. *Terminology*

In CLASSICAL OR TRADITIONAL LOGIC (Aristotelian), JUDGEMENT is the mental act of considering something, whereas the PROPOSITION is that which is considered and may be one of several thoughts expressed in the form of a linguistic construction. In MODERN LOGIC, however, propositions are stated much more concisely, and are called SENTENTIAL FORMULAE.

Propositions — and sentential formulae, of course — have the property of being either TRUE OF FALSE. It is with the study of this particular property that sentential logic is concerned.

The sentential variables p, q, etc. representing true or false propositions are called PROPOSITIONAL VARIABLES.

The signs $-$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, are called SENTENTIAL FUNCTORS, LOGICAL CONSTANTS, OF CONNECTIVES. They express relations, operations, etc. between propositional variables, thereby producing compound propositions.

Propositional variables are assigned truth values such as 'true', 'false', etc.

The truth or falsity of a proposition submitted to a particular logical operation (expressed by the respective functor) is a LOGICAL FUNCTION of the truth values of the variables. The truth or falsity of a compound proposition is the logical function of the truth or falsity of the COMPONENT PROPOSITIONS.

A logic employing only TWO VALUES — true and false — is called BIVALENT.

A statement without a conjunction is a non-compound proposition, and is called an ATOMIC statement, e.g. *John is a student.*

A statement containing a conjunction is a compound proposition and is termed MOLECULAR, e.g. *John is a student and Mary is a teacher.*

A compound proposition written in symbols, results in a SENTENTIAL FORMULA.

2. *Logical Negation*

This is an operation carried out on a statement.

Example 1: *John is a man* p
John is not a man $-p$ (or: $\bar{p}$, $\sim p$)

The sign $-p$ means 'not p'.

The negation of a falsehood:

Example 2: *John is a woman* $-p$
It is not true that John is a woman $-(-p) = p$

Square brackets, parentheses and braces are termed TECHNICAL SYMBOLS.

The relation between the truth and falsity of propositions may be determined by TRUTH TABLES: Thus, examples (1) and (2) above would appear as:

p	$-p$
t	f
f	t

Here the problem of 'double-negation' presents itself. This occurs in some natural languages, such as Spanish:

Example 3: *No he estado nunca en Nueva York.*

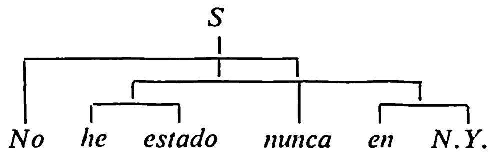
The double-negation disappears if the adverb is placed before the verb:

Nunca he estado en Nueva York.

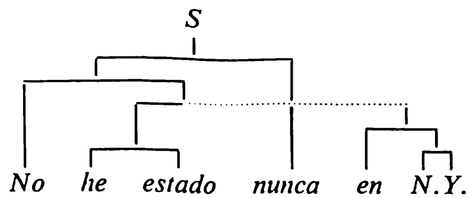
The situation here is the following: If it is true that, *no he estado nunca en Nueva York*, then the variable *p* has the value *t*, and its negation the value *f*. Now, if I should say, *No he estado nunca en Nueva York*, in spite of the fact that I had been there, then I would not be telling the truth, so the denial of my assertion would be perfectly true, as clearly shown by the truth table.

As regards the adverb *nunca* (from Latin *numquam*), this does not carry the logical value of negation, but the LINGUISTIC FUNCTION of providing emphasis when occurring in a postverbal position.

The following tree represents the immediate constituents of the statement, *No he estado nunca en Nueva York*:



It would thus seem more satisfactory to introduce a multiple constituent division at the first stage of the analysis rather than to make a binary division. Thus *nunca* is seen to be descriptive but not defining. The above tree is therefore to be proferred to the following:



Example 4: French: *Je crains qu'elle ne soit malade.*

The English translation of this sentence contains no negative particle: 'I believe she's ill'.

The French *ne* does not convey either linguistic or logical negation in this case, but serves to reflect the emotional state of the speaker who, despite clear indication that the person spoken about is ill, includes the hope that she is not. Here the negative particle has a PSYCHOLOGICAL function.

If we do not consider this psychological nuance then the variable *p* may be designated 'false', its logical negation being the truth or that which apparently corresponds to the truth, viz. 'It seems that she's ill'.

In the following example, the French particles *ne* and *pas* provide a linguistic negation:

Example 5: *Je* ne *crains* pas *qu'elle ait perdu l'argent.*
'I don't believe she has lost her money.'

DOUBLE-NEGATION in the LOGICAL sense does NOT change the original value of the variable:

Example 6:

p	-*p*	−(-*p*)
Juan es varón: v	*Juan no es varón:* f	*No es verdad que Juan no sea varón:* v
Juan es mujer: f	*No es verdad que Juan sea mujer:* v	*No es verdad que Juan no sea mujer:* f

In this Spanish example, if we replace *Juan es varón* by *Juan es hombre* (also interpreted as the former, unless the adverb *muy* occurs before the noun *hombre*), then the negative form, *Juan no es hombre*, would have the different meaning of, *Juan no es valiente*, which is the negation of *Juan es muy hombre.* This points to the advantage of formulating logical negations verbally by placing 'it is not true that' in front of the respective statement. Of course, ordinary language is not carried on in this way, for the simple denial of a statement involves the process of transformation. From the standpoint of linguistics, the negation of *Juan es hombre* would then be *Juan no es varón.*

There are other languages, however, that do not accept double-negation:

Example 7: German: **Ich war niemals nicht in New York.*
**Ich habe kein Buch nicht gekauft.*

The reason why double-negation is acceptable in some languages but not in others, does not seem to lie in the argument that the latter are 'more logical' than the former, but that certain factors of linguistic structure have to be taken into account, such as the intonation curve, the position of the negative particle, the distribution of stress, and so on. Consider also, the following Spanish sentences:

Example 8: (a) *No ha ofrecido ninguna ayuda.*
(b) *No ha ofrecido ayuda alguna.*
(c) *No sabe decir ni siquiera "Buenos días" en inglés.*
(d) *Ni siquiera "Buenos días" sabe decir en inglés.*

Despite the fact that there is linguistic double-negation in sentences (a) and (c), this is less emphatic than the simple negation of sentences (b) and (d). On the other hand, **No ha ofrecido alguna ayuda* would be incorrect.

3. *Logical Conjunction* (*Logical Product*)

Two or more propositions combined by the connective *and* (symbolized: $\wedge$) produce a logical CONJUNCTION OR PRODUCT. The formula denotes the co-existence of the two elements represented by the variables. The respective logical functions are seen in the following truth table:

p	q	$p \wedge q$
v	v	v
v	f	f
f	v	f
f	f	f

Example 9:

Hace frío. p
Está nevando. q
Hace frío y está nevando. $p \wedge q$

In natural language, when the predicate is the same in both cases, the compound proposition is usually stated more concisely, as in (b) below:

Example 10: (a) *John is intelligent and Mary is intelligent.*
(b) *John and Mary are both intelligent.*

Here the problem arises of whether or not it is meaningful to assign the value 'false' to the logical function of a compound proposition such as:

Example 11: (a) *John is intelligent and Mary is lazy,*

for if we designate the first variable as 'false', then an assertion has been made which is not in keeping with the truth. In example 10(b) the adjective clearly applies to both John and Mary. Therefore, if either of the two variables should have the value 'false', then the transformed co-ordinate sentence (b) should also be regarded as 'false'. However, in example 11(a) there are two distinct predicates, so the same transformation cannot be applied without change of meaning because:

Example 11: (b) *John and Mary are intelligent and lazy*
implies that both adjectives refer to Mary as well as to John.

The term LOGICAL PRODUCT may also be explained by assigning the values 1 and 0 to the variables p and q:

p	q	$p \cdot q$
1	1	1
1	0	0
0	1	0
0	0	0

Example 12: If we let 0 stand for the 1st and 2nd grammatical persons, and 1 for the 3rd person, then the combinations can be arranged according to the truth table that corresponds to the logical product.[2]

[2] B. Pottier, "Vers une sémantique moderne."

p	q	$p \wedge q$
yo: 0	*tú*: 0	*nosotros*: 0
yo: 0	*él*: 1	*nosotros*: 0
tú: 0	*él*: 1	*vosotros*: 0
él: 1	*ella*: 1	*ellos*: 1

If the 2nd and 3rd persons are symbolized by the value 1, and the 1st person by 0, then the combinations thus formed will not agree with the logical product.

4. *Logical Disjunction*

The combination of two or more propositions by means of the connective *or* is called a LOGICAL DISJUNCTION.

Example 13: The statement:

> *All the teachers in this school are university graduates or have been to training college*

contains two propositions:

(a) Some of the school's teachers have graduated from university: p

(b) Some of the school's teachers have been to training college: q

The statement would be false if there were teachers in the school who had neither graduated from university nor studied at a training college. This will be seen in the following truth table, in which the connective is symbolized by the initial letter of the Latin conjunction *vel*:

p	q	$p \vee q$
v	*v*	*v*
v	*f*	*v*
f	*v*	*v*
f	*f*	*f*

The table clearly reveals that the proposition is false only when both variables are 'false'. On the other hand, the compound proposition would be 'true' even if only one of the two variables were 'true'. This kind of disjunction is termed INCLUSIVE, and corresponds to the connective *or* in normal language.

Example 14: The sentence:

The class begins at 8 or 9 o'clock tomorrow morning

explicitly asserts that the class will begin at only one of the two times mentioned. The proposition: *The class begins at 8 o'clock* (p) is obviously not in agreement with the proposition: *The class begins at 9 o'clock* (q). This is shown in the following table:

p	q	$p \not\equiv q$
v	v	f
v	f	v
f	v	v
f	f	f

This kind of disjunction is called EXCLUSIVE. As $\vee$ represents the inclusive disjunction (*vel* as opposed to *aut* – *aut* in Latin), it is replaced by the symbol $\not\equiv$.

In ordinary language the connective *either* – *or* is usually employed.

The inclusive disjunction is also known as the LOGICAL SUM, the operation of adding up being carried out in accordance with the rules governing the binary system: $0+0=0$, $0+1=1$, $1+0=1$, $1+1=1$.

Example 15: By comparing the following pairs of sentences, we may see that another linguistic difference between the two kinds of disjunction exists in Spanish — the singular form of the verb in the exclusive disjunction, and the plural form in the inclusive:

(a) *El Dr. Pérez o el Sr. Ramírez* será *el próxime Rector de nuestra Universidad.*

(b) *Juan o María* llegarán *mañana.*

The exclusive disjunction can be symbolized with the help of the constants ∧, –, ∨:

$$(p \wedge -q) \vee (q \wedge -p)$$

Logical disjunction does not demand that there should be a connection in meaning between the two constituent propositions; thus statements such as the following are perfectly acceptable:

Either I'm going to the cinema tonight, or there'll be a full moon.

However, we would normally regard such an utterance as meaningless insofar as it appears contrary to common sense.

Another aspect of inclusive logical disjunction which should be mentioned is that it is not necessary to list all the possibilities in any given case.

Example 16: Let us suppose that a riverside village can be reached by road as well as by river and air. If we ask, *How can you get to the village?*, the answer *Either by road or by air* is incomplete, and one can therefore justly maintain that it is 'false'. However, according to our truth table, the proposition is 'true'. The assertion becomes linguistically sound merely by inserting the adverb *only*:

You can reach the village only by road or by air.

The following inclusive disjunction has a meaning in natural language which is not considered in formal logic:

Tomorrow I'll either pass or fail my examination; that's all I can say.

The additional meaning here lies in the recognition of the fact that my knowledge is so unsure that I cannot say in advance with any degree of certainty what the result of tomorrow's examination will be. This doubt is not taken into account in formal logic.

Similarly, formal logic does not consider whether or not the speaker's intention is in agreement with what he says. If, for example, a debtor has no intention whatsoever of settling debts with his creditor, but nevertheless promises:

> *Tomorrow I'll pay what I owe you, or I'll accept a bill of exchange.*

and the next day he accepts the bill of exchange, then he has given a false promise. However, as a true proposition has here been formally combined with a false one, the logical disjunction has the value 'true'.

Example 17: In the phrases:

(a) *to be in the room*
(b) *to play in the room*

there are two kinds of verbs: *to be* expressing something static, and *to play* indicating movement. In both examples the preposition *in* is used. However, in Japanese this proposition would have two distinct forms, according to whether the verb indicated something static or dynamic:

(a) *heya*	*ni*	*iru*
room	in	to be
(b) *heya*	*de*	*asobu*
	in	to play

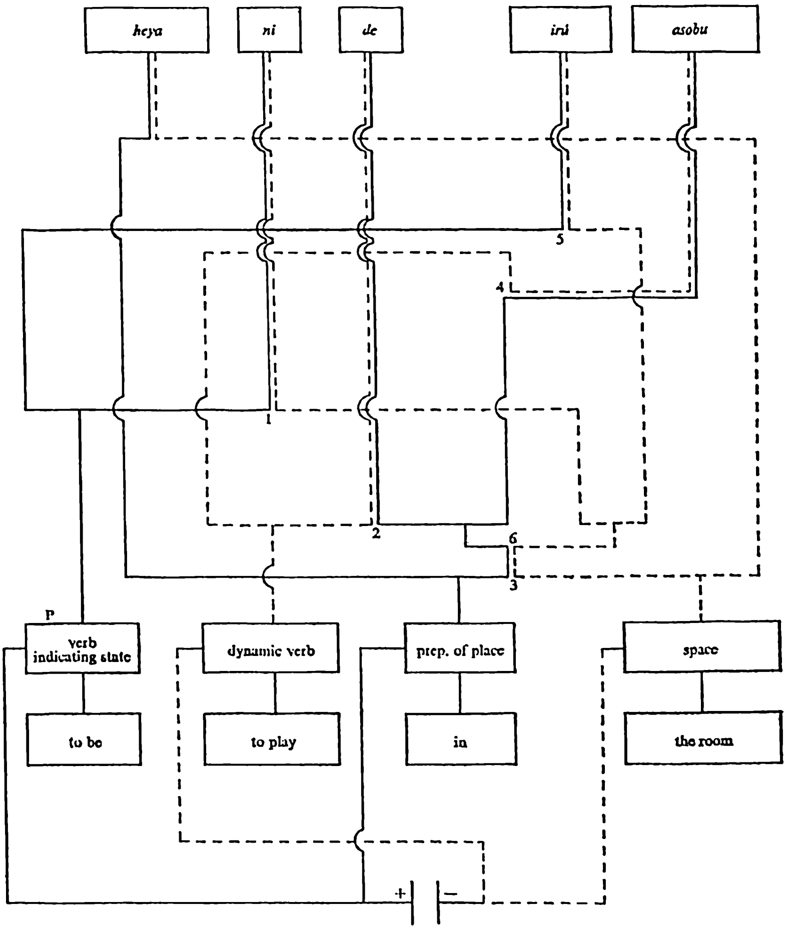
Moreover, the static situation or the activity must refer to a limited space, which is specified in the above example by the word *room*. The analogy between the expressions in English and Japanese is illustrated by the diagram on p. 21 in the form of an electrical circuit. The following are the characteristics of the model represented by the diagram:

(a) Each Japanese word is accompanied by a battery-powered light.

(b) The circuit is so arranged that on contact only those words are illuminated which would provide a correct statement when grouped together.

(c) In the diagram, the continuous line (———) represents the current from the positive pole (+), and the broken line (- - - - -) that from the negative pole (−).

(d) To switch on any one of the lights, a connection must be made with both poles of the battery. This is indicated by the two



lines (a continuous and a broken one) terminating at each box enclosing a word.

(e) The points marked 1, 2, 3, 4, 5 are to be regarded as conjunctions, whereas 6 is a disjunction.

(f) Depending on the 'off' or 'on' position of the switch at 6, either the words *ni* and *iru*, or *de* and *asobu* will be lit up. The dotted broken line (-·--·-) reveals the current flow illuminating *ni* and *iru*.

In the truth table on p. 22, the 1 in the last column denotes that the connections are such that they permit the production of the Japanese sentences.

The factors: 'verb indicating state', 'dynamic verb', 'preposition of place', and 'space' are to be considered as variables.

S	R	Q	P	$S \wedge R$	$Q \wedge (S \wedge R)$	$P \wedge (S \wedge R)$	$[Q \wedge (S \wedge R)] \vee [P \wedge (S \wedge R)]$
1	1	1	0	1	1	0	1
1	1	0	1	1	0	1	1
1	0	1	0	0	0	0	0

etc.

Regarding the above example, it should be pointed out that more factors actually occur in the process of linguistic formulation than those considered here.[3] The interaction of the many content factors required for the construction of a statement can be represented by a formula based on prior linguistic analysis. A comprehensive set of such formulae for several languages, and their subsequent co-ordination, might well serve as the basis for machine translation from one language to another.

5. *The Principle of Logical Contradiction*

The fact that nothing, at one and the same time, and in similar aspect, may possess a quality and yet not possess it, is called the PRINCIPLE OF LOGICAL CONTRADICTION. The fourth column in the following table accords with this principle:

p	$-p$	$p \wedge -p$	$-(p \wedge -p)$
v	f	f	v
f	v	f	v

[3] A. Hoppe, "Der sprachliche Formulierungsprozeß als Grundlage automatischer Hin- und Herübersetzung", *Neuere Ergebnisse der Kybernetik*, (München, 1964). *Idem*, "Der sprachliche Formulierungsprozeß in den Funktionsebenen der Sprache", *Beiträge zur Sprachkunde und Informationsverarbeitung* 4 (1964).

A compound proposition that always has the value 'false' — as in the third column of the above table — is called a CONTRADICTION; and that which maintains the value 'true' — as in the fourth column of the same table — is called a TAUTOLOGY.

In the principle of logical contradiction, the conditions 'at one and the same time' and 'in similar aspect' are important to allow for the DIALECTICAL CONTRADICTION which cannot be obtained formally but must be drawn from reality.[4]

The rules governing the functioning of a language at its various levels abide by the principle of logical contradiction, provided that they are formulated with reference to synchronic criteria. But as an individual language develops, so many of these rules have to be discarded or modified. A dialectical contradiction occurs when, in the encroachment of one language upon another, a particular linguistic phenomenon of the host language receives the impact of another phenomenon of the same kind, but with an opposite meaning, resulting in a new phenomenon. Many cases of this kind are known in diachronic pnonology and lexicology. On the other hand, statements made by general grammar (linguistic universals) also maintain diachronic validity.[5]

6. *The Principle of 'Tertium Non Datur'*

The fact that two contradictory propositions may not have the value 'true' in common, is called the principle of *tertium non datur*, illustrated in the following table.

p	$-p$	$p \vee -p$
v	f	v
f	v	v

[4] See: G. Klaus, *Moderne Logik* (Berlin, 1964), pp. 50ff.

[5] See: J. H. Greenberg (ed.), *Universals of Language* (Cambridge, Mass., 1963); P. Hartmann, *Allgemeinste Strukturgesetze in Sprache und Grammatik* (The Hague, 1961).

Here again there is tautology because the compound proposition in this case always has the value 'true'.

In formulating statements concerning future events, the principle of 'tertium non datur' is valid only if a clear distinction is made between the logical truth and certainty existing as a psychological nuance, and between the truth itself and the problem of if, how, and when it can be verified. If this distinction is not made, then one must have recourse to a TRIVALENT LOGIC, in which the 3rd value is 'possible', thereby invalidating the principle of 'tertium non datur'. In the table below, *m* symbolizes 'possible':

p	-*p*
v	*f*
f	*v*
m	*m*

In natural language a polivalent logic is in force, for language, being a human phenomenon, is naturally subject to the influence of psychological and social factors; the complexities of life cannot be drastically reduced to the all-inclusive judgements of 'true' and 'false'. For those cases which in formal logic do not agree with the principle of 'tertium non datur', every language offers various and often distinct means of expressing degrees of certainty.

Example 18: There is rarely a one-to-one correspondence between any two languages as regards means of expression:

		Form	*Certainty*
Sp.:	*Mañana iré al cine.*	future	high degree
Engl.:	*Tomorrow I shall go to the movies.*	future with *shall*	
Jap.:	*Ashita eiga ni ikimasu.*	present	
Sp.:	*Puede ser que vaya al cine mañana.*	periphrasis with subjunctive	minor degree

	Form	Certainty
Engl.: *I might go to the movies tomorrow.*	periphrasis with *might*	
Jap.: *Ashita ega ni iku ka mo shirimasen.*	periphrasis	

7. *Logical Implication*

Two propositions combined by the connective *if*, symbolized → or sometimes ⊃, form a LOGICAL IMPLICATION. This usually occurs in a compound sentence consisting of a conditional clause (subordinate) and a main clause. The conditional clause is traditionally called the ANTECEDENT, and the main clause, which is often preceded by the correlatum *then*, the CONSEQUENT.

(a) *If I get back early tonight, then we'll go to the cinema.*

The relation between the two combined propositions may be of several kinds, as shown in the following examples:

(b) *If you sprinkle any more of that salt on your dinner, you're going to get terribly thirsty.*
(c) *If the two variables of a logical disjunction have the value 'false', then the overall value of the proposition is also 'false'.*
(d) *If John ever manages to become Rector of this University, then I'll eat my hat.*

In sentence (b) the relation between the antecedent clause and the consequent is a causal nexus, the effect invariably following upon the cause. In sentence (c) the consequent clause is logically derived from its antecedent, but there is no causal nexus, and therefore no time connection between them. In the three sentences, (a), (b) and (c), there exists a relationship in meaning between the antecedent clause and the consequent. On the other hand, in example (d) there is no nexus of this kind whatsoever between the two combined propositions, the nexus between them being entirely logical. In (d) the speaker starts from the assumption that 'John will never manage to become the Rector of this University', and affirms it by the facetious promise to eat his own hat should he be subsequently proved wrong — a penalty he certainly does not expect to be called upon to perform. The compound proposition, however, has the value 'true'. This kind of compound proposition is also known as a MATERIAL IMPLICATION.

The table below presents the values of this logical function for the following cases:

(1) Examples (a), (b), (c)
(2) *If today is Monday, then tomorrow will be Sunday.*
(3) *If today is Monday* (being really Saturday), *then tomorrow will be Sunday* (tomorrow being really Sunday).
(4) Example (d)

p	q	$p \rightarrow q$
v	v	v
v	f	f
f	v	v
f	f	v

Natural language also offers other connectives for combining two propositions to form an implication. However, these variations bring with them certain changes in meaning or in syntax.

Example 19: The Spanish sentence

(a) *Si regresas temprano, entonces puedes venir con nosotros*

can be transformed into the following, with the omission of the word *entonces*:

(b) *Puedes venir con nosotros si regresas temprano.*

In this sentence, the phrase *con tal que* may be substituted for the connective *si*, but in this case it will be necessary to use the subjunctive:

(c) *Puedes venir con nosotros con tal que regreses temprano.*

This substitution, however, is not always quite correct: The sentence

(d) **El libro caerá al suelo con tal que lo sueltes*

would be rejected in favour of

(e) *El libro caerá al suelo si lo sueltas.*

Example 20: In the Spanish sentence:

(a) *Si estudias bien este problema, sabrás la verdad*

the conditional clause may be converted into a construction consisting of the infinitive preceded by the preposition *de*:

(b) *De estudiar bien este problema, sabrá la verdad.*

But this particular substitution also has limited application: The sentence:

(c) *De desear, puedes ir con nosotros*

would in fact be normally replaced by:

(d) *Si deseas, puedes ir con nosotros.*

It would seem that the construction: *de* + verb (infinitive) is given preference only when the verb has some complement (direct, indirect, adverbial).

Example 21: In natural language, whether spoken or written, the following conditional forms occur:

(a) 'realis': *If John comes, he'll tell you all about it himself.*
(b) 'potentialis': *If John came, he would tell you all about it himself.*
(c) 'irrealis': *If John had come, he would have told you all about it himself.*

Case (c) above would correspond to the last row of the truth table for implication, while (a) and (b) reveal an implication in which both variables have the value 'true'.

Example 22: In some languages, as in German, for example, it is possible to form implications without explicit connectives. In such cases the implication is expressed in the form of a particular structural pattern with the optional presence of the correlatum. Thus the sentence:

(a) *Wenn der Motor versagt, dann stürzt das Flugzeug ab*
'If the engine fails, then the plane will crash'

can be transformed into

(b) *Versagt der Motor, (dann) stürzt das Flugzeug ab.*

The same sentence can also be transformed into a construction with the modal verb *sollen*:

(c) *Sollte der Motor versagen, dann stürzt das Flugzeug ab.*

8. *Logical Equivalence*

In the compound proposition:

Lima is the capital of Peru if, and only if, Buenos Aires is the capital of Argentina

the connective, also termed the bi-conditional, *if, and only if* establishes an implication in two directions at the same time: $p \rightarrow q$ and $q \rightarrow p$. Consequently, it can be formulated as follows:

$$(p \rightarrow q) \wedge (q \rightarrow p)$$

This compound proposition, called a logical equivalence, and symbolized by $\leftrightarrow$, appears in the following table:

p	q	$p \leftrightarrow q$
v	v	v
v	f	f
f	v	f
f	f	v

Example 23: In German, the verb in a subordinate clause cannot precede the subject:

..., *dass er abreise* ('..., that he should leave')

Let S stand for the set made up of the subordinating conjunctions c, of the nouns or pronouns p acting as subject, and of the finite verbs w, then in this set there is a relation of antecedence A:

However: $\forall c \in S \wedge \forall p \in S \wedge \forall w \in S \wedge c\,A\,p \wedge p\,A\,w \leftrightarrow -(c\,A\,w)$

9. *The Utterance and Semiotics*

In any utterance we may consider (a) the signs in relation to the objects and phenomena to which they refer, (b) the relationship between the signs themselves, and (c) the signs in their relation to the person using them. (a) is concerned with SEMANTICS; (b) with SYNTAX; and (c) with PRAGMATICS. These three disciplines make up SEMIOTICS.

As regards syntax, a distinction should be drawn between linguistic syntax and the syntax of formal logic. The former examines the various linguistic structures and attempts to provide an adequate set of rules to determine the formation of the linguistic constructions characterizing the respective language; the latter is concerned with the LOGICAL STRUCTURE of an utterance. When dealing with the logical structure of the formalized language of science, the term LOGICAL SYNTAX is usually employed.

Though an utterance may be expressed by different linguistic structures, it can nevertheless have only one logical structure:

Example 24: The German sentence

(a) *Wenn er heute nicht kommt, dann ist er beschäftigt*
'If he doesn't come today, then he'll be busy'

can be transformed to

(b) *Kommt er heute nicht, dann (so) ist er beschäftigt.*

In this transformation, structural changes have taken place according to fixed rules of linguistic syntax. The logical structure, however, has not changed.

Within semantics, several LEVELS can be recognized. ZERO LEVEL is the term applied to objects, relationships, qualities, phenomena, etc. belonging to OBJECTIVE REALITY, but WITHOUT SIGNS. The signs used to designate the objects of zero level belong to an OBJECT LANGUAGE or a FIRST LEVEL LANGUAGE. The language containing the signs with which we designate the object language is called a METALANGUAGE or SECOND LEVEL LANGUAGE. The language containing the signs with which we designate the second level language is called metalanguage 2, and so on. Example:

Object language:	"Two swallows do not make a summer"
Metalanguage 1:	"'Two swallows do not make a summer' is a complete statement."
Metalanguage 2:	"" 'Two swallows do not make a summer' is a complete statement" is a true utterance.""

Two sentential formulae are SEMANTICALLY EQUIVALENT if they contain the SAME variables and if the values of their LOGICAL FUNCTIONS are equal, i.e. if they have the same truth table in common.

Example: The formula $p \leftrightarrow q$ is semantically equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$, which is expressed as:

$$p \leftrightarrow q \text{ formula } (p \rightarrow p) \wedge (q \rightarrow p).$$

The qualities of sentential formulae, the relationships between them and their meaning is studied in METALOGIC and is framed in a metalanguage. A statement of metalogic would be, for example:

> The sentential formulae E_1 and E_2 are semantically equivalent.

Here it should be stressed that the term SEMANTICS is often variously employed:

(a) In GENERAL LINGUISTICS semantics is applied to the study of the meaning of words.

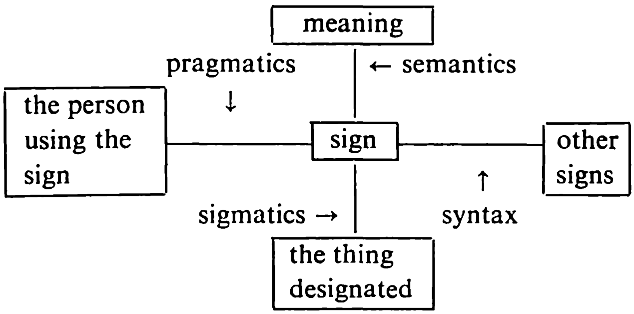
(b) PHILOSOPHICAL SEMANTICS deals with problems in that area of the theory of knowledge which is concerned with the relationship between words and the concepts conditioning their meanings.

(c) In BASIC RESEARCH IN MATHEMATICS semantics refers to the field dealing with problems of truth, general validity of statements, verification, etc.

The term SEMANTICS is also used in SEMIOTICS to refer to the study of the relationship between signs and their meanings, and the term SIGMATICS denotes the study of the connection between THE SIGN AND THE THING SIGNIFIED.[6]

Semiotics is not restricted to the study of natural languages but also studies languages in general, particularly formalized languages. In this sense, semiotics conceives a language as consisting of a collection of signs together with the rules indicating how they are to be combined to form words. In this case words are replaced by abstract expressions such as a formula of sentential logic.

The four dimensions of semiotics are illustrated in the following diagram.[7]



10. *The Laws of Commutation, Association, Distribution and Transitivity*

(a) Laws of commutation

(1) $p \wedge q \leftrightarrow q \wedge p$ (2) $p \vee q \leftrightarrow q \vee p$

[6] See: G. Klaus (Ed.): *Worterbuch der Kybernetik* (Berlin 1967), p. 561.
[7] See: G. Klaus, *Moderne Logik*, p. 565.

Commutation, which in logic deals only with the verifiable values of variables, is somewhat limited in ordinary language.

Example 25: The sentence:

(a) *John and his sister have arrived*

on being transformed to:

(b) *His sister and John have arrived*

changes its meaning, as the possessive always refers to an element previously specified in the context. The sister in the commuted sentence (b) above, is not identical with the sister referred to in sentence (a).

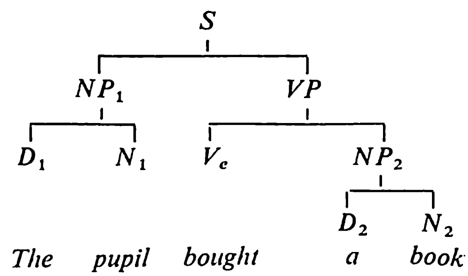
(b) Laws of association

(1) $p \wedge (q \wedge r) \leftrightarrow (p \wedge q) \wedge r$
(2) $p \vee (q \vee r) \leftrightarrow (p \vee q) \vee r$

Example 26: The following phrase structure rules and the accompanying tree represent in diagrammatic form the sentence

The pupil bought a book.

$S \rightarrow NP_1 + VP$
$NP_1 \rightarrow D_1 + N_1$
$VP \rightarrow Ve + NP_2$
$NP_2 \rightarrow D_2 + N_2$



In the truth table below the logical constant should be interpreted as "is immediate co-constituent with". The brackets indicate the hierarchy:

$NP_1 \wedge Ve$	$Ve \wedge NP_2$	$NP_1 \wedge (Ve \wedge NP_2)$	$(NP_1 \wedge Ve) \wedge NP_2$
0	1	1	0

The analysis is not in accordance with the law of association because NP_1 is not immediate co-constituent with *Ve* but only with a structure of higher order, of which *Ve* forms part.

(c) Laws of distribution

$$(1)\ p \vee (q \wedge r) \leftrightarrow (p \vee q) \wedge (p \vee r)$$
$$(2)\ p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$$

Example 27: The word *fast* will fit the slots of only the last two of the following examples:

(a) *John is 20 years* _____.
(b) *John is a* _____ *runner.*
(c) *John runs* _____.

If we assert that:

(a) *fast* fits in sentence (a): p
(b) *fast* fits in sentence (b): q
(c) *fast* fits in sentence (c): r

then, on drawing up the following table, we see that here the law of distribution can be applied.

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee r) \wedge (p \vee r)$
f	v	v	v	v	v	v	v

(d) The law of transitivity

$$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$$

Example 28: In English there is a class of adjectives traditionally termed 'indefinite', which may occur only in a certain order. This leads to the setting up of various subclasses: subclass A: adjectives occurring only before adjectives of subclass B; subclass B: those that cannot be placed before members of subclass A, etc. In the following phrase:

(a) *all my first attempts*

all belongs to class A, whereas *my* is assigned to class B because it may not precede *all*. *First* belongs to the third class, C, because it is not found before members of either class A or B.

(b) *all first offenders*

In this phrase, *all* may be placed immediately before *first*.

These examples all conform to the law transitivity:

$$((x = a \in A \rightarrow \prec b \in B) \wedge (y = b \in B \rightarrow \prec c \in C)) \rightarrow$$
$$(x = a \in A \rightarrow \prec c \in C)$$

The sign $\prec$ means 'may precede'.

The validity of this formula is, of course, based on a prior linguistic analysis of the three classes A, B, and C. Moreover, the behaviour of the elements in the three classes may also depend on the noun referred to. If the word *books* replaces *offenders*, then *all* does not normally occur immediately before *first* but is separated by some other element, e.g. the definite article:

(c) *all the first books*

This is explained by the fact that the deep-structure is not the same in both cases: (b) is the result of the transformation:

(b′) *all those who offended for the first time*

i.e. it specifies a certain action. (c) could not be the result of such a transformation.

(e) Other formulae of sentential logic

The connectives of natural languages are different from those of formal logic, even though they may convey the same logical meaning.

Example 29: The Spanish sentence:

(a) *Ni Juan ni María han venido*

corresponds to the formula:

$-p \wedge -q$

This sentence may be transformed to:

(b) *No han venido ni Juan ni María*

with the same formula as above, and the table having exactly the same logical functions: *f*, *f*, *f*, *t*.

Example 30: The statement

(a) *John has arrived but Mary hasn't*

is expressed by the formula

$p \wedge -q$

The following sentence has the same formula:

Example 31:

(a) *Es verdad que Juan es inteligente, pero no es aplicado.*
German: *Hans ist zwar intelligent, aber er ist nicht fleißig.*

The truth table for these two cases is:

p	q	$-q$	$p \wedge -q$
v	v	f	f
v	f	v	v
f	v	f	f
f	f	v	f

Comparing the values of this compound proposition with those corresponding to the table for implication, we see that the combination $p \wedge -q$ is an inversion of the implication. There is, therefore, the SEMANTIC EQUIVALENCE

$-(p \rightarrow q)$ eqsem $p \wedge -q$

In natural languages, and particularly in Spanish, compound connectives are often used where in formal logic a simple connective would be quite sufficient:

Example 32: In the Spanish statement,

Tanto Juan como María son mis amigos

the connective *tanto - como* corresponds to the logical constant $\wedge$. The formula for this would be

$$p \wedge q$$

Example 33: Also, in the sentence,

John hasn't come, but Mary has

the connective *but* corresponds to the constant $\wedge$

$$-p \wedge q$$

Example 34: Despite the fact that the compound connective in the statement,

Not only John has come, but Mary has too

contains a negative particle, it does not involve a negation, the corresponding logical constant being:

$$p \wedge q$$

The great diversity of connectives in ordinary language is due to the presence of such factors as emphasis, degree of certainty, emotion, etc., which have no place in formal logic.

(f) Other semantic equivalents

In addition to the semantic equivalents given in the previous sections, the following are also important:

(1) $p \wedge q$ eqsem $-(-p \vee -q)$
(2) $p \rightarrow q$ eqsem $-p \vee q$
(3) $p \leftrightarrow q$ eqsem $-(-(-p \vee q) \vee -(-q \vee p))$

In these transformations, equivalent formulae have been obtained containing only negations and disjunctions. While the transformed formulae (1) and (2) can be stated in ordinary language, a verbal rendering of transformation (3) would encounter difficulties. The same could be maintained of transformation (6) below, where the equivalent formulae contain only conjunctions and negations:

(4) $p \vee q$ eqsem $-(-p \wedge -q)$
(5) $p \rightarrow q$ eqsem $-(p \wedge -q)$
(6) $p \leftrightarrow q$ eqsem $-(p \wedge -q) \wedge -(q \wedge -p)$

The following semantic equivalents exist also:

(7) $p \vee q$ eqsem $-p \rightarrow q$
(8) $p \vee q$ eqsem $-q \rightarrow p$
(9) $p \vee q$ eqsem $(p \rightarrow q) \rightarrow q$
(10) $p \vee q$ esqem $(q \rightarrow p) \rightarrow p$
(11) $-(p \wedge q)$ eqsem $-p \vee -q$
(12) $-(p \vee q)$ eqsem $-p \wedge -q$

11. *Normal Forms in Formal Logic*

By means of semantically equivalent sentential formulae, it is possible to effect a series of transformations.

Those transformations which remove all the constants except the connectives $\vee$ and $\wedge$ and at the same time permit negations only of single members, but not of combined members, produce formulae which are called NORMAL FORMS in sentential logic. The normal form is CONJUNCTIVE if its members combine by means of the connective $\wedge$; and it is DISJUNCTIVE (or ALTERNATIVE) if they combine by means of the connective $\vee$.

The normal form allows us to judge more easily the truth value of the compound proposition that contained $\rightarrow$ and $\leftrightarrow$ as logical constants.

The example below shows how an expression with the connective $\leftrightarrow$ is transformed to the respective normal form.

Example 35: The difference between the 'nomen actionis', e.g. *increase* in the phrase

the increase in crime

and the 'nomen infinitum', i.e. the verbal noun as in Spanish:

El amar es una virtud

can be analysed on the basis of Hoppe's theory[8] as follows.

An expression is conditioned by the interaction of content factors: For the 'nomen actionis' (*the increase*):

(a) Taken in its most elementary sense, this word denotes an EVENT (E).

(b) In this case the event is conceived as a MAGNITUDE (M). Grammatically, magnitude functions as a noun, and as such it requires the presence of the grammatical factors of number (n_n), gender (g), and person (p_p).

(c) The inclusion of the factor of ABSTRACTIVE CONCEPTION (A_s) excludes those of time (t), aspect (a), and voice (v) which are normally present if the event is conceived in terms of action rather than abstraction.

The problem would therefore be set out in ordinary language as:

The word *increase* is a 'nomen actionis' if, and only if, the following conditions are met:

(a) that in the word *increase* the event is conceived as a magnitude which is the case if, and only if, the word has number, gender and person; and

(b) it cannot be varied as regards ASPECT, TIME and VOICE, which is the case if, and only if, the noun is conceived abstractively.

This is expressed in formal logic by the following sentential formula in which NA symbolizes 'nomen actionis'; EM, the event conceived as a magnitude; n_v, number of the verb; p_v, person of verb; m, mood; n_n, number of noun; g, gender of noun; p_n, person of noun; A_c, conceived abstractively; a, aspect; t, time; and v, voice. Negation is indicated by the dash placed over the respective variable or expression.

$$NA \leftrightarrow \{(AM \leftrightarrow [(\bar{n}_v \wedge \bar{p}_v \wedge \overline{m}) \wedge (n_n \wedge g \wedge p_n)]) \wedge C_s \leftrightarrow (\bar{a} \wedge \bar{t} \wedge \bar{v})\} \qquad (1)$$

To simplify the operations, we make the following substitutions in formula (1):

[8] See: A. Hoppe, "Der sprachliche Formulierungsprozeß".

Substitution (S) 1: $\bar{n}_v \wedge p_v \wedge \bar{m} = U$
S2: $n_n \wedge g \wedge p_n = X$
S3: $\bar{a} \wedge \bar{\imath} \wedge \bar{v} = Y$

$$NA \leftrightarrow \{[AM \leftrightarrow (U \wedge X)] \wedge (Cs \leftrightarrow Y)\} \tag{2}$$

For further simplification, we effect the following substitutions in (2):

S4: $AM \leftrightarrow (U \wedge X) = Z$
S5: $Cs \leftrightarrow Y = W$

$$NA \leftrightarrow Z \wedge W \tag{3}$$

Now we apply semantic equivalence (6) to formula (3):

$$[\overline{NA \wedge \overline{(Z \wedge W)}}] \wedge [\overline{(Z \wedge W) \wedge \overline{NA}}] \tag{4}$$

We apply semantic equivalence (11) twice to (4):

$$[\overline{NA} \vee \overline{\overline{(Z \wedge W)}}] \wedge [\overline{(Z \wedge W)} \vee \overline{\overline{NA}}] \tag{5}$$

Removing the double negation we obtain:

$$[\overline{NA} \wedge (Z \wedge W)] \wedge [\overline{(Z \wedge W)} \vee NA] \tag{6}$$

As $\overline{NA}$ does not belong to formula (1), we remove it from (6):

$$(Z \wedge W) \wedge [\overline{(Z \wedge W)} \vee NA] \tag{7}$$

We apply semantic equivalence (11) to (7):

$$(Z \wedge W) \wedge [(\bar{Z} \vee \bar{W}) \vee NA] \tag{8}$$

Now we return to Z:

$$Z = AM \leftrightarrow (U \wedge X) \qquad \text{(S4)}$$

Applying semantic equivalence (6) and removing the double negation, we obtain:

$$[\overline{AM} \vee (U \wedge X)] \wedge [\overline{(U \wedge X)} \vee AM] \qquad \text{(S4.1)}$$

Applying equivalence (11), we obtain:

$$[\overline{AM} \vee (U \wedge X)] \wedge [(\bar{U} \vee \bar{X}) \vee AM)] \qquad \text{(S4.2)}$$

Removing $\overline{AM}$, which does not occur in formula (1), and also $\overline{U}$ and $\overline{X}$ whose equivalents neither occur in (1), we obtain:

$$(U \wedge X) \wedge AM = Z \qquad \text{(S4.3)}$$

Now we return to the equivalent of W:

$$W = Y \leftrightarrow Cs \qquad \text{(S5)}$$

Applying semantic equivalence (6), we obtain:

$$\overline{(Y \wedge \overline{Cs})} \wedge \overline{(Cs \wedge \overline{Y})} \qquad \text{(S5.1)}$$

We apply semantic equivalent (11):

$$(\overline{Y} \vee \overline{\overline{Cs}}) \wedge (\overline{Cs} \vee \overline{\overline{Y}}) \qquad \text{(S5.2)}$$

Removing the double negation $\overline{Y}$ and $\overline{Cs}$, as they do not occur in (1), we obtain:

$$W = Cs \wedge Y \qquad \text{(S5.3)}$$

Given that the equivalents of $\overline{W}$ and $\overline{Z}$ do not occur in (1), we remove them from (8):

$$(Z \wedge W) \wedge NA \qquad (9)$$

Now, in (9) we replace Z and W by their equivalents (S4.3) and (S5.3) respectively:

$$\{[(U \wedge X) \wedge AM] \wedge (Cs \wedge Y)\} \wedge NA \qquad (10)$$

In (10) we replace U, X and Y by their respective equivalents, S1, S2, and S3:

$$(\{[(\bar{n}_v \wedge \bar{p}_v \wedge \overline{m}) \wedge (n_n \wedge g \wedge p_n)] \wedge AM\} \wedge [Cs \wedge (\bar{a} \wedge \bar{\imath} \wedge \bar{v})]) \wedge NA \qquad (11)$$

It must be understood that the variables, in spite of possessing negative values, represent true utterances, since 'it is true' that these factors cannot be present when the event is considered as a magnitude and is conceived abstractively.

In formula (11) — the normal conjunctive form of sentential formula (1) — the bi-conditional connective does not appear, enabling us to see immediately if the compound proposition is 'true' or 'false'.

Another formula would be needed for the 'nomen infinitum', in which the factor of 'abstractive conception' would be replaced by that of 'actional conception'. This key factor requires, in its turn, the presence of 'time', 'aspect', and 'voice', seeing that the following Spanish forms are possible:

(a) *El amar es una virtud.*: active voice, imperfective aspect, present time,
(b) *El haber amado le sirve a uno de experiencia.*: active voice, perfective aspect, past time
(c) *El ser amado es el deseo de todas las mujeres.*: passive voice, imperfective aspect, present time
(d) *El haber sido amado le ha servido de mucha experiencia.*: passive voice, perfective aspect, past time

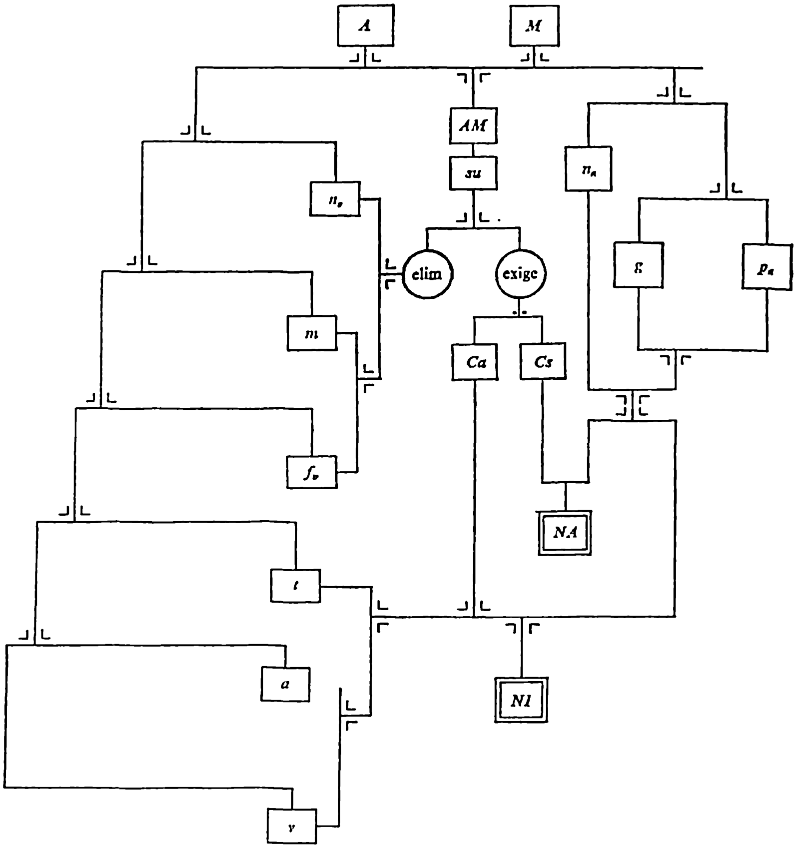
The interrelation of the various content factors necessary for the production of the two kinds of nouns will be seen in the simplified diagram appearing on p. 42.[9] Here, *A* signifies 'actional conception'; *N* 'noun'; and *NI* 'nomen infinitum'. At the points of branching, indicated by the sign $A \overset{C}{\leftarrow\!\mid\!\rightarrow} B$, the information proceeding from *C* continues in both directions (*A*, *B*); on the other hand, the sign $A \overset{C}{-\!\!\uparrow\!\!-} B$ shows that the information continues to *C* only if it proceeds from BOTH directions (*A*, *B*) — a flow of information from right to left, or vice versa, is not permitted.

Whether the noun is a 'nomen actionis' or a 'nomen infinitum' depends on the alternative decision taken at the point marked .

It should be mentioned that the 'voice' factor does not occur only in verbal nouns. Compare the following Spanish sentences:

(a) *El aumento de la escasez de gasolina es considerable.*
(b) *El aumento del precio de la gasolina por el Gobierno fue criticado por la opinión pública.*

[9] See: A. Hoppe, "Der sprachliche Formulierungsprozeß".



The factor of 'voice' does not enter in sentence (a) because it contains no agent, but this is not the case in sentence (b) where *the Government* acts as agent, the noun *aumento* expressing the action taken by the agent which affects the price of petrol; so here the factor of 'voice', specifically 'active voice', is present.

A. ARGUMENT AND PREDICATE

In the predicate calculus the statement is divided into,

(a) one or several ARGUMENTS (the term used instead of 'logical subject') and

(b) one or several PREDICATES.

In the statement

The phoneme /a/ is a vowel

'the phoneme /a/' is the argument, and 'is a vowel', the predicate. The grammatical subject does not always coincide with the argument (logical subject), nor the grammatical predicate with the logical predicate:

Example 36: The statement

The members of the Droxford Bowling Club will be electing their new president this week-end

would be analysed in traditional grammar as:

(1) subject:	*The members*
(2) attribute:	*of the Droxford Bowling Club*
(3) predicate:	*will be electing*
(4) adverbial phrase:	*this week-end*
(5) direct object:	*their new president*

In logic, *the members* and *of the Droxford Bowling Club* together form the argument or logical subject. The other parts of the sentence make up the logical predicate.

Example 37: In the English and Spanish sentences

(a) *I like this dress*
(b) *Este vestido me gusta*

there are two distinct grammatical subjects: *I* and *este vestido.* However, the logical subject is the same in both cases: (a) *I* and (b) *me.* Only in the English sentence are the logical and grammatical subject the same.

Arguments are generally symbolized by small letters, called VARIABLES — w, x, y, z — and predicates by capital letters. The formula $F(x)$ would express the above statement, in which x represents 'the phoneme /a/', and F 'is a vowel'.

When it is necessary to be more specific, additional individual constants may be introduced:

Example 38: *John fetched the book for Mary.* $F(xyz)$, where x signifies *John*, y *the book*, and z *Mary.*

A formula containing no logical constant is called an ATOMIC PREDICATE FORMULA, MONADIC if it has only one variable and POLIADIC with more than one.

By using the constants of sentential logic, atomic formulae may be combined to form molecular formulae:

Example 39: The sentence

John studies and Mary works

has the formula: $F(x) \wedge G(y)$.

B. THE QUANTIFIERS 'ALL' AND 'EXISTS'

The definition,

All vowels are voiced

contains the variable 'vowels' (x). This may be set out as:

For every x it holds that: If x is a vowel, then x is voiced.

The assertion 'for every x' is symbolized $\forall(x)$, this symbol is known as the ALLOPERATOR or UNIVERSAL QUANTIFIER. The above definition may thus be expressed by the formula:

$\forall(x)\ F(x)$, which means: for every x, $F(x)$.

If we replace the variable x by a, b, c, ... i.e. by all the vowels, then we have[10]

$$F(a) \wedge F(b) \wedge F(c) \wedge \ldots$$

This can be written as,

$$\bigwedge{}_x F(x)$$

in which the symbol $\wedge$ is equivalent to the previously used symbol $\forall$.

It is, however, customary to omit these symbols and present the formula simply as,

$$(x)\ F(x)$$

The statement,

Some verbs require a direct object

can be transformed to,

There is an x such that x is a verb that requires a direct object.

The expression 'there is an x' is written as $\exists(x)$. This symbol is called the EXISTENTIAL QUANTIFIER.

The following formula corresponds to the above statement: $\exists(x)$ $F(x)$ and is to be read, 'there is an x such that $F(x)$'.

For this expression to be 'true', at least one of the statements in which the variable x is replaced by the individual verbs a, b, c, ..., e.g., $F(a)$, $F(b)$, $F(c)$ ... has to have the value 'true'. This means that the disjunction:

[10] N.B. *a*, *b*, *c* are symbols representing vowels like a, e, æ; so they are no phonetic symbols nor graphemes corresponding to vowels, but symbols *ad hoc*.

$$F(a) \vee F(b) \vee F(c) \vee \ldots$$

would be assigned the value 'true'. Instead of this formula, the following may also be employed.

$$\bigvee{}_x F(c),$$

in which $\bigvee$ is equivalent to the symbol $\exists$ occurring previously. The statement,

> For every adjective ending in *-o* in Spanish there exists a corresponding adjective ending in *-a*

can be written,

$$\forall(y)\,\exists(x),$$

in which x stands for 'a corresponding adjective ending in *-a*' and y 'every adjective ending in *-o*'.

Using the logical negation, expressions are obtained which in ordinary language are equivalent to: 'not every x': $-\forall(x)$; and 'there is no x such that': $-\exists(x)$.

Every quantification has its own range which depends on whether the quantification refers to the whole formula or only to a part of it.

Example 40: In English there are words which, without changing form, function as adjectives or adverbs. For example, the blanks in sentences (a) and (b) below may be filled by the same form, *fast*:

(a) *John is a _____ runner.*
(b) *John runs _____.*

This can be stated as: $\exists(x)\,(Fx \rightarrow Gx)$. In this case the quantification includes the whole formula and for this reason it is enclosed in brackets.

Example 41: In Spanish, adjectives can be classified according to their behaviour:

- those that may serve as either attributes or predicates
- those that function only as attributes
- those that function only as predicates.

Also, of course, it is necessary to note the kind of noun to which the adjective refers.

In Spanish, one can say:

(a) *el régimen alimenticio*, but not
(a′) **el régimen es alimenticio*;
(b) *el observatorio solar*, but not
(b′) **el observatorio es solar*.

From this we can draw up the following rule:

Not all adjectives occupy the final position of the structural pattern

Noun + copula + _____ (a fact which we symbolize as predicate F).

If this principle is true, and if the adjective *solar* belongs to this class of adjectives, it cannot be inserted in sentence (b′) (predicate G). Our assertion can then be written as:

$$(x)\, F(x) \rightarrow G(x)$$

When a variable is within the 'range' of a quantifier, it is BOUND; if not, then it is FREE. Consequently, those formulae in which the variables are bound, are called CLOSED, and when at least one variable is free, they are called OPEN.

C. SEMANTIC EQUIVALENCE

We may also set up logical equivalences in the predicate calculus e.g.

(E1) $\forall(x)\, F(x) \leftrightarrow -\exists(x) - F(x)$
(E2) $\exists(x)\, F(x) \leftrightarrow -\forall(x) - F(x)$
(E3) $-\forall(x)\, F(x) \leftrightarrow \exists(x) - F(x)$
(E4) $-\exists(x)\, F(x) \leftrightarrow \forall(x) - F(x)$
(E5) $\forall(x)\, F(x) \rightarrow F(x)$
(E6) $F(y) \rightarrow \exists(x)\, F(x)$
(E7) $-\exists(x)\, F(x) \leftrightarrow -F(a) \wedge -F(v) \wedge -F(c) \wedge \ldots$
(E8) $-\forall(x)\, F(x) \leftrightarrow -F(a) \vee -F(b) \vee -F(c) \vee \ldots$

Example 42: Let us suppose we wish to give a negative meaning to the following English sentence:

(a) *All these students are willing to do the work.*

The following possibilities are open to us:

(b) *All these students are unwilling to do the work.*
(c) *None of these students are willing to do the work.*
(d) *Not all of these students are willing to do the work.*
(e) *All these students are not willing to do this work.*

Let the problem be: to choose among sentences (b) to (e) that which represents the logical negation of sentence (a):

The formula for (a) is:

$$\forall(x)\ F(x) \tag{1}$$

and its negation:

$$-\forall(x)\ F(x) \tag{2}$$

In accordance with equivalence (E8) we can convert (2) into:

$$-F(a) \vee -F(b) \vee -F(c) \vee \ldots \tag{3}$$

This expression has the value 'true' if there is at least one x such that $-F(x)$, that is

$$\exists(x)\ -F(x) \tag{4}$$

Consequently

$$[-\forall(x)\ F(x)] \rightarrow [\exists(x) - F(x)] \tag{5}$$

is true.

This expression corresponds to sentence (d), and shows that statement (a) is negated if there is only one student who is *unwilling to do the work.*

The following expression:

$$-F(a) \wedge -F(b) \wedge -F(c) \wedge \ldots,$$

corresponds to sentence (b) and has (E7) as an equivalent:

$$-\exists(x)\, F(x)$$

Sentence (c) is another version of sentence (b), its logical content being the same.

The expression:

$$-[F(a) \quad F(b) \quad F(c)] \ldots, \tag{1}$$

corresponds to sentence (e) and its equivalent (11) in sentential logic:

$$-F(a) \vee -F(b) \wedge -F(c) \vee \ldots \tag{2}$$

Equivalent (E8) corresponds to (2):

$$-\forall(x)\, F(x) \tag{3}$$

Consequently, there is an x, such that $-F(x)$, that is to say, the formula

$$[-\forall(x)\, F(x)] \rightarrow [\exists(x)\, -F(x)] \tag{4}$$

is true.

It will be seen that statement (e) is logically equivalent to (d). However, in spoken language it is possible to give (e) a distinct meaning by placing emphasis on the word *all*, which then carries the information load.[11] The formula for this will be identical with that corresponding to (b) and (c).

D. LOGIC OF CLASSES

1. *The Notion of Class and Classification*

The classification of objects, phenomena, and relations on the basis of properties held in common, results in a particular CLASS. The common property must be ESSENTIAL. The classification may be NATURAL: the only problem here is to discover the members; but it

[11] See: M.A.K. Halliday, "Notes on transitivity and theme in English", *Journal of Linguistics*, vol. 3, no. 2 (October 1967).

can also be ARTIFICIAL. The criterion on which the classification is based is selected *ad hoc*, e.g. classifying pupils according to their surnames and christian names. In the process of classifying, the following rules need to be observed:

(a) If the class is subdivided, the resulting subclasses must form the original class when recombined.

(b) There should be no overlapping of the subclasses, so that their intersection is ZERO.

The ORDERING OF CONCEPTS is a similar procedure to classifying: It concerns the discovery of SUBORDINATE concepts, i.e. subordinate to a main concept which is usually generic. In this procedure one must take care to avoid mixing the criteria governing the order.

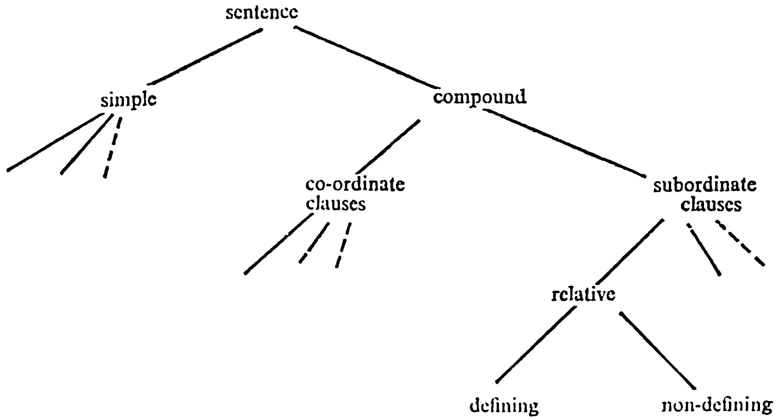
Example 43: Let 'the vowel [i]' be the given concept. If we start from the assumption that this vowel is characterized by quantity (duration), then distinctions can be made such as: long *i*, short *i*, etc. This can be elaborated by reference to other languages in which the vowel *i* occurs: the Spanish *i*, the German *i*, etc. If we arrange the concept 'the vowel [i]' into: long *i*, the German *i*, etc., this will involve confusion of criteria, seeing that our classification would not cover the whole range of the concept, and the overlapping of any two partial concepts, e.g. long *i*, and the German *i* would not ensure zero intersection. There is an *i* that belongs to the class of long *i*'s and also to the class of German *i*'s, a language in which the *i*'s are distinguished by quantity.

Another requisite is that the concepts must be ordered without omitting pertinent criteria, and care taken to observe the natural order.

Example 44: If statements are divided into:

simple sentences
sentences with a defining relative clause
sentences with a non-defining relative clause

then this would not agree with the principle just stated since it would involve a concept of lower rank being co-ordinated with one of higher status. A satisfactory classification would be:



2. *Extension and Intension of a Concept*

Every concept is arrived at by an abstraction of things or phenomena. Thus the concept 'vowel' lays emphasis on that which is common to all vowels and reflects the class of vowels. The reflexion of such a class by the concept is called the EXTENSION of this concept.

The extension of a concept, however, does not exhaustively define the concept, since a particular class of things or phenomena may be reflected by various concepts. Thus the concepts, 'an element which modifies an adjective' and 'an element which modifies an adverb' are different, but they can have the same extension, which is the case when both refer to the class of adverbs, and only to this class; they are not differentiated as regards extension but only as to their INTENSION. The difference between the two concepts in this case is not extensional but intensional.

3. *Class Identity*

A class A is identical with a class B if each element of A is an element of B, and each element of B is an element of A:

$$A = B =_{\mathrm{def}} \forall(x)\,[(x \in A) \leftrightarrow (x \in B)]$$

4. *Inclusion*

A class A is included in a class B, when every element of A is an element of B:

$$A \subset B =_{\text{def}} \forall(x)\,(x \in A \rightarrow x \in B)$$

Example 45: The difference between the following statements:

(a) *a, b and c are members of the Governing Board* and
(b) *a, b and c are the members of the Governing Board*

lies in the fact that (a) deals with the inclusion of one class in another, whereas (b) denotes class-identity. The difference is signalled by the presence (or absence) of the definite article.

The following classes can be established:

Class A_1: Governing Board with more than 3 members (=m)
Class A_2: Governing Board with only 3 members
Class B: The 3 members designated a, b and c respectively.

For statement (a) it holds that:

$$B \subset A_1 \leftrightarrow (a)(b)(c) \in B \rightarrow (a)(b)(c) \in A_1,$$

and for statement (b):

$$A_2 = B \leftrightarrow \forall(m)\,[(m \in A_2) \leftrightarrow (m \in B)]$$
$$A_2 = B \leftrightarrow (A_2 \in B) \wedge (B \in A_2)$$

In languages which have no articles such distinctions would have to be upheld by other syntactic, morphological or lexical means. Thus, in Japanese this case would reveal a variation of the particle accompanying the subject:

(a) 'a' to 'b' to 'c' *ra wa* ...
(b) 'a' to 'b' to 'c' *ga* ...

It will be noticed that the difference is not marked by the grammatical predicate but by the grammatical subject.

5. *Sum of Classes*

A class C is the logical sum or the union of classes A and B when its elements belong to at least one of the classes A and B. An element x is a member of the logical sum $A \cup B$ if, and only if, it is a member of A or of B:

$$x \in (A \cup B) =_{def} \in A \vee x \in B$$

6. *Product of Classes*

A class C is the product of classes A and B when C contains all the elements that belong to both A and B:

$$x \in (A \cap B) =_{def} x \in A \wedge x \in B$$

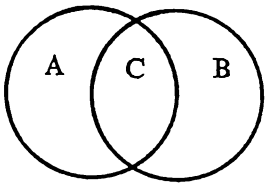
Example 46: The following classes of verbs can be set up:

Class A: Verbs which do not take a direct object
e.g. *Charles came.*

Class B: Verbs which require a direct object
e.g. *John likes apples.*

Class C: Verbs which may or may not take a direct object
e.g. *John is playing.*
John is playing marbles.

The following diagram of Venn[12] illustrates the situation:



The logical sum of classes A and C, $A \cup C$, would form a new class of verbs which do not strictly require a direct object. The logical sum of classes B and C, $B \cup C$, would form a new class of verbs which do not exclude a direct object.

This can be described in the following way:

Predicate F: 'requires a direct object'
Predicate G: 'does not take a direct object'
Predicate H: 'can take a direct object'

For the class $A \cup C$ it holds that:

$$-\exists(x \in A \cup C)\, F(x) \qquad (1)$$

[12] J. Venn was an English mathematician of the 19th century.

or also:

$$\forall(x \in A \cup C)\text{-}F(x) \tag{2}$$

For the class $B \cup C$ it holds that:

$$\text{-}\exists(x \in B \cup C)\ G(x) \tag{3}$$

or also:

$$\forall(x \in B \cup C)\text{-}G(x) \tag{4}$$

For the class $C = A \cap B$ it holds that:

$$\forall(x \in A \cap B)\ H(x) \tag{5}$$

But as $A \cap B \subset A \cup C$, $A \cap B \subset B \cup C$, then the conditions expressed by (1) to (4) must also be accepted, i.e. (5) can be replaced by:

$$\text{-}\exists(x \in A \cap B)\ F(x) \tag{6}$$

or also by:

$$\text{-}\exists(x \in A \cap B)\ G(x)$$

7. *Complement of a Class*

The complement of a class A signifies the class A' of all those items not belonging to class A, such that:

$$x \in A' =_{\text{def}} -(x \in A)$$

It will be observed that the product of these two classes is the null (or empty) class:

$$A \cap A' = \emptyset$$

This law conforms to the principle of contradiction.

In producing the sum A and A' it is necessary to be more specific.

Example 47: If we form a class of all the auxiliary verbs, there will be two complementary classes:

A_1': the class of all non-auxiliaries; this is a negation of the concept underlying class A.

A'_2: the class of all other words. The total vocabulary L of a language would then be:

$$L = A \cup A'_2$$

This law is in keeping with the principle of 'tertium non datur'. Class L in this case is called the universal class with respect to the vocabulary of the given language.

E. LOGIC OF RELATIONS

In the statement, 'the adjective precedes the noun' we can substitute 'the adjective' and 'the noun' by x and y respectively:

x precedes y

The variables x and y, which belong to two different classes, are in a mutually fixed relation, which in this case is defined by precedence.[13] x and y are called the SUBJECTS.[14] In the example given, the relationship is BINARY as there are only two subjects.

Example 48: Within the general class of Spanish adjectives, various subclasses can be formed by applying transformations. The resulting classification will then be based on the semantic behaviour of the adjectives in these transformations.

There is a class of adjectives which, when used with the copula *ser* or *estar*, function structurally and semantically like the adjective *ciego* in transformations (a) and (b) below:

(a) *El muchacho es ciego* → *el muchacho ciego*
(b) *El muchacho está ciego* → *el ciego muchacho*

If we let n stand for the noun, a for the adjective belonging to this subclass, the sign $\prec$ for precedence, and use subscripts to distinguish between the two meanings, then the syntactic and semantic behaviour of these adjectives can be set out as follows:

[13] Other kinds of relations can also be defined.
[14] For the difference between logical and grammatical subject, see under III, A.

$$\forall(a\in A)\{[(n \prec a_1) \wedge (a_2 \prec n)] \rightarrow [(\text{ser} + a_1) \wedge (\text{estar} + a_2)]\}$$

Verbally: For each adjective *a* element of subclass *A* it is the case that the noun *n* precedes the adjective *a* with the meaning 1; if it follows the same adjective with meaning 2, then the *ser* occurs with the adjective when it has meaning 1, and the copula *estar* with the adjective having meaning 2.

Example 49: There is a subclass of Spanish adjectives which, when attributes of certain nouns, may have two distinct meanings according to their position, but which may occur as predicates only with the copula *ser*:

(a) *el jefe alto* → *El jefe es alto.*
(b) *el alto jefe* → ∅

$$\forall(b\in B)\,\forall(n^{(1)} \in N^{(1)})\,\{[(n^{(1)} \prec b_1) \wedge (b_2 \prec n^{(1)})] \rightarrow (\text{ser} + b_1)\}$$

Verbally: For every adjective *b* element of subclass *B*, and for every noun $n^{(1)}$ element of the noun subclass $N^{(1)}$, it holds that if the noun $n^{(1)}$ precedes the adjective *b* with meaning 1, and if the noun follows the same adjective with meaning 2, then the predicative construction is possible only with the copula *ser*, the adjective conveying meaning 1.

As will be realized, the symbolism of quantificational logic discloses nothing new for linguistics in these examples. Its value consists merely in furnishing a more accurate system of notation.

1. *Inclusion of Relations*

A relation *R* is included in a relation *S*, when *S* relates two elements *x* and *y* whenever *R* relates the same two elements:

$$R \subset S =_{\text{def}} (x)(y)\,(xRy \rightarrow xSy)$$

Example 50: The terms for family relationships in Japanese take into account factors such as, 'seniority', 'juniority', 'courtesy', etc. Thus to express, 'is the brother of', one needs to consider if this concerns a younger or an elder brother, and whether my own brother or someone else's is being referred to.

If the general relation S is, 'is the brother of', then it includes the following relations:

R_1: 'is the elder brother of': *oniisan*
R_2: 'is the younger brother of' or
'is my younger brother': *otooto*
R_3: 'is my elder brother': *ani*

2. *Sum of Relations*

A relation Q is the sum of two relations R and S, when Q is the relation of all the elements x with all the elements y such that R relates x with y or S relates x with y, or both. In other words: Two elements x, y are in union $R \cup S$ of two relations R, S, if between them there exists at least one of the relations R, S:

$$\forall(x)\, \forall(y)\, [(P \cup S)\,(s,y) \leftrightarrow P(x,y) \vee S(x,y)]$$

The relation symbol can also be placed in front of the related elements, i.e. $R(x,y)$ instead of xRy.

Example 51: If R signifies the relation 'to be the brother of', and S 'to be the sister of', their combination is expressed in German as *verschwistert sein mit.*

In everyday English there is no special expression for indicating this combination (the word *sibling* is restricted in usage), as can be seen by comparing the following examples:

(a) English: *John and Mary are brother and sister.*
(b) German: *Hans und Maria sind verschwistert* =
Hans und Maria sind Geschwister.
(c) Spanish: *Juan y María son hermanos.*

It can be seen that neither has Spanish a special term for indicating this sum of relations, so in cases as the above it uses the word for 'brothers' (*hermanos*) to include that for 'sisters' (*hermanas*).

Example 52: The sum of the three relations in Example 50 is exactly the relation 'to be the brother of', i.e. $R_1 \cup R_2 \cup R_3$.

3. *Product of Relations*

A relation Q is the product of two relations R and S when Q is the relation of all the elements x with all the elements y, such that R relates x to y, and S relates x to y:

$$\forall(x)\ \forall(y)\ \left[(R \cap S)(x,y) \leftrightarrow R(x,y) \wedge S(x,y)\right]$$

Example 53: as we have seen in Example 50, the relation 'is the younger brother of' has a special expression in Japanese: *otooto*. It is the product of the relations, 'my brother' and 'younger than'.

4. *Identical Relations*

Two relations R and S are identical when an element x has together with an element y the relation R if, and only if, x together with y has the relation S:

$$R = S =_{\text{def}} \forall(x)\ \forall(y)\ (xRy \leftrightarrow xSy)$$

Example 54: Natural language often distinguishes among relations which are not differentiated by logic. As we have already noted in Example 49, the relation,

(a) 'x is the brother of y'

cannot be expressed in Japanese unless an elder or a younger brother is specified:

(b) *x wa y no niisan desu* 'x is y's elder brother'
(c) *x wa y no otooto desu*: 'x is y's younger brother'

There exists an identity between the relations (a) and (b), and (a) and (c).

5. *The Complement of a Relation*

The complement R' of a relation R is the relation of every x with every y such that it is not the case that R relates x to y:

$$R' =_{\text{def}} \forall(x)\ \forall(y)\ [R'(x,y) \leftrightarrow -R(x,y)]$$

Example 55: If R is the relation 'to be the brother or sister of', R' will be 'not to be the brother or sister of'. In German, as we saw in Example 51, there is a special expression for the relation 'to be the brother or sister of' in which the sex is not distinguished:

(a) *verschwistert sein mit*

The negation of this would be:

(b) *nicht verschwistert sein mit.*

6. *The Universal Relation*

The relation which everything has with everything else is called the UNIVERSAL RELATION:

$$\forall(x)\,\forall(y)\,(R \cup R')(x,y)$$

7. *The Null Relation*

The relation that nothing has with anything else is called the NULL RELATION:

$$-\exists(x)\,\exists(y)\,[R(x,y) \wedge R'(x,y)]$$

8. *The Converse of a Relation*

The converse of a relation R is the relation of every x with every y such that $R(y,x)$:

$$\forall(x)\,\forall(y)\,[R(x,y) \leftrightarrow \breve{R}(y,x)]$$

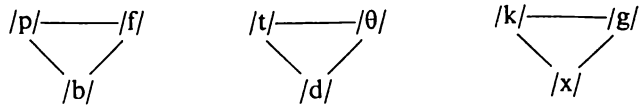
Example 56: If R is the relation 'to govern grammatically', then $\breve{R}$ will be 'to be governed grammatically by'.

9. *The Relative Product of Relations*

Two relations may be linked, which results in the RELATIVE PRODUCT: The linked relation R/S holds for a pair of entities — $R/S(x,y)$ — if, and only if, there is an entity u such that $R(x,u)$ and $S(u,y)$:

$$\forall(x)\ \forall(y)\ \{R/S(x,y) \leftrightarrow [R(x,u) \wedge S(u,y)]\}$$

Example 57: In the phonological system of peninsular Spanish, the following correlated groups occur, based on the correlations of voiced/voiceless and plosive/fricative:



Let us first define the three classes of phonemes:

Class U, formed by the elements u_1, u_2, u_3 which are the voiceless plosives /p/, /k/, /t/.

Class X, formed by the elements x_1, x_2, x_3 which are the voiced plosives /b/, /g/, /d/.

Class Y, formed by the elements y_1, y_2, y_3 which are the voiceless fricatives /f/, /x/, /θ/.

Now let us define the following two relations:

R: ... is different from ... by the contrast voiced/voiceless
S: ... is different from ... by the contrast plosive/fricative

We then have the following situations:

$$\forall(x \in X), \quad \forall(u \in U), \quad R(x,u), \text{ i.e.:}$$

For every x and for every u it holds that: Every x is different from every u by the contrast voiced/voiceless. (Note: It is NOT maintained 'is different ONLY by ...')

$$\forall(u \in U), \quad \forall(y \in Y), \quad S(u,x)$$

This satisfies the following condition:

$$\forall(u)\ [R(x,u) \wedge S(u,y)]$$

The elements of class X differ from those of class Y as much by the voiced/voiceless contrast as that of plosive/fricative, so that every x and every y is in distinct contrast to every u, i.e.

$$\forall(x)\ \forall(y)\ [R/S(x,y)]$$

10. *The Properties of Relations*

Relations may be classified according to the following properties:

(a) Reflexive relation

A relation is reflexive within a given class when,

$$\forall(x \in X)\ (x\ R\ x)$$

Example 58: Whenever in a given language a particular phoneme is (re)produced, an acceptable reproduction may not vary beyond certain recognized limits. We can say then, that within the class of phonemes of a given language there exists a reflexive relation which is essential if the phonemes are to carry out their linguistic function.

(b) Irreflexive relation

A relation R is irreflexive within a given class when,

$$\forall(x \in X)\ \text{-}(x\ R\ x)$$

Example 59: In the class of syntactic structures the relation between the immediate constituents and their respective constitute is irreflexive: The constitute,

(a) *John sings*

is made up of the immediate constituents *John* and *sings*, and not vice versa.

Example 60: Within the class of phonemes of a given language the phonemes must be distinct from one another to carry out their linguistic function of producing the necessary distinctions. It follows, therefore, that between any pair of phonemes their exists the relation of irreflexibility.

(c) Symmetrical relation

A relation R is symmetrical within a given class when

$$\forall(x)\ \forall(y)\ (x\ R\ y) \rightarrow (y\ R\ x)$$

Example 61: The relation 'to be brother(s) and sister(s)' is a symmetrical relation in Spanish:

(a) *María y Juan son hermanos.*

We have already seen in Example 51 that there are languages like German in which there is a special expression for this kind of family relationship: 'verschwistert sein'.

(d) Asymmetrical relation

This relation is expressed in the following formula:

$$\forall(x)\,\forall(y)\,(x\ R\ y) \rightarrow -(y\ R\ x)$$

Example 62: When particular phenomena in a linguistic community occupy a place of importance, then the language provides the means to emphasize them. This can be demonstrated, for example, by Japanese family relationships which are more highly differentiated linguistically than is customary in Indo-European language:[15]

brother:	elder:	*niisan*
	younger:	*otōto*
sister:	elder:	*nēsan*
	younger:	*imōto*, etc.

In each of these relations the asymmetric relationship is reflected linguistically.

(e) Transitive and intransitive relation

A relation is transitive when,

$$\forall(x)\,\forall(y)\,\forall(z)\,[(x\ R\ y) \wedge (y\ R\ z) \rightarrow (x\ R\ z)]$$

and it is intransitive when

$$\forall(x)\,\forall(y)\,\forall(z)\,[(x\ R\ y) \wedge (y\ R\ z) \rightarrow -(x\ R\ z)]$$

Example 63: There are cases in which transitivity does not operate in the relations of precedence in the spoken language:

(a) *Half the team fell sick.*
(b) **Half team fell sick.*

In English there exists a class of indefinite adjectives such as 'half' which cannot occur immediately before a noun.

[15] See K. Li, "Comparison in Componential Analysis of English and Japanese Kinship Words", *Lenguaje y Ciencias* 21 (1966).

(f) Equivalence relations

A relation that is reflexive, symmetrical and transitive is called equivalent.

Example 64: In the process of verbal communication the phonemes carry out their linguistic function on the basis of a relation of equivalence: (1) Each phoneme, on being uttered, must not vary beyond certain limits, generally imposed by the system of allophones (condition of reflexivity); (2) a phoneme x produced by speaker A must be reasonably similar to the same phoneme produced by speaker B and vice versa, i.e. it must belong to a common stock of phonemes (condition of symmetry); and (3) if a phoneme x belongs to the stock common to the speakers, the same phoneme produced by speaker A is also understood by speaker C, i.e. it belongs to the stock common to A and C, which makes communication possible within a linguistic community (condition of transitivity).

IV. HIGHER PREDICATE CALCULUS

1. *Preliminary Observations*

In the predicate calculus discussed in the previous chapter, only the quantification of arguments was considered, so that it is also called ELEMENTARY PREDICATE CALCULUS. If a quantification of the predicates is also considered, we obtain a HIGHER PREDICATE CALCULUS.

Example 65: General linguistics states a series of 'universals'. Thus in phonology it holds that the two members of a phonological opposition have at least one phonetic feature in common (F), which constitutes what is called the common base for the comparison of the phonemes x and y:

$$\exists(F)[F(x) \wedge F(y)]$$

Example 66: Voicing (F) is not a phonetic property of all the phonemes (x):

$$\exists(F) \text{-} \forall(x)\, F(x)$$

Within higher predicate calculus there are also cases where an expression is formulated with the predicate of another predicate. In this way predicates of 2nd, 3rd level, etc. are obtained. Of course, an argument must be implicit in the respective statement.

Example 67: The Spanish sentence,

(a) *La generosidad de María es excesiva*

would have the following logical structure:

$G^2(F(x))$.

In this example we can observe the difference between the LOGICAL DETERMINATION and the GRAMMATICAL DETERMINATION: The former refers to the MEANING of the argument (logical subject: x), in this case *María*; and subsequently to the meaning of the predicate (F). On the other hand, in the grammatical determination the logical subject determines the FORM of the predicate: *la generosidad* understood as an attribute of *María* requires the preposition *de* as a connective. This determination is shown more clearly in the following transformation in Spanish:

(b) *María es generosa. En ello es excesiva,*

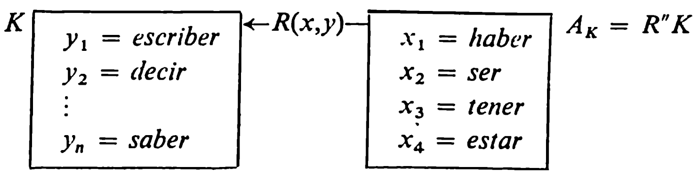
where the adjective endings show that they are determined by the logical subject *María*, which here turns out to be also the grammatical subject.

2. *Mapping*

We call the mapping of a class K with respect to a relation R, the class A_K such that each of its elements x is in relation $R(x,y)$ with one or several elements y of a class K. The class A_K is called the R-mapping of K: $R''K$. This can be formulated as:

$$\forall(x)\,\forall(K)\,\forall(R)\,\{x \in R''K \leftrightarrow \exists(y)\,[R(x,y) \wedge y \in K]\}$$

Example 68: If we group all Spanish verbs under K, with y as the main verbs *(escribir, decir,* etc.) and let 'to be an auxiliary verb of' be the relation R, then $R''K$ is the class of auxiliary verbs that correspond to the main verbs. This is represented in diagram form as:



This example does not stipulate that every auxiliary can be combined with every main verb.

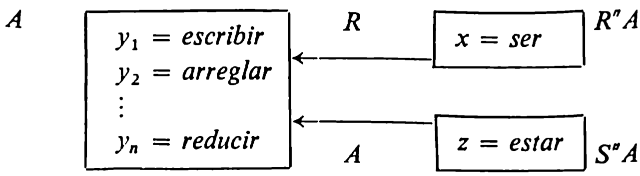
Example 69: In many languages, e.g. in Spanish, there are two kinds of passive voice: One form which expresses the occurrence of the action itself, and another which denotes the result of the action:

(a) *La puerta es abierta a las 8 de la mañana*
(b) *La puerta está abierta a las 8 de la mañana*

If A is the class formed by the main verbs which take the passive voice with *estar* and *ser*, R the relation 'to be the auxiliary of ... for the passive voice which expresses the occurrence of an action', and S the relation 'to be the auxiliary of ... for the passive voice which indicates the result of the action', then we have two more classes: $R''A$, the class formed by the element *ser*, and the class $S''A$ by the element *estar*. The sum $R \cup S$ indicates 'to be the auxiliary of ... for the formation of the passive voice', and the elements which participate in this sum must be elements of either $R''A$ or of $S''A$:

$$\forall(A)\ \forall(R)\ \forall(S)\,[(R \cup S)''B = R''B \cup S''B]$$

This situation would be represented diagrammatically as follows:



3. *The Notion of Heredity*

From the biological idea of 'heredity', 'hereditary law', etc. the general notion of heredity can be obtained by abstraction. This concept will be used in the following example:

Example 70: Let X be the class formed by the voiceless plosives termed x, Y the class of voiced plosives termed y, A the predicate 'to be a Spanish phoneme', and the relation R, 'to be in phonological opposition to'. It then holds that

$$\forall(x)\,\forall(y)\,\{[A(x) \wedge R(x,y)] \rightarrow A(y)\},$$

since in Spanish there exists the correlation of voiced/voiceless. One may then say that the quality of 'being a Spanish phoneme' is HEREDITARY regarding the relation 'to be in phonological opposition to'.

In sentential logic 'deduction' is understood as the obtaining of new knowledge. The procedure consists of arriving at a new formula, called the conclusion, by means of particular rules of INFERENCE, starting from one or several formulae called PREMISSES.

A classical example of logical deduction is called the 'modus ponens', which can be expressed as:

If S_1	$\vdash S_1$
and: If S_1, then S_2,	$\vdash S_1 \rightarrow S_2$
then also S_2	$\vdash S_2$

The general formula of a deduction would then be:

$$S_1, S_2, \ldots, S_n \vdash S_n$$

In formal logic, particularly in metalogic, the 'modus ponens' is called the RULE OF DETACHMENT.

This rule of inference cannot always be reversed.

Example 71: 'If the phoneme /t/ is replaced by the phoneme /d/in the word *mat*, the meaning will obviously be changed.' If the antecedent is symbolized by p, and the consequent by q, then the following conclusion can be deduced:

$$\frac{\begin{array}{c} p \\ p \rightarrow q \end{array}}{q}$$

However, if I know that both $p \rightarrow q$ and q are true statements, I cannot infer the truth of p from this, seeing that the change in meaning of the word *mat* could also be effected by replacing the phoneme /t/ by a phoneme other then /d/, for example by /p/, resulting in *map*.

A permitted reversal of the 'modus ponens' is called the 'modus tollens', whereby the truth of $-p$ is inferred from the truths of $-q$ and $p \rightarrow q$. Applying the 'modus tollens' to Example 71, we may conclude that if the conditional is true, and it is also true that it does not change the meaning of the word ($-q$), then it has undergone no substitution whatsoever ($-p$).

The negative form of the rule $p \rightarrow q$ is called the LAW OF CONTRAPOSITION:

If $p \rightarrow q$, then $-q \rightarrow -p$

The validity of the Law of Inference can be seen in the following table:

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$[p \wedge (p \rightarrow)] \rightarrow q$
v	v	v	v	v
v	f	f	f	v
f	v	v	f	v
f	f	v	f	v

In this table the compound premiss of the 3rd column is combined with q, giving the result in the 4th column that the conclusion is correct even when the combining of the premisses is false, which means that it is possible to obtain correct conclusions even though the premisses may be false.

The SYLLOGISM of traditional logic — Aristotelian — provides a special case of deduction consistent with sentential logic. In its respective statements a clear distinction is made between the argument and the predicate.

Example 72: 'If the sounds [p] and [b] are not interchangeable in the English word *pack* without changing the meaning, then they are phonemes'.

'If [p] and [b] are phonemes in English, they are not allophones of the same phoneme'.

Consequently:
'If the sounds [p] and [b] are not interchangeable in the word *pack* without altering the meaning, then they are not allophones of the same English phoneme'.

This syllogism is expressed as:

$$\begin{array}{l} -R(x,y) \rightarrow (Fx \wedge Fy) \\ \underline{(Fx \wedge Fy) \rightarrow (-Gx \wedge -Gy)} \\ -R(x,y) \rightarrow (-Gx \wedge -Gy) \end{array}$$

The symbols in these formulae are to be interpreted as follows:

The relation R:	'are interchangeable in the word *pack* without changing the meaning', which becomes the predicate of
arguments x, y:	the sounds [p] and [b]
Predicate F:	'are phonemes'
Predicate G:	'are allophones of the same English phoneme'

As can be seen, this HYPOTHETIC SYLLOGISM has been formulated according to the Law of Transitivity.

The CATEGORICAL HYPOTHETIC SYLLOGISM can be expressed by a sentential formula corresponding to the 'modus tollens':

Example 73: 'If in a given language the sounds x and y cannot occur in the same environment, then they are allophones'
'The two sounds x and y are not allophones'

Therefore, x and y can occur in the same environment.

$$[(p \rightarrow q) \wedge -q] \rightarrow -p$$

In the following example there is a DISJUNCTIVE SYLLOGISM:

Example 74: 'Every German sentence beginning with a finite verb is a conditional, interrogative or imperative sentence'
'The sentence *Bring mir das Buch, das auf dem Tisch liegt* is neither a conditional nor an interrogative sentence'

'Therefore, the given sentence is imperative'

$$\forall(a \in A)\,[(Fa \vee Ga \vee Ha) \wedge (\text{-}Fa_t \wedge \text{-}Ga_t)] \rightarrow Ha_t,$$

in which a is a German sentence beginning with a finite verb, and A represents all German sentences of this kind. The other symbols are self-explanatory.

In formalized logical deduction the LAW OF SUBSTITUTION is also important; this permits the replacement of a sentential variable by any well-formed formula. Thus the expression H' is derived from expression H by means of substitution if H' can be obtained from H by substituting in H a particular expression H^* for a variable p_t wherever the latter is found in H.

We have met this rule before and applied it in dealing with normal forms.

1. *Explication and Definition*

In a scientific discipline the term 'explication' refers to the process of making expressions more precise.

Every explication consists of two expressions joined by the sign $=_{ex}$. The expression to the left of the sign is called the 'explicandum', and to the right, the 'explicans'.

Example 75: To say that: '/i/ is a Spanish sound' would be regarded as an explanation in normal language; but to state that: '/i/ is a Spanish vowel which has the property of being fronted and raised' is a scientific explanation (= explication) which can be reduced to a formula:

$$[\mathrm{i}] =_{ex} F(i) \wedge G(i) \wedge H(i) \wedge \ldots$$

in which the predicate terms mean 'vowel', 'fronted', 'raised' respectively. The 'explicans' has been obtained empirically and strictly-speaking is not complete, at least not theoretically, for additional features may be present.

A LOGICAL DEFINITION, which also has two expressions: the 'definiendum' and the 'definiens', joined together by the sign '$=_{def}$', defines the 'definiendum' — a non-primitive sign — with respect to certain primitive signs given.

Example 76: In phonemic analysis the following two rules, among others, are usually employed:

(a) Two sounds x and y are phonemes in a language if they are not interchangeable in the same environment without modifying the meaning of the respective word.

Example: /t/:/d/ : *mat*:*mad*

(b) When two phonetically similar sounds x and y never occur in the same environment, they are allophones of the same phoneme.

Example: The aspirated /pʰ/ of *pit*, and the unaspirated /p/ of *spit*.

Let us now introduce the following primitive signs (already defined previously by a different procedure):

argument: x, y
predicate: G: to occur in an environment Z
relation: H: to be interchangeable in a particular word without modifying its meaning
relation: R: to be in opposition to
constants: $-, \vee, \wedge, \rightarrow$

With the help of these signs the definition can be formulated as follows:

$$\{[G(x) \wedge G(y)] \wedge -(xHy)\} \rightarrow (xRy)$$
$$=_{\text{def}} [G(x) \wedge -G(y)] \rightarrow -(xRy)$$

On the basis of semantic equivalence $p \rightarrow q$ formula $-p \vee q$ rule (a) can also be defined as:

$$\{[G(x) \wedge G(y)] \wedge -(xHy)\} \rightarrow (xRy)$$
$$=_{\text{def}} - \{[G(x) \wedge G(y)] \wedge -(xHy)\} \vee (xRy)$$

2. *Sentential Calculus*

A language considered with reference to its SYNTACTIC STRUCTURE is called a CALCULUS.

A sentential calculus consists of the following parts:

(a) the LANGUAGE OF THE CALCULUS, which comprises:[16]

[16] See: H. Behnke, R. Remmert, H.-G. Steiner, and H. Tietz, *Mathematik* I (Frankfurt am Main, 1964), p. 204.

(aa) the PRIMITIVE SIGNS which are those not defined, and include the logical constants;
(bb) the RULES OF FORMATION according to which certain pertinent expressions, especially RELEVANT SENTENTIAL FORMULAE, will be constructed on the basis of the primitive signs.

(b) the DEDUCTIVE APPARATUS, which comprises:
(aa) the relevant AXIOMS;
(bb) the RULES OF INFERENCE which will enable the relevant formulae to be deduced as theorems from the axioms.

3. *Axiomatization*

To axiomatize a scientific theory means to present it in such a way that certain statements are selected as axioms, from which all the other theorems of the theory are deduced.

A deductive system, to be rigorously axiomatized, must fulfil the following conditions:[17]

(a) It must be consistent:
(aa) SEMANTICALLY: Each expression deduced within the system has general validity;
(bb) CLASSICALLY: From two expressions *H* and *-H* only one can be deduced.
(cc) SYNTACTICALLY: Within the system only certain expressions can be deduced.

(b) The axioms must be independent, i.e. no axiom can be deduced from any other in the system.

(c) The axiomatized deductive system must be COMPLETE:
(aa) SEMANTICALLY: All the expressions of general validity can be deduced within the system;
(bb) CLASSICALLY: For any expression within the respective field it holds that it, or its negation, is deducible;
(cc) SYNTACTICALLY: The system would no longer be syntactic

[17] See: G. Klaus, *Moderne Logik*, pp. 322-326.

if an expression not deducible from the system were added to it.

A system of things and relations which satisfy an axiomatic system is also called a MODEL. An axiomatic system which has no model lacks scientific value.

Example 77: Let us suppose that a mathematician interested in linguistics were to maintain that for the adequate analysis of a language, and for the preparation of a transformational grammar of that language, only a theory that used the methods of mathematics was of any real value. Such a claim could be written as:

> 'For every linguistic theory it holds that if it is not compatible with "mathematical linguistics", then neither is it compatible with the elaboration of an adequate transformational grammar':
>
> (1) $\forall T\,(\overline{L \wedge T} \rightarrow \overline{Tr \wedge T})$, where: T = linguistic theories not using mathematical methods; L = mathematical linguistics; Tr = transformational grammar

This expression can be submitted to a series of transformations, borrowing the rules of sentential calculus:[18]

(2) $\forall T\,[(L \wedge T) \vee \overline{Tr \wedge T}]$	eqsem (2); $\overline{\overline{p}}$ eqsem p
(3) $\forall T\,(L \wedge T) \vee (\overline{Tr} \vee \overline{T})$	eqsem (11)
(4) $\forall T\,[(L \vee \overline{Tr} \vee \overline{T}) \wedge (T \vee \overline{Tr} \vee \overline{T})]$	distr. law (1)
(5) $\forall T\,[(\overline{Tr} \vee L \vee \overline{T}) \wedge (T \vee \overline{T} \vee \overline{Tr})]$	interchange of the terms connected by $\vee$
(6) $\forall T\,[(\overline{Tr} \vee L \vee \overline{T}) \wedge (t \vee \overline{Tr})]$	$p \vee \overline{p}$ eqsem t (true)
(7) $\forall T\,[(\overline{Tr} \vee L \vee \overline{T}) \wedge t]$	$t \vee q$ eqsem t
(8) $\forall T\,[\overline{Tr} \vee L \vee \overline{T}]$	$q \vee t$ eqsem q
(9) $\forall T\,[(Tr \rightarrow L) \vee \overline{T}]$	eqsem (2)

[18] See page 35,36.

The quantification $\forall T$ indicates that within its range the term T must be replaced by all the existing linguistic theories different from mathematical linguistics, and the symbols representing them must be joined by the constant $\wedge$. The conjunctions thus obtained are sententially identical for every true T. The same holds for every false T. Let us symbolize the true conjunctions by t, and the false by f, and then apply the rule $p \wedge p$ eqsem p:

(10) $[(Tr \rightarrow L) \vee \bar{t}] \wedge [(Tr \rightarrow L) \vee \bar{f}]$
(11) $[(Tr \rightarrow L) \vee f] \wedge [(Tr \rightarrow L) \vee t]$ $\bar{f}$ eqsem t; $\bar{t}$ eqsem f
(12) $(Tr \rightarrow L) \wedge t$ $p \vee f$ eqsem p; $p \vee t$ eqsem t
(13) $Tr \rightarrow L$ $p \wedge t$ eqsem p

According to implication (13), mathematical linguistics is the consequent of the 'grammatical transformational' antecedent. Thus the linguistics necessary for the preparation of a transformational grammar need NOT be mathematical linguistics. However, if, when using competent methods, we are forced into a mathematical approach, then this should be accepted:

$t \rightarrow t$ eqsem t

On the other hand, if results are arrived at which are contrary to employing mathematical linguistics, the original postulate would have to be rejected.

$t \rightarrow f$ eqsem f

This example shows that the rather categorical statements of some schools no longer appear so categorical when they are subjected to the procedures of sentential calculus.

Example 78: The difference between defining and non-defining relative clauses is shown by different formulae:

(a) *The boys, who were tired, went to bed early.*
(b) *The boys who were tired went to bed early.*

The difference in meaning between (a) and (b) is revealed orthographically by commas, and in speech by a pause before and after the relative clause in (a). Sentence (a) can be transformed to

(a') *The boys were tired; they went to bed*

without radical change of meaning — in both constructions it is clear that all the boys were tired and that they all went to bed.

The corresponding formula would be:

$$\forall(x)\,[F(x) \wedge G(x)]$$

Here there are two bound individual constants and for this reason two square brackets have been used to embrace both terms.

Sentence (b) cannot be transformed in the same way as (a) without changing the situation expressed by the former, seeing that it implicitly states that only the boys who were tired went to bed, i.e. not all the boys retired. The formula for this would be the following:

$$\forall(x)\,[F(x) \leftrightarrow G(x)]$$

Example 79: The articles of a language can have different functions, and there is often no similarity between one language and another in this respect. In the Spanish sentences (a) and (b) below, the definite article has a generalizing function:

(a) El *gato es un animal doméstico*
(b) Los *gatos son animales domésticos*

Apparently, (a) identifies an entity as belonging to a class, whereas (b) implies the inclusion of one class in another ((b)):

$$\exists(x)\ x \in A$$
$$G \subset A$$

However, both sentences mention the genus *cat*, and therefore they deal with the inclusion of one class in another, i.e. the second formula would be the adequate one.

In both German and English the INDEFINITE article may also have a generalizing function:

(c) *Die Katze ist ein Haustier.*
(d) *Eine Katze ist ein Haustier.*
(e) *The cat is a domestic animal.*
(f) *A cat is a domestic animal.*

As regards the plural, the definite article has a PARTICULARIZING function in English, and can have a generalizing function in German:

(g) *Cats are domestic animals.*
(h) *The cats on the roof are mine.*
(i) *Die Katzen sind Haustiere.*
(j) *Katzen sind Haustiere.*

When there is no attempt at classifying, as in sentences (k) and (l) below, the indefinite article expresses INDIVIDUALIZATION, and the definite article PARTICULARIZATION:

(k) *I gave John a cat.*
(l) *I gave the cat to John.*

The same difference exists in German:

(m) *Ich gab Hans eine Katze.*
(n) *Ich gab die Katze Hans.*

Regarding these last two sentences, it should perhaps be mentioned that (m) cannot be transformed to

(m′) **Ich gab eine Katze Hans.*

In German, the part carrying the greatest informative load must be expressed by the syntax or by means of stress and intonation. If we shift the emphases of (n) then it can be transformed to

(n′) *Ich gab Hans die Katze.*

with attention directed to *Hans*, when this statement is a reply to the question: *Wem gaben Sie die Katze?*

In German then, there is a contrast between the definite and indefinite articles which is neutralized in expressing a generalization.

In the case of the examples which contain an individualizing article the formula would be the following:

$$\exists(x) \quad F(x),$$

in which F stands for 'is given to John as a present'.

When the article is particularizing, then the following formula will be appropriate:

$$\exists(x) \quad [G(x) \leftrightarrow F(x)],$$

where G represents 'previously introduced in the context' and F again means 'is given as a present to'.

Example 80: The syntagmeme 'noun + noun' can express various kinds of semantic relation.[19] In the following Spanish examples the first member is symbolized by x, and the second by y:

(a) $x \ldots y$
estado policía
médico-jefe
The proposed formula: $G(y) \quad [F(x)]$

[19] See: C. Wu de Zierer, "El sintagma 'nombre + nombre' en el español moderno", *Lenguaje y Ciencias* 23 (1967).

(b) x has an essential feature of y:
ciudad jardín
viaje relámpago

$$F_k(x) \in [\sum_{i=1}^{n} F_i(y) \wedge (F_i(y) \leftrightarrow y)]$$

(c) x has a characteristic of y:
papel periódico

$$F_k(x) \in \sum_{i=1}^{n} F_i(y)$$

(d) y contains an essential feature of x; x is the means or instrument of y:
radio periódico

$$F_i(y) \in [\sum_{i=1}^{n} F_i(x) \wedge (F_i(x) \leftrightarrow x)]$$

(e) y is the effect of x:
cine fórum
Formula: $x \rightarrow y$

(f) x has the effect of y:
decreto ley

In the following formula, E stands for 'has the necessary legal effects':

$$[(y \rightarrow E) \in \sum_{i=1}^{n} F_i(y)]\ (x)$$

In this formula the expression in square brackets represents the predicate of x.

(g) x and b combined:
autor-escritor

In the following formula X and Y represent the classes of *autores* and *escritores* respectively:

$$\exists(x)\ Z \in X \cap Y$$

(h) y makes a distinction between x and an equal to x:
Juan padre

$$x \in Y \wedge y \in X$$

It should be pointed out here that the expression *Juan padre* would be used in a context where the expression *Juan hijo* also occurs. *Juan padre* is identical with the father of *Juan hijo*, i.e. *Juan* in the given expression belongs to a class of fathers (Y). In this context, the *padre* belongs to the class of *Juanes* (X).

(i) x has a feature of that which is produced by y: *vestido sastre*

In the following formula $y\ R\ z$ symbolizes 'y produces z':

$$[y\ R\ z \rightarrow F_k(z) \in \sum_{i=1}^{n} F_i(z)] \rightarrow F_k(x)$$

Example 81: The Spanish adversative conjunction *pero* 'but' has two distinct equivalents in Russian: а, но

(a) *El habla el ruso con rapidez*, pero *yo lentamente.*
Он говорит по-русски быстро, а я медленно.

(b) *El habla el ruso con rapidez*, pero *con faltas.*
Он говорит по-русски быстро, но с ошибками.

Mi hermano me ha telefoneado, pero *no me dijo nada sobre esto.*
Сестра звонила мне, но ничего не сказала об зтом.

In (a) above, the conjunction introduces a clause with a logical predicate that would be the negation of the previous clause:

$$F(x) \wedge -F(y)$$

In the sentences of (b) the second clause contains a predicate contrasted with the first, but without denying it. The argument of the second clause does not necessarily have to be identical with that of the first:

$$F(x) \wedge [G(x) \vee G(y)]$$

As will be realized, in both cases the psychological nuance contained in the conjunction *pero* is reflected in a combination of the terms by means of the negation and the connectives $\wedge$ and $\vee$ characteristic of each of the situations.

Example 82: The Spanish word *quien* has the following uses:

(a) interrogative: *¿Quién dijo esto?*
(b) exclamatory-optative: *¡Quién tuviera tanto dinero!*
(c) relative: *Felipe, quien es muy inteligente, es mi amigo*
(d) indefinite: *Quien trabaja tiene alhaja*

In the first use the interrogation refers to all those individuals involved in the predicate of (a). In the corresponding formula the argument has to be bound by the constant (or operator) of interrogation:

$$\forall(x)\,(?x)\,F(x)$$

Use (b) results in the following formula:

$$\exists(x)\,(!x)\,F(x)$$

The relative explicative sentence (c) can be rendered as a conjunction:

$$\exists(x)\,[F(x) \wedge G(x)]$$

Sentence (d) is converted to an implication:

$$\forall(x)\,[F(x) \rightarrow G(x)]$$

Comparing the four formulae, one sees a transition from the general to the particular and then again to the general.[20]

Pottier accepts intermediate positions.[21] Thus between the particular case *Felipe, quien* ... and the general *Quien trabaja* ... he includes *no hay quien* ....

We shall interpret the dotted line in this statement in the sense that they can be replaced by any construction which completes the the sentence, e.g.

[20] This tallies with the result of the analysis of 'quien' made by B. Pottier, *Lingüística moderna y filología hispánica* (Madrid, Ed. Gredos, 1968), p. 85ff.
[21] See: B. Pottier, *Lingüística moderna*, pp. 87-88.

(e) *No hay quien trabaja y que no tenga alhaja*

Let us assume: If this statement is semantically equivalent to sentence (d) then it does not take up an intermediate position.

Instead of $F(x)$ and $G(x)$ we write p and q respectively.

(d): $\forall(x)\,(p \to q)$ (1d)
(e): $-\exists(x)\,(p \wedge \bar{q})$ (1e)

Affirmation: $\forall(x)\,(p \to q)$ eqsem $-\exists(x)\,(p \wedge \bar{q})$

(2e) $-\exists(x)\,\overline{p \to q}$ / $p \wedge \bar{q}$ eqsem $\overline{p \to q}$

(3e) $\forall(x)\,\overline{\overline{p \to q}}$ / $-\exists(x)-F(x)$ eqsem $\forall(x)F(x)$

(4e) $\forall(x)\,(p \to q)$ / $\bar{\bar{p}}$ eqsem p

i.e. the formulae corresponding to sentences (d) and (e) are semantically equivalent. Thus, from a strictly logical point of view, both sentences should occupy the same position in Pottier's scheme.

The case becomes more interesting when the expression *No hay quien* ... is completed with ... *no tenga alhaja si trabaja.*

This can be represented by the following formula:

(1f) $-\exists(x)\,(p \to \bar{q})$

(2f) $-\exists(x)\,\overline{p \wedge q}$ / $p \to \bar{q}$ eqsem $\overline{p \wedge q}$

(3f) $\forall(x)\,\overline{\overline{p \wedge q}}$ / $-\exists(x)F(x)$ eqsem $\forall(x)-F(x)$

(4f) $\forall(x)\,(p \wedge q)$ / $\bar{\bar{p}}$ eqsem p

Obviously, (4f) is not semantically equivalent to (1d). However, in ordinary language we consider sentences (e) and (f) to be semantically equivalent. On the other hand, if formula (4f) were converted into natural language, it would not be regarded as being the equivalent of (e):

(g) *Todos los x trabajan y tienen alhaja*

The implication corresponding to sentence (d) has the value 'false' only if q is also 'false'. Statement (d) is valid under normal conditions, i.e. except under social systems where the subject is so exploited that no matter how hard he works, he will never raise his standard of living.

Example 83: Phonemic changes are often the result of a series of displacements. For example, in keeping with the general shift affecting the double consonants in western vulgar Latin, the phoneme /tt/ became more and more similar to the phoneme /t/ owing to their phonetic resemblance (voiceless, alveolar). The latter, in order to maintain its distinctive function, moved towards the phoneme /d/, and assumed its phonetic features. The original phoneme /d/ in its turn then became converted to the fricative [đ].[22] This provides a compound proposition:

$$(p \rightarrow q) \rightarrow r,$$

in which p ... /tt/ moves towards /t/
q ... /t/ moves towards /d/
r ... /d/ becomes a fricative.

Example 84: Japanese requires at least four verbs for the English *to give*. Selecting the appropriate verb in any one case is determined by the following factors: grammatical person of the giver (d), grammatical person of the receiver (r), and the social relationship between the two. The verbs are: *ageru*, *yaru*, *kureru*, *kudasaru*.

$ageru_1$: (*Watashi wa*) *anata ni hon wo agemashita.*[23]
(I) you gave the book

Let us represent the action 'x gives y' by xRy (asymmetrical relation), and the relation 'd is of higher social standing than r' by dKr. The formula corresponding to the verb $ageru_1$ would then be:

$ageru_2$: (*Watashi wa*) *sensei ni hon wo agemashita.*
'(I) gave the book to the teacher.'
Formula: $(1R3 \wedge rKd) \rightarrow ageru_2$

[22] See E. Alarcos Llorach, *Fonología española* (Madrid, Ed. Gredos, 1954), pp. 103-04.
[23] Translations of the Japanese words are given on pp. 85, 86.

$ageru_3$:	*Anata wa sensei ni hon wo agemashita.* 'You gave the book to the teacher.' Formula: $(2R3 \wedge rKd) \rightarrow ageru_3$
$ageru_4$:	*Seito wa sensei ni hon wo agemashita.* 'The pupil gave the book to the teacher.' Formula: $(3R3 \wedge rKd) \rightarrow ageru_4$
$yaru_1$:	*(Boku wa) kimi ni hon wo yarimashita.* 'I gave you the book.' Formula: $(1R2 \wedge dKr) \rightarrow yaru_1$
$yaru_2$:	*(Watashi wa) seito ni hon wo yarimashita.* '(I) gave the book to the pupil' Formula: $(1R3 \wedge dKr) \rightarrow yaru_2$
$yaru_3$:	*Anata wa seito ni hon wo yarimashita.* 'You gave the book to the pupil.' Formula: $(2R3 \wedge dKr) \rightarrow yaru_3$
$yaru_4$:	*Sensei wa seito ni hon wo yarimashita.* 'The teacher gave the book to the pupil.' Formula: $(3R3 \wedge dKr) \rightarrow yaru_4$
$kureru_1$:	*Kimi wa boku ni hon wo kuremashita.* 'You gave me the book.' Formula: $(2R1 \wedge rKd) \rightarrow kureru_1$
$kureru_2$:	*Seito wa (watashi ni) hon wo kuremashita.* 'The pupil gave me the book.' Formula: $(3R1 \wedge rKd) \rightarrow kureru_2$
$kudasaru_1$:	*Anata wa (watashi ni) hon wo kudasaimashita.* 'You gave me the book.' Formula: $(2R1 \wedge dKr) \rightarrow kudasaru_1$
$kudasaru_2$:	*Sensei wa (watashi ni) hon wo kudasaimashita.* 'The teacher gave me the book.' Formula: $(3R1 \wedge dKr) \rightarrow kudasaru_2$

Vocabulary:

watashi	'I'
wa	subject particle

anata	'you'
ni	indirect object particle
hon	'book'
wo	direct object particle
agemashita	past tense form of the verb *ageru*
sensei	teacher, master
boku	'I' (familiar)
kimi	'you' (familiar)
yarimashita	past tense form of the verb *yaru*
seito	'pupil'
kuremashita	past tense form of the verb *kureru*
kudasaimashita	past tense form of the verb *kudasaru*

If the corresponding formulae are combined with each of the 4 equivalents, we obtain:

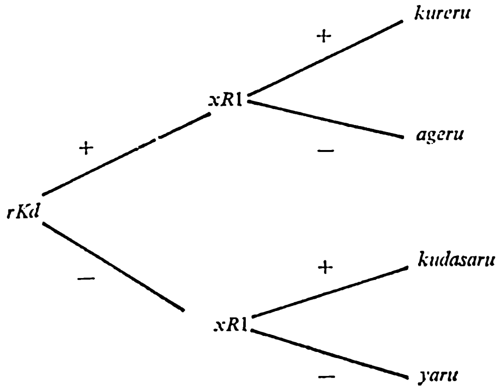
(a) $[(1R2 \vee 1R3 \vee 2R3 \vee 3R3) \wedge rKd] \rightarrow ageru$
(b) $[(1R2 \vee 1R3 \vee 2R3 \vee 3R3) \wedge dKr] \rightarrow yaru$
(c) $[(2R1 \vee 3R1) \wedge rKd] \rightarrow kureru$
(d) $[(2R1 \vee 3R1) \wedge dKr] \rightarrow kudasaru$

Let us now effect the following simplification: The term before R is replaced by x. If the second term is 1, it will be re-written as $xR1$, and if different from 1, then it will be transcribed as $\overline{xR1}$. This new representation will be written just once. Expressions (a) to (d) are then converted into the following:

(a′) $(rKd \wedge \overline{xR1}) \rightarrow ageru$
(b′) $(dKr \wedge \overline{xR1}) \rightarrow yaru$
(c′) $(dKr \wedge xR1) \rightarrow kureru$
(d′) $(rKd \wedge xR1) \rightarrow kudasaru$

The tree below shows the binary coding of the four Japanese verbs. Each node occupied by a formula represents the point of an exclusive alternative decision, i.e. 'either ... or' but not 'both'. If the alternative is 'no' then formulae rKd and $xR1$ are interpreted as dKr and $\overline{xR1}$ respectively.

Here again, in keeping with the definition given previously, $\overline{xR1}$



does not mean 'false' in the conjunction corresponding to each verb, but simply denotes the negation of $xR1$.

Example 85: A large number of asseverative adjectives in Spanish can be classified in three groups:

(1) Adjectives followed only by a clause introduced by the conjunction *que*:

(a) *Es indiscutible que Juan tenga razón.*

(2) Adjectives followed only by an infinitive clause:

(b) *Es fácil traducir esta carta.*

(3) Adjectives accepting both kinds of clause:

(c) *Es posible que Juan venga.*
(d) *Es posible solucionar el problema.*

The distinctive semantic feature seems to be the difference between 'fact' and 'action', which is neutralized in the 3rd group. The classification conforms to the logical function of disjunction:

action	fact	action ∨ fact	group
v	*v*	*v*	3
v	*f*	*v*	2
f	*v*	*v*	1
f	*f*	*f*	∅

It seems that in any language, classifications can be made according to the formula:

(1) *A* or *B*
(2) Only *A*
(3) Only *B*.[24]

Example 86: When two *sememes* (= the meaning of a lexical morpheme (= lexeme)) are compared, their intersection gives the ARCHISEMEME, i.e. the group of semes (minimum distinctive semantic features of a lexeme) common to both. There often exists a lexeme whose sememe is exactly the archisememe of two other lexemes. Such a lexeme is called an ARCHILEXEME. The archisememe becomes included in each of the two sememes forming the logical product. To this situation can be applied the logical implication:[25]

The terms *ophthalmologist*, *pediatrician*, *gynaecologist* refer to medical specialists, whereas general practitioners are usually called just *doctors*. The following statements can therefore be made:

(a) If *A* is an ophthalmologist, he is a doctor:	'true'
(b) If *A* is an ophthalmologist, he is not a doctor:	'false'
(c) If *A* is not an ophthalmologist, he is a doctor:	'true'
(d) If *A* is not an ophthalmologist, he is not a doctor:	'true'

As will be noticed, sentences (a) and (d) conform to the truth table corresponding to the logical implication.

Example 87: As mentioned previously, there are predicates with the argument 0: e.g. the Spanish *llueve*:

with 1 argument, e.g. *Juan duerme*;
with 2 arguments, e.g. *Juan compró un libro*; etc.

In natural language statements are often formed about statements, e.g. *Es probable que Juan tenga razón.*

A statement about another statement is called a REFLEXION. There are statements about statements which, in their turn, are

[24] See: G. Kaufmann, "Sprachforschung und Datenverarbeitung", *Deutschunterricht für Ausländer* 1 (1966).
[25] See: B. Pottier, p. 117.

statements about other statements; this gives reflexions of various degrees.

The logic of predicates permits conversion, e.g. of a negative statement into an EXPLICIT REFLEXIVE FORM: The Spanish negative sentence

(a) *Juan no compró el libro*

can be converted into

(a′) *No es verdad que Juan comprara este libro.*

The negation is a reflexive predicate.

Other reflexive predicates are the conjunction, disjunction, implication and quantification, which all represent truth functions. In natural language there also operate propositional functions of a psychological kind reflected in such expressions as, *creo, puede ser, muchos, pocos*, etc.

Most grammatical elements accept explicit reflexive forms.

(b) *Juan compró un libro ayer*

can be transformed to

(b′) *Fue ayer que Juan compró un libro.*

This Gallicism would not be stylistically acceptable, nevertheless, it will be noted that here the adverb is a statement about another statement.

Sentences and even full texts can be analyzed as regards their reflexive structure. Let us, for example, analyze the following Spanish sentence:

(c) *Carlos cree que Juan llamará hoy o María no enviará el dinero mañana.*

This sentence can be transcribed in symbols in such a way that the reflexions will remain explicit, using brackets to indicate the range of the predicates. A predicate before the brackets relates to all the terms separated by a comma within the brackets. The following symbols will be used:

Arguments:

c ... *Carlos* j ... *Juan* m ... *María*
d ... *dinero*

Predicates with 1 argument:

H ... *hoy* L ... *llamar*
M ... *mañana* - ... *negación*

Predicates with 2 arguments:

C .. *creer* E .. *enviar*
V .. *o*

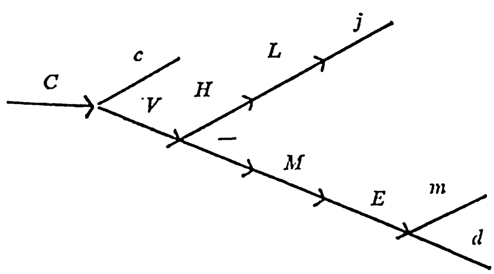
We then have the following formula:

$$C\,\{c,\ V[H\ L(j),\ \text{-}ME(m,d)]\}$$

We now replace each argument by the letter *a*, and each predicate by *P*. Using superscripts for series of more than one *P*:

$$P\,\{a,\ P[P^2a,\ P^3(a,a)]\}$$

This structure can be shown by a graph in which the arrows indicate the predicates and the lines the arguments. Where two lines originate from a node, the arrow terminating at this node represents a predicate of two arguments:[26]



In the graph there is a maximum sequence of 5 arrows, for which reason we say that this is a 5-degree structure of reflexion.

Stylistic features can be determined by this kind of analysis.

[26] See: G. Frey, "Reflexionsanalysen von Texten", *Studium Generale* 19:7 (1966).

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