XI

NUMERALS AND THEIR NOTATIONAL SYSTEMS

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I

PART I: THE COMMERCIAL NOTATION OR *MA TZU*

THE Chinese people have two systems in current and concurrent use for writing their Numerals.

Of these, one is the ordinary notation with characters of pictographic origin, except the first three cyphers.

The other has been called the "commercial" notation, and, though now invaded by members of the first, was once predominantly, and still remains essentially, a series of tally-strokes, or counters symbolized. The distinguishing feature of this notation is the collocation of vertical and horizontal strokes. These combinations have been known to the Chinese from ancient time as *ma tzŭ*, a term which I shall discuss later, and in view of their configuration I have in this paper called them "rod-and-bar" groups, as a conveniently descriptive name.

I propose to examine both these systems. The questions which disclose themselves are neither few nor free from difficulties, but with the help of recent research and discoveries, I hope to solve some of these, to make suggestions as to others, and where neither course is possible, to show in more precise outlines the obstructions barring the advance of historical exploration.

At this point I wish to make a small confession. Much the ground covered in my paper had been previously traversed by the late M. Terrien de Lacouperie, in an essay contributed to the *Numismatic Chronicle* in 1883, under the title "The Old Numerals, the Counting-Rods,

and the Swan-Pan in China". That essay is one of the sounder and more sober efforts of the French writer, and contains much with which I agree, much also from which I greatly differ. But had I re-read it (after some thirty years interval) before, instead of after, commencing the independent inquiry for the present paper, I doubt if I should have delved again in this particular field. But a generation has passed since then, new material has accumulated, and I hope a fresh investigation will not prove quite unfruitful.

It will be convenient to examine first the Commercial numerals, and to trace them backwards to the earliest point accessible to us, through several phases of development and modification. In this study I have in the main followed the guidance of a native work, the 金石契 *Chin Shih Ch'i*, or Texts on Metal and Stone, by 張燕昌 Chang Yen-ch'ang, a translation of whose valuable notes on the subject will appear below in full. But here I shall introduce the special facts brought to light by the author in a rather more concentrated shape than that of their presentation in Chang's original Chinese text.

Immediately below are set out in their modern form the members of the two notations for reference and comparison :—

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
Normal.	一	二	三	四	五	六	七	八	九	十
Commercial.	𝍠	𝍡	𝍢	𝍣	𝍤	𝍥	𝍦	𝍧	𝍨	𝍩, and the zero 〇.

Let us note first of all the presence of a zero in the commercial notation. Giles gives no representative of Ten in his list of the commercial numerals, and it might be reasonably inferred that as with our "Arabic" numerals, so with these, Ten would be written 10. But it is not. Instead, the sign 𝍩 is borrowed from the normal series, though in the numbers from 11 to 19 the vertical stroke is often used.

Next observe that where necessary the vertical strokes for 1, 2, and 3 can be replaced by horizontal lines. To quote Professor Giles (*Chinese–English Dictionary*, 1st ed., p. 1385), "when 1 and 2 or 3 come together, they are written alternately vertically and horizontally. Thus 12,332 would be written 𝍠𝍡𝍢𝍣𝍤."

The cyphers for the numerals 4, 5, and 9 require special attention. They are difficult to explain, and do not conform to the earlier signs standing for those numbers in this system. The first two of the three may *possibly* be naturalized aliens, as, it would seem, the zero also must be. The case of 𝍥 for 4 is the more singular inasmuch as this cruciform sign is a well-known ancient Chinese form of 5. As we shall see, the author of the *Chin Shih Ch'i* considers it frankly inexplicable. It is indeed hard to understand how, if of indigenous origin, it could have been perverted to denote 4. In a suggestive and ingenious paper on "The Evolution of Modern Numerals from Ancient Tally Marks", published in the *American Mathematical Monthly* for August–September, 1909, and kindly sent me by the author, Major Charles E. Woodruff, A.M., M.D., U.S. Army, the latter writes, "The X which represents the commercial four is a direct descendant of the four vertical tally marks." I confess I do not see how 𝍥 can be directly, or even indirectly, descended from 𝍧, and prefer the blank ignorance of the writer of the *Texts on Metal and Stone.*

The use of 𝍦 for 5 is less difficult to account for. (We may note in passing the curious likeness to 𝍪, an old form of an Indian cypher for 4.) Lacouperie thought this Chinese sign "a cursive form of the regular numeral for five".[1] But there is no such cursive form. Major Woodruff, on the other hand,[2] writes, "The commercial five, which looks like our eight, is also a direct descendant of a very old form of five strokes which early replaced

[1] "The Old Numerals, etc., in China," p. 29. [2] Loc. cit., p. 131.

the five parallel ones—the transition forms given by Chalfant being quite conclusive as to this point." I demur to this statement, or rather all these statements. To my eyes there is little resemblance between 〥 and 8. I do not gather which of the transition forms given by Chalfant, Major Woodruff refers to. They are 𠄡, 㐅, 㐅, 乂, and 𠄡. But this last example is an error. It is taken from a coin, one of the Ming knives 明 刀 series,[1] and is clearly due, if not to poor printing, to a badly cut or badly worn 𠄡. No second example exists. I do not believe the commercial five descends from any of what Major Woodruff thinks "transition forms". Far more probable seems the explanation given by Chang Yen-ch'ang, the author of the *Chin Shih Ch'i*, that the sign in question is simply the "grass" or cursive form of the homophonous character 午 *wu*, with which in fact it is almost exactly identical; thus, 〥 = 午, and 〥 = 五.

There may be some doubt as to the origin of the sign 〩 for 9. It *may* be, as suggested in the extract from the *Chin Shih Ch'i*, a form slightly altered for convenience of writing, of the homophonous character 久 *chiu*, long-lasting. But in that case the question arises at once why, if the rod-and-bar combination for 9 was to be abandoned as too cumbrous, should not the simple numeral character 九 *chiu* itself have been adopted, a character quite as easy to write as its homophone *chiu*, long-lasting? A satisfactory answer is not easy to give.

It will be well, before proceeding further, to illustrate by actual and authentic examples the use of the present-day *ma tzŭ* or commercial numerals, because they exhibit some features not to be anticipated from a survey of the individual cyphers.

I have already said that 十, not 10, is the figure used to write Ten. This holds good also when that numeral is

[1] See the *Ku Ch'üan Hui* 古 泉 匯, Section Hêng 亨, p. 2.

part of a number of higher value. Thus 610 is written 〦十 (where the subscribed 〩 stands for 百 *pê* = 100).

Again, the numbers between Ten and Twenty, whether alone or in higher combinations, are usually not written with the appropriate strokes following a vertical stroke (though this mode is sometimes employed), but the strokes follow the cruciform 十. Thus, 1019 is 〡〇十〩 (where the subscribed ノ, or more fully 千, stands for *ch'ien* = thousand). Twenty and Thirty are written 〹 and 〺. Thus we have 〡〇〹一 for 1021, and 〩〺三 for 933.

These two groups also illustrate the use of the alternative horizontal forms of 1, 2, and 3, when it is desired to avoid any confusion with vertical strokes preceding on the left. In some cases it is indifferent whether the vertical or horizontal stroke is written. For instance, the list from which all these examples are copied[1] has 〩〥〡 for 951, but 〩〥二 for 952. And again, 〡〇〇〡 for 1001, but 〡〇〇二 for 1002. A further point shown in these latter groups is the vertical, and not horizontal, succession of two zeros coming together. With round numbers in the hundreds or thousands, the zeros are not used. Thus, 600 is 〦〩, and 1000 is 〡千, where we have a contracted form of 百 *pê* = hundred, and the full form of 千 *ch'ien* = thousand, preferred.

And lastly should be noticed the survival in this system of a very ancient substitutional character for 一 *i*, one, viz. its homophone 乙 *i*, the second of the Ten Cyclical Stems. Thus, 〥十乙 is 511, and 〤〥乙 stands for 451.

We have now to trace backwards, as far as we can, this notation of numerals as an integral system. When it first assumed its present aspect, there seems no evidence to

[1] It is a list of the different documents recovered by Sir Aurel Stein, numbered in *ma tzŭ* by his Chinese secretary.

show. It is not mentioned in the Index to the great *T'u Shu Chi Ch'êng* Cyclopedia, as Dr. Lionel Giles informs me. Being an organ of the counting-house and the workshop it was not found worthy of record in the library or the study. Neither is it known why these *ma tzŭ* are specially associated with the great city of Soochow in Kiangsu Province.

When we next meet the notation, it is in such an altered garb that, at first, it might seem not to be an earlier stage at all, but a distinct and independent scheme. Yet it really is essentially the same, possessing the key-character of the rod-and-bar combination for the digits between 5 and 10.

Kublai Khan is not a name that associates itself exactly with the peaceful methods of the mathematician. But it was during his lifetime, in the thirteenth century, that there flourished and faded two such scholars, by name Li Yeh 李冶 and Ch'in Chiu-shao 秦九韶. Both of them employed an identical notational apparatus, which is consistent, convenient, and clear, and is marked by alternative arrangements of the rod-and-bar groups, by the Chinese styled the "vertical" and the "horizontal" schemes. Thus, they wrote for the cyphers from 1 to 9, either 𝍠, 𝍡, 𝍢, 𝍣, 𝍤, 𝍥, 𝍦, 𝍧, 𝍨, or 𝍩, 𝍪, 𝍫, 𝍬, 𝍭, 𝍮, 𝍯, 𝍰, 𝍱. The object of these alternative schemes was the avoidance of confusion, and, as can be easily seen, was absolutely necessary. But it may be asked, why is 𝍥 considered "vertical", and its reversed form 𝍮 "horizontal"? It is because in 𝍥 and the following digits the vertical strokes represent 1, 2, etc., and the horizontal is 5, while in 𝍮 it is the vertical line that has the value of 5, and the horizontal strokes that are units. It should also be noted that this is the only stage of the rod-and-bar system where the digit Five is represented by five vertical or horizontal lines.

There is an interval of two centuries before we come

on the mathematical notation again, when it reappears in the work entitled *Ch'ien Hsü*, by Ssŭ-ma Kuang (b. 1009, d. 1086). Here are the cyphers as employed by this author: 𝍠, 𝍡, 𝍢, 𝍣, 𝍤, 𝍥, 𝍦, 𝍧, and 𝍨. How Ten was written is not expressly stated.

Notice the curious transfer of the same sign 𝍤 from the place of the fifth digit in the eleventh century to that of the fourth digit in the modern *ma tzŭ* or commercial figures. The notation thus appearing in this Sung dynasty writer is very like that displayed on a series of ten Square-footed *Pu* coins issued by the Usurper Wang Mang during his reign from A.D. 9 to A.D. 23. The coins, however, have the digits from 1 to 4, written with horizontal, not vertical strokes, and Five is expressed by 𝍩, a unique variant of 𝍪, the ordinary old pictographic symbol for that numeral.

The Usurper was, we are told, "in all things an imitator of antiquity," and doubtless had documentary warrant for the numeral forms he selected for this monetary issue. And, indeed, we can find them—except the Five—in the bronze *Pu* coins and knife coins of the third, fourth, and fifth centuries B.C., issued at various mints in North China.

The notation in which they occur, and of which they seem to be only an inconsiderable part, may be studied in the pages of the *Ku Ch'üan Hui* 古泉滙, or Thesaurus of Ancient Coins, vols. iii to vii (Sections Yuan 元 and Hêng 𝍫). The system there abundantly illustrated, but not fully elucidated in the text, presents a problem which, to me at least, proves as difficult to solve as it is impossible to ignore. Chinese scholars at any rate have not solved it. The numismatist has felt it right to make it over to the epigrapher. The epigrapher, with a sigh, has handed it back to the numismatist. They do, however, agree that the enigmatic notation is of the nature of *ma tzŭ*, our "commercial" system.

Of Western writers the only one to essay a solution has been the late Terrien de Lacouperie, who published a Table of "Numerals from the Chinese Coins of the Fourth and Third Century B.C.", where the numerical equivalencies of the signs in question are set out as he supposed them to be. I shall return to this Table later.

The coinage on which this system of notation is found is of that peculiar shape known as *Pu* 布, principally in the "Pointed-foot" variety of that class, though the "Square-foot" type and knife coins also afford numerous examples. The accompanying illustration shows the type.

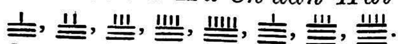
The outline rather suggests something which the authors of its being had intended to be a spade, but that itself inclining to an easier and more artistic career, it wished to become a tuning-fork, and in the end, having lost the robust simplicity of its origin, without achieving the elegant attenuation of its desires, had lapsed into the indeterminate imagery of a coin.

On one side of these pieces is written, commonly in negligent and contracted style, the name of the city of issue. The reverse displays (see the above illustration) what is by general admission a numeral.

Unfortunately nothing on the coins themselves indicates to what this numeral refers, and it is this silence that renders the numerical equivalence of many of the rod-and-bar combinations so hard to unravel. It can hardly be a unit of value, as we should naturally expect, because different numerals occur on specimens of the same size, weight, material, and issue.

One interesting feature met with in examining the coins of these series is the apparent indifference with which the numerals of the two categories, the normal and the commercial, were used. There appear to be no prejudices; sometimes the figures of one class were written, sometimes of the other. Thus, on the Ming Knife coins

we find both  and  for 6, the first being of the *ma tzŭ* or commercial series, the other a common old variant of the normal or pictographic numerals. So on the same set of coins we have both  and  for 7. So again, on a sequence of Square-foot coins (*Ku Ch'üan Hui*, Section Yuan 元, iv, p. 11), both  and  stand for 8. But on the whole, apart from the ambiguous compounds about to be discussed, the normal forms of the digits up to Ten are far the commoner.

Now if the facts stopped here, the matter would be much simpler and less perplexing than it is. We should have traced our commercial series in its salient and fundamental elements to the earliest point provided by the documents, and should have reached a clear-cut terminus. But the facts disclosed do not stop there, but on the contrary introduce us to further material of the most puzzling nature. The native numismatists, competent scholars as they are, have been baffled, and the only solution hitherto attempted, so far as I am aware, is that of the ingenious French investigator Terrien de Lacouperie. The new material referred to is furnished by the frequent occurrence of other analogous rod-and-bar combinations, but differing from the  , standing for 6, 7, 8, and 9, by having *more than one* vertical stroke in the upper register. Here is a sequence of such groups collected from the *Pu* and Knife coins figured in vols. iii to vii of the *Ku Ch'üan Hui* :   . It will be seen that $\frac{4}{2}$ and $\frac{2}{4}$ are missing, and that there is only a single example of five vertical strokes.

How ought we to express these groups of strokes in terms of our arithmetic ?

Probably we can narrow this question down to one of smaller scope, What is the value of the vertical stroke in any of these groups ? Does it stand for 1, or for 5, or for 10 ?

After considerable study of the figures of the *Thesaurus of Ancient Coins* in which these rod-and-bar combinations appear, I must confess they have beaten me, and while unable at present to adopt Lacouperie's solution I can produce no better one.

Lacouperie's Table assumes the value of 10 for each vertical stroke, and on this assumption he has constructed a series of numbers from 1 to 65, recorded on fourth and third century B.C. coins, from which only the numbers 17, 46, 51, 54, 56, 58, and 59 are absent. Now in favour of this interpretation are the facts that on the ancient bronzes often, and on the Honan bones nearly always, a vertical stroke does stand for the numeral 10. And Lacouperie seems to have the valuable support of Lo Chên-yü, who asserts in his recent illuminating work, the *Yin Hsü Shu Ch'i K'ao Shih* 殷虛書契考釋, p. 16, that on the Honan bones, as on the smaller Pointed-foot *Pu* coins, 10 is always written | (a statement which, as regards these *Pu*, I must demur to).

But against this must be weighed the following objections. The equation of the vertical stroke with 10 would be an obvious one, and cannot have failed to occur to the Chinese numismatists. Yet so recent an authority as the author of the *Ku Ch'üan Hui* ignores it. Another difficulty is this. The numerals on these coins that we can identify with certainty are, in by far the most cases, the digits up to and including Ten, with a certain residue between Ten and Twenty. It would be difficult to account for the sporadic presence of numerals of so high a value as is required by Lacouperie's hypothesis, among groups having so small an average value. More serious than either of these objections, however, is the fact that Lacouperie's Table attributes a double value to  and  , which are made to equal in one case, both 8 and 13, and in the other both 9 and 14. How could any notational system tolerate such an inconsistency ?

Escaping from this atmosphere of uncertainty, we can emerge into the definite and precise knowledge that no trace of the commercial notation is to be discovered in the far older inscriptions of the Honan bones.

The subjoined translation of an extract from the *Chin Shih Ch'i* 金石契 of Chang Yen-ch'ang 張燕昌, published in 1778, will, I hope, be found to justify the outline of the system traced above, and to contain several points of interest to students of Chinese antiquity. The text, being intended by a Chinese scholar for other Chinese scholars, assumes a knowledge of many things on the part of his readers that the Occidental student does not usually possess. I have therefore thought it advisable to add such explanations and comments of my own as my limited competence allows, enclosed in square brackets, which cause less discomfort than the jerky dislocation of the vision involved by footnotes.

EXTRACT FROM THE *CHIN SHIH CH'I*, SECTION 角.

(Translation.)

With regard to the numbering of the Ten *Pu*-coin series, the Hsiao Pu, 100, the Yao Pu, 200, the Yu Pu, 300, and the Hsü Pu, 400, these all use aggregations of straight strokes, corresponding to the inscriptions of the round coins known as 泉貨 *ch'üan huo* [also issued by Wang Mang]. The Ch'a Pu, 500, also corresponds to the character on these in being written 𠫓. But the 丅百, 600, of the Chung Pu, the 丌百, 700, of the Chuang Pu, the 丗百, 800, of the Ti Pu, and the 丗丨百, 900, of the Tz'ŭ Pu, correspond to the present-day 號碼, *hao ma*, or sign-marks [the so-called Soochow numerals, or commercial series], in vulgar use. [But note that they are *analogous*, not identical, for 丄 is the modern 6, not 丅.] These are rarely found in old writings. I propose to examine the question.

Characters for the numerals were used by the ancients

to record counters. Accordingly the written forms resemble the shape of counters. *Suan* 算, or counter, is equivalent to *shu* 數, or reckoning.

(*Original note* [by the author Chang Yen-ch'ang].—Based on the *Erh Ya*, the *Shuo Wên*, the *I Li*, the *Li Chi*, and the *Lun Yü*, as annotated by Chêng K'ang-ch'êng.)

The *Han Shu, History of the Han Dynasty*, writes, Reckoning starts with the unit, and proceeds through Ten, a Hundred, a Thousand, to Ten Thousand, and is the means for counting actions and objects.

The *Shih Pên* 世本 states that "Li Shou was the first to make calculations".

"The Courtier" 舍人 [*alias* 劉歆 Liu Hsin], in his commentary on the *Erh Ya* remarks, As to the use of the word *suan* 算 to explain the word *shu* 數, in counting numbers certain objects are used. The name of these objects is *suan* or counters. Hence, in the *District Archery Record* of the *I Li*, we have the phrase "contains six *suan* or counters". In the *T'ou Hu* 投壺 Chapter of the *Ta Tai Li* and the *Li Chi*, we have the words "The *suan* or counters are 1 foot 2 inches in length". Chêng K'ang-ch'êng comments, "The *suan* were 1 foot in length and had 握 *wo*. *Wo* is equivalent to 數 *shu*, a fixed number." [The words 有握 *yu wo*, are otherwise translated by Couvreur, "and have a handle."]

(*Original note.*—The *Han Shu* has the following passage: "In calculating, bamboos were used one-tenth of an inch in diameter and 6 inches in length. Of these there were 271 in all, composing an hexagonal handful or bundle.")

The *suan* was also called 籌 *ch'ou*. In the *Ta Shê Li* section of the *I Li* occurs the passage, "The bow and arrows of the guest, with the cylinder 中, *chung*, and the counters, *ch'ou*, were all retained below the western hall." And the *Hsiang Shê Li* has the words "Eighty bamboo counters, *ch'ou*", on which Chêng K'ang-ch'êng annotates,

"*Ch'ou* is equivalent to *suan*." In the *Liu Hou Shih Chia*, 留侯世家, chapter of the *Shih Chi*, the phrase 運籌 *yün ch'ou*, to shift the counters, is equivalent to the phrase 運筭籌 *yün suan ch'ou*, of works on magic, 術書 *shu shu*. [*Suan* or *ch'ou* is] also interchanged with the term 疇 *ch'ou*. The expression 疇人子弟, *ch'ou jen tzŭ ti*, is equivalent to 算人之子弟 *suan jên chih tzŭ ti*, the descendants of the calculators.

(*Original note.*—Ju Shun's comment is, "Their hereditary calling was handed down generation after generation for the calculation of the almanack. For twenty-three successions the official calculators each followed their father's science," 家業世世相傳疇年二十三傳之疇官各從其父學.)

[Chavannes, in a note on p. 326 of vol. iii of his *Mémoires historiques de Se-ma Ts'ien*, writes, "Remarquer l'expression 疇人 qui s'est conservé jusqu'à nos jours pour désigner les astronomes et les mathématiciens." I do not quite understand why Chavannes, in the text itself, instead of rendering the words 疇人子弟 *ch'ou jên tzŭ ti* by "les descendants des mathématiciens", paraphrases, or rather periphrases, the expression by "les descendants des hommes dont la fonction était héréditaire". It is to be noted that the Kanghsi editors, though not defining *ch'ou* as hereditary, yet have quoted only part of Ju Shun's comment, ending at the first *ch'ou*, omitting the following 年 *nien*, year, the object of the verb, and inserting 爲 *wei*, before *ch'ou*, thus misleading the reader into supposing that what Ju Shun meant was, "a calling handed down generation after generation is *ch'ou*." But this was tampering with his text.]

The *suan* was also called 馬 *ma*, horse. [At least this is the face value of the character. I discuss the point further on.] In the *T'ou Hu* chapter of the *Li Chi* occurs the passage, "The Director of the game begs permission to set up a horse for the winner," 司射請爲勝者立馬,

corresponding to the "one horse follows two horses", 一馬從二馬, previous to the Director's "depositing a counter", 釋算 [which the text says he did for each successful throw in the later part of the match]. Chêng K'ang-ch'êng observes on this, "The third horse was the winning counter 勝算. They distinguished this third counter so as to mark the winning counter. The reason for calling it *ma*, horse, was as much as to say, 'With skill such as this, you could be a General and ride a charger.'" [Such an explanation is absurd, neither does it explain the phrase *i ma ts'ung erh ma*, one horse following two horses.]

The *Shuo Wên* under the Radical *chu*, bamboo, has two [contiguous] entries, 筭 *suan* and 算 *suan*. The first is explained as follows: "Six inches in length, and used for making calculations. Composed with *chu*, bamboo, and 弄 *lung*, to handle, expressing that by constant practice [*lit.* handling] mistakes are not made."

Under 算 *suan*, the *Shuo Wên* writes, "To count up, 數 *shu*. Composed with *chu*, bamboo, and 具 *chü*, instrument. Read like 筭 *suan*." And under the Radical 丌 *shih*, to display, there is a separate entry 祘 *suan*, which is explained as "To regard clearly so as to reckon up. Composed of 丌 *shih*, doubled. The lost Books of Chou have the words [here follows a passage of eleven characters, which, as the editor of Wang Yün's edition of the *Shuo Wên* considers them "obscure", I shall not attempt to translate, but 祘 *suan* occurs twice in it]. Read like 算 *suan*". The *I Ch'ieh Ching Yin I* regards the character 祘 as the ancient form of 算 *suan*, for the ancient form [would] correctly depict the shape of *suan* or counters. [Incidentally too, we may notice, it would depict two of the rod-and-bar groups.]

The author of the *Chin Shih Ch'i* then continues: My own humble opinion is this. The ancients, when they laid out counters, at first only placed them horizontally.

When thus placed to the amount of five, their number led to the risk of error, and so two counters were crossed diagonally, thus X.

(*Original note.*—The *Shuo Wên* gives X as "the ancient form" of Five, and says that it symbolizes the crossing of the Yin and the Yang.)

For Six, one counter was laid horizontally, and one vertically, thus 丅, without crossing. From this point, the numeral Five became — [viz. a horizontal line], and the numeral One, | [a vertical line], and for Seven, Eight, and Nine, successive lines were added vertically to differentiate them [thus 𝍠 = 7, 𝍡 = 8, 𝍢 = 9]. When the full tale of Ten was thus reached, two counters were taken, one vertical and one horizontal, and crossed at right angles. This is expressed by the *Shuo Wên's* words, "Ten is the completion of the numerals. The stroke — stands for East and West, | for North and South, thus completing the Four Quarters and the Centre." And the *Shuo Wên* is right, for there are five numerals affected to Heaven and five to Earth. And in writing, a horizontal stroke above stands for Heaven, and one at the bottom for Earth. Hence —, One, can take the place of Five. [The foregoing explanation, as well as the *Shuo Wên's* as to Five and Ten, are pure will-o'-the wisp symbolism, that secular obsession of the Chinese mind.]

The use of the signs 丅, 𝍠, 𝍡, and 𝍢, for 6, 7, 8, and 9, first appears in the 潛虛 *Ch'ien Hsü* of Ssŭ-ma Kuang of the Sung dynasty. In using X for 5, the *Ch'ien Hsü* employs the regular ancient cross-sign, but is singular in writing vertically 𝍦, 𝍧, 𝍨, and 𝍩 for —, 𝌆, 𝌇, and 𝌈, 1, 2, 3, and 4.

In the calculations of the 測圓海鏡 *Ts'é Yuan Hai Ching* of 李冶 Li Yeh of the Yuan dynasty, there are both vertical and horizontal digits. Thus we find, written vertically, 𝍦, 𝍧, 𝍨, 𝍩, 𝍪, 丅, 𝍠, 𝍡, 𝍢, and horizontally, —, 𝌆, 𝌇, 𝌈, 𝌉, 丄, 𝌊, 𝌋, 𝌌. Note that 丄 is

called "horizontal" because its horizontal line here denotes One, whereas in 丅 it denotes Five.]

With these counter combinations, when the leading one is written horizontally, the next is vertical, the next horizontal again. Thus, 123 is written 一𝍠𝍡. If the leading group is vertical, then the next is horizontal, and the next again vertical. Thus 678 is written 丅𝍢𝍣. We may infer that the reason of this was the fear of confusion if the same disposition of the strokes were used for all the digits [viz. if *all* were written either vertically or horizontally], and so a distinction was made in this way.

Contrasted with the method of the *Ch'ien Hsü*, we have in this scheme the digit 5 written with that number of strokes, and the digits below 5 optionally written with horizontal strokes.

With regard to the modern system in vulgar use known as "Marking figures", 號碼 *hao ma*, which runs thus, 𝍠 (1), 𝍡 (2), 𝍣 (3), 𝍤 (4), 𝍥 (5), 𝍦 (6), 𝍧 (7), 𝍨 (8), and 𝍩 (9), the digit 4 is not written with four vertical strokes. [By an obvious slip the text prints 八, instead of 四 or perhaps 𝍤, for 4.] The sign 𝍥 for 5 is in my humble view the "Grass character" form of 午 *wu*, cross [a homophone of *wu*, five]. As to the sign 𝍩 for 9, my friend and neighbour, Mr. Wu Ling-yün, thinks it is just the character 久 *chiu*, long-lasting [a homophone of *chiu*, 9].

(*Original note.*—The present-day adoption of 玖 *chiu* [said to be a kind of quartz] as the Majuscule [or "Bankers"] form, 大字 *ta tzŭ*, of 9, goes back as far as the 五經文字 *Wu Ching Wên Tzŭ*, of T'ang dynasty times [which was first published in A.D. 876].)

But the use of the form 𝍤 to represent 4 is impossible to understand [especially as it is one of the old forms of 5]. Besides, while in imitation of the ancients, it certainly has not the ancient significance [which was 5. I have translated, as I understand them, the author's words, which are 且與古人相殺必非古義].

With my shallow learning and dull intelligence, I am not inclined to forced views or glossing interpretations, but I venture to observe that Ssŭ-ma Kuang being, as he was, a Sung dynasty statesman of great repute, and widely versed in historical literature, these numerals of his must have had documentary sanction. Hence I have taken occasion from the Ten *Pu* coins to examine the question, with the hope that if my views should come to the notice of any learned scholar he will correct any errors into which I may have fallen.

(*Original note.*—Mr. Li Jui 李銳, of Soochow [died A.D. 1818, see Wylie's *Notes on Chinese Literature*, pp. 99–100, who calls him "probably the most distinguished writer on mathematics during the present century"], was deeply versed in mathematical processes, and he has confirmed the opinions expressed above, by evidence not within my access, which I now append below. Mr. Li Jui writes:—)

The arithmetical methods followed by the Sung dynasty writer Ch'in Chiu-shao 秦九韶, in his *Shu Hsüo Chiu Chang* 數學九章 [dated A.D. 1247, see Wylie, p. 93, who cites the work with 書 *shu* in place of 學 *hsüo*], do not differ from those of Li Yeh in his *Ts'ê Yuan Hai Ching* and *I Ku Yen Tuan* 益古演段 [dated A.D. 1248 and 1282 respectively]. The notation is by alternate vertical and horizontal strokes — what is known as "the recumbent and the erect counters", 臥算 豎算 *wo suan shu suan.* Although ancient works contain no examples of arithmetical notation, yet we have in the *Sun Tzŭ Suan Ching* 孫子算經 [see Wylie, p. 91, "nothing is known of the author Sun tsze, but it is supposed to have been written about the third century"] the phrase 五不隻六不積 *wu pu chih liu pu chi*, Five not single six not accumulated. Here the words "Five not single" mean that up to and including Five, each digit must have a corresponding

number of strokes. [Thus 1 has one stroke, 2 has two strokes, and so on.] The words "Six not accumulated" mean that from and above Six, one of the strokes must have the value of the numeral 5. [Thus in 丅 = 6 the horizontal stroke = 5, the vertical = 1; conversely, when 丄 = 6 it is the vertical stroke that = 5 and the horizontal that = 1], thus agreeing with the notational methods of Ch'in Chiu-shao and Li Yeh. We find also in the same work the passage 凡算之法一縱十橫百立千僵十千相望萬百相當 *fan suan chih fa i tsung shih hêng pê li ch'ien chiang shih ch'ien hsiang wang wan pê hsiang tang*. Here, by *i*, one, is meant what nowadays is called 單立 *tan li*, a single upright [viz. 1]. The word *li* [the tenth character in the above passage] is equivalent to 縱 *tsung*, vertical. The word *chiang* is equivalent to 橫 *hêng*, horizontal. *Shih*, ten, and *ch'ien*, thousand, are separated by one place, as are *pê*, hundred, and *wan*, ten-thousand [viz. in the notation of large numbers where "position" is in question]. This is the same system as "the alternate vertical and horizontal strokes" of both authors, Ch'in and Li. [We can now translate the above quoted passage, which is in rhyme in the Chinese, as follows: "In all calculations the method is that One is vertical, Ten is horizontal, Hundred stands erect, Thousand lies prostrate. Ten and Thousand look across at each other. Ten-thousand and Hundred mutually correspond."]

It is not known in what period Sun Tzŭ lived. Perhaps he may be the same as 孫武 Sun Wu [sixth century B.C.]. The important point is that he lived before the time of the Wei dynasty [A.D. 220–64].

Thus far Chang Yen-ch'ang and Li Jui. I will conclude this part of my paper by devoting a few sentences to the meaning of the term *ma*, now denoting the cyphers of the commercial numerals, and variously written 碼, 瑪, and 馬.

If the extract translated from the *Chin Shih Ch'i* is examined, it will be seen that the author does not explicitly identify the syllable *ma* in its modern use with the word which in the *T'ou Hu* or Arrow-pitching chapter of the *Li Chi* is written as *ma*, horse. But there can be little doubt that the same *word* is in question in both cases. In the Arrow-pitching chapter, however, the native commentators agree that the word there means a bamboo marking-counter, some holding that it was a mere *alias* of *suan*, others that it was not so, but a marker of a special kind. After carefully examining the several passages in which it occurs, I find that they are all—except perhaps the last—compatible with the sense of "score" or "mark", made on a surface, for the purpose in this case, of recording a winning throw. The last instance of the word perhaps rather militates against this acceptation. The Director of the game is said, on the termination of the match, to ask permission "to remove the *ma*", 徹 馬 *ch'é ma*, an expression, it may be thought, unlikely to be used of a marked-up score. On the other hand, a little earlier in the chapter we have the words 請 立 馬。馬 各 直 其 算 *ch'ing li ma, ma ko chih ch'i suan*, (the Director) "requests leave to set up the *ma*, each *ma* is equivalent to a counter". If a *ma*, in fact, *was* a counter, would it be said to be equivalent to or count as one? However this may be, we can safely regard the word, in its ancient and its modern use, as one and the same, and so bring to an end Part I.

(*To be continued.*)