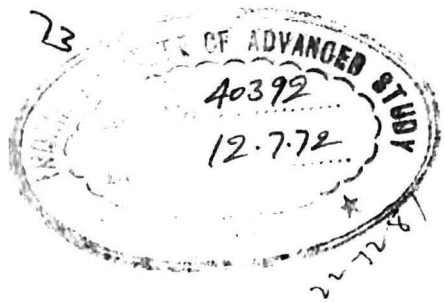
UNIVERSITY OF RAJASTHAN STUDIES

PHYSICS

JAIPUR

1965



CONTENTS

THERMAL CONDUCTIVITY OF MULTICOMPONENT MIXTURES OF INERT GASES II

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Recently we[1] have discussed the four commonly used methods of calculating the thermal conductivity of mixtures of monatomic gases. A critical assessment of the relative merits of these methods was presented on the basis of numerical calculations on mixtures of inert gases. These procedures are refered as rigorous, approximate, empirical and semiempirical. Since the publication of this work, considerable work has been done in this direction in this laboratory as well as at a few other research centres. The purpose of this article is to discuss all these different methods with a view to throw some light on their relative merits and appropriateness.

All the four methods of computing thermal conductivity of mixtures of monatomic gases, λ_{mix}, have already been described in detail by Saxena and Gandhi[1]. We, however, present here a brief description for relevance to proper reference and understanding. In the rigorous method the theoretical expression derived by Muckenfuss and Curtiss[2] as modified by Mason and Saxena[3] is used. This rigorous expression has been cast into the following form by Mason and Saxena[4] :

$$\lambda_{mix} = \sum_{i=1}^{n} \lambda_i \left[1 + \sum_{\substack{k=1 \\ k \neq i}}^{n} G_{ik} \frac{x_k}{x_i} \right]^{-1} \qquad \text{.....(1)}$$

Here λ_i is the thermal conductivity of the pure component i, x_i is the molefraction of the $i-th$ component, and G_{ik} is given by an expression which involves only pure thermal conductivities and molecular weights, M, of the various components of the multicomponent mixture. This is approximate method. In the empirical method G_{ik} of *Eq.* (1) are treated as disposable parameters and are

determined from the knowledge of λ_{mix} values at two composi tions. In the semiempirical method one λ_{mix} value and the relation

$$\frac{G_{ik}}{G_{ki}} = \frac{\lambda_i}{\lambda_k}, \qquad \text{.....(2)}$$

are used to determine G_{ik} and G_{ki} of Eq. (1)

In view of the fact that the λ_{mix} expression given by Eq. (1) forms the basis of so many procedures for calculating λmix, it will be useful if the relationship of Eq. (1) with the rigorous expression may be better understood than that given by Mason and Saxena.[4] Recently Gambhir and Saxena[5] have shown that the rigorous λmix expression can be expressed into the form of Eq. (1) with the following relation for G_{ik}:

$$G_{ik} = \frac{B_{ik} - Bx_i \left(x_k + B_{ki} x_i\right)^{-1}}{1 + Bx_k \left(x_k + B_{ki} x_i\right)^{-1}}, \qquad \text{.....(3)}$$

where

$$B_{ik} + B_{ki}^{-1} = b_{ik},$$

$$B_{ik}\, B_{ki}^{-1} = \beta_k,$$

$$B = B_{ki}\, \beta_{ik} \left(\lambda_i + \lambda_k\right)^{-1} - 1, \text{ and}$$

$$b_{ik} = \left(\frac{4}{\lambda_k} - \gamma\, \frac{\lambda i}{4}\right)\beta^{-1} + \alpha \frac{\lambda i}{4}$$

$$\alpha = \frac{16T}{25p} \frac{(15/2)\, M_i^2 + (25/4 - 3\, B_{ik}^*)\, M_k^2 + 4A_{ik}^*\, M_i M_k}{(M_i + M_k)^2\, D_{ik}},$$

$$\beta = \frac{16T}{25p} \frac{(15/2)\, M_k^2 + (25/4 - 3\, B_{ik}^*)\, M_i^2 + 4A_{ik}^*\, M_i M_k}{(M_i + M_k)^2\, D_{ik}},$$

$$\gamma = \frac{16T}{25p} \frac{(55/4 - 4A_{ik}^* - 3B_{ik}^*)\, M_i M_k}{(M_i + M_k)^2\, D_{ik}},$$

$$\beta_k = (\lambda_i/\lambda_k)\,(\alpha/\beta),$$

and

$$\beta_{ik} = (4+2\gamma\lambda_i \ +4\lambda_i/\lambda_k) \beta^{-1} .$$

D_{ik} is the mutual diffusion coefficient of components i and k, p the pressure, T the temperature, A^*_{ik} and B^*_{ik} are functions involving collision integrals[6]. The expression for G_{ki} is obtained from G_{ik} by the interchange of subscripts refering to molecular species. Gambhir and Saxena[5] have shown that the temperature and composition dependence of G_{ik} is small and they derived simpler expressions for those binary systems where $M_i >> M_k$. Equation (3) then simplifies to

$$G_{ik} \simeq B_{ik} .$$

This relation can be further simplified for such system so that

$$G_{ik} = \sqrt{\beta_k} \text{ , and } G_{ki} = 1/\sqrt{\beta_k} ,$$

$$\text{or } \frac{G_{ik}}{G_{ki}} = \beta_k = \frac{\lambda i}{\lambda k} \cdot \frac{59M^2+88M+150}{150M^2+88M+59} . \qquad \text{.....(4)}$$

Here A^*_{ik} and B^*_{ik} are assigned a constant value of 1.10 and $M = M_k/M_i$.

Thus, Gambhir and Saxena[5] Suggest relation (4) along with one λ_{mix} value to evaluate G_{ik} and G_{ki}. This has since been successfully tested by Saksena and Saxena.[7]

Saxena and Gambhir[8] have also shown that G_{ik}'s obey the following relation :

$$\frac{G_{ik}}{G_{ki}} = \frac{\eta_i}{\eta_k}\left(\frac{M_k}{M_i}\right)^{0.85} = \frac{\lambda_i \text{ (Trans)}}{\lambda_k \text{ (Trans)}}\left(\frac{M_k}{M_i}\right)^{-0.15} . \qquad \text{.....(5)}$$

Here η_i stands for the viscosity of the i-th component. Consequently the relation (5) and one λ_{mix} value can be used to determine G_{ik}. Detailed calculations of Saxena and Gambhir[9] confirm the adequacy of this semi-empirical procedure.

Thus, we have now two other versions of the semiempirical method given by Eqs. (1) and (4), and Eqs. (1) and (5) in addition to the one given by Eqs. (1) and (2) for calculating λ_{mix}. It will be

interesting to compare the accuracies of all these methods to know their relative adequacies. For this purpose we consider the experimental data of Srivastava and Saxena[10], and Saxena[11] for six binary systems at 38°C and at 39 compositions. The average absolute deviations for the rigorous, approximate, empirical and semi-empirical procedures are 2.5, 2.4, 1.1, and 1.1 percent, respectively. The semi-empirical method refers to Eqs. (1) and (2) and we will call this method as semiempirical method I. The semiempirical methods based on Eqs. (1) and (4), and (1) and (5) will be refered as semi-empirical methods-II and III, respectiveiy. These two semi-empirical procedures yield the average absolute deviations for the above thirty nine compositions as 1.1 and 0.4% respectively. Thus all the three semi-empirical procedures seem equally good and also compete in merit with the other methods.

For He-Xe system, which offers a wide range in the λ values, calculations were also performed for the data of other workers. The percentage average absolute deviations for the different methods of calculations are shown in Table 1. In all cases it will be seen that the semiempirical procedures are adequate and dependable. More critical remarks are possible only when the accurate experimental data become available.

All these methods are competent to yield multicomponent thermal conductivity values on the basis of Eq. (1). We will test this only for those ternary systems where experimental data are available. We will consider the experimental data of Srivastava and Saxena[10] on Ne-Ar-Kr, Saxena[11] on He-Ar-Xe, both at 38°C, and von Ubisch[12] on He-Kr-Xe at 520°C all as a function of composition. In this case we get for the percentage average absolute deviations in the rigorous, approximate, empirical and semiempirical procedure I, II and III respectively as 1.4, 1.4, 1.3, 1.1, 1.1 and 1.2 for Ne-Ar-Kr, 0.4, 4.1 0.7, 0.6, 0.5 and 0.5 for He-Ar-Xe and 2.5. 3.9, 2.7, 2.6, 4.6 and 2.9 for He-Kr-Xe. Thus, we find that for ternary systems also these formulae are satisfactory and almost equally good. Nothing every precise can be said in view of the great uncertainty associated with these measurements, particularly of von Ubisch[12]. It will be of great interest to have accurate measurements on the thermal conductivity of multicompanent gas mixtures as a function of temperature.

A very interesting application of the approximate, empirical and semiempirical proeedures has been done in the prediction of

λ_{mix} values at high temperatures. This method of course depends for its success on the fact that G_{ik} are essentially temperature and composition independent. This was shown by Gambhir and Saxena[5] and since then this fact has been used by a number of workers in predicting λ_{mix} values at high temperatures. It will be relevant to quote here the successes obtained according to different procedures in actual cases. The figures of percentage deviations given here refer to the data of von Ubisch[12] on ten binary systems and at all the compositions at which he has given the experimental values. In each case G_{ik} were evaluated at 29°C and these values were used to calculate λ_{mix} at 520°C. The percentage average absolute deviations are : 2.3 for approximate, 3.4 for empirical, 3,0 for semiempirical I, 3.9 for semiempirical II, and 2.5 for semiempirical III. For the He-Kr-Xe system of von Ubisch[12] similar calculations were performed and the percentage average absolute deviations are : 2.0 for approximate, 6.3 for empirical, 3.8 for semiemperical I, 4.6 for semiempirical II, and 2.8 for semiempirical III. Thus, here again we find that all the methods are almost equally successful and any critical relative assessment will await the availability of enough accurate data on several systems.

Table : Percentage average absolute deviations of the calculated thermal conductivity values from experimental data for He-Xe system by various methods.

Reference of the Expt. data.	Temp. °C	Method of calculation: Rig.	Approx.	Empir.	Semi-empir. I	Semi-empir. II	Semi-empir. III
Thoronton	18	7.3	6.8	8.8	4.4	—	4.8
yon Ubisch	29	6.3	4.9	2.6	2.7	2.4	—
von Ubisch	520	4.1	7.7	0.2	2.1	—	1.2
Saxena	38	6.0	1.4	2.0	1.3	1.1	0.3

We can thus sum up with the following conclusions on the basis of this detailed comparison of theory and experiment :

1. All the six methods of computing thermal conductivity of multicomponent gas mixtures are almost equally accurate and reliable. The choice of a particular procedure will depend upon the amount of initial input data available and the computational effort that can be put.

2. The prediction of multicomponent λ_{mix} values on the basis of binary λ_{mix} values by all the five methods (i. e, except rigorous) is dependable. This is one of the very potential methods for supplying the multicomponent λ_{mix} values for which very limited experimental data are available.

3. The calculation of high temperature λ_{mix} values uslng G_{ik} determined at a lower temperature from any of the five methods seems fairly accurate and encouraging. In view of the great lack of data at high temperatures and their important applications in a number of very useful problems viz., reactor design, flame propagation, explosions and detonation etc., these procedures will continue to be the only source of providing the requisite information.

Acknowledgments

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REFERENCES

1. S. C. Saxena and J. M. Gandhi, Revs. Mod. Phys. *35*, 1022 (1963).
2. C. Muckenfuss and C. F. Curtiss, J. Chem. Phys. *29*, 1273 (1958).
3. E. A. Mason and S. C. Saxena, J. Chem. Phys. *31*. 511 (1958).
4. E. A. Mason and S. C. Saxena, Phys. Fluids *1*, 361 (1958).
5. R. S. Gambhir and S. C. Saxena, Trans. Faraday. Soc. *60*, 38 (1964).
6. J. O. Hirschfelder, C. F. Curtiss and R. B. Bird, Molecular Theory of Gases and Liquids, John Wiley, New York, 1954.
7. M. P. Saksena and S. C. Saxena, Natl. Inst. Sci, to be published.
8. S. C. Saxena and R. S. Gambhir, Proc. Phys. Soc. (London) *81*, 788 (1963).
9. S. C. Saxena and R. S. Gambhir, Indian J. Pure & Applied Phys. *1*, 318 (1963).
10. B. N. Srivastava and S. C. Saxena, Proc. Phys. Soc., Lond., *70 B*, 369 (1957).
11. S. C. Saxena, Indian J. Phys., *31*, 597 (1957).
12. H. Von Ubisch, Ark. Fys., *16*, 93 (1959).

THERMAL CONDUCTIVITY OF MONATOMIC GASES AND BINARY GAS MIXTURES

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ABSTRACT

All the available experimental thermal conductivity data on pure noble gases have been considered with a view to determine their relative accuracies which also enable the assessment of the comparative merits of the different methods used in measurements. Some very interesting conclusions follow from this study. The experimental data on the binary inert gas mixtures are also examined and it turns out that there are several notable discrepancies in the data of different workers. The accuracy, in particular, of von Ubisch data is poor and this over-all study suggests that there is a general deficiency of the experimental data. Attempts are beinig made in this laboratory to supplement this information and resolve the discrepancies posed by the existing data as revealed by the present work. Following a procedure earlier suggested by Srivastava and and Saxena the present work has made possible to predict and estimate the thermal conductivity values of Rn and of its mixtures with other inert gases as a function of composition.

Thermal conductivity data of gases like many other equilibrium and non-equilibrium properties provide a very adequate opportunity for exploring the appropriateness of the formulation of kinetie theory[1,2]. In fact the extraordinary sensitivity of thermal conductivity to the presence of internal degrees of freedom, rotation and vibration, makes it some-what preferable over other properties. The knowledge of accurate thermal conductivity values as a function of temperature is basic for such a study. Unfortunately, there is a general paucity of the accurate data of this type. The endeavour of the present article is to pool together all the available data on pure gases as a function of temperature and also of binary mixtures as a function of temperature and composition. There are several very useful purposes at the back of such a laborious though straight forward effort and these only have provided incentive to the present work. We enumerate them here in brief.

Recently we[3] interpreted the thermal conductivity data on binary mixtures, λ_{mix}, in terms of the rigorous kinetic theory and

three other procedures of calculation. Since then many other methods have been suggested [4-7] for estimating λ_{mix}. For the success of these works it is essential to know the λ_{mix} values with a fair degree of reliance. The plan of present work will provide such information. Further all the discrepancies in the existing data will get into light and this will enable experimentalists to plan suitable experiments. Also till that time we will have the knowledge of the correct assessment of the existing data from present study. Lastly this work also permits an estimation of the thermal conductivity of pure radon and its combinations with other inert gases, which are not known otherwise.

We now consider the data on pure gases. Helium in this connection is of special interest, for a large amount of data of different workers obtained from different techniques are available. Kannuluik and Carman[8], Srivastava and Saxena[9] have used the "thick-wire" variant of the "hot-wire" method; while Von Ubisch[10], and Johnston and Grilly[11] have used the "thin-wire" variant of the "hot-wire" method. Keyes[12] and Cheung, Bromley and Wilke[13] have exploited the concentric cylinders method while Blais and Mann[14] have developed a thermal diffusion column method to determine thermal conductivity at high temperatures. Most of these data were plotted and considered by Saxena and Agrawal[15] and they showed that Blais and Mann[14] values are probably consistently higher than the true values. Consequently we have not considered this data while rest of the data [8-13,16] are shown plotted in Fig. 1, which also includes Ne. The experimental λ data for Ar, Kr and Xe are plotted in Fig. 2. A critical examination of these figures leads to the following interesting and useful observations :

The thermal conductivity values obtained using either variant of the hot-wire cell are consistent with each other. This conclusion though may be regarded at present as partly prematured, for the elaborate data using the thin-wire variant are available only for He[11]. The only other set of data available are those of von Ubisch[10] but nothing reliable can be said for these are known to be consistently larger than the true values. It is important to note that elaborate data are not available even on the thick-wire variant of two independent workers to permit any conclusion about the possible relative consistencies. Efforts directed to produce such

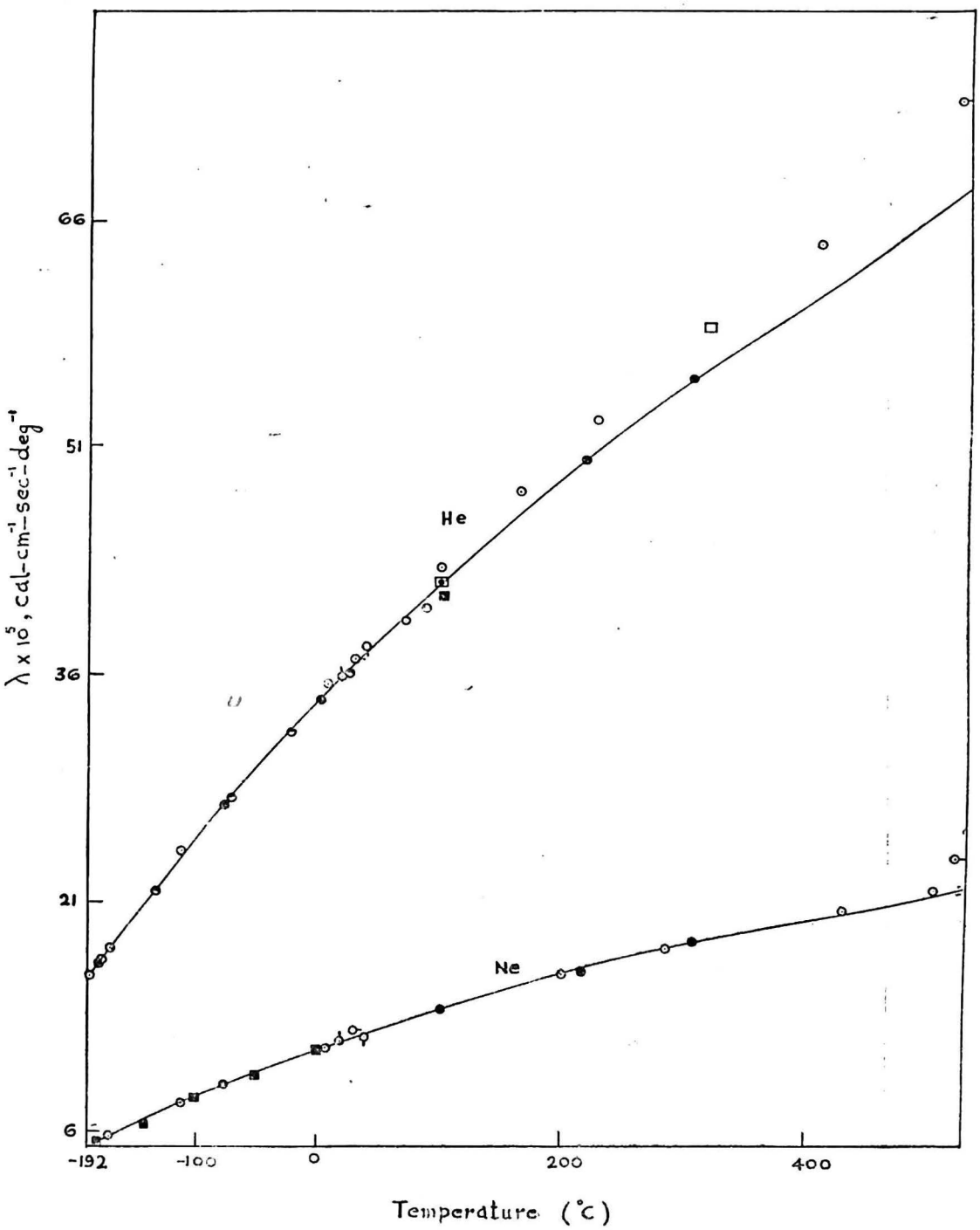

66
51
36
21
6
λ x 10^5, cal-cm^-1-sec^-1-deg^-1
He
Ne
-192
-100
0
200
400
Temperature (°C)

Fig. 1

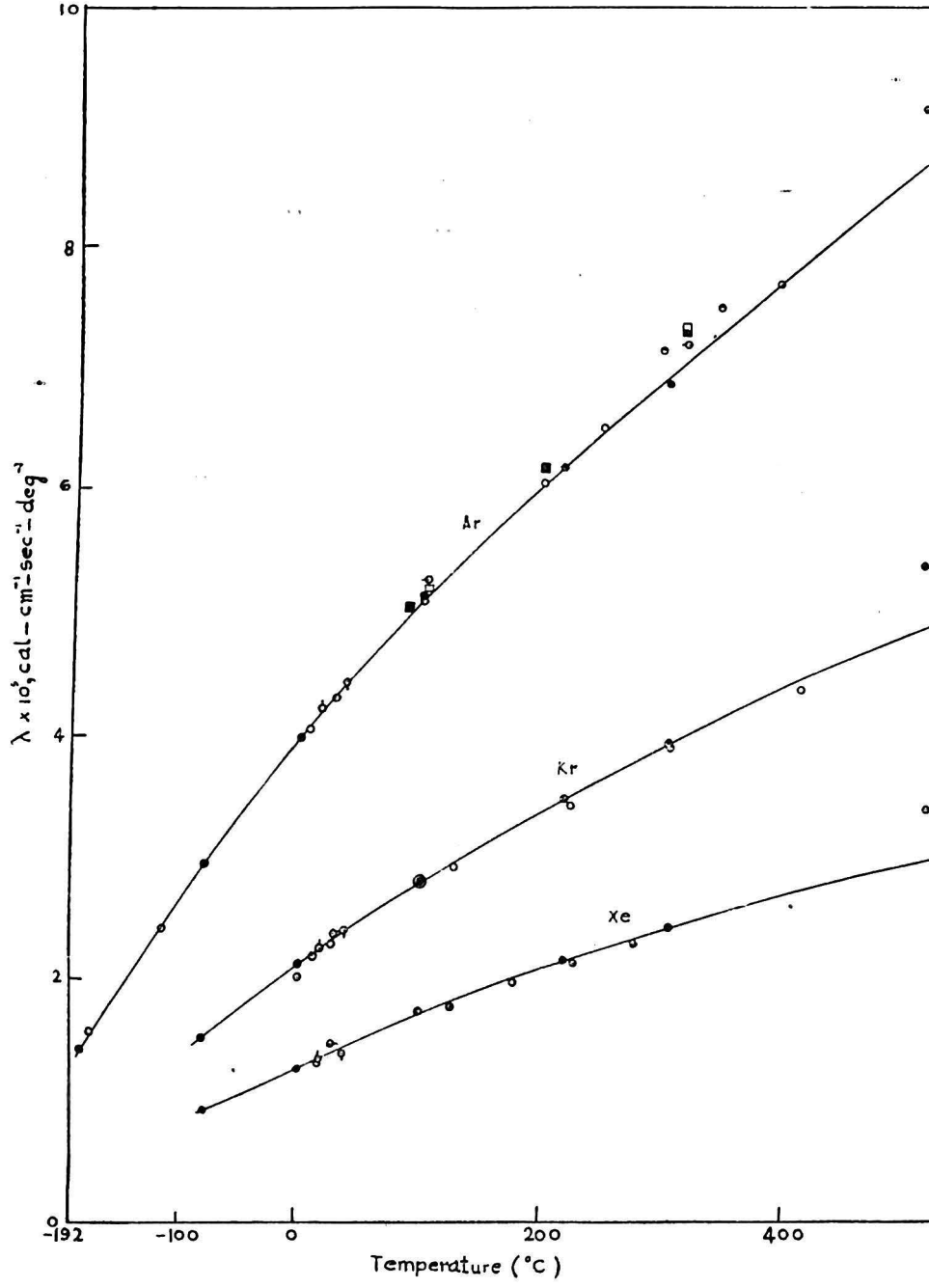

λ x 10⁵, cal-cm⁻¹-sec⁻¹-deg⁻¹
Temperature (°C)
Ar
Kr
Xe
10
8
6
4
2
0
-192
-100
0
200
400

Fig. 2

data are highly desirable and valuable to derive conclusions regarding the attainable accuracies.

Using concentric cylinders type of conductivity cell Keyes[12] have reported values for He, Ne and Ar, and Cheung, Bromley and Wilke[13] for He and Ar. Keyes[12] values are in agreement with Kannuluik and Carman[8] values for Ne but are in disagreement in the case of Ar. The data of Cheung et al.[13] and of Schottky[17] for Ar also favour the values of Keyes[12], all suggesting the data of Kannuluik and Carman[8] to be somewhat low at temperatures above approximately 100°C. This conclusion is also atleast partly substantiated from the measurements of Cheung et al.[13] for He. It is unfortunate that no other date are available to further confirm this possibility and it will be highly useful if additional measurements are planned for temperatures above 100°C to resolve this discrepancy.

Thornton[16] values have been obtained using a katharometer which as pointed out by Srivastava and Saxena[9] is inherently incapable of yielding accurate values. His values refer to 18°C and in all the five gases these are in satisfactory agreement with the existing values as he has corrected his values on the basis of Kannulik et al.[8] Srivastava and Saxena[9] values at 38°C are also in reasonable agreement with the rest of the data

Another indirect method of testing the accuracy of values is possible on the basis of viscosity data (η). λ can he easily generated from the corresponding η values using the simple relation of the kinetic theory[1,2] viz.,

$$\lambda = \frac{15\,R}{4\,M}\,\eta \qquad ,$$

where R is the familiar gas constant and M the molecular weight. The λ values so obtained are shown in Figs. 1 and 2 in all the cases. The agreement of these indirect λ values with the directly obtained values is satisfactory in all cases except He. This again shows the possibility of Kannuluik and Carman[8] values to be systematically lower at higher temperatures.

In both these figures the continuous curves have been obtained by smoothly joining the points of Kannuluik and Carman[8]. We do not thereby imply any preference for these data except this facilitates the visual comparison with other data.

The experimental λ_{mix} data on binary mixtures of five noble gases have been reported by Thornton[16] at 18°C, von Ubisch[10] at 29° and 520°C, and Srivastava and Saxena[9] at 38°C. All these measurements at a particular temperature have been reported as a function of composition and we represent them graphically in Figs. 3—12. We will discuss all these data in terms of their relative consistency and absolute accuracy so that we may know for definite what data with what reliance already exist, and what should be planned in the immediate future to enable a thorough and complete study of thermal conductivity on binary mixtures. The knowledge of λ_{mix} on binary systems has become now a days still more important for it has been shown by Srivastava and Saxena[18], and Saxena[19] as to how the λ_{mix} data of binary systems can be used to predict multicomponent λ_{mix} values with fair accuracy. Mason and Saxena[20] have derived these formulae also from rigorous theory by well defined approximations and since then the scope of these calculations have been further enlarged to cover high temperatures also by Gambhir and Saxena[6], and Gandhi and Saxena[21]. We now report some important conclusions and comments based on these plots.

In general from these plots we find that λ_{mix} values of von Ubisch[10] at 520°C are reasonably consistent in as much as in all cases we get a smooth plot of λ_{mix} as a function of composition. Nothing definite can be said about their absolute accuracy as there are no other data availabie for comparison. If pure λ values as compared from the extrapolated values of Kannuluik and Carman[8] are any guide von Ubisch[10] λ_{mix} values may be higher than the true values by a couple of per cent. In all cases there is a considerable confusion in the λ_{mix} values at lower temperatures and we present here a critical analysis for each system.

For He–Xe system all the λ_{mix} data are shown in Eig. 3. All the three sets of measurements seem to be reasonably consistent and the agreement becomes still better if it is recalled that von Ubisch[10] measurements are higher by a couple of per cent. In the case of He–Kr system, Fig. 4, the Thornton[16] values are inconsistent with those of von Ubisch[10] and the discrepancy gets further enhanced if the data of von Ubisch[10] are corrected for the general trend of being higher by a couple of per cent than the correct values. One is thus tempted to think that Thornton[16] data may be systematically higher. This latter possibility also receives some support from the results on

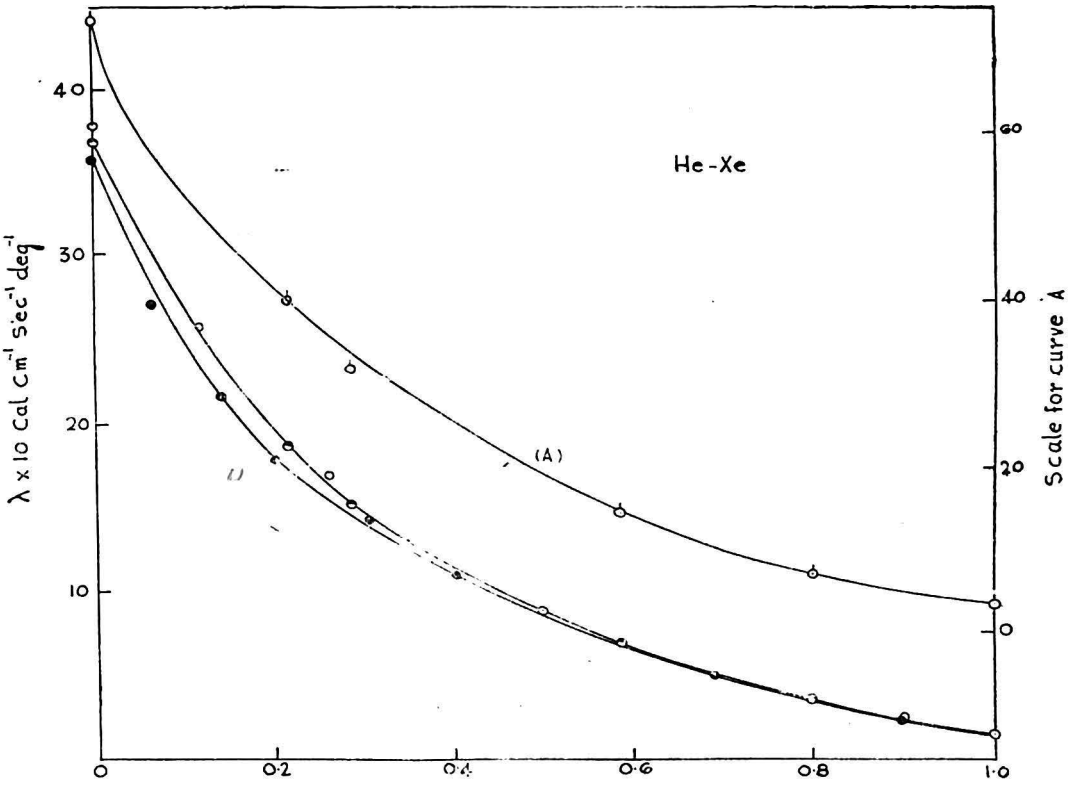

He-Xe
(A)
λ x 10 Cal Cm⁻¹ Sec⁻¹ deg⁻¹
Scale for curve A
40
30
20
10
0
60
40
20
0
0·2
0·4
0·6
0·8
1·0
Mole Fraction Xe

Fig. 3

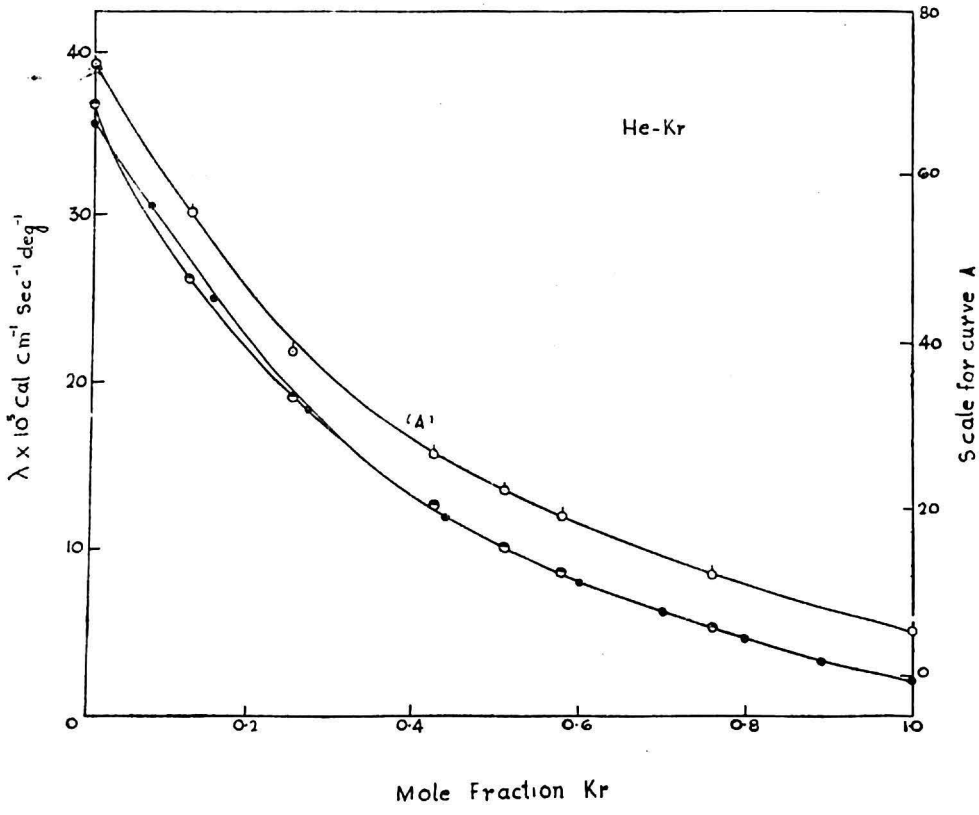

He-Kr
'A'
λ x 10⁵ cal cm⁻¹ sec⁻¹ deg⁻¹
Scale for curve A
Mole Fraction Kr
0
10
20
30
40
0·2
0·4
0·6
0·8
1·0
0
20
40
60
80

Fig 4

other systems, as discussed below. Additional measurements will be useful to clarify this point.

Measurements of all the three groups for Ar–He are plotted in Fig. 5, and these data also do not seem to be consistent. von Ubisch[10] values are again higher and so are of Thornton[16]. The former values when corrected will be at the appropriate place relative to measurements of Srivastava and Saxena[9], while those of Thornton[16] which refer to the lowest temperature appear somewhat greater than the correct values. Further measurements therefore seem reasonable.

Available data on He–Ne system are shown in Fig. 6 and the two sets of measurements are completely inconsistent with each other. Thornton[16] values which refer to a lower temperature are greater than von Ubisch[10] values. The discrepancy gets further enhanced if von Ubisch[10] data are corrected which will reduce the plotted values. This again shows that Thornton[16] λ values may be greater than the correct value.

In Fig. 7 are shown the λ_{mix} values for the Ar–Ne system. If reliance is given to the values of Srivastava and Saxena[9] one again finds that Thornton[16] values are somewhat higher. This view is further substantiated if von Ubisch[10] values are corrected, which then will fall below those of thornton[16] values inspite of the fact that these refer to a higher temperatue. To resolve the discrepancy posed by these measurements additional data are required. The experimental data on Ne–Kr system are shown in Fig. 8 and the same trends and conclusions as observed in the case of Fig. 7 are also valid in this case.

The date on the Ne–Xe system are shown in Fig. 9. The measurements as they are, appear to be qualitatively in the right order. However, if von Ubisch[10] measurements are corrected, these values almost merge into those of Thornton[16]. This again indicates the possibility of Thornton[16] values being systematically higher. New measurements are therefore valuable for this system.

Experimental data on Ar–Kr and Ar–Xe systems are plotted in Figs. 10 and 11 respectively. In both the cases measurements of Thornton[16] and Srivastava and Saxena[9] seem to be consistent with

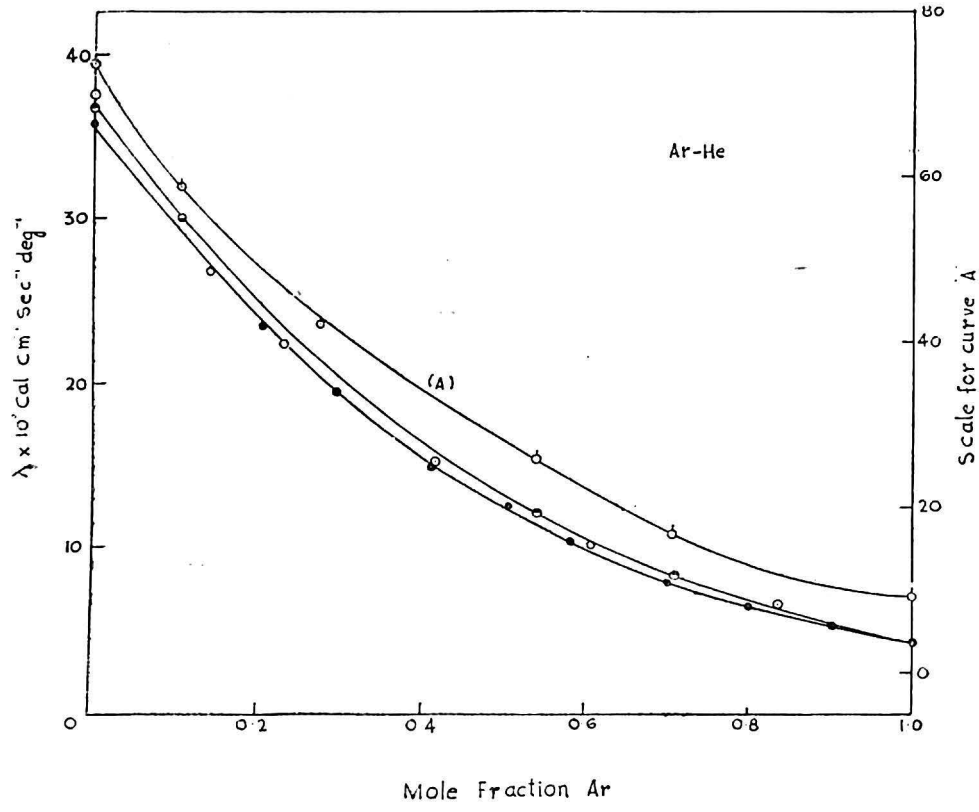

Ar-He
(A)
λ x 10³ Cal Cm⁻¹ Sec⁻¹ deg⁻¹
Scale for curve A
Mole Fraction Ar

Fig. 5

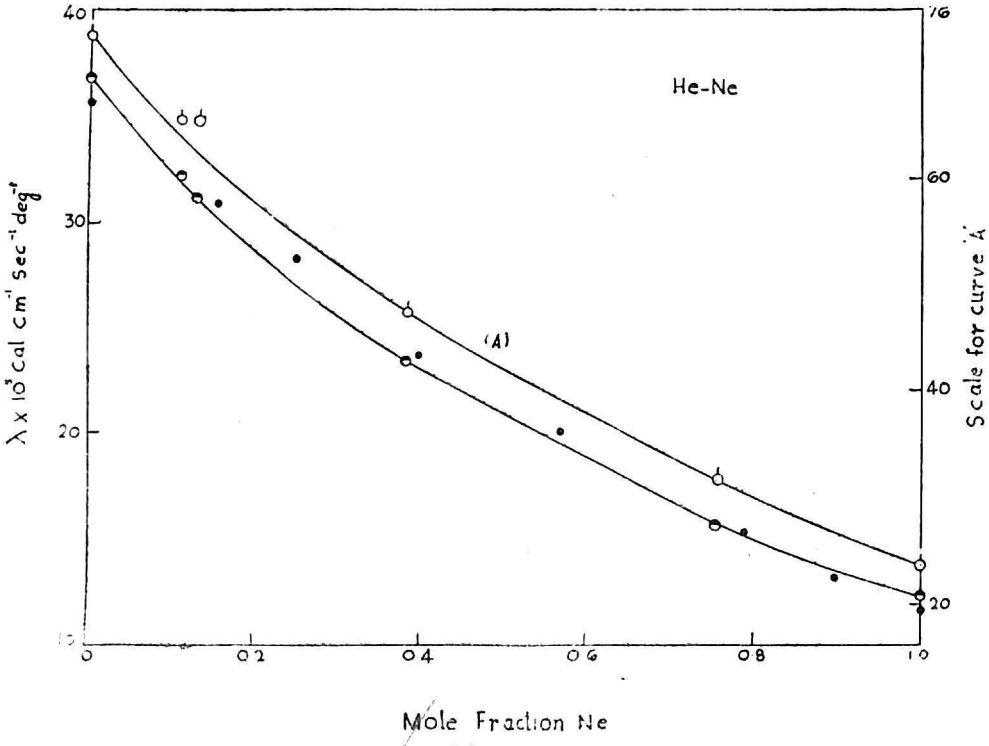

He-Ne
(A)
λ x 10³ Cal Cm⁻¹ Sec⁻¹ deg⁻¹
Scale for curve A
Mole Fraction Ne

Fig. 6

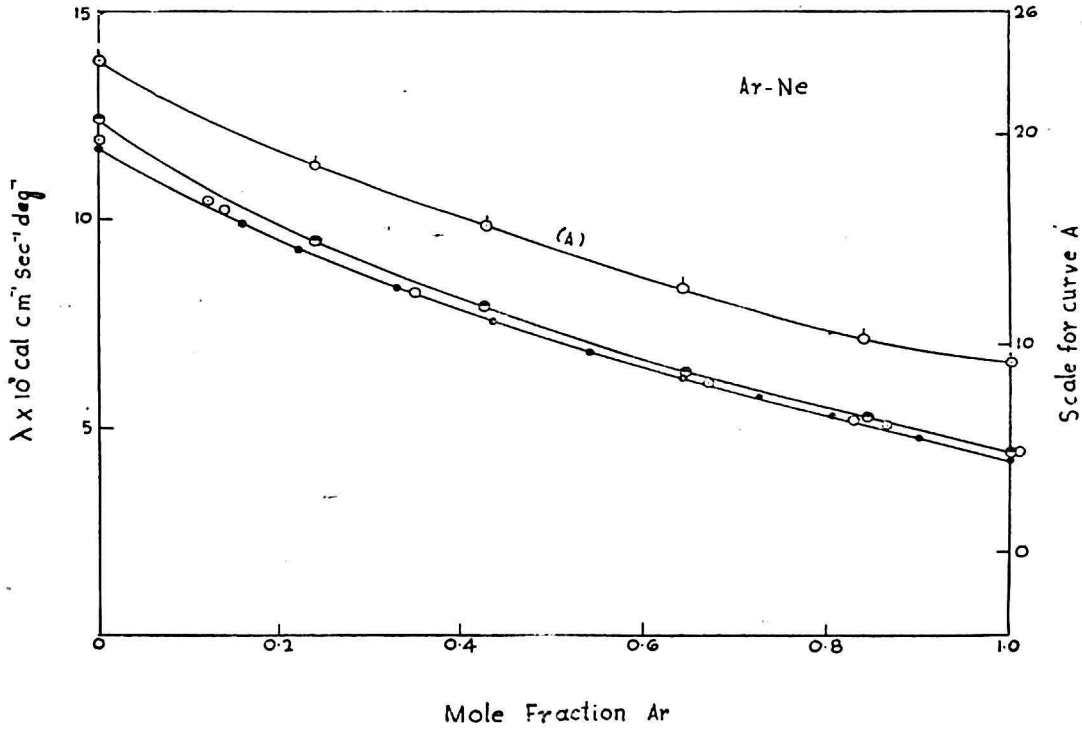

Ar-Ne
(A)
λ x 10⁵ cal cm⁻¹ Sec⁻¹ deg⁻¹
Scale for curve A
Mole Fraction Ar

Fig. 7

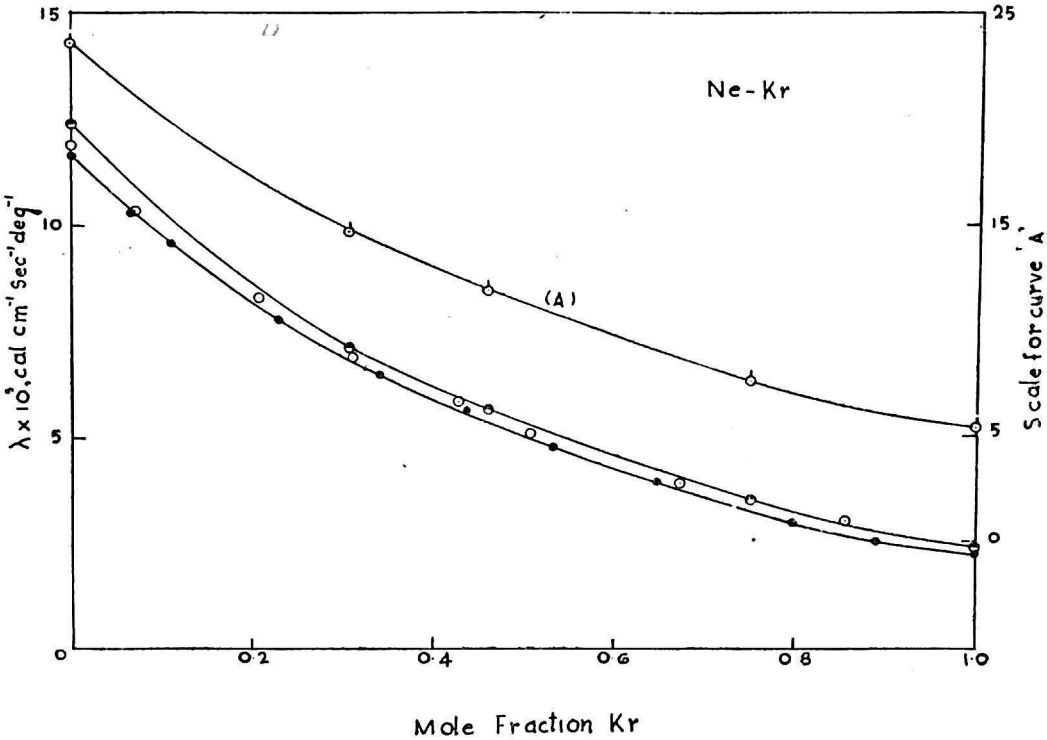

Ne-Kr
(A)
λ x 10⁵ cal cm⁻¹ Sec⁻¹ deg⁻¹
Scale for curve A
Mole Fraction Kr

Fig. 8

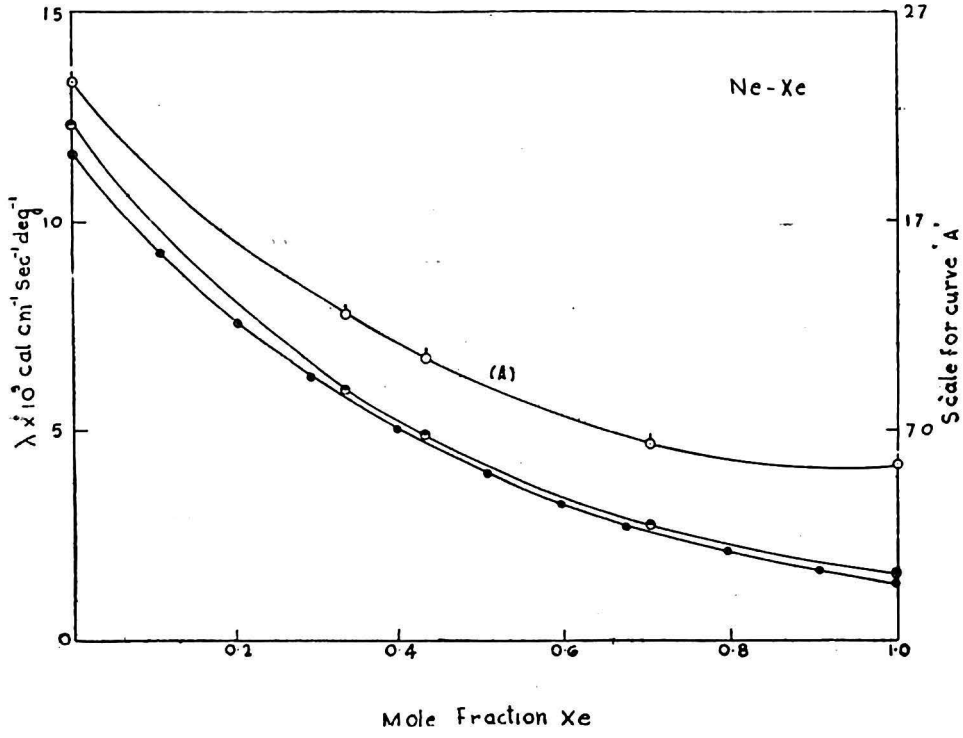

Ne-Xe
(A)
λ x 10⁵ cal cm⁻¹ Sec⁻¹ deg⁻¹
Scale for curve 'A'
Mole Fraction Xe

Fig. 9

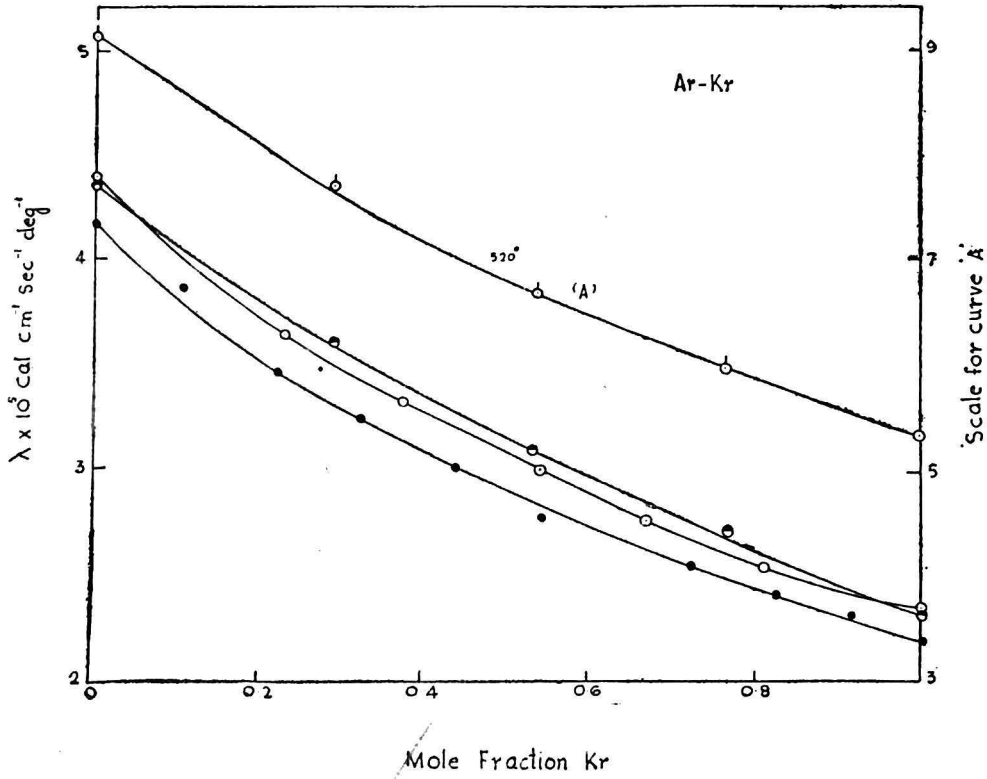

Ar-Kr
320°
(A)
λ x 10⁵ cal cm⁻¹ Sec⁻¹ deg⁻¹
Scale for curve 'A'
Mole Fraction Kr

Fig. 10

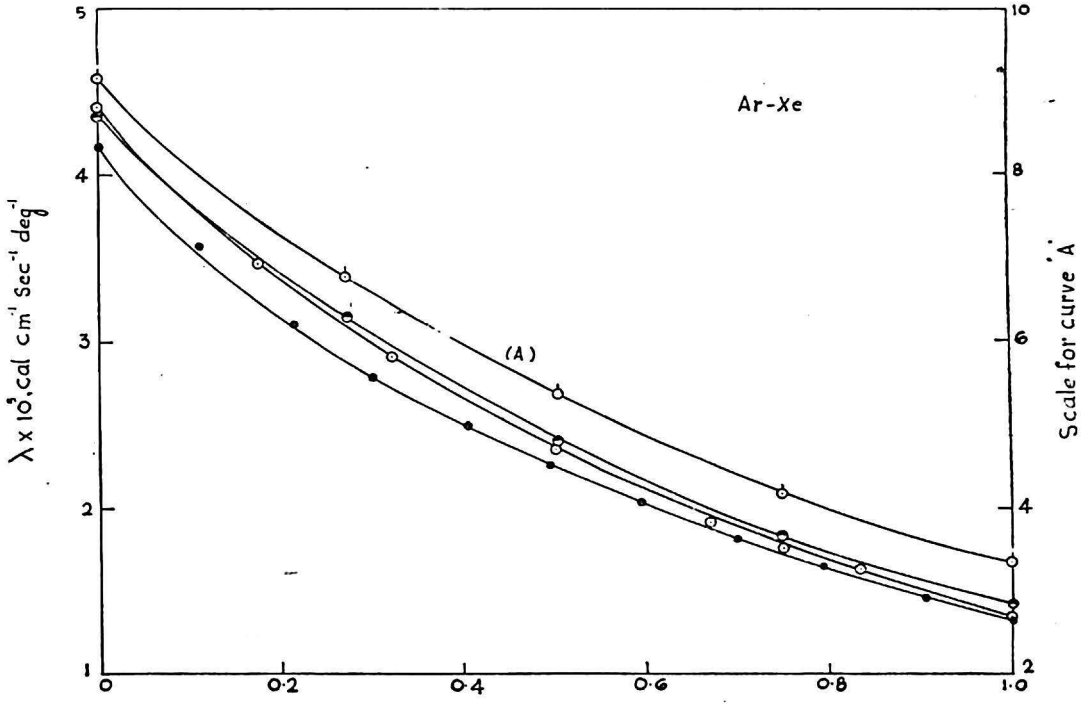

Ar-Xe
(A)
λ x 10⁵, cal cm⁻¹ Sec⁻¹ deg⁻¹
Scale for curve 'A'
Mole Fraction Xe

Fig. 11

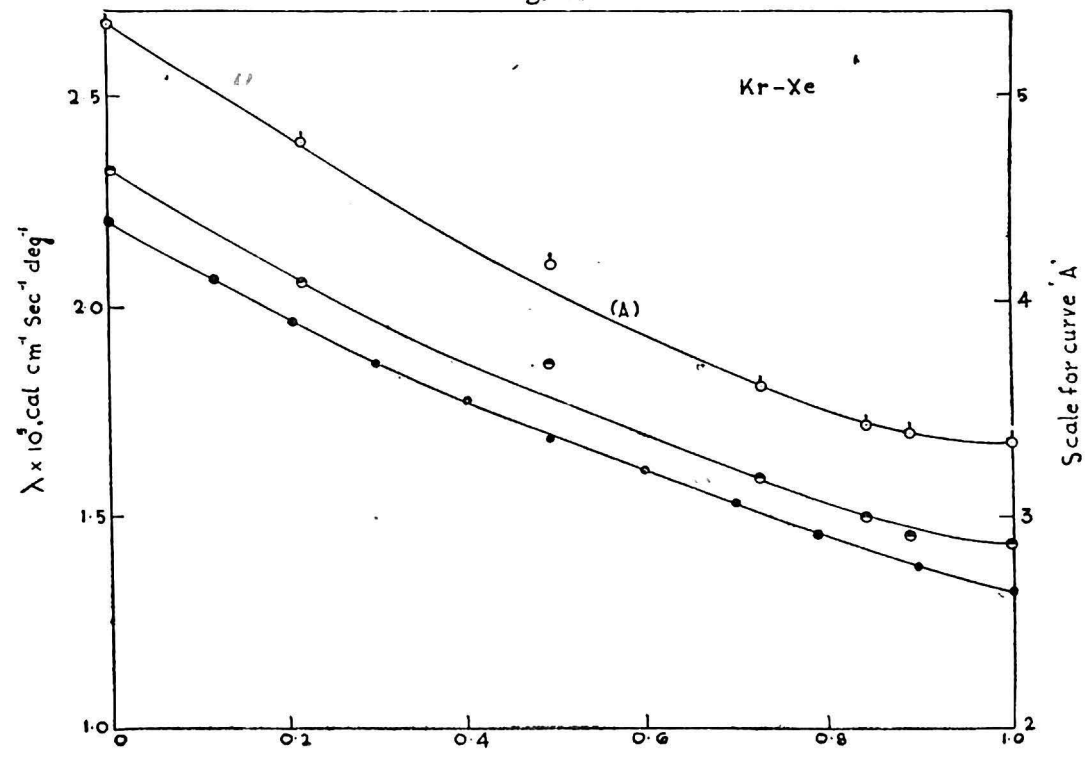

Kr-Xe
(A)
λ x 10⁵, cal cm⁻¹ Sec⁻¹ deg⁻¹
Scale for curve 'A'
Mole Fraction Xe

Fig. 12

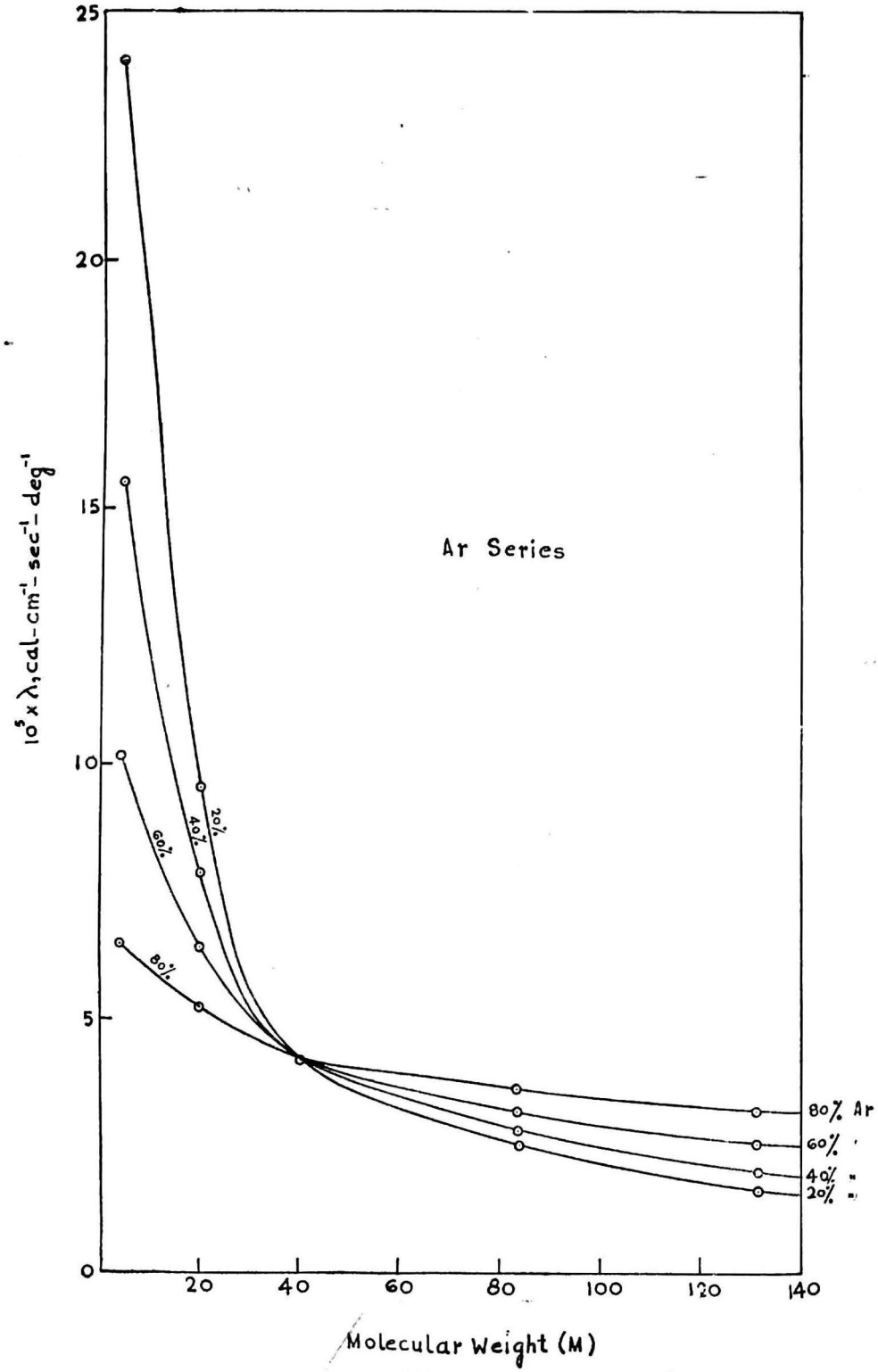

25
20
15
10
5
0
20
40
60
80
100
120
140
$10^5 \times \lambda$, cal-cm^{-1}-sec^{-1}-deg^{-1}
Molecular Weight (M)
Ar Series
20%
40%
60%
80%
80% Ar
60% "
40% "
20% "

Fig. 13

each other while those of von Ubisch[10] also become in line when corrected. Figure 12 shows the results for the Kr–Xe system and the results seem to be satisfactory.

Thus, we can sum up the over-all picture by the following general remarks :

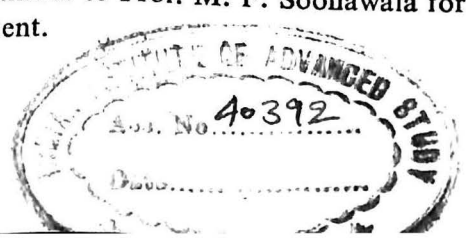
(a) The data of von Ubisch[10] seem to be consistently higher than the correct values.

(b) The data of Thornton[16] also do not seem to be very accurate and there are evidences to support the view that his data may also be systematically higher.

(c) For a number of systems the three sets of measurements present serious discrepancies and new reliable measurements seem highly desirable.

Another very interesting use of the present study lies in the possibility of predicting the thermal conductivity values of Rn and its mixtures with other inert gases following Srivastava and Saxena,[22] and Saxena[23]. From Figs. 3–12, λ_{mix} values were read for Thornton[16] data at arbitrarily chosen compositions. The values corresponding to different systems at a fixed composition of the common constituent for each series were plotted versus the molecular weight. On extrapolation these curves yield the value corresponding to the binary mixture of the common constituent of the series with Rn at that composition. One such representative set of curves for the argon series is reproduced in Fig. 13. In the table we list the λ_{mix} values for the various combinations of Rn with other inert gases as a function of composition. The pure Rn value reported in this table is obtained by plotting the pure values of all the gases at 18°C against the molecular weight and extrapolating the curve upto the molecular weight of Rn. These values are only rough estimates and may be uncertain by as much as even 20% on the average. Inspite of this large uncertainty we are tempted to report them for there are no direct values available.

We are thankful to Prof. M. F. Soonawala for his kind interest and encouragement.



REFERENCES

1. S. Chapman and T. G. Cowling, The Mathematical Theory of Non-uniform Gases (Cambridge University Press, London, (1952), second edition.
2. J. O. Hirschfelder, C. F. Curtiss and R. B. Bird, Molecular Theory of Gases and Liquids (John Wiley and Sons, Inc., New York, 1954).
3. S. C. Saxena and J. M. Gandhi, Revs. Mod. Phys. *35*, 1022 (1963).
4. P. G. Wright and P. Gray, Trans. Faraday Soc. *58*, 1 (1962).
5. S. C. Saxena and R. S. Gambhir, Proc. Phys. Soc. *81*,788 (1963); Indian J. Pure & Applied Phys. *1*, 318 (1963).
6. R. S. Gambhir and S. C. Saxena, Trans. Faraday Soc. *60*, 38 (1964); M. P. Saksena and S. C. Saxena, Proc. Natl. Inst. Sci.,
7. J. M. Gandhi and S. C. Saxena, Indian J. Pure and Applied Phys., 2, 83(1964); See also S. C. Saxena and R. S. Gambhir, British J. Applied Phys. *14*, 436 (1963).
8. W. G. Kannuluik and E. H. Carman, Proc. Phys. Soc. (London) *65 B*, 701 (1952).
9. B. N. Srivastava and S. C. Saxena, Proc. Phys. Soc. (London) *70 B*, 369 (1957); S. C. Saxena, Indian J. Phys. *31*, 597 (1957).
10. H. von Ubisch. Ark. Fys. *16*, 93 (1959); See also E. A. Mason and H. von Ubisch, Phys. Fluids 3, 355 (1960).
11. H. L. Johnston and E. R. Grilly, J. Chem. Phys. *14*, 233 (1946).
12. F. G. Keyes, A. S. M. E. *76*, 809 (1954).
13. H. Cheung, L. A. Bromley and C. R. Wilke, U. C. R. L. Rept. No. 8230 Rev. (1959).
14. N. C. Blais and J. B. Mann, J. Chem. Phys. *32*, 1459 (1960).
15. S. C. Saxena and J. P. Agrawal, J. Chem. Phys. *35*, 2107 (1961).
16. E. Thornton, Proc. Phys. Soc. (London) *76*, 104 (1960); *77*, 1166 (1961); E. Thornton and W. A. D. Baker, ibid. *80*, 1171 (1962).
17. W. F. Schottky, Z. Electrochem. *56*, 889 (1952).
18. B. N. Srivastava and S. C. Saxena, J. Chem. Phys. *27*, 583 (1957).
19. S. C. Saxena, J. Chem. Phys. *25*, 360 (1956).
20. E. A. Mason and S. C. Saxena, Phys. Fluids *1*, 361 (1958).
21. J. M. Gandhi and S. C. Saxena, Indian J. Pure and Applied Phys., 3,312 (1965).

22. B. N. Srivastava and S. C. Saxena, Physica *22*, 253 (1956).

23. S. C. Saxena, Physica *22*, 1242 (1956).

Table : λ_{mix} values of Rn with other inert gases.

% of Rn	$10^5 \times \lambda_{mix}$, Cal/cm-see-deg				
	He	Ne	Ar	Kr	Xe
100	0.6	0.6	0.6	0.6	0.6
80	2.4	1.5	1.3	0.6	0.6
60	5.1	2.7	1.7	1.0	0.8
40	7.9	4.7	2.2	1.3	1.0
20	13.0	7.0	3.0	1.9	1.2
0	35.6	11.6	4.2	2.2	1.3

Figure Captions

Figure 1. λ of He and Ne as a function of temperature. Experimental points : ● Kannuluik and Carman, σ von Ubisch, Ψ Srivastava and Saxena, $\flat$ Thornton, ◙ Keyes, ▯ Cheung et al., Δ Schottky, ○ Calculated from η, and ● Johnston and Grilly.

Figure 2. The legend is same as in figure 1 except the values refer to Ar, Kr and Xe.

Figure 3. λ_{mix} values for He–Xe mixture as a function of composition. Experimental points : δ von Ubisch at 520°C, ● von Ubisch at 29°C, ● Thornton, and ○ Srivastava and Saxena.

Figure 4. The legend is same as in Fig. 3. except it refers to He-Kr system.

Figure 5. The legend is same as in Fig. 3 except it refers to He-Ar system.

Figure 6. The legend is same as in Fig. 3 except it refers to He-Ne system.

Figure 7. The legend is same as in Fig. 3 except it refers to Ar-Ne system.

Figure 8. The legend is same as in Fig. 3 except it refers to Ne-Kr system.

Figure 9. The legend is same as in Fig. 3 except it refers to Ne-Xe system.

Figure 10. The legend is same as in Fig. 3 except it refers to Ar-Kr system.

Figure 11. The legend is same as in Fig. 3 except it refers to Ar-Xe system.

Figure 12. The legend is same as in Fig. 3 except it refors to Kr-Xe system.

Figure 13. Plots of λ_{mix} versus M for argon series at various concentrations of argon.

WAVES IN A HEAVY, VISCOUS, INCOMPRESSIBLE ELECTRICALLY CONDUCTING FLUID OF VARIABLE DENSITY IN THE PRESENCE OF A MAGNETIC FIELD

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ABSTRACT

An equation, relating the growth rate n, and the wave number k, which charactrizes the equilibrium of a heavy viscous, incompressible stratified fluid of finite depth in the presence of a horizontal magnetic field, derived by Gandhi has been studied. The properties of the magnetohydrodynamic waves generated under certain conditions, have been discussed in detail.

INTRODUCTION

Gandhi (1964) studied the properties of waves generated in a heavy, viscous, incompressible, electrically conducting fluid of variable density, in the presence of a magnetic field. He had enunciated a variational principal for the problem. While discussing the applications of variational principals, he studied the case of a continuously stratified fluid of a finite depth, Measuring k (wave number of propogation) and n (the growth rate) in non-dimensional parameters of the two different kinds, he had derived, the following equations :—

$$y^2+2y\,(1+x^2)-\frac{4Gx^2}{1+x_2}+\frac{Qx^2y}{y+2\rho(1+x^2)}=0 \qquad (1)$$

where

$$x = \frac{kd}{\pi s} \text{ and } y = n \frac{2d^2}{\nu \pi^2 s^2} \tag{2}$$

and

$$G \equiv \frac{g\beta d^4}{\pi^4 \nu^2 s^4} \tag{3}$$

$$Q \equiv \frac{KH^2\alpha^2}{\pi^3 \rho_1 \nu^2 s^2} \tag{4}$$

$$P = \frac{\eta}{\nu} \tag{5}$$

Measuring

$$x = \frac{kd}{\pi s},\ y = n \frac{\alpha}{\pi s}\left(\frac{4\pi\rho_1}{KH^2}\right)^{\frac{1}{2}} \tag{6}$$

equation (1) takes the form :—

$$y^2 + 2y(1+x^2)S = \frac{Bx^2}{1+x^2} + \frac{yx^2}{y+2R(1+x^2)} = 0 \tag{7}$$

where

$$R = \tfrac{1}{2}\eta \frac{\pi s}{d} \frac{1}{v_A} \tag{8}$$

$$B = \frac{g\beta d^2}{\pi^2 s^2 v_A^2} \tag{9}$$

$$S = \tfrac{1}{2}\nu\left(\frac{\pi s}{d}\right)\frac{1}{v_A} \tag{10}$$

Gandhi discussed equation (7) and did not study equation (1) on the ground that in this particular form of the equation (equation 1) we can not discuss cases with facility when $\nu = 0$. It may be observed that in the form of the equation as given by (7), we cannot discuss cases when the field $H = 0$. As we are studing the effect of a vertical magnetic field, it will be advisable to discuss equation (1) also, so that we can compare the various cases in the presence as well as in the absence of the magnetic field. Hence we discuss equation (1) in detail.

Equation (1) is cubic in y but reduces to a quadratic equation in two cases, namely $G = 0$ and $P = 0$. Below we deal with them separately.

WAVES IN THE ABSENCE OF BUOYANCY FORCES

This is the case when $G=0$ in equation (1) which now becomes :—

$$y^2+2y\,(1+x^2)\,(1+P)\,+\,4P\,(1+x^2)^2\,+\,Qx^2=0 \qquad (12)$$

provided $y\neq 0$.

The solutious of the last equation are

$$y\,=\,-(1+P)\,(1+x^2)\,\pm\,i\,\{Q^2-(1-P)^2(1+x^2)^2\}^{\frac{1}{2}} \qquad (13)$$

From the last equation we can readily see that when $o\leqslant x\leqslant x_c$, where

$$1\,=\,(1-P)^2\left(\frac{1+x_c^{\,2}}{x_c}\right)^2\,\frac{1}{Q} \qquad (14)$$

the solution corruspounds to a damped oscillation. The damping co-efficient

$$-R(y)\,=\,(1+P)\,(1+x^2) \qquad (15)$$

and the frequency

$$\frac{\&}{\partial}\,(y)\,=\,\pm\,\{\,Qx^2-(1-P)^2\,(1+x^2)^2\}^{\frac{1}{2}} \qquad (16)$$

These oscillations give rise to horizontally propagated magnetohydrodynamic wave.

The waves are propogated with phase velocity V_w and group velocity V_g where

$$V_w^{\,2}=\frac{[\,\&(y)\,]}{\partial\,x^2}=\left\{\,Q\,-\,(1-P)^2\left(\frac{1+x^2}{x}\right)^2\right\} \qquad (17)$$

$$V_g^{\,2}=\left[\frac{d}{dx}\,\frac{\&}{\partial}(y)\right]^2=\frac{\{\,Q-2(1-P)^2\,(1+x^2)\,\}^2}{V_w^{\,2}} \qquad (18)$$

When the wave number x exceeds the critical wave number x_c, the quantity under the radical sign in equation (13) becomes negative. Thus when $x<x_c$ the motion is aperiodically damped with damping co-efficients

$$-\,R\,(\,y\,)=(1+P)\,(1+x^2)\,\pm\,\{\,(1-P)^2\,(1+x^2)^2\,-\,Qx^2\,\}^{\frac{1}{2}} \qquad (19)$$

The foregoing discussion of the behaviour of the roots of the equation (12) is illustrated by figures 1, 2 and 3.

Waves in an Ideal Conductor in the presence of Buoyancy Forces :

This is the case when $P=0$, so that the equation (1) simplifies to

$$y^2+2y(1+x^2)+Qx^2-\frac{4Gx^2}{1+x^2}=0 \tag{20}$$

The solution of which is

$$y=-(1+x^2)\pm\left\{(1+x^2)-Qx^2+\frac{4Gx^2}{1+x^2}\right\}^{\frac{1}{2}} \tag{21}$$

Unstable Stratification :-

When $\beta>0$, $G>0$ and according to equation (21) when $x>x_o$, x_o being the real root of the equation

$$\left\{\frac{4G}{1+x_o^2}-Q\right\}=0 \tag{22}$$

The value of y correspounding to the upper sign is real and positive. The equilibrium in these circumstances is unstable as the disturbance grows (aperiodically) with time.

When $x>x_o$, y no longer has a positive value, so that over this wave number range no amplified motion is possible. The mathematically stable modes of this case of unstable stratification are of no physical interest, hence we conclude this case here.

Stable Stratification :—

When $\beta<0$, $G<o$ and by equation (21) y never has a positive real part, so that the equilibrium is stable. Whether the equilibrium is restored periodically or aperiodically depends on the sign of the quantity under the radical sign. In order to discuss the conditions under which one or the other of these types of flow arises, it will be convenient to introduce the quantity

$$Z=\frac{x^2}{(1+x^2)^2}+\frac{4G_1x^2}{Q(1+x^2)^3} \tag{23}$$

where $G_1=-G$, G_1 being positive in all the following discussions. It follows from equation (21) that the motion is periodically or aperiodically damped according as

$$1 \lessgtr QZ \tag{24}$$

In figure (4) Z is plotted as a function of x for several values of (G_1/Q). According to the criterion (24) we only have oscillattons at those values of x for which a line dtawn parallel to the x axis at a distance $\frac{1}{Q}$ above it lies below the curve Z, otherwise the equilibrium is restored aperiodically. It is easily seen from the figure (4) for all values of β_1, z rises from $x=0$ untll it reaches

$$Z_{max} = \frac{-\frac{4G_1}{Q} \pm \left(\frac{16G_1^2}{Q^2} + \frac{4G_1}{Q} + 1\right)^{\frac{1}{2}}}{\left(1 - \frac{4G_1}{G}\right) \pm \left(\frac{16G_1^2}{Q^2} + \frac{4G_1}{Q} + 1\right)^{\frac{1}{2}}} \left\{ 1 + \frac{\frac{4G_1}{Q}}{\left(1 - \frac{4G_1}{Q}\right) \pm \left(\frac{16G_1^2}{Q^2} + \frac{4G_1}{Q} + 1\right)^{\frac{1}{2}}} \right\} \quad (25)$$

and

$$X_{max} = \left\{ -\frac{4G_1}{Q} \pm \left(\frac{16G_1^2}{Q^2} + \frac{4G_1}{Q} + 1\right)^{\frac{1}{2}} \right\}^{\frac{1}{2}} \quad (26)$$

from this point on z falls monotonically with z, the curve approaching the x axis osymptolically.

It remains now to specify the properties of the motion. In the case of aperiodic damping there are two damping co-efficients (see equation 21)

$$-y = (1+x^2) \pm \left\{ (1+x^2)^2 - \left(Q + \frac{4G_1}{1+x^2} \right) x_2 \right\}^{\frac{1}{2}} \quad (27)$$

In the case of oscillatory motion there is only one damping coefficient

$$-R(y) = (1+x^2) \quad (28)$$

the angular frequency of the oscillation

$$\frac{\&}{d}(y) = \pm \left\{ Qx^2 + \frac{4G_1x^2}{1+x^2} - (1+x^2)^2 \right\}^{\frac{1}{2}} \quad (29)$$

The corrosponding wave and group velocities V_w & V_g satisfies the equations

$$V^2_w = \frac{\left[\frac{\&}{d}(y)\right]^2}{x^2} = \left\{ Q + \frac{4G_1}{1+x^2} - \left(\frac{1+x^2}{x}\right)^2 \right\} \quad (30)$$

and

$$V_g^2 = \left[\frac{\alpha}{\alpha x} \overset{\&}{\mathrm{d}} (y)\right]^2 = \frac{\left\{ Q + \frac{4G_1 x}{(1+x^2)^2} - 2(1+x^2) \right\}^{\frac{1}{2}}}{V_w^2} \qquad (31)$$

These equations are illustrated by figures 5 and 6.

ACKNOWLEDGEMENTS

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REFERENCES

1. Gandhi, J. M. (1964) Stability Problems in Hydromagneties (A Ph. D. Thesis), University of Rajasthan, Jaipur.
2. Hide, R (1956)—Waves in a conducting fluid of variable density. Proc. Roy. Soc. A, 233, 376–396.

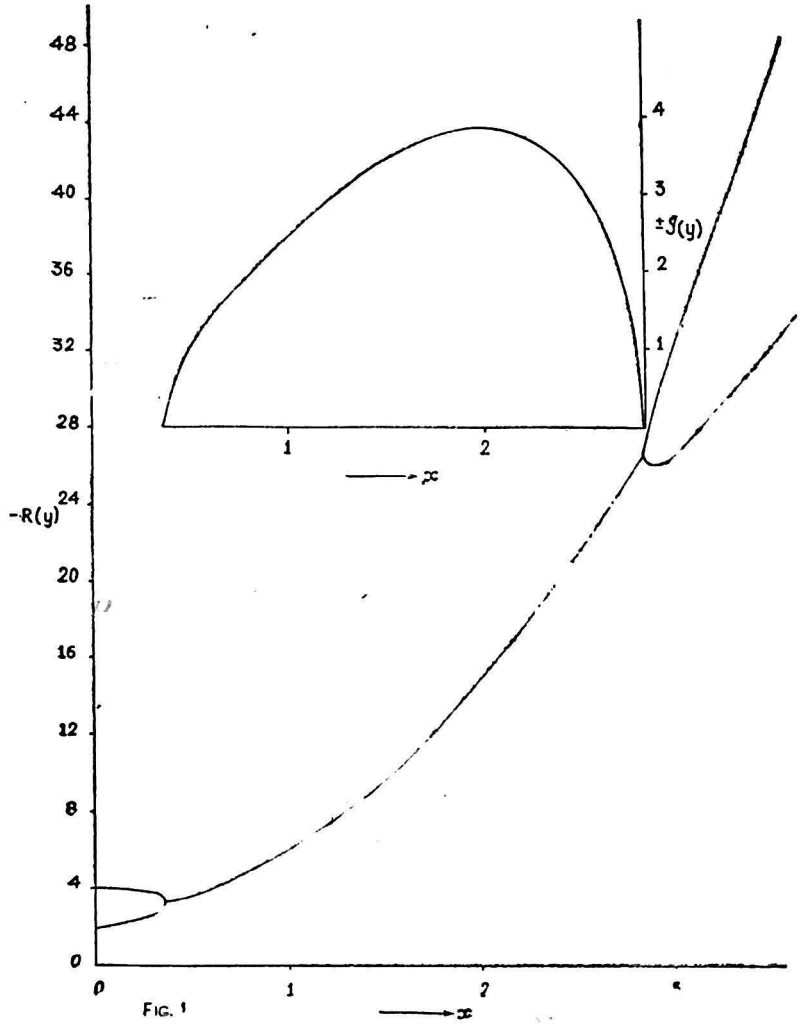

FIG. 1

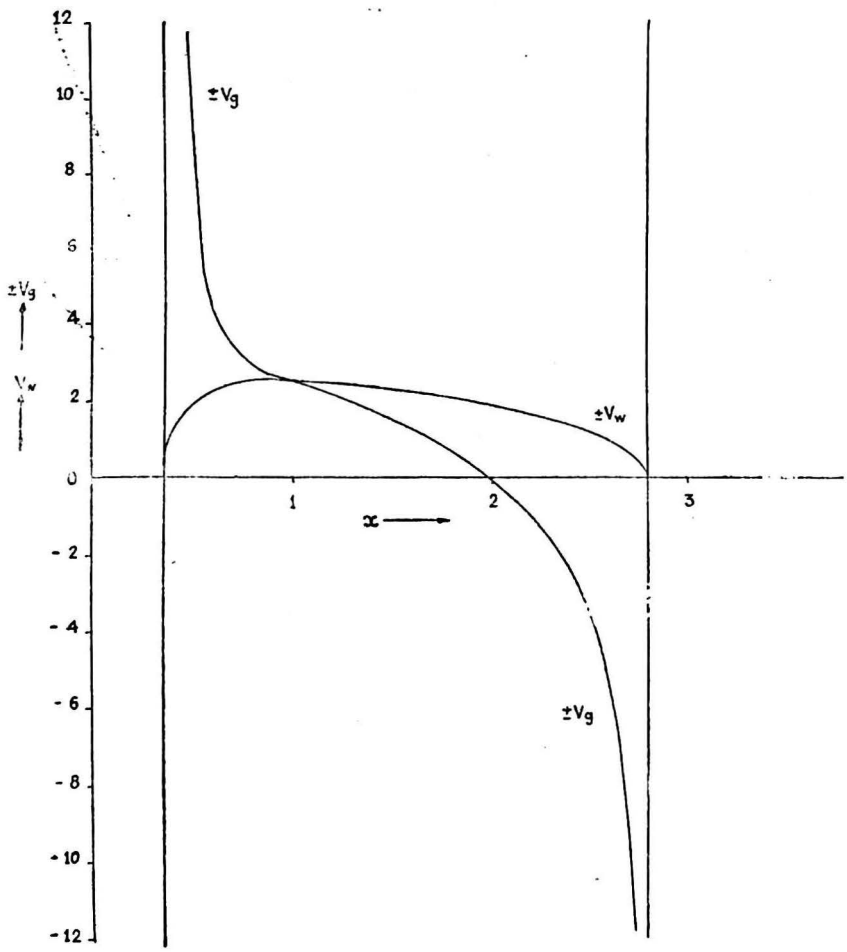

FIG. 2

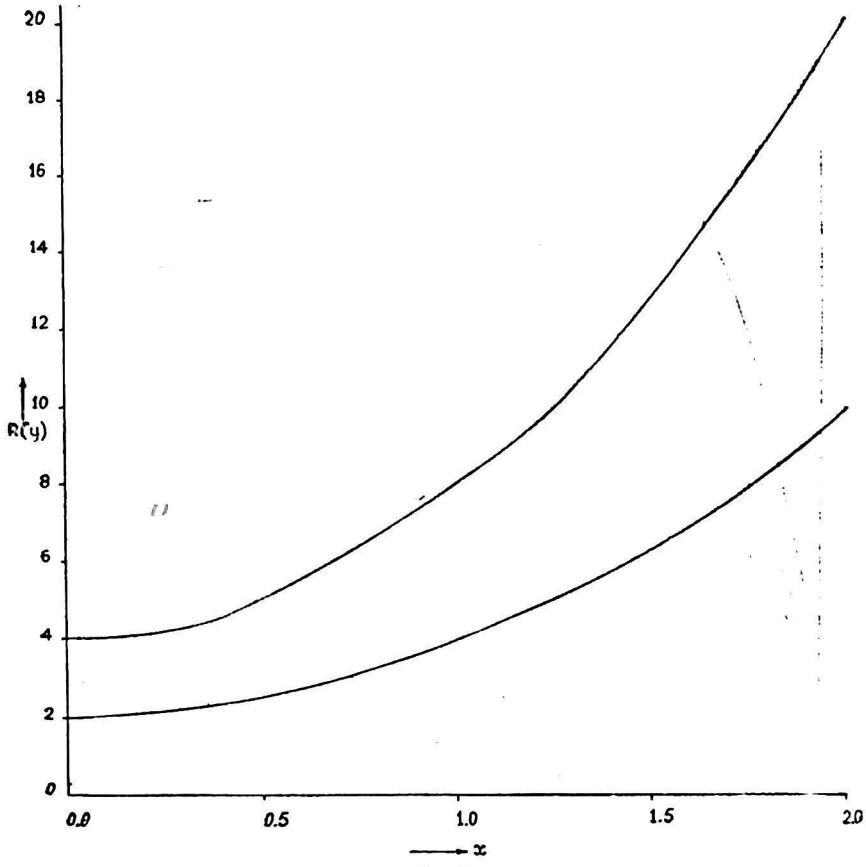

FIG. 3

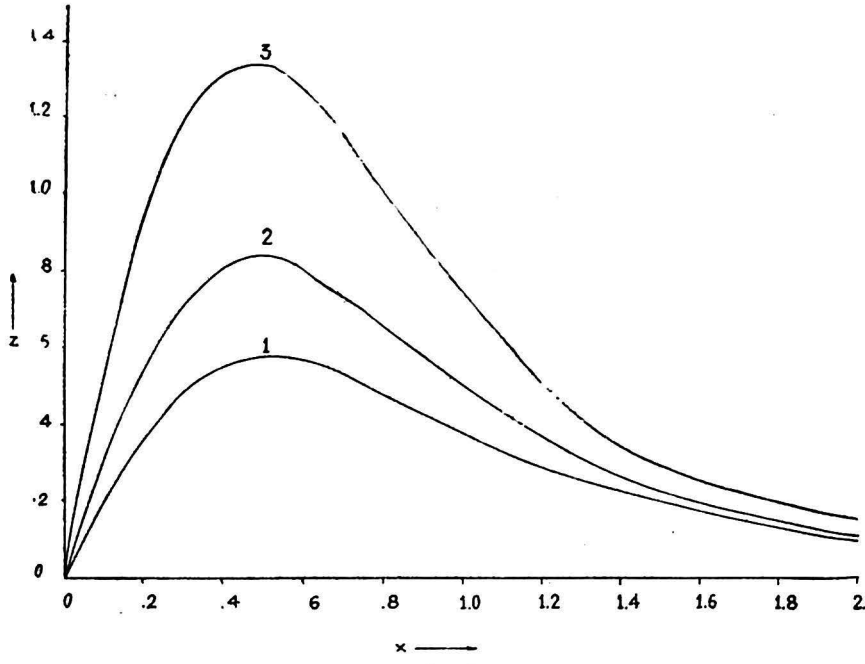

Fig. 4.

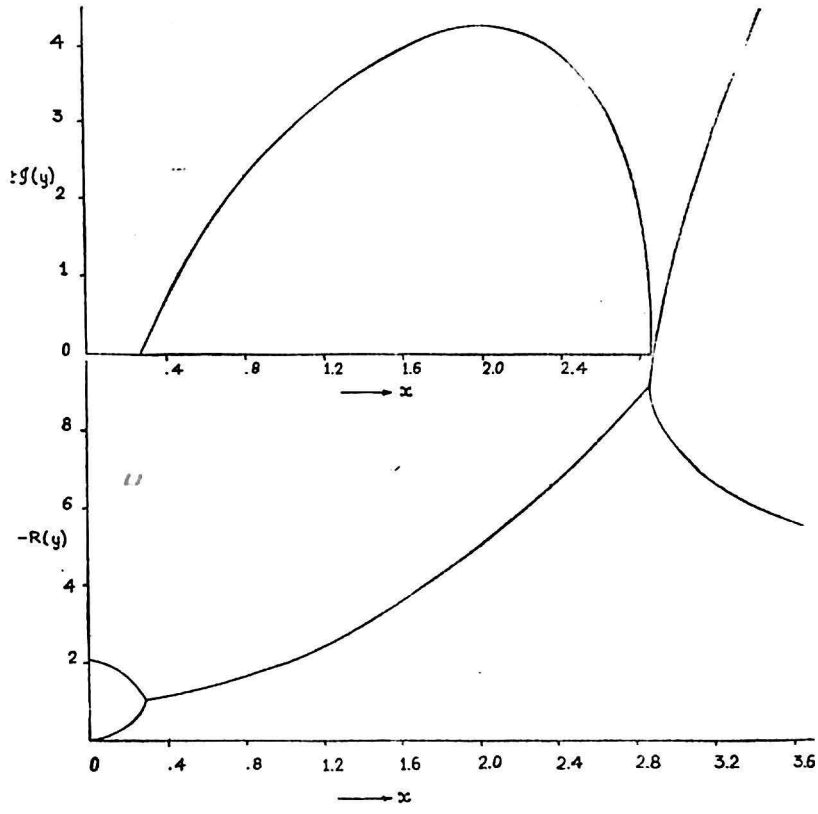

Fig. 5

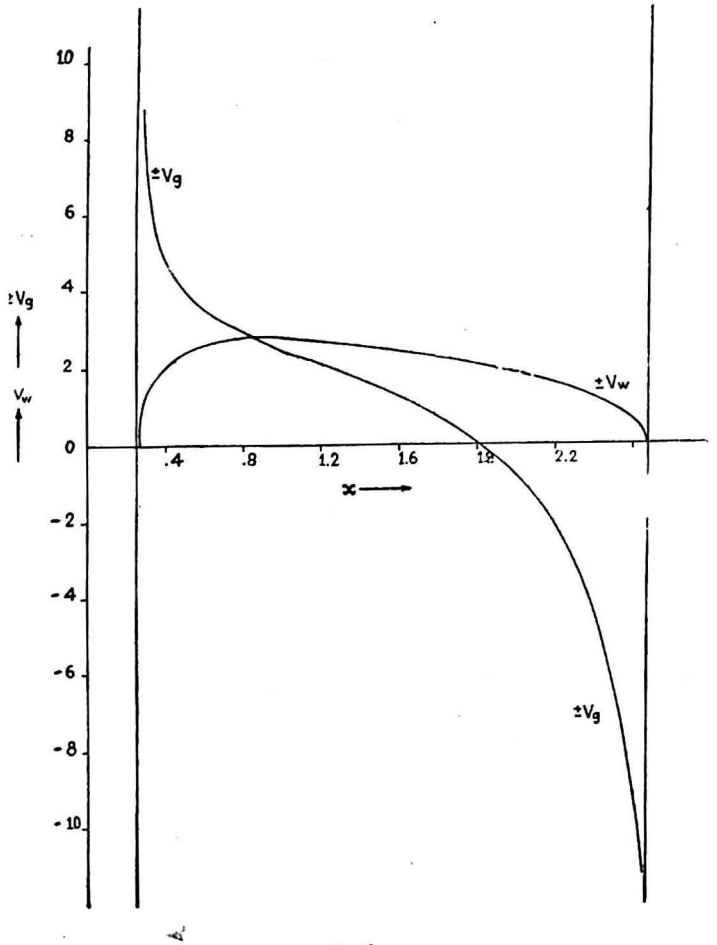

FIG. 6

A VARIATIONAL PRINCIPLE FOR A VISCOUS COMPRESSIBLE FLUID WITH A PREVELENT MAGNETIC FIELD

S. R. SHARMA AND J. M. GANDHI

A variational principle for treating the stability of viscous compressible fluid with a prevelent magnetic field is discussed. It is shown that the self-adjoint character of the problem provides the basis for a variational treatment of the problem. A necessary and sufficient condition of stability is also derived.

INTRODUCTION

Variational principles charactering the stability of hydromagnetic systems have been discussed by various authors [1—4]. The purpose of the present paper is to give a most general variational principle for a hydromagnetic system where the electrically conducting fluid is assumed to be viscous and compressible. (Separate variational principles for the compressible but non viscous fluid and for incompressible but viscous fluid were discussed by Chandrasekhar[5].) It is shown that the problem possesses a self adjoint character and the system forms the basic of a variational treatment of the problem. A condition for the stability of the hydromagnetic system is also derived.

Let us consider a viscous compressible and infinitely conducting fluid occupying a volume V enclosed in a surface S which is surrounded by vacuum. Let us suppose that in equilibrium a magnetic field $\vec{H}$ and material pressure p prevailes. In equilibrium, inside the fluid we have

$$\frac{\partial R}{\delta x_i} = \frac{1}{4\pi} \frac{\partial}{\partial x_k} (H_i H_k) \tag{1}$$

where R is the total scalar psessure,

$$R = p + H^2/8\pi \tag{2}$$

On the bounding surface normal stress must be continuous hence

$$P^{(in)} + \frac{|H^{(in)}|^2}{8\pi} = \frac{|H^{ex}|^2}{8\pi} \tag{3}$$

Further since we have supposed that the system is surrounded by vacuum, the external magnetic field must be curl free.

$$\text{Curl } \vec{H}^{ex} = o \tag{4}$$

further we must have everywhere on the bounding surface

$$\vec{H} \cdot \vec{ds} = o \tag{5}$$

so that the magnetic lines of force lie on the surface and this is a necessary condition if infinite accelerations are not to arise on the surface.

Now we shall consider the perturbed state of the system. The equations governing the first order perturbations of the initial state can be written as

$$\rho \frac{\partial u_i}{\partial t} = -\frac{\partial \omega_{ij}}{\partial x_i} + \frac{1}{4\pi}(h_i H_j + h_j H_i) \tag{6}$$

$$\text{and } \frac{\partial h_i}{\partial t} = \frac{\partial}{\partial x_j}(H_j u_i - u_j H_i) \tag{7}$$

where u_i and h_i denote the perturbed velocity and magnetic fields, p is the density of the medium, and

$$\omega_{ij} = \left(\delta p + \frac{H_k h_k}{4\pi}\right)\delta_{ij} - p_{ij} \tag{8}$$

with

$$p_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij} \tag{9}$$

in which μ is the coefficient of viscosity of the fluid,

We now introduce a displacement ξ which is connected with the velocity field by

$$u_i = \frac{\partial \xi_i}{\partial t} \tag{10}$$

Substitution of (10) in (7) immediately yields

$$h_i = \frac{\partial}{\sigma x_i} (H_j \ \xi_l - \xi_j \ H_i) \tag{11}$$

As usual, we consider the Perturbation quantities having exponential time dependence, $e^{i \sigma t}$, then the perturbed equation of motion gives,

$$-\sigma^2 \rho \xi_i = -\frac{\partial \omega_{ij}}{\partial x_j} + \frac{1}{4\pi} \frac{\partial}{\partial x_j} (H_j \ h_i + h_j \ H_i) \tag{12}$$

The perturbed fields outside V must satisfy the relations

$$\text{Curl } h^{ex} = o \tag{13}$$

$$\text{Curl } E^{ex} = -i\sigma h^{ex} \tag{14}$$

On the displaced boundary $S+\delta S$ the conditions which must be satisfied are ([5])

$$N_j \ \triangle \ (P_{ij}) = o \tag{15}$$

$$\vec{N} \ . \ \vec{H} = o \tag{16}$$

$$\vec{N} \times \triangle \ (\vec{E}) = (\vec{N} \ \vec{u}) \ \triangle \ \vec{H} = i\sigma \ (\vec{N} \ \vec{\xi}) \ \triangle \ \vec{H} \tag{17}$$

where $\triangle \ (f)$ represents the jump in the quantity f on $S+\delta S$ and $\vec{N}$ is the outward drawn normal to this surface. In (15)-(17) $\vec{H}$ stands for the total magnetic field, the perturbed plus unperturbed and Eq. (17) results from the relation

$$\vec{E} + \vec{u} \times \vec{H} = o \tag{18}$$

Equations (12) and (14) constitute a characteristic value problem for σ^2 with the boundary conditions expressed by means of equations (15) to (17).

Consider equation (12) belonging to $\sigma^{(\lambda)}$ multiplied (12) by $\xi^{(\mu)}$ belonging to defferent characteristic value $\sigma^{(\mu)}$, and integrate over the volume V, eq. (12) becomes

$$-\left(\sigma^{(\lambda)}\right)^2 \int_v \rho \xi_i^{(\lambda)} \xi_i^{(\mu)} dV = -\int_v \frac{\partial_{ij}^{(\lambda)}}{\partial x_j} \xi_i^{(\mu)} dV +$$

$$\frac{1}{4\pi}\int_v H_i \frac{\partial h_i^{(\lambda)}}{\partial x_j} \xi_i^{(\mu)} dV + \frac{1}{4\pi}\int_v h_j^{(\lambda)} \frac{\partial H_i}{\partial x_j} \xi_i^{(\mu)} dV. \quad (19)$$

Integrating the second term in eq. (19) by parts, we have

$$-[\sigma^{(\gamma)}]^2 \int_v \rho \xi_i^{(\lambda)} \xi_i^{(\lambda)} dV = -\int_v \frac{\partial \omega_{ij}^{(\lambda)}}{\partial x_j} \xi_i^{(\lambda)} dV -$$

$$\int_v \left(H_j h_i^{(\lambda)} \frac{\partial \xi_i^{(\lambda)}}{\partial x_j} - h_j^{(\lambda)} \frac{\partial H_i}{\partial x_j} \xi_i^{(\mu)} \right) dV. \quad (20)$$

substituting the value of from eq. (11) after a little simplification and making use of eq. (1) we get

$$-(\sigma^{(\lambda)})^2 \int_v \xi_i^{(\lambda)} \xi_i^{(\mu)} dV = -\int \frac{\partial \omega_{ij}}{\partial x_j} \xi_i^{(\mu)} dV -$$

$$-\frac{1}{4\pi}\int \left(H_i \frac{\partial \xi_i^{(\mu)}}{\partial x_j} \right) \left(H_k \frac{\partial \xi_i^{(\lambda)}}{\partial x_k} \right) dV.$$

$$-\int \xi_k^{(\lambda)} \xi_i^{(\mu)} \frac{\partial^2 R}{\partial x_k \, \rho x_i} dV + \frac{1}{4\pi}\int \left(H_j H_i \frac{\partial \xi_i^{(\mu)}}{\partial x_j} \frac{\partial \xi_i^{(\lambda)}}{\partial x_k} - \right.$$

$$\left. H_j \xi_i^{(\mu)} \frac{\partial \xi_k^{(\lambda)}}{\partial x_k} \frac{\partial H_i}{\partial x_j} \right) dV. \quad (21)$$

Now consider the integral

$$-\int \frac{\omega \partial_{ij}^{(\lambda)}}{\partial x_j} \xi_i^{(\mu)} dV = -\int_s \omega_{ij}^{(\lambda)} \xi_i^{(\mu)} dS_j +$$

$$\int_v \omega_{ij}^{(\lambda)} \frac{\partial \xi_i^{(\mu)}}{\partial x_j} dV. \quad (22)$$

We now expand eq. (14), regarding the continuity of normal stress tensor, expressing it in the surface integral.

$$\left[\omega^{(ij)}{}^{(\lambda)}\right]_s = \frac{H_k{}^{ex}\, h_k{}^{ex,\,\lambda}}{4\pi}\left(\vec{N}\,\vec{\xi}^{(\lambda)}\right)\left[\vec{N}\ grad\ \triangle_s\,(P_{ij})\right] \qquad (23)$$

where $\triangle_s\,(f)$ denotes the jump a quantity f experiences on s. Substituting (23) in the first integral on the right hand side of (22), and using the relation (cf. eq. 17)

$$\vec{N} \times E^{(ea,\mu)} = i_{\sigma^-}{}^{(\mu)}\left(\vec{N}.\,\xi^{(\mu)}\right) H^{ex} \qquad (24)$$

we get

$$-\int_s \omega_{ij}{}^{(\lambda)}\,\xi^{(\mu)}\,ds_j = \int\left(\vec{N}.\,\xi^{(\lambda)}\right)\left(\vec{N}.\,\xi^{(\mu)}\right)\left[\vec{N}.\,grad\,\triangle_s\,(P_{ij})\right]$$

$$ds - \frac{1}{4\pi\, i_{\sigma^-}{}^{(\mu)}}\int h^{(ex,\lambda)}.\left(\vec{N}\times E^{(ex,\mu)}\right) ds \qquad (25)$$

Now the second integral on the right hand side of eq. (25) can be changed to volume integral exterior to $\triangledown$ by Gauss's theorem;

$$-\frac{1}{4\pi\, i_{\sigma^-}{}^{(\mu)}}\int_s h^{(ex,\lambda)}.\left(\vec{N}\times E^{(ex,\mu)}\right) ds$$

$$= +\frac{1}{4\pi\, i_{\sigma^-}{}^{(\mu)}}\int_s \varepsilon_{ijk}\, ds_i\, h_j{}^{(ex,\lambda)}\, E_k{}^{(ex,\mu)}$$

$$= -\frac{1}{4\pi\, i_{\sigma^-}{}^{(\mu)}}\int_{V^{ex}} \vec{h}^{(\lambda)}\ curl\ E^{(\mu)}\, dv$$

$$= -\frac{1}{4\pi}\int_{V^{ex}} h_i{}^{(\lambda)}\ h_i{}^{(\mu)}\, dv. \qquad (26)$$

Hence

$$-\int \omega_{ij}{}^{(\lambda)}\,\xi_i{}^{(\mu)}\,ds_j = -\int_s \left(N.\,\xi^{(\lambda)}\right)\left(N.\,\xi^{(\mu)}\right)$$

$$\left[N.\,grad\,\triangle_s\,(P_{ij})\right]ds - \frac{1}{4\pi}\int_{V}{}^{ex}\, h_i{}^{(\mu)}\ h_i{}^{(\lambda)}\, dv. \qquad (27)$$

Now consider the second integral of (22).

$$\int_v \omega_{ij}{}^{(\lambda)}\frac{\partial \xi_i{}^{(\mu)}}{\partial x_j}\, dv = \int_v (\delta p^{(\lambda)} + \left(\frac{H_k\, h_k{}^{(\lambda)}}{4\pi}\right)\frac{\partial \xi_i{}^{(\mu)}}{\partial x_i} dv -$$

$$\int p_{ij}{}^{(\lambda)}\,\frac{\partial \xi_i{}^{(\mu)}}{\partial x_j} dv \qquad (28)$$

Let us assume that the variation in pressure and density take place adiabatically as we follow the fluid element. This amounts to the equation for perturbed state

$$\frac{\partial}{\partial t}\delta p + u_j \frac{\partial p}{\partial x_j} = \frac{rp}{\rho}\left(\frac{\partial}{\partial t}\delta\rho + u_j \frac{\partial \rho}{\partial x_j}\right) \tag{29}$$

where r is the ratio of the specific heats and p is the unperturbed pressure. From the equation of continuity we have

$$\frac{\partial}{\partial t}\delta\rho + u_j \frac{\partial \rho}{\partial x_j} = -\rho \frac{\partial u_j}{\partial x_j} \tag{30}$$

using eq. (10) we get

$$\delta\rho = -\xi_k \frac{\partial \rho}{\partial x_k} - \rho \frac{\partial \xi_k}{\partial x_k} \tag{31}$$

and

$$\delta p = -\xi_j \frac{\partial p}{\partial x_j} - C^2 \rho \frac{\partial \xi_k}{\partial x_k} \tag{32}$$

where c is the velocity of sound in the fluid

$$c^2 = \frac{rp}{\rho} \tag{33}$$

Using (32) we have

$$\int \delta p^{(\lambda)} \frac{\partial \xi_i^{(\mu)}}{\partial x_j} dv = -\int \left(\xi_j^{(\lambda)} \frac{\partial p}{\partial x_j} + c^2 \rho \frac{\partial \xi_k^{(\lambda)}}{\partial x k} \right) \frac{\partial \xi_i^{(\mu)}}{\partial x i} dv \tag{34}$$

Also we have used eq. (11) and simplifying

$$\int \frac{H_k h_k^{(\lambda)}}{4\pi} \frac{\partial \xi_i^{(\mu)}}{\partial x i} dv = -\frac{1}{4\pi}\int (H) \frac{2\,\partial \xi_i^{(\mu)}}{\partial x i} \frac{\partial \xi_j^{(\lambda)} dv}{\partial x j}$$

$$-\int_v \xi_i^{(\lambda)} \frac{\partial \xi_i^{(\mu)}}{\partial x i} \frac{\partial}{\partial x i}\left(\frac{H^2}{8\pi}\right) dv + \frac{1}{4\pi}\int H_j H_k \frac{\partial \xi_i^{(\mu)}}{\partial x i} \frac{\partial \xi_k^{(\lambda)}}{\partial x j} dv \tag{35}$$

Now consider the second intergal on the R.H.S. of eq. (28), using eq. (10) it becomes

$$\int p_{ij}^{(\lambda)} \frac{\partial \xi_i^{(\mu)}}{\partial x j} dv = \left(i_{\sigma}{}^{(\lambda)}\right)\left[\int \mu \left(\frac{\partial \xi_i^{(\lambda)}}{\partial x j} + \frac{\partial \xi_j^{(\lambda)}}{\partial x i}\right) \frac{\partial \xi_i^{(\mu)}}{\partial x j} dv \right.$$

$$\left. -\int \frac{2\lambda}{3}\left(\frac{\partial \xi_k^{(\lambda)}}{\partial x k}\right)\left(\frac{\partial \xi_i^{(\mu)}}{\partial x i}\right) dv \right. \tag{36}$$

Finally using equations (22), (27), (34), (35) and (36), and after a little simplification we can write equ. 21 as

$$(\sigma^{(\lambda)})^2 \int \rho\, \xi_i^{(\lambda)}\ \xi_i^{(\mu)}\, dV = \int_s \left(N.\xi^{(\lambda)} \right) \left(N.\xi^{(\mu)} \right) \left[N \text{ grad } \triangle_s (P_{ij}) \right] dS$$

$$+ \frac{1}{4\pi} \int_v \left\{ \left(H_k \frac{\partial \xi_i^{(\lambda)}}{\partial x_k} \right) \left(H_j \frac{\partial \xi_i^{(\mu)}}{\partial x_j} \right) + 4\pi\, \xi_k^{(\lambda)}\ \xi_k^{(\mu)} \frac{\partial^2 R}{\partial x_k\ \partial x_c} \right\} dV$$

$$+ \int_v \left(C^2\rho + \frac{H^2}{8\pi} \right) \frac{\partial \xi_i^{(\mu)}}{\partial x_i} \frac{\partial \xi_j^{(\lambda)}}{\partial x_j} dV + \frac{1}{4\pi} \int_{vex} h_i^{(\lambda)}\ h_i^{(\mu)} dV$$

$$+ \frac{1}{4\pi} \int_v \left\{ H_k \frac{\partial H_j}{\partial x_k} \left(\xi_j^{(\mu)} \frac{\partial \xi_i^{(\lambda)}}{\partial x_j} + \xi_j^{(\lambda)} \frac{\partial \xi_i^{(\mu)}}{\partial x_j} \right) \right.$$

$$- H_j\ H_k \left(\frac{\partial \xi_k^{(\lambda)}}{\partial x_j} \frac{\partial \xi_i^{(\mu)}}{\partial x_i} + \frac{\partial \xi_i^{(\lambda)}}{\partial x_j} \frac{\partial \xi_k^{(\mu)}}{\partial x_j} \right) dV$$

$$+ i\sigma \int \left[\mu \left(\frac{\partial \xi_i^{(\lambda)}}{\partial x_j} + \frac{\partial \xi_j^{(\lambda)}}{\partial x_i} \right) \frac{\partial \xi_i^{(\mu)}}{\partial x_j} \right.$$

$$\left. - \frac{2\mu}{3} \frac{\partial \xi_k^{(\lambda)}}{\partial x_k} \frac{\partial \xi_i^{(\mu)}}{\partial x_j} \right] dV \qquad (37)$$

we observe that the expression on the right hand side of the equation (37) is symmetrical in λ and μ and therefore

$$\int_v \rho\, \xi_i^{(\lambda)}\ \xi_i^{(\mu)}\, dV = o\ (\lambda \neq \mu) \qquad (38)$$

Hence self adjoint character of the characteristic value problem is proved.

If we replace $\xi_i^{(\mu)}$ and $h_i^{(\mu)}$ by the complex conjugate of $\xi_i^{(\lambda)}$ and

$h_i^{(\lambda)}$ respectively, we obtain.

$$\sigma^2 \int \rho |\xi|^2 \, dV = \int_s (N.\xi|^2 \left[N.\ \text{grad}\ \triangle_s\ (P_{ij}) \right] dS$$

$$+ \frac{1}{4\pi} \int_v \left\{ |H_j \frac{\partial \xi}{\partial x_j}|^2 + 4\pi\ \xi_i\ \xi_j^*\ \frac{\partial^2 R}{\partial x_i\ \partial x_j} \right\} dV$$

$$+ \int \left(c^2 \rho + \frac{|\vec{H}|^2}{4\pi} \right) \left| \frac{\partial \xi_i}{\partial x_i} \right|^2 dV + \frac{1}{4\pi} \int_{vex} \left| \vec{h} \right|^2 dV$$

$$+ \frac{1}{2\pi} \int_v \left\{ H_k\ \frac{\partial H_j}{\partial x_k} Re \left(\xi_j^*\ \frac{\partial \xi_i}{\partial x_j} \right) \right.$$

$$\left. - H_j\ H_k\ Re \left(\frac{\partial \xi_k^*}{\partial x_j}\ \frac{\partial \xi_i}{\partial x_i} \right) \right\} dV$$

$$+ (i\sigma) \int \left[\mu \left(\frac{\partial \xi_i}{\partial x_j} + \frac{\partial \xi_j}{\partial x_i} \right) \frac{\partial \xi_i^*}{\partial x_j} \right.$$

$$\left. - \frac{2\mu}{3} \left(\frac{\partial \xi_k}{\partial x_k} \right) \frac{\partial \xi_i^*}{\partial x_i} \right] dV \qquad (39)$$

or symbolically we can write

$$(i\sigma)^2\ I - (i\sigma)\ \phi - \Sigma = O \qquad (40)$$

The roots of this equation are

$$i\sigma = \frac{1}{2I} \left\{ -\phi \pm \sqrt{\phi^2 - 4\Sigma I} \right\} \qquad (41)$$

Since I is necessarily positive, a necesary and sufficient condition for stability is that Σ^1 must be positive. When the system is stable i.e. $\Sigma^1 > o$, the magnitude of ϕ will determine the modes of oscillation of the system.

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A NOTE ON LEXELL'S PROOF OF FERMAT'S LAST THEOREM FOR THE POWER 5.

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We, in thit note show that Lexell's proof for the impossibility of the equation $a^5+b^5=c^5$ [2] is incorrect. Firstly we reproduce L.E. Dickson's report [1] Page 732, on Lexell's proof for the impossibility of the above equation.

"A.J. Lexell considered $a^5+b^5=c^5$.

Set $x+y=a^5$, $x-y=b^5$, then

$(x^2-y^2)/4x^2=(z/x)^5=a^5\ b^5/c^{10}$

$x^6-4xz^5=x^4\ y^2=\square$

Since the factors are relatively prime,

$x=p^2$, $x^5-4z^5=q^2$, hence

$p^{10}-q^2=4r^5\ s^5$ or $p^5+q=2r^5$

$p^5-q=2s^5$, $p^5=r^5+s^5$."

Then we show that the above proof is incorrect.

Since $x+y=a^5$, $x-y=b^5$

$2x=a^5+b^5=c^5$,

which shows that x is necessarily even. Also it can be easily seen that z is even and hence the factors x and x^5-4z^5 are not relatively prime as was assumed by Lexell.

It is interesting to note that the assumption of the truth of Lexell's proof implies the proof of F.L.T. for all primes, for substitute the power 5 by p and all the results will remain as they are, except that the prime 5 in all the results will now be replaced by p.

However if Lexell's proof can be corrected without changing his outlines of the proof, a complete proof of Fermat's Last Theorem may probably follow.

REFERENCES

1. L.E. Dickson—History of Theory of numbers. Vol. II Chelsea Pub. Co. (1952).
2. Euler's Opera Postuma, 1962. 231–232. (about 1768)

COMMENT ON LOUI'S S. MANN'S PAPER

"A REMARK ON F.L.T."

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In this note we show that the results obtained by Mann in his paper [1] regarding Fermat's Last Theorem, are not correct.

$$\text{Consider } a^n+b^n=c^n \tag{1}$$

Suppose (1) factors into

$$\left(c^{n/2}-b^{n/2}-a^{p}\right)\left(c^{n/2}+b^{n/2}-a^{n-p}\right)=0 \tag{2}$$

Expanding (2) we have

$$c^n-b^n+a^n-a^{n-p}\left(c^{n/2}-b^{n/2}\right) -a^p\left(c^{n/2}+b^{n/2}\right)=0. \tag{3}$$

In order that (1) and (3) be identical, we must have (as is argued by Mann)

$$p=0,\ c^{n/2}-b^{n/2}=1,\ c^{n/2}+b^{n/2}=a^n \tag{4}$$

This seems to be incorrect.

In general (3) will be identical with (1) when

$$p=k,\ c^{n/2}-b^{n/2}=a^k,\ c^{n/2}+b^{n/2}=a^{n-k} \tag{5}$$

k is any integer.

Putting p=0 means assuming one impossible factor of (1), since then (1) will read as

$$\left(c^{n/2}-b^{n/2}-1\right)\left(c^{n/2}+b^{n/2}-a^{n}\right)=0$$

and we must have

$$c^{n/2}-b^{n/2}=1 \tag{6}$$

and $c^{n/2} + b^{n/2} = a^n$ (7)

Evidently $c^{n/2} - b^{n/2} = 1$ is impossible when $n > 2$. Similarly consider

$$a^n + b^n + c^n \quad (8)$$

Let $a = U^n V^n$ (9)

so that $U^n V^n + b^n = c^n$ (10)

Now Mann factorizes (10) as

$$\left(c^{n/2} - b^{n/2} - V^n\right)\left(c^{n/2} + b^{n/2} - U^n\right) = 0 \quad (11)$$

This assumption itself is not correct, since the truth of this implies that

$$c^{n/2} - b^{n/2} = V^n \quad (12)$$

and $c^{n/2} + b^{n/2} = U^n$ (13)

As discussed by Mann, equation (12) and (13) lead to impossible results when $n > 2$, therefore (10) cannot be factorized as (11), and thus Mann's method of attacking Fermat's Last Theorem does not succeed.

REFERENCES

(1) L. S. Mann., A remark on F.L.T. Maths. Mag. Vol. 28 (3), (1955) 153.