NEW LIGHT ON CHILDREN'S IDEAS OF NUMBER

The work of Professor Piaget

NATHAN ISAACS

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By the same author

THE GROWTH OF UNDERSTANDING IN THE YOUNG CHILD
A brief introduction to Piaget's work

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INTRODUCTION

DURING THE LAST FEW DECADES there has been a radical new approach to the problem of what number means for young children—an approach supported by a large body of most interesting and revealing experiment. Furthermore this forms part of a far wider plan of research, covering all the child's most basic ways of learning and providing the most valuable new insight into his mental growth as a whole. I propose here to give a broad sketch of this new approach to number which we owe to Professor Piaget, but in a way which will, I hope, convey also something of its much wider background.

Perhaps two purely personal anecdotes may help me to strike the right keynote. The first might be called "Getting the knack of Arithmetic".

As a small boy I took quite kindly to counting and elementary number ideas; but at some time in my first few school years I started falling behind. I remember vididly puzzling my head about the *reasons* for what we were being taught, and being unable to see them; but it did not seem to occur to the teacher to stop and explain, so my mind just stayed blank and bewildered, and I failed to do my sums. It worried me to be unable to keep up with the other boys, and all the more because I liked figures; but there I was. Then one day came the blinding flash of the obvious. It suddenly occurred to me that perhaps one could *learn the rules* for doing sums without stopping to understand them first. Perhaps that was what the other boys, or anyway some of them, did. I tried this out and it worked like magic. I was soon holding my own with most of the others, and after that, in the matter of arithmetical performance, I never looked back. But I recall that for some time a haunting sense of having *cheated* remained.

My second anecdote might perhaps bear the title: "What does number mean to the adult?" It may also help to illustrate, in a very contemporary setting, that the other boys may not in fact have understood any better than myself what this number business was really about. I found myself recently discussing with a Training College lecturer in the teaching of science the large theme of introducing some first scientific ideas into junior education. Presently something led to the not unusual question how far, for that purpose, mathematics was a science. What exactly was in fact its status or relation to the experimental sciences, or, if one liked, to all the other sciences? My lecturer friend thereupon tentatively defined mathematics as the making of certain assumptions and the working out of their consequences. That seemed right enough as far as it went, but obviously was only half a statement. I wanted to get a clear view of the other half which, it seemed, was bound to be in my friend's mind, so I asked him, "What assumptions, or assumptions about what?" His sole answer was: "Ah, that's philosophy." There the discussion ended. That is, without more than a statement that by itself meant strictly nothing. In fact just half a thought, which cried out logically for the missing other half.

Of course it is quite true that this way philosophy and its unending argument lies. And speaking for myself, I have spent so much time and effort on the argument with such small satisfaction that I could certainly not criticise anyone for shying away from it. But there does seem to be something wrong if a teacher of teachers of science can rest content with a formula which, *as it stands*, is not so much debatable as just meaningless. Perhaps, on that question of what number really means he, along with most of us, has not after all advanced so very much further than we earlier youngsters did—or anyway not further than from a knack-learning to a formula-learning stage?

But now, from this anecdotage, let me plunge into my real theme. I have long thought that there could possibly be an alternative, or third course, out of our past dilemma of either having to balk on some illogical threshold of philosophy, or else being swept right over into it. Might not the science of psychology be able to throw a measure of *factual* light on the question of how we arrive at the idea of number—what it means for us—how we build up its rules—and on what its extraordinary potency and usefulness rests? But the trouble was that until a few years ago it seemed impossible to point to any psychological work of which one could say: "Ah, that is what was needed. That is what we had been waiting for." And this fact seemed to give support to the philosophers, who mostly poured scorn on the very idea that a mere empirical science like psychology could have any real light to throw on the idea or meaning of number.

Now, however, the situation has, I believe, radically changed. Professor Piaget's work has come along and in my view does fill the bill. It shows that when the approach and planning are right, the natural science of psychology *can* offer the most illuminating light firstly on the development of the idea of number in young children, and secondly on its relation to the growth of the child's mental capacities as a whole. And this then, since it gives us the way in which we ourselves came by the notion of number, yields us a new insight into its make-up and working in our own minds. So that, if teaching is our job, we can now hope to teach arithmetic with understanding, and can perhaps even aspire to teach the understanding of arithmetic. We need not suppose that Professor Piaget's labours, or those of any psychologist, can solve any ultimate philosophic problems. And even from a more modest angle some of his conclusions no doubt remain controversial. But he does provide at least a comprehensive working model, resting on facts and capable of being further tested, which we can follow through as far as it will take us. And that, I believe, is vastly further than anyone has taken us before.

That then is the basis for this invitation to plunge into a not too familiar or easy terrain. I shall try to offer a fairly careful survey of the main gist of Professor Piaget's volume, *The Child's Conception of Number*, which fully sets out both his theoretical conclusions on this theme and the experimental work on which they rest. But I should say at once that Number by no means exhausts the scope of this book. Whilst it is primarily a study of the first building up in the minds of children of 4–7 years of the idea of number—as distinct from the earlier mere knack of counting and the later mere knack of school arithmetic—it also aims at a much larger target. It links up numerical thinking with *logical* thinking in the widest sense, and seeks to show that their development is most closely related and indeed each depends on the other. The basic unity of mathematics and logic has been the theme of many mathematical philosophers and logicians for some decades. But Piaget has, I believe, been the first to turn this theme into practical experimental psychology, with direct bearings on the mental growth, and so the education, of the young child. In the present study my chief topic is intended to be number, and the new understanding and possibly the new methods in the teaching of arithmetic, to which Piaget's work might lead. However I shall also try to bring out something of the link-up with logic, which may perhaps add an extra touch of novelty and even provocativeness to the discussion.

I propose to begin with a thumbnail sketch of Piaget and his labours as a whole, with a brief note on their present scientific status. Then, setting out from our common assumptions about number and arithmetic, I shall seek to show in broad general terms how his findings impinge on these. After that will follow a summary of some of Piaget's main experiments and their results, together with a short discussion of the latter. Finally, I shall offer some few comments on the chief educational implications of Piaget's results, anyway as I see them.

I

PIAGET AND HIS WORK AS A WHOLE

1. THE MAN AND HIS WRITINGS

FIRST OF ALL, then, Piaget himself. He is a French-Swiss genetic psychologist, born in 1896. He started with a training in biology, and still carries this with him, but he soon became more and more interested first in the philosophy and then in the psychology of knowledge, which in fact became his main lifework. For he had come to see in genetic psychology the key to the growth of the human capacity for knowledge and understanding as such. This meant for him, in essence, the growth of logical, mathematical and scientific thinking, and everything to which they have led. And by the age of 25 or so, he had worked out a great plan of experimental enquiry into the processes of intellectual development in children from their beginnings to maturity. Together with a large team of collaborators and pupils, he has been realising this plan ever since.

From the early 1920's onward he published a series of volumes on the language and thought of children, their judgment and reasoning, their ideas of causality and their notion of the physical world. These were followed by an illuminating excursion into the development of moral judgment in the child. Then came an intensive study of his own three infants from birth onward, recorded in two remarkable volumes: *The Origin of Intelligence in the Child*, and *The Child's Construction of Reality*. After that he poured forth studies of *Play, Dreams and Imitation in Childhood*; of the growth of the notions of number, physical quantity, space, time, movement and speed; of the development of the child's logic and capacity for abstract thought, etc., etc. As his findings took full theoretical shape, he also published some more general works on the psychology of intelligence and on logic, and furthermore a three-volume treatise called *An Introduction to Genetic Epistemology*, which is a comprehensive analysis of the development of the main type of knowledge in both the race and the individual. A number of his books has not been translated yet, or anyway has not yet appeared in translation. Unfortunately also much of Piaget's work is at best not too easy to read, whether in the original or in its English rendering, and in one or two cases the latter has not served its author too well.

I should add here that in the opinion of some of us the first group of writings, though very stimulating, was open to deep-reaching criticisms. However, Piaget subsequently modified his procedure in the light partly of these criticisms, but still more of his intensive study of his own children and of all he learnt from them; and his later results in my view carry substantial conviction. The book on Number belongs to this later period. It was first published in French in 1941; the English translation came out in 1952. It is only fair to say that whilst the latter shows Professor Piaget as sole author, the French original joins with him a colleague, Mlle. Alina Szeminska, whose name should equally have appeared in the translation. However, it remains true that the theoretical inspiration was obviously Piaget's, as part of his total research plan, which had already gone on for many years. And he had always mobilised the labours of a considerable number of helpers, above all experimental; that in fact is one of the important sources of strength of his work. Their contributions have thus been material and should not by any means be underrated. (One of them, Mlle. Inhelder, is now a permanent close colleague who speaks with an authority second only to Piaget's own.) Yet the master-plan remains his and he stays the true architect of the great structure of new knowledge and insight linked with his name. I shall therefore continue, if only for simplicity's sake, to refer solely to him.

2. HIS OVERALL VIEW OF MENTAL DEVELOPMENT: ACTION AS THE KEY EVEN TO MATHEMATICAL AND LOGICAL THOUGHT

If now we look at his work as a whole, the first point to note is that we have here a vast series of ingenious and searching experiments, spread over more than a generation and over most major aspects of intellectual, development, all leading to mutually supporting results. The general viewpoint which Piaget formulated at an early stage has in fact, in the further course of his labours, been steadily confirmed, elaborated more fully, and again confirmed.

The essence of that view is this. The starting-point and crux of the child's intellectual growth is not—as it was long the fashion to assume—sensory perception or anything else passively impressed on him from outside, but *his own action*. And action in the most literal, physical sense of the term. From the beginning it is patterns of active behaviour that govern his life. Through these he takes in ever new experiences which become worked into his action-patterns and continually help to expand their range and scope. It is through actively turning to look or listen, through following and repeating, through exploring by touch and handling and manipulating, through striving to walk and talk, through dramatic play and the mastery of every sort of new activity and skill, that he goes on all the time both enlarging his world and organising it. His own physical activity thus enters from the outset into his whole world-scheme and indeed fashions it, supports it and provides the master-key to it.

In effect thought itself is now simply an internal version or development of outward action. It is action which becomes *progressively* internalised through the child's acquisition of language and his growing use of imagination and representation. It then goes on expanding under the guidance partly of social life, partly of the physical world, till it culminates in a great organised scheme of mental *operations*. This is governed by certain rules of *mobile* equilibrium that allow us to make the most flexible use of our knowledge and to regulate our thought-life to our utmost adaptive advantage. These rules form themselves into two closely related patterns, intimately interacting with one another and probably at bottom one, which we call logic and mathematics respectively. Their operation represents our intelligence at its most effective level. They can both be clearly seen at work in those great notion-systems whereby we order all our experience: space—time—objects—causality, and so on. Piaget traces the development of each of these systems from its beginnings until it becomes fully operational. And the key all the way, up to the *most abstruse forms of logical and mathematical thought*, remains *action*. The child stays an organism or person continually interacting with his environment and striving by ever more complex procedures partly to fit himself into his world, partly to fit it to himself, physically and socially.

So condensed a sketch may not convey overmuch at the present stage; but it is only intended as a first backcloth and I hope will gather further meaning as we go along. I should only make clear again that I am not putting Piaget's work forward as *fully* established but rather as a point of view which is tremendously worth following through, at least as a working hypothesis. And since over most of its range it has strong experimental support, we should either have to find some major flaw in this, or else be ready to treat it as something with which we must come to terms.

I should add here, regarding the view taken of Piaget's work by contemporary British psychologists, that it is still somewhat early days for any definite summing-up. Only within the last few years has widespread attention been brought to bear on this work, and active scientific research focused on it. There had previously been a tendency to treat it as interesting, but rather off the main line of advance of modern psychology. It was criticised as too philosophic or not sufficiently scientific, not properly standardised and controlled, not satisfactorily presented and badly lacking in any statistical foundation, etc. Some of the latter criticism is not to be gainsaid, as I shall note. However, in spite of all this, the sheer calibre and weight of the steadily mounting work has more recently begun to win through. In a number of places experimental psychologists have seriously started checking up on Piaget's findings, repeating this or that part of his investigations, organising closely related researches, and so on. Much of this work is still uncompleted, or unreported, or anyway unpublished. However, it can be said that several broad confirmations of his results, both as regards number and in other fields, have already been obtained. One particularly interesting instance is an enquiry recently carried through at Aden on the number-ideas of local schoolchildren representing the most diverse races, where the investigator was fascinated to obtain from Arab and Somali children just the same kind of responses as Piaget has reported on his European, that is Genevan subjects.

The broad confirmations found do not exclude points of difference, and it may well emerge that both his concrete findings and his theory need some qualification, above all in the direction of greater flexibility. On the other hand there is still much misconception about the meaning and effect of some of his views. Once this is corrected, I think one can fairly sum up that such confirmation as has already accrued, together with the cumulative and cross-checking force of Piaget's own evidence, has now established his work as a development of major importance that demands the closest attention.

II

PIAGET ON NUMBER—VERSUS OUR COMMON ASSUMPTIONS

3. OUR COMMON ASSUMPTIONS: COUNTING AS THE SOURCE OF NUMBER AND OF ARITHMETIC

COMING BACK to the book on Number, we can see now how close this is to the heart of his general theory. For it is directly concerned with both mathematics and logic, and with the child's first efforts to enter into these two basic ways of organising his thought. But before I turn to the perspectives which Piaget opens up, let me try first to formulate what most of us would ordinarily tend to believe about the meaning of number and arithmetic. We can then clearly assess just where his contribution comes in.

Supposing we start in the time-honoured way from a dictionary definition which, if it is nothing else, is at least an express attempt to formulate what we all *think* we mean. The *Concise Oxford Dictionary* gives as one of the basic senses of the word "number" the following: "Tale, count, sum, company or aggregate of persons or things or abstract units" and also "symbol or figure representing such aggregate". That certainly covers what most of us would regard as the important arithmetical sense. I imagine however that we should at once want to pick out the counting element and put the main stress on it. We should think of number as the result of any process of putting together one by one. We might then add that arithmetic beings when each successive term of a counting operation, from "one" onward, is represented by a written symbol forming part of a regular scheme by which all such terms can be represented. A number thus is, for a start, any member of a systematic counting scheme which begins from one and proceeds one by one; and it is *the* number of all sets or collections that can be formed by the same process carried to the same point.

This of course deals only with the natural whole numbers and does not pretend to be anything like a final, word-perfect definition. It can, however, pass, I hope, as a first attempt to refine out with a little care just what we do mean by number, in the arithmetical sense. And it does lead straight on to the further stages of counting in groups, instead of one by one; combining groups and separating them; combining sets of groups and dividing them; and generally moving backward and forward with complete freedom within our systematic scheme and working out the rules of all the different operations we can perform within it, whilst maintaining its *basic* character of one-by-one countability. The sum of these operational rules within that scheme would then be what, at least to begin with, we mean by arithmetic.

The basic pattern is thus very simple and counting may well appear all the open sesame the child needs, anyway for his initial entry into the field. When he has learnt to count he has, it seems, the main secret of number as such. The rest after that is just following through and elaborating, but above all doing the necessary grind. That is, learning the names of the larger numbers, learning the scheme for symbolising and arranging them, learning the rules for manipulating more and more complicated models and throughout all this, practice, practice, practice. Nowadays of course we realise the need for a broad basis of concrete number *experience*, and also for keeping our number-work in touch with the practical things that interest the child. Thus we help him to see the countless ways in which numerical problems constantly arise; the indispensable need for arithmetic in practical life; and its vast potency and value for successfully coping with the most varied tasks. In other words, we have more and more come to recognise that the child must somehow be *interested*, and must be kept interested, as the stages of the grind proceed.

But we also know full well that these are only stages on the way to real arithmetic. It is not in fact such until it has been *freed* from these concrete entanglements and distractions and turned into the science or skill of pure calculation based on pure numbers. It only becomes applicable, as it should be, to virtually everything, when it is in fact applied to nothing, and thus becomes true arithmetic and no other thing besides. However, we are likewise aware that the nearer we approach to this goal, the more liable we are to lose our hold on many of those we are trying to guide there. The further we penetrate into real arithmetic, the more we come up against children who dejectedly feel that they are no good at figures, or frankly detest them, or are bored or frightened by them. Many of these then only go on because they have no choice; but they do so on stay-in strike lines, and shed the burden the first moment they can, and if possible for the rest of their lives.

4. ARITHMETIC AS AN EDUCATIONAL PROBLEM: WHAT LIGHT DOES PIAGET THROW ON THIS?

In all this there seems so far to be no particular mystery or *theoretical* difficulty. We might reasonably conclude that numbers are a special interest or even a special gift, and that by and large we must accept the frequent lack, or low level, of this interest or gift as we find it. Since arithmetic is so essential a tool for most of the practical purposes of life we must somehow go on coaxing our young people into some minimal ability to handle it, by whatever ingenuity or skill we can muster. We must east and lighten the grind and employ whatever adventitious aids we can devise. And above all, we must, as already stated, strive to enlist and maintain their interest. Even so, it all remains, with too many of our children, a very uphill business and even among those teachers who have in the main accepted modern ways, some may perhaps, in this matter of figures, look back with a little nostalgia to earlier days. That is, to the days when children were expected to make every *effort* to learn, however uninterested, what *we* knew they would need, and if they dragged their feet, could have salutary pressure applied to them. And of course many teachers still think this the only possible way, certainly with figures, if not with everything else.

The pity of it is however that for so many small children counting, when first learnt, is fun, but arithmetic is not. Yet the latter could be all the *games* one can play with counting. But that does not get through to the child. For him arithmetic is something quite different, namely just the grind of "sums"!

What light now does Piaget's work throw on this situation? Can he help us to understand better what happens in our children? Can he enable us so to handle the transition from counting to arithmetic that the latter will remain *alive* for the child? And how does he view the relation of the two, and indeed the nature of arithmetic? Here we may find ourselves plunging into somewhat deep waters. But let us turn now to what Piaget actually tells us.

III

THE OUTCOME OF THE EXPERIMENTS

5. GULF BETWEEN CHILD'S ABILITY TO COUNT, AND THE "IDEA" OF NUMBER

THE FIRST and most startling thing which Piaget demonstrates is the great gulf there is for the young child between being able to count and even the most rudimentary real numerical idea. Counting for him does not generate number. It is an enjoyable minor skill which he readily learns up to let us say "ten" or even higher, and then goes on performing for its own fun's sake. But he may not have any glimmering that a number, once counted, has any existence or status of its own, or is equal to another number similarly counted, or that it cannot grow or shrink by turns, or even do both at the same time.

And there is no reason to think that these are things he learns through the arithmetic he does at school. He does move on to them, often within a matter of a few months, but this may well happen before his school arithmetic has begun. Children down to 5 years may already be in possession of true number ideas. But older ones, however assiduously "taught", may not have them yet.

What must actually take place, as Piaget's work shows, is an inward course of growth, a process of organisation and structuring, as a result of which an idea that did not exist before is presently found functioning and in clear control. The child now *behaves* quite differently from the way he did before. What had completely baffled him a few months earlier has become self-evident and a matter of course. Contradictions and absurdities at which he had not turned a hair, he now dismisses with adult scorn.

That does not mean—as we shall see—that this inward process goes on irrespective of what happens to him in the outer world. School arithmetic may not have much to do with it, but that is another question. The important point for the moment is that whatever may or may not happen from outside, there is a great *psychological* distance to be spanned between the child's learning to perform counts, however proficiently, and his attaining the first genuine, working idea of number in his mind. Piaget demonstrates this by first evidencing in a dozen different ways the complete *absence* of any such idea at the initial stage of his experiments, and then in the same dozen different ways its full *presence* in children who are some months or perhaps up to a year older. Thus from the ability to perform counts to the possession of the idea of number a spanning process does and must occur within the child's mind. At his half-way stage, in the same dozen different situations, Piaget shows the process actually happening.

What is particularly interesting, of course, is the manner in which, by this progressive experimental analysis, he brings out all that enters into the make-up of the genuine idea of number when it *is* there—in the child or *in us*. Counting remains the final key, but it is the key to a far more complex psychic structure than my earlier account of our ordinary assumptions might have suggested. What the bearing of this may be on the problem of education in arithmetic is a separate issue. The first need is to acknowledge the facts and to try to understand them. The complex psychic structure which we designate as the proper idea of number may, as I have already noted, be present even in a 5 year old. But it is vital to realise that it is present only as a functioning structure, not as an explicit concept, and the child is quite incapable of giving an account of it in language, as Piaget has done for *us*. That indeed is likely to remain true for the rest of his career, and for that of most of us, even if we become the most expert arithmeticians, or possibly even mathematicians. The trouble is perhaps precisely that this functioning psychic structure gets formed so early and then goes on functioning *so automatically and unawarely*. Thus may be created the great psychological gap between the child's first achievement of the idea of number and what seems to so many children the dead and meaningless grind of their school arithmetic. Perhaps the value of Piaget may be to enable the teacher, with a new understanding of the child and even of himself, really to cope with this gap. But of that more later. Let me only acknowledge here for myself that Piaget's way of taking the idea of number to pieces and re-assembling it, and better still showing us how it gets put together in the first place, has made the concept come far more *alive*, as well as far more clearly articulated, in my own mind than it ever was before.

6. NUMBERS AS "PERSISTING" PRODUCTS OF COUNTS, AND MEMBERS OF A REALM OF NUMBERS

To return to the child's own unsuspected long voyage from the mastery of counting to the idea of number, the crucial fact is that, in the initial phase, his interest lies in the activity, not in its product. The latter, the resulting number, is just not conserved as such. Counting signifies not numbers but *merely* counts. If the child is challenged or tested on numbers as such, even when he has just generated them himself by counting, we find that they have dissolved immediately into a general blur, in which they are fused with size, shape, spacing and their perceived context generally, and it is one or another of these directly perceived elements that dominates, whilst the counting as such counts for nothing.

What must evolve gradually instead is that the product of the count somehow begins to persist for the child, as itself an object for attention and interest and, as it were, now an entity existing in its own right. It must remain linked with the count and controlled by this alone; but different countings should engender in the child's mind the idea of a whole realm of such count-generated entities; and gradually an ordered realm, all of the same kind, in which one can pass from one member to another and eventually from any member to any other, by regulated repetitions of the same counting activity.

This realm must thereupon be more and more completely separated—always in the child's mind—from the context of perception, from size and shape and spacing and arrangement and all their changes and vicissitudes. It must be turned into the notion of a world apart, which, in contrast to perception, is up to a point entirely under one's own control. In that world indeed one can generate numbers just by one's own counting, even if one has nothing tangible to count, or anyway nothing but tokens or imaginary units. But one's control is now merely that of the points of entry into this world. As the child begins to attend to the numbers he has counted up as members of *their* own world, he comes to see that they have their own nature and properties, all linked up with one another. And thus counting becomes for him merely the way into this special realm, which one must study and learn about, just as with the physical world. He will then be ready to find that there is a subject, a school-subject, arithmetic, which consists in just that learning. But by that time he will probably have lost the original and controlling link with counting; this will be a very elementary activity which for most purposes he has left behind; and the world of numbers and arithmetic as such will most often seem a very dead and boring one, which means nothing whatever to him.

That is something like the cycle through which too many of us pass. If, through one cause or another, we do stay interested, we can recover eventually the counting-key by which the whole world of number is formed and controlled. And if we turn to Piaget we shall find that for his theory this does indeed remain the master-key linked up with his whole view of human action, operation and mental development.

7. SURPRISING NATURE OF PIAGET'S FINDINGS

Thus the wheel comes full circle and Piaget shows how and why it does so. At the stage of the small child, however, all that concerns us is that before he can *handle* even the most elementary number-situations, he must first form a properly structured idea, or functioning schema, of number and numbers. And for this purpose he must somehow get the order or system of number, as a separate self-existing realm controllable only by counting, into his own system.

How utterly remote he is initially from this, we shall see from the behaviour of Piaget's first stage children in his actual experimental situations. Most readers are completely taken by surprise by the extent of the failures, the contradictions and absurdities, the blank incomprehensions, into which Piaget's youngest group, however much at home in counting, appear to fall. Many people are indeed strongly inclined to discount the experimental reports and to question the validity of work that seems to lead to such incredible results. They suggest that the questions are badly put or misleading, and that anyway the children have obviously failed to understand them, or else cannot express their answers properly. After all, as they say, in stage 1, these infants are for the most part only 4–5 years old. But this sort of negative attitude cannot really be sustained in the face of all the detailed discussions with so many individual children, the diversity of experiments and the changes rung on each one, and the various forms of concrete help given by the experimenters. Furthermore, as I have suggested, the case is most strongly clinched by the evidence of a half-way stage in which the children can be *seen* feeling out towards something they have not quite reached yet, scoring partial successes in easy cases, now advancing tentatively and gropingly, and now falling back. And so that no one can say that the grasp of number which is being tested is far above the heads of small children anyway, we have already noted the further fact that at an average age of only a year or so more, there is the most complete antithesis to the first-stage picture—a set of answers by children of 6–8 years as rational and adequate as an ordinary adult's, and as *confident* as his could possibly be.

Thus the very surprise and incredulity aroused by Piaget's first-stage results is a measure of the novelty, the value and revealingness of his work. The great gulf between counting-ability and even the most rudimentary working grasp of number is both proved to be there, and proved to be successfully bridged. The process by which the small child does so is in truth one of internal growth—for what else could it be? But, as I have indicated, we must be on our guard against jumping to the conclusion of so many readers of Piaget that he must mean a *purely* inward growth or maturation; that is, one which takes place quite independently of the child's outward life. That does not follow in the least. Piaget's own interest lies in laying bare the nature of the total inward evolution of the child's mind, and he does not concern himself with outward circumstances. But when occasion arises he does expressly acknowledge the effect they could have on the detailed rate of different children's mental growth. And above all one should remember that his very model of such growth turns on the child's active relations with the world around him, and his continual interchange with this through his action upon it and its reaction on him. His basic pattern of advance is continuous varied activity bringing the child further and further experience, and then the embodiment or assimilation of this in his action-patterns which in just that way expand into ever wider scope and richness. That is no model of purely inward or maturational growth. It may on the contrary entail an educational approach for which only the self-education pivoted on the child's own activities and active experiencing is psychologically real. That would be very much more in line with Piaget's own educational sympathies and outlook. But this is a theme to which I must return in my final discussion.

IV

THE PRESENTATION OF THE EXPERIMENTS—AND THE EXPERIMENTS THEMSELVES

8. DIFFICULTIES AND STUMBLING BLOCKS

I COME NOW at length to the actual tenor of Piaget's work on Number and his detailed experiments. I have spent much time on the theoretical build-up, since without it most of the value of Piaget's findings must inevitably get lost. In fact the book itself is distinctly difficult to read, and even more so to consider and appraise. I naturally do not wish to discourage anyone from trying it; my object, on the contrary, is to praise Caesar, not to bury him. But one must be well-warned for a somewhat laborious enterprise, for which the author himself does not afford over-much help. And just because nevertheless it *is* so important and enlightening, I feel I must add some more cautionary comments before I pass on to the detailed contents of the book.

(1) Most of the theoretical part—as distinct from the experimental material—is written in an abstract and often highly technical vocabulary which Piaget does little to explain. Thus he draws freely on the language of modern formal logic, and also rather seems to assume that his readers will be already familiar with his own theoretical thought. One can still follow most of his thought, even if one is not equipped in either of these ways; but the going is undoubtedly hard.

(2) He fails to give reasonable introductory information about the actual place and circumstances of his tests, or even about the numbers of children tested. I *believe* the work was done partly at a nursery school in Geneva with which Piaget had long been associated, and partly at Genevan infant and primary schools. The children were not sorted out in any way as regards intelligence level, and no particulars in this respect are provided.

(3) Piaget exhibits his results, as already indicated, in terms of three stages: a first one showing the total lack of any idea of number—a second or intermediate one which exhibits some groping and uncertain progress—and a third one where normal adult-level responses are produced as a matter of course. But these are not in the main three stages in the growth of the *same children*. There are a few cases where the same child does crop up again, a stage further on. By and large, however, the subjects are mostly different, not only for each experiment, but for each

stage. All that really happens is that Piaget cites, in each instance, some total-flop responses, some half-way ones, and some that are confident and correct. These he marshals as his three stages. The only direct link with the observable facts of individual growth is that the average ages of the children in each successive group show a *progressive increase* of a year or so.

This emerges in spite of much overlap, with stage 3 responses from some children in their 6th year and stage 1 reactions from some between 7 and 8. That of course is just what one would expect from unselected mixed groups, in which some 5 year olds might have a mental age in advance of that of other children aged 7. Probably the progression would have come out far more sharply and impressively if mental ages had been determined and these instead of chronological ages had been correlated with the stages. Also people would not be misled into linking particular stages with particular chronological ages, and either trying to refute Piaget by challenging these linkages, or else drawing quite unwarranted educational conclusions from them.

On the other hand one would have liked to know more about the children of about 7 years who were still in the total-blank stage. They, and the slow movers generally, might well provide a large proportion of those who later on could not get on with figures—perhaps *because* they never had the right active experiences for the vital first step. But these are speculations in which one gets little aid from the way the data are presented, or perhaps have been collected.

All one can say on the whole theme is that Piaget seems to rely mainly on the *internal evidence* of progression between his three stages and although such reasoning *could* be dangerous, in fact his evidence seems to me extremely convincing. Moreover the advance of each stage in average age does provide noteworthy extra support. I might perhaps add that the number of children whose similar and surprising total-flop responses are given is 77—quite a respectable figure.

(4) The actual sequence of experiments, though in the main logical enough in the light of Piaget's own theoretical thought, seems to entail one or two anomalies even by this standard. In any case, however, it presents additional stumbling blocks to those not familiar in advance with his thought. He starts from cardinal numbers, but oddly enough *begins* with some experiments in *continuous* quantities, i.e. with liquids, which seem rather off his main target. Then he considers at great length ordinal numbers and their relation to cardinal. After that come some experiments in pure logic and much discussion of these. Finally there is a return to cardinal numbers in a more developed form, illuminated, as Piaget holds, by both the ordinal and the logical discussions. This progression involves some points of theory which I personally regard as controversial, but cannot try to cope with here. I shall chiefly focus on the cardinal number sections, which I think are what most of us have in mind in connection with arithmetic, and can only refer to the ordinal-

number work in passing, but I do want to leave some little space for the vital and most suggestive cross-reference to logic.

Furthermore, I shall depart from Piaget's own arrangement in another way, but this time simply for the sake of properly bringing out what I myself found his most dramatic effect. Piaget, for each experiment, gives in sequence the "stage 1", "stage 2" and "stage 3" responses. I shall group together all the experiments on cardinal numbers and then present all the stage 1 responses, in order to demonstrate the full reality and consistency and unshakable non-comprehension of this stage. I shall then more briefly illustrate the stage 2 and stage 3 answers, and proceed in the same way as regards the logical experiments.

9. THE EXPERIMENTS AS PLAY SITUATIONS

With this I come to the experiments themselves. They are all put in the form of *play* situations into which the children generally seem to enter with interest and even zest and, up to near the limit of their capacity, with ready co-operation. I have no space here for the details of the "pretend" build-up, but great ingenuity as well as understanding of small children's ways has gone into devising most of the situations, so that they should come as naturally as possible to the children, anyway to begin with, and carry them along.—In the case of cardinal numbers one main way in which Piaget and his helpers tested whether any idea of number as such existed was to vary the shape, apparent size, spacing and arrangement of a group the children themselves had counted, and then to see whether they stood by their counted number or not. Another way was to try if they could do something as simple as matching a given counted group with another equal in number, either one by one or any way they liked. A third order of tests was whether they could re-arrange two unequal heaps into two equal ones, or appreciate something so elementary as that two equal sets of things, even if thereafter subdivided differently, would still stay equal. In other words, by all these tests, had the child really advanced from counting to the *idea* of a number? Could he think in terms of this, or *with* it, or *use* it as an idea? Did a number as a number have any meaning for him yet? Did he naturally turn to counting as a check on it? And where he counted one by one, had he any notion yet of a unit as a unit, of a number as made up of units, and of two equal numbers as made up of corresponding units?

With these questions well in mind, we can let the following brief summary of the actual sequence of cardinal number experiments and their results tell its own tale.

10. DESCRIPTION OF CHIEF "CARDINAL NUMBER" EXPERIMENTS

(1) Two equal lots of beads are counted out into two similar containers where they reach the same level, and the children see them to be equal in every way. One lot is then put into two differently shaped containers,

first into a wide and shallow one, then into a tall and narrow one, and they are asked whether there is the same number of beads in the new vessel as in the untouched original one, whether it would make a necklace of the same length, etc.

A similar experiment was carried out with two quantities of liquid filling two exactly similar glasses to the same level and then poured into different shaped vessels, and also into two or more small ones.

(2) (*a*) The children are requested to match various sets of objects with another set which would naturally go with them: bottles with glasses, vases with flowers, egg-cups with eggs. Or to use a given number of coins to buy sweets at the rate of a sweet per coin. If the child manages this, the experimenter then spreads out or closes up one set or the other, so that they are no longer the same length, and puts the question each time: are they still the same number?

(*b*) Since the natural link between these sets might be providing non-numerical help, the children are given piles of counters and simply asked to pick out from these the same number as there is, first, in another lot put down anyhow, then in a set pattern, then in various closed figures, simple or more complicated, then in a row.

(3) (*a*) Two equal lots of sweets are arranged first as four each to be eaten in the morning and afternoon of two days, and then as four each to be eaten in the morning and afternoon of one day, but only one for the morning and seven for the afternoon of the following day. The children are asked whether they would be eating the same number each of the two days.

(*b*) They are handed two unequal piles of counters and asked to make them equal.

(*c*) They are supplied with a single pile and requested to divide this up equally between two friends.

(4) (*a*) After they have seen a set of say 6 flowers matched to 6 vases, and then the 6 vases matched to another set of 6 flowers, they are asked if they think the first set (of 6) is the same number as the third; and also if two of the sets together are two times the third.

(*b*) They are given various lots of liquids in different vessels with the query whether these are the same quantity or different, whilst at the same time they are offered a glass and other empty vessels to help them find out. The point then is whether they will take in that they can use the glass as a unit or measure, for comparing the quantities. And also whether they will see that using the glass twice gives two times the quantity, and using it three times gives three times the quantity. That is, how far the idea of *measure* and *units* means anything to them.

II. TYPICAL FINDINGS: STAGE 1

Now in the case of every one of these experiments and every variation of them there was a number of children, classified by Piaget as at stage one, mostly 4–5 years old, but some older, who proved hopelessly at sea. But there were others who could give the right replies where they were helped by perception or trial and error, though they went back to stage one as soon as appearances went against them, or anyway went badly against them. These, mostly 5–6 years old, but some younger as well as some older, represent Piaget's transitional stage 2. Finally a number, mostly 6–7 years old, showed that they really had the idea of what number means by giving the obvious answers, as child's play.

For the authentic flavour of the first stage, one must read Piaget's detailed account of the replies of each child to the succession of questions put to him. But I shall try to indicate at least their typical pattern.

(1) *First, the conservation experiments with beads and liquids*

Most of the stage 1 children thought that when one set of beads was put into a taller but narrower container, it became more because it reached higher. Some, however, held that there were more when they were put in the shallower but wider container, because they were spread wider. It did not even occur to them that the number would or could still be the same. And even if the beads were put back in the original container and once more seen to be the same, they became different again as soon as they were retransferred to the different shaped vessels. And this was *not* merely a question of getting mixed up between number and height: when a child was asked whether the numbers would be the same or different if the beads were poured out on to the table, the reply was that there would be more of those poured out of the taller glass, because they came *from* a taller glass. The replies were still the same even if the child himself put the beads one by one in turns into the two different containers; the taller one, or else the wider one, was still said to have more beads in it. When it was suggested to the children, in order to help them further, to imagine the beads being strung into a necklace, they pictured this out in detail but remained convinced that the beads from one of the containers would produce a longer necklace.

The pouring of a given quantity of liquid from one vessel to another yielded exactly parallel results. As an interesting variant, a child who had decided that there was more liquid in the taller vessel than in the original ones, was asked to mark the level which he thought each liquid would reach when poured into similar larger glasses. He indicated two very different levels and was greatly astonished when the pouring had been done and he saw they were the same. The child was so convinced that the quantity of liquid had become different that when he observed the same levels, he suggested that in the case of the original lower level glass, some liquid must have been *added*.

(2) *The matching experiments*

(*a*) First of all, the stage 1 children, who could to all appearance count, just could not match 6 glasses to 6 bottles, or 6 flowers to 6 vases, or 6 eggs to 6 egg cups. They set up some sort of a row and then floundered. One took 12 glasses, put them close together, thus got a shorter row than the 6 bottles and thereupon said there were more bottles than glasses. Another matched 7 glasses to 6 bottles and when shown by their being arranged one to one that there was a glass over, he asked for another bottle. When he had this, however, he put the bottle at the end of the row, so that he now had a bottle over at one end and a glass at the other, and said they were *not* the same number. A third child held that 6 bottles were less than 5 glasses because the latter made a longer row, but then brought the 5 glasses closer together to make them less, and so *equal* to 6 bottles. Another child had matched 4 eggs widely spread out to 7 egg cups, and manifested real surprise when he came to put the eggs inside the egg cups and found he had *not enough*. He then got hold of 12 eggs which he put close together to match the 7 egg cups, and was again very surprised when he came to put the eggs in the cups and found he had *too many*. Exactly the same type of result was obtained when the situation was repeated in terms of flowers and vases, or varied in terms of coins exchanged one by one for sweets. These contrasts between the ability to count, apparently like ourselves, and such utter failure to grasp what counting means and does are surely cumulative and astonishing. Employing it as a way of verifying a number, or of comparing two sets of things, or finding out if they are the same, just does not come into the children's minds.

(*b*) When asked to put out counters equal in number to a random group or a pattern, or a closed figure, or a row, they again behave in exactly the same way. They make a rough total guess, or they try to imitate the pattern or figure, and only succeed in getting the number right when the figure happens to be a simple and familiar one, such as a square with one in the middle. Counting again does not occur to them. In the case of the row, they try to match not its number but its length or closeness together. One child happens to get his number right, but his row comes out longer, so he says "that's not right", and takes some away. Another child puts 10 coins close together to match a row of 6 sweets, but even so the row of 10 is shorter, so he adds 2 more to make them the *same* number. Still another first matches 6 with 7, and says that the row of 7 is more because it is a longer row, but then corrects himself and says that the row of 6 is more because they are so close together. Once more, number is just not number yet, has nothing to do with counting, and is nothing more than "a-lotness" or "a-fewness", "moreness" or "lessness", according to one aspect or another of its *appearance*.

(3) *Splitting-up and equalising experiments*

(*a*) The stage 1 children (who in the two cases quoted are actually 5¾ years and 6 years 11 months respectively), do not begin to understand, and even with help and prompting cannot be got to understand, that 1 plus 7 sweets are the same as 4 plus 4, and that the total remains the same. Both children insist that they will be eating more sweets the second day, because seven is such a lot. They stick to this even when the sweets are shifted backward and forward between 4 plus 4 and 7 plus 1 in front of their eyes. Thus again, no trace of number as number yet, and no thought of counting.

(*b*) Children shown two unequal lots of 14 and 8 counters and asked to rearrange them so that they are both the same, shift them around haphazardly. One turns the 14:8 grouping first into 16:6, then into 7:15, then again 16:6, then 5:17. Another ends up with 13:9, and so on.

(*c*) Children asked to divide 18 counters between themselves and the experimenter, so that they each have the same, take a shot at splitting the heap into two equal lots, but simply by sight. A child chances to get two lots of 9 each, but one of them takes up more space. So he decides that he has gone wrong—and just shifts the two lots over bodily to take one another's place. Another very carefully and correctly distributes the original heap one by one between himself and the experimenter, but then decides he is wrong and the two lots are not the same, because one is spread more. A third child puts the counters one by one in separate boxes, and declares them equal, but when the two boxes are emptied out and one lot comes out closer together he considers it fewer.

(4) *Experiments to test the idea of numerical equality as such, that of unit or measure, and that of the simplest multiple relations, such as two times or three times*

(*a*) There is as usual a group which registers total failure. Out of two who have just put one red and one blue flower each in several vases, one is very *uncertain* whether the number of red and blue flowers is the same or not. The other is sure they are not, because one lot as set out takes up more space. In a variation of the experiment, a third child, who has "bought" successively the same number of blue and pink flowers for the same number of coins, denies that he has the same number of each because the pink ones came from a bigger heap of flowers held by the experimenter. When the pink and the blue flowers are each matched one to one with the pennies he says: "Ah yes, they are the same." But as soon as the separate buying begins again, he returns to his belief that there are more pink ones. In another variation a fourth child has put *two* lots of 10 flowers each in 10 vases. He is given some small flower-holders and shown that they will only take one flower each. He is then requested to take enough of these small holders for all the flowers. He thereupon gets hold of ten, which he places one to each vase. When the question is put whether he has enough for all the flowers, he takes another four. He is invited to try them out and towards the end adds another two, but the idea of two holders to each vase never dawns on him. A fifth child also starts with ten holders and then adds 5 or 6 more. A sixth one, who has made up two sets of 10 flowers to go with 10 vases, is asked: "If I want to put all these flowers into these vases, how many must I put in each vase?" He answers: "One" After trying this out on 5 or 6 he discovers he needs more and finds at length that he has put in two each. But this puzzles him and he enquires: "*Why* does one have to put in two?" He fails in the same way as the other children with the one-flower holders, of which he sets up 10 to take the 20 flowers. When he finds he has not enough, he adds first 4, then 3 and then another 3, and thus gets all the flowers placed; but once more he enquires *why* he needs more holders than vases. Only with still more help does he at last rise to the idea that two holders are needed for each vase.

(*b*) The experiments on the ideas of units or measures or simple multiples are completely above the heads of the lowest age-range group. The children get so far as to pour the liquids backward and forward, by way of showing their general sense of the problem, but they expect quite different levels for two equal amounts of liquid, can make nothing of their equal findings, entangle themselves in contradictions, suggest that merely pouring a quantity into another vessel makes it more, and generally have no glimmering of the relations involved.

12. TYPICAL FINDINGS: STAGES 2 AND 3

So much for the stage 1 reactions, which are very much alike in all the experiments. Once more let me emphasise that since the children are for the most part not the same, all that can strictly be said is that for every experiment there is a number of children who behave in a certain way which Piaget calls pre-numerical or stage 1. Thus if we talk of stage 1 children producing those reactions, this is only shorthand for the fact that there are children who show that reaction, which we then classify as stage 1.

The same applies to stages 2 and 3. In each case there are children who give transitional responses and others who produce matured and perfectly correct ones. The former are grouped as stage 2, the latter as stage 3. Yet it is reasonable to *infer* on every sort of ground that in fact all children must start from what Piaget calls the first stage where no idea of number is present yet, and must pass through some sort of intermediate or half-way phase such as Piaget calls stage 2, before they can attain the level of the fully-formed idea of number, which Piaget designates as stage 3. Furthermore we can also legitimately conclude that each stage shows the general characteristics which emerge from the responses to the experiments and are taken up into Piaget's general developmental theory. Stage 2 is of course in the nature of the case much less clearly defined than either 1 or 3. There is every kind of intermediate performance between the wholly negative extreme of 1 and the wholly positive one of 3; Piaget himself cites some responses which he calls transitional between stage 1 and 2, or between stage 2 and 3. However, it is possible to place in relief something like a characteristic picture of the *midway* region between stages 1 and 3, and I shall now give a range of illustrations to bring this out. And in each case I shall go on briefly to stage 3, obvious though this is, simply to show how the story rounds itself off.

(1) *The conservation experiments with beads and liquids*

The stage 2 children dealing with the beads could get the necklace answer right, because they had only to think of length. They went wrong, however, about the different shaped containers, because they could not attend to height and breadth at the same time, but were overborne by the perceptual effect of one or the other. Similarly with the liquid poured from one vessel into smaller or different shaped ones. Here they could hold on to conservation in the case of transfer to two smaller glasses or to vessels not too greatly different in level or breadth, but fell down on three or more glasses or large differences in shape. Thus it is plain that they still had not really mastered the principle of the thing—though one or two pulled themselves up after having gone wrong and in the end produced a stage 3 solution.

Children fully in stage 3 give the right reply as a matter of course and say at once: "It's always the same thing." Or: "I saw it was the same thing." Or, in the case of the liquid: "We've only poured from one glass into another" or ". . . into some others." Or: "There seems to be less in this glass because it's wider, but it's the same thing." Or: "This is narrower, so we must fill it up more."

(2) *The matching experiments*

(*a*) Stage 2 children can do the actual matching of glasses to bottles, flowers to vases, and so on without difficulty, or anyway very soon after a first false start. But there is still no real notion of *numerical* equivalence or constancy: overall appearance still carries the day against counting, and after testifying to six bottles and six glasses because, as the child himself says, he has counted them, he succumbs to either wider spacing or serried closeness, and affirms more bottles than glasses or vice versa. Similarly with the flowers and vases, and the coins exchanged for sweets. The stage 3 children say boldly: "Nothing is changed, you've merely put the glasses closer together." Or: "Spreading makes no difference, because the flowers were in those vases." Or: "Same thing," and when asked why: "Because one can match."

(*b*) Putting out counters equal to a random group, a figure, or a row. In stage 2 the pattern is more or less correctly reproduced, and amended if necessary till the numbers do correspond. But if the original collection is then spread out or otherwise changed, the children cannot sustain the equivalence. In some cases, however, particularly in the relatively simple ones of the rows, the child himself restores his sense of equivalence by bringing the spacing back to exact correspondence. Stage 3 children break up the model, if complicated, for easier matching; or do not try to follow it at all but just count freely and place their own counters in a line; or when the experimenter alters the spacing of the rows, one of them comments: "It's the same thing; you've spread out one line and brought the other close."

(3) *Splitting up and equalising experiments*

The stage 2 children still tend to go wrong over the relation of 7 plus 1 to 4 plus 4, but themselves readily notice there is something amiss as they focus attention first on the 7 and then on the 1, with contradictory results. Finally they fully see the point. When asked to equalise two unequal groups, they try to turn them into two similar figures or patterns which they can then adjust by transfer till they exactly match. Similarly with the division of a collection into two equal quantities. But they do not just *count*, and change of spacing or arrangement or even of orientation of the figures, or if they themselves start with a too complicated one, soon throws them out. Stage 3 children explain precisely and at once why the seven plus one are the same as the four plus four, pointing to three out of the seven as accounting for there being only one left. They equalise the unequal quantities by forming a simple pattern or straight line, and matching. And they divide the collection equally by splitting it 1 by 1 or 2 by 2. In each case, moreover, nothing which the experimenter may say can move them off the equality they have established.

(4) *Experiments on equivalence of three or more groups, on units of measurement, and on simplest relations of two to one or three to one, or one to two or one to three*

Children in stage 2 in this set of experiments may already be quite firm about the equality of a given set of flowers and a set of vases, and also of another set of flowers and the same vases, but are still liable to prove very shaky about the equality of the two sets of flowers to one another, or may positively deny it. However, they themselves may then think of checking up by direct matching and so reach the correct conclusion, though only on rule-of-thumb grounds. Differences of spacing may upset this again, but the upset may be rectified by bringing the three sets into exact correspondence, and counting may be brought in to clinch the matter. In the experiments which involve matching two narrow flower holders against each vase to take *two* lots of 10 flowers each, the stage 2 children proceed by trial and error and eventually arrive at the correct solution. Similarly, in the experiments to test children's ability to use a unit measure to compare the equalities or inequalities of quantities of liquids in different shapes of vessels, those in stage 2 oscillate a great deal and contradict themselves, but tend to pour backward and forward experimentally and finally they do thus arrive at a somewhat sketchy form of the notion of using a given glass as a unit of comparison and measurement. They make also some attempt at co-ordination of level and width, but whilst they have the right idea, they are uncertain how to give effect to it. More generally, they start in tentative fashion to try and work things out by reasoning, on the basis of what they already know, but have no great confidence in this and usually decide that the only sound way of reaching the right conclusion is by practical trial and error.

The stage 3 children know exactly what they are about. The two sets of flowers in the 10 vases are the same number, because the child has counted the one set of 10 and he knows that the other matching set must also be 10 without even counting them. Or he says he saw what was in the vases, and then simply counts by these. Again, the children at once recognise that it needs two of the one-flower holders to each vase to hold the same number of flowers previously arranged two in a vase.—When it comes to measuring, one child answers that he thinks the quantities in two vessels are the same, but he will have to measure. He does so, and confirms his judgment. Another rightaway starts measuring, makes a slip, but immediately corrects himself.—In the further experiments involving simple proportions, the stage 3 children reason the answers out, and one adds that measuring would show the same thing.

13. EXPERIMENTS ON ORDINAL NUMBER IDEAS AND THEIR RELATION TO CARDINAL

This is a long and detailed section which, for reasons of space, I must unfortunately pass over very briefly. Piaget holds that the child's notion of ordinal number develops in the closest relation with his cardinal number ideas and in fact each depends on the growth of the other, in the same way as both inter-depend with the growth of the child's logic. He and his co-workers have carried out a sequence of highly ingenious and interesting experiments based, first, on ordinal numbers and series as such, and then on their relation to cardinals. The results are very closely parallel to those already described; there are the same typical responses of total failure—of some very imperfect and qualified successes, based on easy cases—and of instant, matter-of-course solutions, just like an adult's. These different levels of response are spread out in time, over approximately the same age-range, as for the cardinal experiments.*

I am not sure whether Piaget establishes in fact more than that children's understanding of ordinal numbers and relations depends on their developing grasp of cardinal ones. He, however, attaches considerable importance to his own view of reciprocal dependence, for the purposes of his theory of logico-mathematical development at large. I cannot pursue that issue here, beyond acknowledging that I do not find this

* Those interested in a short account of these experiments may be referred to the brochure *Some Aspects of Piaget's Work*, obtainable from the National Froebel Foundation, 2 Manchester Square, London W.1 (price 2*s*. 0*d*. plus postage). This contains, *inter alia*, a detailed résumé of the entire Number Volume, which follows Piaget's own order of presentation, and may in general be found useful as a supplement to the present study.

part of his thesis wholly convincing and adding that it does not seem to me so very important for his standpoint as a whole. But in any case the experiments remain most interesting at the least as a further contribution to the total picture and additional confirmation of this.

14. EXPERIMENTS ON CHILDREN'S GRASP OF SIMPLE "LOGICAL" RELATIONS

We have here a chapter for which most ordinary readers would probably be least prepared in a work on the development of the child's conception of number. It comes moreover in the middle of the latter theme and as a vital part of the entire structure. However, we have already seen that this springs directly from Piaget's basic conception of the intimate relation of mathematics and logic and their joint controlling role throughout the process of human intellectual development. And such a conception leads inevitably to the question how the growth of the child's simplest notions of logical relations fits in with that of his elementary arithmetical ideas. Accordingly Piaget takes the purely logical relation which comes closest to a numerical one, namely that of part to whole. This is exemplified by the typical and far-reaching relation of any sub-class or sub-classes to some wider class in which they are included, equivalent to the familiar logical antithesis of *some* and *all*. Piaget constructs a number of experimental situations involving that relation, to see how far young children have grasped it and can handle it.

Thus, for a start, he has a box containing wooden beads, mostly brown, but two *white*, and the children are asked: "Are there more wooden or more brown beads?" To make the question easier and more intelligible for 5–6 year olds, he tries to help them to picture it out by asking which would be longer, a necklace made from the wooden beads or one made from the brown ones? To assist them further, he provides two empty boxes beside the full one and enquires in succession: "If I take out the brown beads and put them in this empty box, will there be any left in the first box?" "And if I take out the wooden beads and put them in the other empty box, will there be any left in the first box?" Only when these questions have been correctly answered does he go on to this further one: "If I make a necklace with all the wooden beads there would be in *this* box, and another with all the brown beads there would be in *that* box, which would be longer?"

The problem is further varied by having all the beads the same colour, but different in shape; mostly square, but some round; or mostly cones, but some round. Or different in size; mostly large but some small. Or, again, it is made still more evident to the eye by having two sets of beads, so that the children can actually visualise the two alternatives at the same time. And furthermore Piaget tries varying the proportions and bringing them much closer together: instead of 18 brown and 2 white beads, he has 20 brown and 18 of another colour. Finally, he tests what difference it would make if instead of artificial classes he worked with natural ones, very familiar to the children: such as a quantity of flowers, of which most are poppies, but 2 or 3 cornflowers; or even children in a school-class, of which most are girls, but a few boys.

However, all these questions, down to their easiest forms, draw completely wrong answers from a number of children between 5 and 7 years. The only effect of the helping hand given in the various ways described is to shift the age range of failure down in the main to 5–6 years. The children who fail, i.e. who are at stage 1, insist, however hard they are pressed, that there are more of the bigger sub-class than of the total class; that the former would make a longer necklace; that there are more poppies than flowers; that there are more girls than children in the schoolroom, and so on. They may describe the wooden necklace as brown *and* white, or may do a drawing of the brown necklace, with all the beads filled in black, and of the wooden necklace, with most of them filled in black, but two left white. Nevertheless, they still declare firmly that the brown bead necklace will be longer than the wooden one. Similarly a child will himself say that if the brown beads are taken out of the full box, the white ones will remain, whilst if the wooden ones are taken out, nothing will remain, but will yet maintain that the brown necklace would be longer. One child even enquires whether only white beads will be used for the wooden necklace and, when answered "No," goes on to ask, "The brown also?" and herself says, "Yes, because they are also wooden." Nevertheless she still insists that the brown necklace will be the longer one.

Parallel results are obtained with all the other versions of the problem. In terms of 13 blue beads, of which 10 are shaped like small cones, and 3 are round, a child can say that there are more cones than blue beads because there are many cones; and when the experimenter asks, "But what about the blue beads?" the child can himself reply, "All are blue," whilst yet when the question is repeated whether there are more blue beads or cones, he affirms once again, "More cones." In other words, many is more than all. In terms of blue beads mostly square but some round, a child who is told that one little girl wants to make a necklace with the square beads and another girl with the blue ones, laughs and says of her own accord, "They're all blue." Nevertheless when she is called on to say which necklace would be longer, she states with conviction, "The one with square beads, because there are more."—In terms of the flower problem, a child who is asked what is left if the poppies are picked, can answer, "The cornflowers"; if the cornflowers are picked, then "The poppies"; and if the flowers are picked (after a pause for reflection): "Nothing." In spite of all this, however, when the experimenter again puts to him whether it is the poppies or the flowers that will make the bigger bunch the child replies: the poppies, because there are such a lot of them.

Thus Piaget once more establishes a stage 1 at which children of the age-range of 5–7 years have not yet *begun* to grasp the nature of the relation of a larger class to the sub-classes included within it, or conversely. In other words the part can still be greater than the whole. This is true even if it is not in fact much larger than the remaining part, and thus does not stand out as the main bulk of the larger class; thus where out of 38 wooden beads, there were 20 brown and 18 green, there was still the same typical insistence on the part of various children that there were more brown beads than wooden ones.

Stage 2 children find their way to the correct answers, but only by intuitive groping, not by reasoning. They begin by going completely wrong, as at stage 1, but then stumble on what must strike them as a good judicial solution, though in fact no less incorrect. A typical child says at first that the brown necklace will be longer because there are more brown beads. He is then asked again: "Are there more wooden beads or more brown ones?" His answer is: "More brown. No, more wooden. No, *both the same*!" Actually three children tried this way out. Eventually, however, with more questioning, they correct themselves (unlike those in stage 1, however much they may be questioned and helped), bring in the other coloured beads, and give the right answer. Even in stage 3, two of the children quoted fell at the first moment into the old error, but quickly and completely rectified this. The two others cited gave the right reply at once and explained why.

In sum, then, Piaget finds that at about the same age at which children have not yet any grasp of even the simplest numerical relations they have equally little grasp of the simplest *logical* ones (in the distinctive sense of this term). He goes on to establish that as the child advances to stage 2 and then to stage 3 in respect of numerical grasp, so he also progresses in the strictly logical field. In these facts he finds confirmation for his view regarding the close kinship between arithmetical and strictly logical operations and their interrelated growth, and he goes on to show, by theoretical analysis, how near to one another they in fact are and how they can only develop together.

V

GENERAL DISCUSSION

15. PIAGET'S THEORY OF NUMBER, AND ITS RELATION TO LOGIC

THE FOREGOING WILL, I think, have given body to the stages by which, according to Piaget's findings, the child proceeds from a first phase of counting ability but complete lack of the *idea* of number to the final one of full functional mastery of it.

I have tried to spare ordinary readers most of Piaget's technical vocabulary about the addition of relations, the multiplication of relations, additive and multiplicative composition of relations and equalisation of differences, etc., etc. But I think I should at least sketch the groundwork of the theory of number which he puts forward and which he believes that his experimental findings have firmly established.

As I have already indicated, this theory is intimately bound up with his view of logic. He holds that number is a synthesis or fusion of the two basic processes that underlie logic: that of *classification* leading to hierarchies of wider and wider classes, such as those of plants and animals, and indeed most other kinds of objects, processes, or situations; and that of *seriation*, or arrangement in a graduated order, which is applicable to most physical qualities and properties and to a vast number of relations, as diverse as those of space, succession, kinship, social rank, etc. Piaget's view, if I understand it correctly, is that whilst classification is based on *similarity* and seriation on *cumulative difference*, number is a form of grouping that arises when these two types of ordering are brought together into a single operation which sheds something of each and fuses the rest. The class of natural whole numbers is a class of which the sub-classes are a series. In fact the class of such numbers is formed by a single process of cumulative seriation. If one takes 2, 5, 28, 103, they are the names of sub-classes of the class of numbers, as spaniels, terriers, bulldogs, and greyhounds are the names of sub-classes of the class of dogs, or gold, copper and cobalt are sub-classes of the class of metals. But the sub-classes of the class of numbers are linked with one another in a continuous series or ladder formed by the repetition of a single process of generating new members by the addition of a like further element. Quality and with it difference are completely eliminated; nothing but the process of seriation and class-formation by seriation remains.

Accordingly the whole scheme is best developed in terms of abstract symbols and in fact of a scheme of such symbols. Numbers are not things, like classes and sub-classes of natural objects; they are products of the mind resulting from a basic process of the mind elaborated into a *system*. Number as such is only grasped when it is seen as a progressive *system* of numbers built up by this process. But when once the system is given, we can set up rules of movement which allow us to move freely within it in all directions, composing and combining members in any order or any clustering, and decomposing or splitting these clusters or *reversing* any previous operation or set of operations *ad lib.*

This system, Piaget insists, has most of its characters and properties in common with the separate systems of classification and seriation which he regards as the groundwork of what he calls qualitative logic. There are some much-controverted questions involved here, but it does seem to me that at the least a striking parallel with logic, both in actual functioning and in psychological history, is brought out.

16. THE ODD EDUCATIONAL SITUATION ABOUT LOGIC

That leads directly to a question to which I imagine few of us ordinarily give much thought, namely that of education in logic, or the teaching of logic as such. The situation here is really quite an odd one. We all agree that arithmetic is one of the great basic subjects which, at least in an elementary way, everyone must be taught. But why is not logic recognised as equally basic and equally needed as a part of every education? Is it really less vital to us than arithmetic throughout the course of our lives? We all have to *reason* as soundly as we can, all the time—in every problem or emergency of our practical life as well as throughout our social one. In particular we continually discuss with others, and they with us, and we with ourselves, courses of action as well as beliefs and views, and we all of us take it for granted that we can usually tell sound reasoning from unsound. We base the whole of our lives on inferences and beliefs which we consider cannot be wrong, and there is hardly a situation in which our safety and success does not depend on our capacity for thinking with at least some approach to logical validity.

There is thus an overwhelming case for holding that logic, as the theory or "subject" of the processes and rules of valid reasoning, should be taught as universally as the three R's. Indeed even reading and writing, to say nothing of arithmetic, would be little use to us if we could not back them up with some grasp of logic and the demands of sound reasoning or inference or argument. If that is so, however, why is logic as a subject hardly ever taught except to budding philosophers or theologians or, in limited measure, to a minority of scientists? The parallelism of the processes and rules of logic to those of arithmetic makes the anomaly of our totally different educational attitude towards them all the more striking.

Of course the answer is that we are so accustomed just to take logic, as a functioning system, for granted. We assume in the main that we find it in ourselves, as it were by the light of nature, and do not have to learn or teach it; and that to some supplementary extent we pick up the finer points of it as we go along in the course of our earlier practical experience of mutual communication and social life. But why then does not the same apply, or why is it not left to happen, in the case of arithmetic?

The interesting light shed by Piaget on these unaccustomed questions is that to a large extent the same process *does* happen, and is left to happen, in arithmetic. The most central rules and principles of this are also not taught to us. Notions like those of conservation, composability, associativity, reversibility, which are the keys to the possession of the very idea of number, never enter into ordinary arithmetic teaching. Here, too, we assume that the roots are somehow present in us natively, and that up to a point the setting of our ordinary upbringing is enough to help them to sprout and grow. The child is taught in the nursery, and picks up as a game, the activity of counting, and that, together with the innate capacity of his mind, is held to set him up with the first elementary idea of number. But of course after that we behave in a way totally different from what we do, or rather fail to do, about logical thought. We realise full well that between the idea of number, which we postulate as soon as the activity of counting has been got going, and any sort of proficiency in arithmetic there is a long and arduous road which can only be travelled by much diligent learning and above all practising. What happens in the parallel case of logical thinking?

The further insight provided by Piaget here is that neither the idea of number nor that of logic are just found in ourselves in the way we tend to assume. Both involve complex processes of inward structural growth, and in his view the two processes of growth, as we have seen, are closely interlinked. Indeed under existing conditions that of logic is a much slower one; children master the basic functional idea of number at around seven, and secure their first clear ideas of the implications of class-inclusion relations, as Piaget shows, at about the same time. But for the ability to handle logical relations with the same freedom and matter-of-courseness with which they manipulate simple numerical ones from 6½–7 years onward, we must wait, in the case of most average children, till they have reached the age of 11–14 years. Yet perhaps this has something to do with our very different educational attitude to the two subjects? I can only raise this question; it is too difficult and complicated to try and consider in detail here.

There are just two comments, however, which may be worth making. First, the working logic to which most of us are led by the supposed light of nature plus our social apprenticeship is hardly very perfect. It is not to be underrated: most people do develop quite a competent capacity for the appraisal even of abstract reasoning and argument, more particularly other people's, and the spotting of fallacies, more particularly other people's. But how much room for improvement there might be!

Secondly, the old traditional logic of the syllogism is not something that most present-day logicians would want to teach, at any rate as their main theme, if their subject were suddenly adopted as a school target, on a par in importance with arithmetic. Nor is this the logic with which Piaget is most concerned and which he exhibits as growing in close correlation with arithmetic. It has taken logicians until the last few decades to discover that logic is, or ought to be, something altogether wider than the scheme we had inherited from Aristotle. For the modern relational point of view syllogistic reasoning is only a limited special case. Thus the problem of why logic is not taught as much as arithmetic could not really have been posed *in adequate terms* until our time; there was no logic to teach comparable in scope with arithmetic. And, in my view, it is only Piaget's work that has given real searchingness and importance to the problem, precisely because it has shown that what matters most is not verbally learning the rules and the unfamiliar polysyllabic terms, but the complex *functional* structure or organisation which has to be built up in our minds, both in the field of logic and in that of arithmetic. Thus the problem now is in effect quite a different one. What is it that really governs the inward building up of this structure, its rate and degree of progress? To what extent can the real inwardness of arithmetic be taught, any more than that of logic? And so we come to the threshold of the question: what are the educational bearings of Piaget's work?

VI

EDUCATIONAL BEARINGS AND QUESTIONS

17. MISUNDERSTANDINGS TO WHICH PIAGET'S WORK LENDS ITSELF

I HAVE ALREADY at various points partly forestalled this theme. But let me try now very briefly to pull the threads together.

(i) First of all, I hope I have largely removed the fundamental misconception to which Piaget's findings have given rise in the minds of many people. He brings out the great process of slow and complex inward growth, spread over the whole field of mental life and all the years from birth to the threshold of adolescence, by which our minds develop into their full functional capacities and organisation. The child's growth into his first functional grasp of the idea of number is one important part of this total process, though a comparatively early one. Many of those who have followed Piaget's work have tended to see this slow inward process in *antithesis* to the action from without of teaching and education. Thus it has seemed as if the latter's scope were being challenged and indeed radically limited by the boundaries now apparently set by the true reality within. What could be taught appeared to become something extraneous and superficial which was meaningful only if it followed in the wake of each stage of inward growth and merely exploited what each of these made possible.

With that tendency went a closely related one to take Piaget's stages and the chronological ages with which he linked them much too literally. The stages again were construed as something inward and almost organic which, if Piaget was right, had to be respected; and the age-ranges established by Piaget then seemed obviously the natural guide to them. Conversely, however, it also appeared as if Piaget could be refuted by attacking this supposed linkage, by showing that the claimed chronological relationship frequently did not hold, by criticising the supposed fixity of the stages, etc., etc.

(ii) I have already suggested that all this is in the main just a profound misunderstanding. It is not an unnatural one and Piaget himself has lent some colour to it, first, by his almost exclusive focus on the study and understanding of the *inward* processes of development and secondly by the manner of presentation of many of his results and even of his theories. One might go further and query the slant of some of the actual theory and I should myself want to alter its balance in some respects. By and large, however, the factor of sheer misunderstanding remains. Piaget's basic view of the very process of inward growth is, as I have pointed out, pivoted on the continual cycle of interchanges between the child and the outward world: his action on that world and its reaction on him. It is this cycle that is the very motor of the child's mental advance, which proceeds by a constant rhythm of in turn assimilating outward reality and accommodating to it, on an ever-widening and ever more effective and powerful scale. Thus outward reality is as all-important for inward growth as the inward impetus in the child himself. As regards the stages, all but the largest divisions are merely convenient ways of breaking up the continuum of growth. And even these must be so interpreted that they are compatible with that underlying continuity; they simply mark certain major rhythms in it: phases of relative consolidation and temporary stability whilst the ground is being prepared psychologically for the next surge forward. The chronological linkages are no more than an approximate method of marking out the *sequence* of the distinguishable phases of growth, the order in which they follow one another.

(iii) Thus in reality the scope of education is not in the least narrowed or threatened by Piaget's work. What becomes all-important is merely the way and the means by which we try to educate. Certainly outward teaching which is not related to inward growth, and to the stage which this has already reached, becomes peculiarly futile and meaningless—as meaningless as progressive educationists have long contended it to be. By the same token any approach which is not based on *clear and full understanding* of that growth must inevitably fail, even if the utmost will to educate from within is there.

But we can now also invoke a positive counterpart to all this. Piaget's work as a whole has made plain all the vital education that goes on in the child quite independently of the set educational processes, and above all in his first few years, before those processes have even begun. Indeed, by far the most important portion of his intellectual growth is achieved by himself, through the direct working of the interchange cycle by which he actively learns to take in all the main features and the general make-up of the physical and social world around him. In that way, though he starts from practically nothing but the familiar "blooming, buzzing confusion" of his first few weeks, there is formed in his mind, by the age of *5–6* years, a far-reaching *functional working model* of his surrounding world. And if we watch all he does and says and clearly understands, even for only a few average days in say his 6th year, and try to work out for ourselves, without any preconceptions, everything which this ordinary round of his performances, practical and intellectual, implies, we shall see how very much that model must be there and how far-reaching it must be. It is only, in effect, because he has this model constantly operating in his mind that he can play the natural, largely spontaneous and actively participant role in the world around him, which we normally take so much as a matter of course.

If that is a reasonably correct and significant picture, what is the scope left for any would-be *theory* of education, that is, theory of *planned* intervention in the child's life to put him in possession of at least the most important historic gains and achievements of the society round him, gains which otherwise he would probably miss, or at best attain much more slowly and imperfectly? Surely all the accent must now fall on putting him in *real* possession, not merely verbal and apparent? And real possession must signify, at the least, not less real than that of the working world-model he has built up for himself and can successfully draw upon for every sort of purpose and contingency of his life. In fact real possession can only mean incorporation into that world-model, thus continually *expanding* it to the point of *transformation*, in exactly the same way as it has been expanded, and transformed by expansion, up to the 5–6-year level. But that then requires that we as educators shall fully understand that process, as it has already happened and is still happening, in all the tremendous sweep of its achievement; and that we shall intervene in it only, as it were, *with* its own current, making use of all its momentum and aiming simply at guiding and helping it on. Its momentum, however, is a direct function of the child's own activity, his exploring, enquiring, forward pressing interests, his wish to extend his knowledge, to understand and to be able to do. These have already carried him incredibly far and can carry him immensely further, up to the true limit of his capacities.

Formal, mainly verbal, teaching, by subjects, as we normally envisage it, has its place in this process, but if it is not to produce mere patter, or the verbal semblance of knowledge and understanding instead of its reality, it must in most fields come in only slowly and relatively late, when the ground is fully prepared for it. In other words, when a broad and solid foundation of wide-ranging active experience actively worked over, and a strongly established, positive, forward-reaching and self-helping attitude, has already been built up. For only thus is the child or young adolescent enabled to meet deliberate subject-teaching half-way, or, still better, three-quarters of the way, with a true capacity to *understand*; which means, to transform verbal material into a real psychic structure in his own mind, and one which can become continuous with the real structure that is already there. (This of course does not exclude the recognition that even in the earlier years some teaching in the conventional sense is unavoidable, since there is much that *can* only be verbally learnt—that is, which is not significant in itself, but solely a means to significant ends, and can only be acquired by memorisation and practice. The one important point is merely that this sort of teaching or "learning" shall not be mistaken for true learning or education—or thrust on the child in place of the real thing, where anything significant is at stake.)

And thus Piaget's work brings us back to the insights and the methods and objectives which progressive educationists have long urged on us—the stress on understanding the laws of the child's true inward growth, and co-operating with these and using them to lead him on and guide him; the value placed on his own active interest and own active thought; and so on. But thanks to Piaget, there is now available a new massive body of actual psychological knowledge which most powerfully supports the vision of the great educational reformers, and at the same time can, we may hope, be drawn upon to realise that vision with a new efficacy and success.

18. POSSIBLE EDUCATIONAL USES OF HIS NUMBER STUDIES

What, in conclusion, can we say of the application of all the foregoing to the specific theme of number—and, if Piaget is right, by the same token to logic? Let me defer the latter for the moment and merely emphasise again that for Piaget the two themes are most intimately related. In fact they constitute for him the twin tools which, above all, the child has to learn to understand and master, in order that he may increasingly be able to organise and to extend, to enjoy and to exploit, all the experience and knowledge which his activities bring to him throughout his life and growth.

What then of number-learning, the practical business of arithmetic and arithmetic teaching, as such? Since I am not a teacher myself, I can only most tentatively venture into the field of practical applications, and must leave this in the main to others better equipped with experience and know-how than myself. A few broad conclusions, however, seem reasonably clear. First of all, in the most general terms, it does look as if there should be something that merits the attention of working teachers in the wealth of actual concrete situations devised by Piaget and his helpers, which so plainly bring out where children stand about number, from nowhere through to full (functional) grasp of the idea and its due working control. Thus these situations can be used, for a start, to *test* how far small children have advanced towards that idea; and we can make sure that they have attained it before we try to impose any formal arithmetical teaching or learning on them. Otherwise this will just leave their minds behind, and will almost inevitably go forward either as mere rule-memorising and knack-learning, or, too often, without even that effect, or any at all.

Secondly, the Piagetian situations actually show us, in terms of his stage 2, some of the typical steps by which children accomplish their advance from sheer helpless floundering to trial-and-error groping and then on from this to grasping the right idea. There is much in Piaget's own reports to indicate that such situations can be fruitfully used to help children on their way; in a recent check-up on his findings this was actually done. Moreover, it would be surprising if all these ingenious and novel, but yet so obvious experiments, if considered by imaginative working teachers, did not suggest endless further variations and developments of a similar kind.

The vital point, however, is the way in which Piaget permits us to see our arithmetic and mathematics—even as our logic—as of one piece with the rest of our intellectual life and growth. What thereupon emerges directly is the pivotal importance of ensuring that this intrinsic unity shall be *preserved* and not *severed* by our educational approach. It may *not* be preserved even if children "learn" their arithmetic successfully in terms of words and figures and get their answers to sums "right". It is not *necessarily* preserved even if they are taught their first arithmetic in terms of concrete objects and familiar practical interests and activities, from shopping onward. It is only preserved in so far as, first of all, children start by forming their own true inward structured idea of number; and secondly, but far more difficult, if the later rule-learning, operation-learning and so on, becomes a living graft on that idea, or rather, is successfully developed as a further stage of its own inward growth. Can we learn to carry most children with us, in full understanding and lively interest, as we equip them to expand the world of numbers progressively for themselves—and thus enable them to do so as an integral part of the total pattern of their growth, continuous with every other, and an organic segment of the whole? And after the initial effort has been made, is there perhaps a chance that this may actually in the long run prove easier than the dead weight of drudgery, lightened only by extraneous artifices, which until now we have kept on inflicting on our children and ourselves?

Whatever the answers to these widest questions, however, I trust that readers of this study will at least feel prompted to look into Piaget's book themselves for the full detail of his actual experimental situations, and the stimulus and starting-points which these might provide.

19. THE ODD QUESTION OF EDUCATION IN LOGIC AGAIN: PERHAPS NOT SO UNPRACTICAL AFTER ALL?

What now, finally, of that other, so unaccustomed question of *logic*? To attempt to deal adequately with this large unfamiliar topic would require a volume to itself. But, by way of the briefest look at the lie of the land, let me suppose that such a volume came to exist and were called, for example, "Logic for the Primary School", or "Logical Thinking for Juniors". The bare word "logic" would surely be enough to frighten everyone off the whole enterprise, or else would cause them to treat it as just another of those crank ideas, to be dismissed with derision. Would this not, however, be a case of being stopped by literally a mere sound-barrier, which in fact we must learn to break through? After all, the word "logic" is not more Greek than that other Greek word "arithmetic" and for that matter, nothing like so long. We have simply become used to the latter, and to everything that goes with it. That may be the whole difference, and I have already suggested grounds why logic may be not less entitled, nor less suited, to assume the same familiar mien. In sober fact, it makes, I believe, sense to say that arithmetic is much the more sophisticated and artificial thought-formation of the two.

It is true that, according to Piaget himself and also the evidence of earlier reasoning and absurdity tests, children do not seem generally ready for strictly logical thinking until around 11–14 years. But that again could merely be due, as I have already suggested, to our past educational attitudes and assumptions, together with the limitations of traditional "logic" itself. In effect there is ample proof available that children *are* capable of cogent logical criticism and logical construction, in favourable surroundings, at far earlier ages. They are continually led to the one or the other if they live in an atmosphere that encourages them to discuss, argue and reason. And above all if this joins on to their usual lively interest in everything around them, animal, vegetable or mineral, mechanical, electrical, and so on, and if they are constantly stimulated to explore and question, to try to think up and test out suppositions or hypotheses, or to seek explanations.

A book like Susan Isaacs's *Intellectual Growth in Young Children* is full of examples of the way in which this natural development in logic works, both critically and constructively, in children down to 4–6 years. Most of those in her school were admittedly above average in intelligence, but according to normal psychological theory one might reasonably expect more average children who were only, say, a couple of years older, to show up similarly *in similar situations*. The key however would be in the real similarity of the situations—that is, they would have had to be all the way through equally stimulating to free and productive activity, with hand, eye and brain. It is not indeed even certain that under such conditions 4–6 years might not prove something like the typical commencing age for the genuine logical interests of children at large.

That is perhaps the right context in which the suggested place of logic in our educational theory should come in. I cannot try to develop this theme any further here. I only want to suggest that it may be *capable* of quite important development; and that it merits thinking about all the more just because it is so novel and unfamiliar and even repugnant to most of our past habits of thought. It may well be these habits of thought that need our fresh attention.

One point more: Of course even books on Arithmetic for the Primary School may come to look distinctly different from the way they do at present if Professor Piaget's work has the value and pregnancy which I am attributing to it. Thus we can draw no conclusions from the pattern of our current arithmetics to a parallel pattern of any hypothetical "Logic for the Primary School". On the other hand we should possibly find that if gradually we develop new types of arithmetical text-books aiming directly at the structural and functional growth of children's own ideas of number and number operations, the need for a largely parallel type of text-book of logic and logical operations would emerge at the same time. The two directions of advance might in fact prove to have much in common all the way through. Children would be continually exercising and developing two complementary sets of working rules governing the activity of their own minds and determining their efficacy and practical success. In each case they would have the same strong natural interest in playing or working at these rules because, under "activity" conditions, they are constantly cropping up in everything the child does, and because things so obviously go wrong if he falls down on them, but go so well and swimmingly if he masters them. The most piquant fact here is that, as I have already suggested, the strictly logical type of interest develops earlier and more strongly than the numerical one (apart from the first mechanical and meaningless game of counting), as can be seen, *inter alia*, from a sufficiently careful study of children's "Why" questions*; so that suitable forms of "Logic for Small Children" may well in the end have far more help to give to arithmetic than vice versa.

Anyway, it does not seem impossible that on this double foundation teachers may in future be able to build up a groundwork of essential intellectual education far more effective and far more capable of further growth by its own resources and momentum than we have known hitherto. Whether this might not now be achievable is the largest question, or perhaps prospect, raised by the new horizons which Piaget's work has opened up.

* I have tried to bring this out in my examination of such questions in Susan Isaacs's *Intellectual Growth in Young Children.*

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