OUTLINES OF MODERN LEGAL LOGIC

BY

ILMAR TAMMELO

FRANZ STEINER VERLAG GMBH · WIESBADEN

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Preface

In 1588 Abraham Fraunce published "*The Lawiers Logike* exemplifying the praecepts of Logike by the practise of the common Lawe". In his poem by which he dedicates the book to Lord Pembrooke, Fraunce says:

I see no reason, why that Law and Logike should not bee
The nearest and the dearest freends, and therefore best agree.

In the same poem he also says:

I sought for Logike in our Law, and found it as I thought.

This work, influential in its epoch, fell into oblivion in the course of time, but nothing can detract from Abraham Fraunce's words of commendation. There have been deprecators of the use of logic in the service of law and what they have said about this matter has been influential; however, their utterances have not succeeded in achieving more than impressing impressionable minds to re-echo misunderstandings about logic and its relation to law or in voicing misdirected objections to logic.

If it is assumed that it is indispensable for any legal system to have some consistency of its component parts, then there has never been and will never be law without logic. The so-called irrationalities of law are really not lack of logic in law or legal thought but rather manifestations of intricacies of the structure of law and reflections of intractabilities or uncertainties of its substance.

It is surprising that after Abraham Fraunce's book, there has been no further book of similar scope in English. Even the recently awakened interest in logic among lawyers in the Anglo-American world has materialised only in various articles written on the application of logic in the lawyer's field of work. The absence of a text-book or even of an adequate introduction in English to legal logic is not to be regarded as a sign that the role of logic in this field is trivial. What it indicates is that it has proved feasible to conduct legal reasoning in a satisfactory manner without explicit recourse to logic as a specific discipline of thought. Thus law schools have been able to afford not to include the study of logic in their ordinary curricula and lawyers have managed to perform their tasks tolerably well by employing a logic embedded in patterns of ordinary ways of thought and expression.

Nevertheless I feel that explicit knowledge and skill in applying logic is important for the lawyer and that anything short of this either will not do at all today or will do only for limited purposes and for lower levels of lawyers' activities. Explicit and sound knowledge of logic brings many benefits to the lawyer. Above all it helps the reasoner to acquire proficiency and self-confidence in reasoning. For logic charts the practicable roads of reasoning and the pitfalls which await those who diverge from these roads. Those who master the principles and methods of logic are capable of quickly discovering valid arguments, defects in the reasoning of their own as well as of their opponents, to expose flaws in any discourse and to dispose of or overcome them efficiently. This gives poise to the reasoner in all argumentative situations.

The present book is an introductory compendium of legal logic. As such it may serve as a key to the understanding of formal aspects of legal reasoning in general and of specialised scholarly works in this area. It is also intended as a groundwork for a more extensive and detailed treatment of logic in the service of law to follow this Compendium. A consideration which has determined its summary character is that only an exposition of the principles and methods of logic which avoids complicated matters as far as possible is likely to offer an access to a rewarding study and to proper explorations of relevant thoughts in depth.

Logic proves to be relevant to legal thought in two main directions, namely in the lawyer's thought about law and in thought conveyed by the expressions of law itself. Statements about law are presumably just as amenable to logical reasoning as are any statements about things or events; hence application of logic in this area would scarcely raise any special problems. In contrast, law itself as a special body of thought has peculiarities which challenge logical endeavour. Since beaten tracks of logic which could be safely followed in this area of legal thought are not available, anyone who is confronted with logical problems here must venture to blaze some paths in what continues to be largely an unpioneered territory.

The present book has arisen from teaching legal logic to undergraduate and postgraduate students in the Law School of the University of Sydney since 1960. It took its first shape in the form of an experimental model issued in mimeographed form in 1966. This work proved to be unsatisfactory in many respects and it had to be completely rewritten. However, it served a good purpose in that it enabled me and some of my Australian and overseas colleagues to scrutinise my proposed exposition of legal logic in order to discover its shortcomings and to find ways to remove them within the intended scope of the work.

I am indebted above all to Mr. Ronald D. Klinger, now my Senior Research Assistant, with whom I have had frequent consultations and who has constantly checked the adequacy and accuracy of the logical expressions in the drafts brought to his attention. Most of his suggestions have been incorporated in the text. His contributions have been particularly valuable to the preparation of the Appendices and to the exposition of the methods of proof. Before this Compendium reached its present version, Mr. Fiori Rinaldi of the Department of Philosophy of the University of Queensland, Dr. Georges Kalinowski of the Centre National de la Researche Scientifique of Paris, and Professor Zigmunt Ziembiński of the Faculty of Law of the University of Poznań offered a number of helpful criticisms and comments relating to the above mentioned precursor of this book. Finally, I wish to thank my colleague Mr. Anthony R. Blackshield in the Department of Jurisprudence and International Law of the University of Sydney, who has assisted me in connection with some jurisprudential and linguistic problems, and Mr. Peter Cornelius, my former postgraduate student, who has made helpful suggestions.

The present Compendium is not only an application of generally accepted principles and methods of logic to legal thought but it also contains some experimentation with ideas both in the area of logic and in that of legal theory. Where I have diverged from conventional ways of expression and thought I have acted out of my own spirit of intellectual adventure for whose products I alone am responsible. Of the distinguished scholars from whose works I have derived stimulus and instruction for the present enterprise, I would like to mention Julius Stone, Hans Kelsen, Karl Engisch, Herbert L. A. Hart, Eduardo García Máynez, Ulrich Klug, Layman E. Allen, Georges Kalinowski, George Henrik von Wright, Irving M. Copi, and Ernest Nagel. What fruit their thought has borne in this book remains for them or for others to judge.

That the publication of this book was possible now instead of at some indefinite future time is largely owed to a subsidy from the Australian Research Grants Committee providing research assistance for my work on an extensive project on the foundations of legal logic. In the execution of this project the present Compendium is the first step.

Sydney, Australia *Ilmar Tammelo*

Introduction

Law is a complex of norms regulating human conduct. It is a system of norms expected to conform to certain standards of rationality and to be applied in an intellectually orderly manner. The intellectual orderliness of legal systems and of the application of law is sometimes rather defective. This defectiveness is compatible with the idea of law only to a certain degree. A complete chaos of legal thought and a completely capricious application of law represents a state of affairs to be called "lawlessness" rather than "law". The virtues and vices of law and its application are judged by reference to criteria among which the principles of correctness of thought are prominent. These principles, in particular the rules according to which self-consistence and mutual consistence of thought-formations are determined, constitute a system which may be conceived as a normative system. Hence it can be said that law, being a normative system, is governed by logic as another normative system, just as it is governed by the normative system of the grammar of the language in which it is expressed.

The significance of logic for law is generally well recognised by lawyers and it is safe to dismiss the denials of this significance as ill-considered views or as expressions of some kind of misunderstanding or of peevish impatience. Nevertheless these denials deserve some notice because occasionally they have come from the highest judicial or scholarly authorities and have thus managed to command undue attention and even to influence approaches and attitudes to the legal process. It appears therefore to be necessary, before a treatment of logic in the service of law may be undertaken, to examine critically certain adverse statements which have been made about the role of logic in the field of law. This, however, is rather awkward in the initial stage of an exposition of legal logic, because the necessary appraisal presupposes a sufficient acquaintance with logic itself and the understanding of its actual and desirable operation in the area of legal reasoning. The present introductory remarks cannot go therefore into requisite details. All that can be done here is to mention main types of challenges to the role of logic in the field of law and to indicate briefly why they must be deemed unsuccessful.

It has been said that the development of law ("the life of law") has not been determined by logic but by other factors, for example, considerations of justice, expediency, and material conditions of the community. This argument is based on a misunderstanding of the nature and role of logic: logic is not concerned with processes and factors which bring about evolution but with intellectual procedures or operations which help to discover order in thought and to assure consistency of thought where this is considered important.

It has been contended that the application of logic in the area of legal thought imposes rigidity on the operation of law ("places law into a straitjacket") and prevents it from serving its important social ends. This argument confuses legal formalism with the application of logic in the field of law. It also mistakenly assumes that logic militates against the achievement of justice, common good, etc. Legal formalism, insofar as it is reprehensible, is an abuse of logic or its misapplication or an application of a pseudo-logic. Because logic is one of the principal means which assures intellectual discipline and integrity, it can, if properly applied, only promote the achievement of desirable social ends.

It has been observed that legal reasoning is not entirely or principally logical reasoning but it involves procedures of thought other than those offered by logic. This argument imports only an apparent denial of the significance of logic in the field of law. Of course, there are procedures of thought in legal reasoning which do not fall within the scope of logic in its ordinary or strict sense. Logic is not primarily concerned with discovering or supplying premisses for legal reasoning; it is primarily concerned with deriving conclusions from them. It may indeed be that actual legal reasoning is not chiefly logical reasoning, but even if this is the case there remains a scope for logic in legal thought; it would then play an important subordinate role in this thought.

Occasionally contemptuous remarks have been made on the use of syllogistic reasoning in the field of law. In some instances these remarks may be justified because there are instances in which syllogistic argument is not appropriate. It is to be noted that syllogistic reasoning is only a part of the methods of logic. Modern logic offers other methods of deductive reasoning which are not syllogistic and which may be appropriate in some instances requiring logical treatment.

To some extent, adverse attitudes to logic in the service of law may be explained by the fact that there is no complete certainty about the subject matter of logic and that, correspondingly, there is uncertainty about the meaning of the word "logic". In common language, this word and its derivatives, especially "logical" and "illogical" are often employed rather loosely. They refer not only to consistency and inconsistency of thought but also to soundness and unsoundness of thought in general. Thus a line of reasoning which is found to be out of touch with reality or is felt to be materially repugnant is frequently branded as illogical. The word "logic" and its derivatives are also used to refer to entities other than thought-formations; thus it is quite idiomatic to speak of the logic of events, to say that certain behaviour is illogical, etc. This loose usage appears also in scholarly expressions. Even reputable scholars have employed phrases such as "transcendental logic" and "material-logical structures" and have designated their books dealing with history, psychology, metaphysics, or cosmology as books on logic.

There is no complete agreement about what "logic" means even among logicians. All logicians agree that a subject matter of logic is propositions and concepts. But there are some among them who also contend that there is a logic of imperatives and even that there is a logic of acts. All logicians agree that logic is concerned with drawing *formally* compelling ("stringent") conclusions from propositions. But there are some among them who include within the scope of logic also arguments which lead only to the establishment of what is reasonable to accept as being well-founded or convincing. In books of logic written even by distinguished logicians problems such as informal fallacies and paradoxes have been discussed as if they were subject matters of logic.

In the context of an introduction to a work on legal logic it is untimely to attempt to offer a strict delimitation of the field of logic, because the preliminaries necessary for arriving at a reasoned decision on this matter will require a discussion which presupposes a good acquaintance with principles and methods of contemporary logic. However, a broad indication of what is the proper meaning of "logic" is called for even here in order to avoid sailing out to a completely uncharted sea. It is advisable to follow the usage of the word "logic" occurring in the works of those scholars who regard themselves as logicians and who are regarded as such in scholarly circles. Accordingly, logic is to be conceived of as a discipline of thought concerned with thought-formations and not with the world at large, in particular not with processes of *thinking*. The central object of logic is inference, and propositions and concepts are matters of principal concern to logic.

The origin of logic thus understood lies in the awakening of man to problems relating to thought and its expression. To arrive at logic it was necessary for thought to reflect upon itself and to discover the standards by which it could be directed and judged as to its *correctness.* In Western civilisation, logical problems were clearly posed and extensively discussed by the sophists and by Socrates. Logical principles found their early remarkable application in the ancient beginnings of geometry. The first systematic treatment of logic that has come down to us was by Aristotle, his main writing on this subject being *Prior Analytics*, the most notable contribution of which is the theory of syllogistic inference.

The scope of logic as found in Aristotelian writings was expanded by megarians and stoics, who addressed themselves to propositional inferences not examined by Aristotle and anticipated modern developments of logic. The Hellenic logic was further developed by mediaeval scholars who added refinements to it and converted it into a discipline of thought applied to the treatment of theological and philosophical problems. Thus preparatory work was done which paved the way for logic to become one of the foundations of the emerging scientific thought and also of modern philosophic and juristic thought. A logic based on this tradition continues to be studied, taught, refined, and employed even today.

Today traditional logic has largely been superseded by modern or symbolic logic and it is increasingly losing its role in theoretical and practical applications of logic. Certain ideas of modern logic have their origin in classical antiquity; notably the principle of minimum conditional (usually called "material implication") was formulated by Philo of Megara. However, the real scope of modern logic was not envisaged until the 18th century by Leibniz, whose ideas of *ars combinatoria*, *characteristica universalis*, and *calculus ratiocinator* were antecedents of its development in works of mathematicians in the 19th century, pre-eminently in those of George Boole, Augustus de Morgan, Gottlob Frege, and Giuseppe Peano. In the beginning of the present century, modern logic found a systematic and comprehensive exposition in the monumental treatise *Principia Mathematica* (1910—13) by Alfred North Whitehead and Bertrand Russell.

Modern logic can be viewed as a development of pure mathematics resulting from mathematicians' efforts to provide a logical foundation for their discipline of thought. This foundation of mathematics is, however, relevant not only to mathematics but has a scope which makes it fundamental to all thought having formal structure. The historical coincidence that mathematics has been the base of departure for modern logic has lent it a formalistic rigour and accounts for the fact that its principles and procedures are expressed in a thoroughgoing symbolic form. This in its turn has been responsible for a far-reaching detachment of modern logic from ordinary language and for its dissociation from intuitions of ordinary thinking. Traditional logic, too, even in its early Aristotelian exposition, employs symbols, but this is rather incidental and not a pervading feature as it is in modern logic. Therefore the name "symbolic logic", by which modern logic is usually known, is quite apposite.

Symbolic logic is a tool of formal reasoning vastly superior to traditional logic. It surpasses the latter not only by greater precision and subtlety but also by a wider scope. Traditional logic can be completely expressed in terms of symbolic logic and can be given various interpretations through it. There are valid inferences which cannot be formulated at all by traditional logic and many others can be formulated by it only in a very tortuous manner. There are parts of modern logic containing ideas which have no correspondents in traditional logic. The superiority of modern logic has given rise to the view that traditional logic has outlived its usefulness and is only of historical interest today.

This view appears to be too sanguine. Traditional logic has not yet reached the end of its career. It retains vitality as a quintessence and refinement of the logic embedded not only in ordinary but also in scholarly ways of thought in various areas of learning. Thus lawyers are still reasoning along the lines of traditional rather than of symbolic logic. Traditional logic can achieve a greater precision and the flaws of its ordinary exposition can be removed through insights gained from modern logic. There are noteworthy attempts to extend and reinterpret it and thereby to adjust it to *some* modern needs. Moreover, presentation of a system of traditional logic can be used as a convenient access to the understanding of rather esoteric ideas of symbolic logic and for the appreciation of the latter's special virtues. Before one may discard traditional logic it is proper first to make some acquaintance with it and before the exclusive company of symbolic logic becomes comfortable it is requisite to readjust ordinary habits of thought, to re-educate ordinary intuitions relating to formal constructs, and to reform ordinary ways of expression. The days of traditional logic are perhaps not yet numbered, but its complete overhaul may well be on the way and therefore it may be wise to be prepared to do without it altogether.

Whatever the contemporary significance of traditional logic for legal thought may be, symbolic logic has become increasingly relevant to it.

Contemporary analytical jurisprudence relies heavily on modern logic. For legal practitioners this logic has proved valuable in directing their way through intricacies of complex legal arguments and in discovering and removing ambiguities and inconsistencies in law. Making really worthwhile use of computers in the field of law is unfeasible without recourse to principles and methods of symbolic logic. Whatever the relative merits of traditional logic and symbolic logic in the service of law may be, there is no reasonable doubt about the value of logical reasoning for all men of law. To derive full benefits from logic, both traditional and modern, it is not sufficient to acquire only proper habits of formal reasoning. It is also necessary to acquire an explicit knowledge of its foundations.

The main aim of this Compendium is to help lawyers to an overall grasp of fundamentals of logic and to its application to legal matters. In order to achieve this end, the number of illustrations has been reduced to a minimum, because it seems that in the course of initiation into logic their wealth tends to sidetrack the mind and divert attention from the essentials of formal issues. For the same reason, entering into controversies on theoretical problems, albeit important, has been avoided here. In order to keep to a possibly straightforward course of exposition of general as well as legal logic, the writer has stated his positions on controversial matters in a terse manner. This may appear, but is not intended, to be dogmatic; he is neither unwilling to argue them out by rational debate, nor resistant to the possibility that he may be wrong. The selection of illustrations in this Compendium has been based on the consideration that they should be telling to ordinary intuitions. Therefore simple and rather uniform illustrations have been chosen, whatever their literary merits or entertainment value.

Especially in works on modern logic, as they are not based on centuries old tradition, terminology is still in an experimental stage and not always settled and satisfactory. The writer has therefore tried to choose among available terminological alternatives those which seemed to him the best and has occasionally even ventured to introduce new terms. Since a systematic work on legal logic in English has not yet been published, it is opportune to make the requisite adjustments in logical terminology at this stage.

The writer has decided to employ Polish notation throughout the exposition of modern logic, even though Italian notation (employed in *Principia Mathematica* and most English works on logic) and German notation (which is rather similar to the latter) may have certain advantages for some purposes. The use of the same system of notation throughout the Compendium has the advantage of exhibiting clearly the common principles underlying all parts of logic as well as the differences between various parts of it. In protological and propositional calculus the notation here chosen has a scarcely surpassable elegance and simplicity and is particularly suitable for devising logic games of great didactic value.

The conception of the scope of logic adopted in this Compendium may seem too narrow to some scholars concerned with problems of legal reasoning. The writer has been reluctant to address here problems of semantics, informal arguments, interpretation, and inductive and statistical methods. There can be no doubt whatsoever about the great significance of these problems for legal reasoning nor can it be disputed that they are intimately connected with logical matters in the total context of this reasoning. However, the aim of the present book is not to be a comprehensive treatise on legal reasoning but a concentrated treatment of one special aspect of it in order that some of its specific features could be apprehended and handled in an appropriate manner.

Chapter I: A System of Traditional Logic

1. The Proposition and Its Components

The system of logic presented in the first chapter of this Compendium is a contemporary form of logic originated by Aristotle and elaborated and refined throughout the centuries after him. The core of this logic is rather uniform all over the civilised world, though its mode of expression and some of its details vary in different schools of thought and in works of individual writers and there are various extensions of it going beyond its original scope.

Traditional logic — or simply "logic" in the context of the present chapter — is concerned with intellectual aspects of propositions, and not with their emotive or conative aspects. A proposition in the sense of logic is a thought-formation which can meaningfully be asserted to be either *true* or *false*. Whether a proposition is either actually true or actually false is immaterial for the logician; what matters for him is that it makes sense to say that a given thought-formation is either true or false. For the purposes of logic, propositions may relate to actual states of affairs, but they may also relate to what does not exist in the real world at all but only in the realm of fantasy, imagination, fairy tales. The system of traditional logic here presented is concerned with categoric, hypothetic, and disjunctive propositions having a certain form and based on certain assumptions later to be specified.

The present exposition employs the word "assertion" to refer to any claim concerning truth or falsity of a proposition. An assertion can be to the following effect: (1) a proposition is *affirmed to be true*, (2) a proposition is *denied to be true*, (3) a proposition is *affirmed to be false*, (4) a proposition is *denied to be false*. To affirm that a proposition is true means the same as to deny that it is false and to deny that a proposition is true means the same as to affirm that it is false. Thus the expressions "affirmation as false" and "denial as false" can be dispensed with and in the subsequent exposition the meaning of "affirmation as true" will be conveyed by "*affirmation*" simply and the meaning of "denial as true" will be conveyed by "*denial*" simply.

Logic serves, of course, the pursuit of truth; however, the end which logical procedures primarily follow is *correctness* in reasoning. The endeavour to assure correct reasoning is governed by the cardinal principles of correct reasoning usually called "the laws of thought". These and other principles of logic can be conceived as norms addressed to the reasoner. The observance of the norms of logic assures *formal* but not material soundness of reasoning.

Keeping in mind that the "laws of thought" have normative force, they can be expressed (in indicative mood) as follows:

(1) *The Principle of Identity:*
Every proposition is equivalent to itself.
(2) *The Principle of Non-contradiction:*
No proposition is both true and false.
(3) *The Principle of Excluded Middle:*
Every proposition is either true or false.

Since it is conceivable neither to affirm nor to deny a proposition (which would mean suspension of judgment), Principle (3) is not as self-evident as are Principles (1) and (2). However, traditional logic works on the assumption that in this case no *logical* operation is performed. There are systems of formal reasoning dispensing with the *tertium non datur* principle, however, they are not systems of traditional logic.

It is to be noted the affirmation of a proposition does not mean that the affirmed proposition is taken *actually* to be true and the denial of a proposition does not mean that the denied proposition is taken *actually* to be false. For what is false can be affirmed (even though wrongly) and what is true can be denied (even though wrongly). The reasoning may still be *correct*, that is, impeccable from the *formal point of view* in both cases, though this *correct* reasoning diverges from what is the case in actual fact. Even propositions which import patent nonsense can be asserted or denied without reasoning becoming incorrect. Usually, of course, logical operations are not performed on nonsensical propositions nor is what is manifestly false affirmed nor is what is manifestly true denied. Therefore illustrations of logical operations which affirm only ostensibly true propositions and deny only ostensibly false propositions provide a good intuitive ground for appreciating these operations. The illustrations in this Compendium are chosen accordingly.

That propositions can be either affirmed or denied distinguishes them from questions, commands, and exclamations, which cannot be said to be either true or false, and hence they are excluded from the scope of traditional logic. Grammatically, questions, commands, and exclamations are usually not in indicative mood in English, whereas propositions invariably are.

Although propositions require language for their expression, for the purposes of logical operations they are detached from their linguistic manifestations. Thus the same proposition can have various linguistic expressions: "He is a thief", "*Er ist ein Dieb*" (German), and "*Il est un voleur*" (French) convey exactly the same proposition. A proposition may be said to be the *meaning* of a declarative sentence; it is not the grammatical expression of this meaning, that is, not a declarative sentence itself. Different sentences in the one language may express the same proposition. For example, "He owns that horse" and "That horse is owned by him", "John is a bachelor" and "John is an unmarried man".

In the remaining part of this section and in the following two sections, only categoric propositions will be considered; hypothetic and disjunctive propositions will be considered in the final section of the present chapter. A categoric proposition — or simply "proposition" in the context of the sections *1*, *2*, and *3* — has the following three concepts as its principal components: a *subject*, a *predicate*, and a *copula*. The subject and the predicate are called "*terms*"; their grammatical counterpart is normally a single noun (or its corresponding pronoun), or a noun (or a pronoun) qualified by adjectives or adjectival phrases or clauses. Adjectives or adjectival phrases also occur as terms, but when they are employed they are to be regarded as elliptic ways of conveying the idea of a noun (for example, "Tigers are ferocious" is an elliptic way of saying that "Tigers are ferocious animals"). The copula is the component of the proposition linking the two terms into a unity of a thought-formation which can meaningfully be either affirmed or denied. Its normal grammatical counterpart is a part of the verb "to be", usually in the present tense. The two terms represent two classes of entities; the copula joins them into a relation.

The present treatment of categoric propositions is based on the assumption that their terms are *non-transcendental* and *referential*. A term is non-transensdental if it refers to something which is not everything whatsoever (as does, for example, "Being", which is a transcendental term). A term is referential (or instantiated) if it refers at least to one entity which is at least postulated to exist. Another way of saying that a term is referential is saying that it is not empty (or void). The subsequent exposition of a system of traditional logic will be chiefly concerned with terms which, in addition to being referential and non-transcendental, are also general and positive.

In the complete logical form of a proposition there appears also a quantifying concept: either "*all*" or "*some*", which is prefixed to the subject. In the logical form of a proposition there may occur also the concept of negation; which is represented either by "*not*" immediately following the copula or by "*no*" immediately preceding the subject. "*All*" and "*some*" are the concepts which determine the *quantity* aspect of the proposition whereas the concept of negation if relating to the whole proposition or the absence of such negation determines the *quality* aspect of the proposition. By their quantity, propositions are divided into *universal propositions* and *particular propositions*. By their quality, propositions are divided into *positive* (usually, but not quite aptly, called "*affirmative*") *propositions* and *negative propositions*. Accordingly, there are the following four propositional forms:

(1) Universal positive propositions
(e. g. "*All* trespassers *are* tortfeasors").
(2) Universal negative propositions
(e. g. "*No* trespassers *are* invitees").
(3) Particular positive propositions
(e. g. "*Some* trespassers *are* burglars").
(4) Particular negative propositions
(e. g. "*Some* trespassers *are not* burglars").

In connection with the particular proposition it is to be noted that "*some*" is a technical term in logic meaning *one at least, possibly more, and possibly all*. Hence these propositions have a scope which may include the corresponding universal propositions. Thus "*Some* trespassers *are* tortfeasors" being true does not exclude the possibility that "*All* trespassers *are* tortfeasors" is true and "*Some* trespassers *are not* invitees" being true does not exclude the possibility that "*No* trespassers *are* invitees" is true. In the given instances it so happens that the latter are in fact the case. In connection with the particular negative propositions it is to be noted that the reason why "*not*" follows "*are*" rather than precedes "*some*" is that "*not some*" is likely to convey the idea of "*none*", which is not intended. It is further to be noted that the reason why "*no*" rather than "*not all*" is employed to form the universal negative proposition is that "*not all*" is likely to convey the idea of "*not all but some*", which is not intended.

It has become conventional to employ certain symbols for signifying the terms and the quantitative and qualitative aspects of the proposition.

The subject is usually signified by S and the predicate by P. The quantitative and qualitative aspects of the proposition are usually signified by the following letters placed between the symbols of the terms: *a* for the universal positive proposition, *e* for the universal negative proposition, *i* for the particular positive proposition, and *o* for the particular negative proposition. The four propositional forms can thus be expressed as follows:

(1) S *a* P, (2) S *e* P, (3) S *i* P, (4) S *o* P.

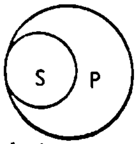
Both terms of the proposition have an extensional and an intensional aspect. The former is called "denotation" and the latter "connotation". *Denotation* means the range of entities to which a term refers (e.g. "men" refers to Englishmen, Germans, Russians, Egyptians, Brazilians, etc., whereas "Europeans" refers to Englishmen, Germans, etc., but not to Egyptians, Brazilians, etc.). *Connotation* means the properties belonging to the range of entities to which a term refers (e.g. the properties of "men" are organism, animal, mammal, etc.). A term may denote an individual entity, in which case there is only one single instance referred to by the term. Those propositions whose subject is such a term are called "singular propositions". For most logical purposes, these propositions can be treated as universal propositions. A term may denote also a "non-existent" entity or "non-existent" entities (e.g. The Snark, unicorns) by referring to nothing that is there *in actual fact*. Because logic is not "ontologically committed" — that is, it need not be restricted to actually existing entities, such terms do not create any vexing logical problem. They are not empty, since in appropriate contexts of thought such entities are *postulated* to exist. Thus "The Snark" is a referential term in Lewis Carroll's literary imagination, "unicorns" is instantiated in mythical thought, and "John Doe" and "Richard Roe" did exist as common law constructs.

The terms in the proposition appear either as *distributed* or as *undistributed*. A term is distributed if it is claimed to relate to the whole range of entities to which it refers. A term is undistributed if it is not claimed to relate to more than a part of the range of entities to which it refers. For example, "crimes" in the proposition "*All* crimes *are* illegal acts" is distributed, whereas "illegal acts" in the proposition "*Some* illegal acts *are* crimes" is undistributed.

The denotations of the subject and the predicate of a proposition may be regarded as constituting *classes* of entities and the relevant class relationships are: (1) *inclusion* of the subject class in the predicate class,

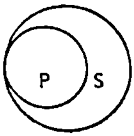
(2) *inclusion* of the predicate class in the subject class, (3) *coextension* of the subject class and the predicate class, (4) *intersection* of the subject class and predicate class, (5) *exclusion* of the subject class from the predicate class (and *vice versa*). These relationships can be diagrammatically represented by the aid of the so-called *Euler's Circles*, which method employs here two circles: one representing the denotation of the subject and the other the denotation of the predicate.

(1) Inclusion of the Subject Class in the Predicate Class



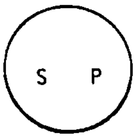
S *a* P (e.g. "*All* crimes *are* illegal acts"
P *o* S and "*Some* illegal acts *are not* crimes")

(2) Inclusion of the Predicate Class in the Subject Class



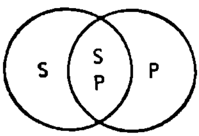
S *o* P (e.g. "*Some* scholars *are not* legal scholars"
P *a* S and "*All* legal scholars *are* scholars")

(3) Coextension of the Subject Class and the Predicate Class



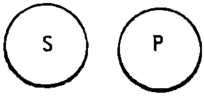
S *a* P (e.g. "*All* spinsters *are* unmarried females"
P *a* S and "*All* unmarried females *are* spinsters")

(4) Intersection of the Subject Class and the Predicate Class



S *i* P (e.g. "*Some* professors *are* legal scholars"
S *o* P and "*Some* professors *are not* legal scholars"
P *o* S and "*Some* legal scholars *are not* professors")

(5) Exclusion of the Subject Class from the Predicate Class



S *e* P (e.g. "*No* invitees *are* trespassers"
P *e* S and "*No* trespassers *are* invitees")

The four propositional forms differ in respect of the distribution of their terms and this may be seen with the aid of the above diagrams. In the case of universal positive propositions (S *a* P), there may be either coextension of S and P or inclusion of S in P. For example, "*All* spinsters *are* unmarried females" and "*All* spinsters *are* females".

In the first example, the whole denotation of the subject is referred to and the whole denotation of the predicate is also referred to. In the second example, the whole denotation of the subject is referred to whereas only a part of the denotation of the predicate is referred to. It is clear that given a proposition in the form of S *a* P, it is not possible to tell without the aid of additional information (i.e. by recourse to a further proposition) which of these relationships is in fact intended. In regard to the subject, the whole denotation of it is referred to in either case, so that the subject term of a universal positive proposition is distributed. It is not possible, however, to assume that the whole of the denotation of the predicate is referred to, as it cannot be told which of the two possibilities in question is here actually intended without the assistance of extrinsic information. Therefore, it is not permissible simply to assume that more than a part of the denotation of the predicate is being referred to and hence the predicate term of a universal positive proposition is treated as undistributed.

In the case of universal negative propositions (S *e* P), the circles representing S and P stand in separation from each other. For example, "*No* invitees *are* trespassers". As the relevant diagram shows, the whole of the denotation of the subject as well as of the predicate is referred to and hence in a universal negative proposition both the subject term and the predicate term are distributed.

In the case of particular positive propositions (S *i* P), the circle representing S either intersects with the circle representing P or is included in it or includes it, or is coextensive with it, that is to say, a particular positive proposition may represent any relationship other than exclusion. For example, "*Some* professors *are* legal scholars", "*Some* crimes *are* illegal acts", "*Some* scholars *are* legal scholars" and "*Some* spinsters *are* unmarried females". In the first and third examples, only a part of the denotation of the subject is referred to, while in the second and fourth examples the whole of the denotation of the subject is referred to. Without further information (i.e. without additional propositions) it is not possible to tell which of these class relationships is in fact intended and therefore it cannot be assumed that the whole denotation is referred to. The minimum possibility is that a part of the denotation of the subject is referred to; hence the subject term of a particular positive proposition is treated as undistributed. Similarly, a part of the denotation of the predicate is referred to in the first and the second examples, while in the third and the fourth examples the whole of the denotation of the predicate is referred to. As before, without further information it is not possible to tell which relationship is intended between the terms and hence only the minimum possibility can be assumed. Therefore the predicate term in a particular positive proposition is treated as undistributed.

In the case of particular negative propositions (S *o* P), the circle representing S may either intersect with the circle representing P, or include it, or stand in separation from it. For example, "*Some* professors *are not* legal scholars", "*Some* scholars *are not* legal scholars", and "*Some* invitees *are not* trespassers". In the first and the second examples, a part of the denotation of the subject is excluded from the denotation of the predicate, while in the third example the whole of the denotation of the subject is so excluded. Without further information (i.e. without additional propositions) it is not possible to tell which of these relationships is intended. Therefore it is not permissible to assume that more than a part of the denotation of the subject is referred to and hence the subject term of a particular negative proposition is treated as undistributed. However, in each example, it is the whole of the denotation of the predicate from which either a part or the whole of the denotation of the subject is excluded. Thus the predicate term of a particular negative proposition is treated as distributed.

The rules of distribution of the terms in the four propositional forms can be summarised as follows:

(1) *The subject of a universal proposition is distributed.*
(2) *The subject of a particular proposition is undistributed.*
(3) *The predicate of a negative proposition is distributed.*
(4) *The predicate of a positive proposition is undistributed.*

There is something unusual in the second and the fourth examples for particular positive propositions, because in such instances the word "*some*" is not ordinarily used but the word "*all*" (namely if one wishes to assert that the relationship in the relevant diagrams exists). However, they are correct from the viewpoint of traditional logic, for (as was said above) any particular proposition includes the possibility of the corresponding universal proposition within its scope ("*some*" does not exclude "*all*"). A similar comment applies to the third example for particular negative propositions.

When putting ordinary language into logical form, it is the meaning behind the words used that must be captured. There are several guidelines in this regard. Metaphorical or poetical language is to be avoided so that, for example, "A stich in time saves nine" becomes in the language of logic "*All* damages which are not repaired quickly *are* damages which become extensive". Similarly, the use of abstract terms is to be avoided and they are to be replaced by concrete terms, for example, "Skill in advocacy requires diligence" can be expressed for logical purposes as "*All* skilful advocates *are* diligent persons". It is also requisite that the subject and the predicate terms are terms complete in themselves. It would not be feasible to put "A man's dog is always devoted to him" into logical form as "*All* dogs possessed by a man *are* creatures devoted to him" as the word "him" refers back to the subject term and thus the predicate term would not be complete in itself. A practicable way would be to say "*All* dogs with male owners *are* creatures devoted to their owners" (where "their" refers to "creatures", which is a concept within the ambit of the predicate term).

As it appears from the above exposition of the theory of propositions, the language of traditional logic is rigid and in some respects unnatural. In ordinary language, stringent reasoning is usually conducted in a smooth, unstrained, and even elegant manner. However, the relevant expressions in ordinary language are elliptical, lack precision, and are thus exposed to the hazard of losing the formal line of reasoning. Therefore they have to be interpreted and reframed in order to give them proper logical form which would assure rigour in formal reasoning. In some instances this can be done relatively easily. For example, instead of saying "All men are mortal" it can be said "*All* men *are* mortals", instead of saying "There are Australians of non-European origin" it can be said "*Some* Australians *are* persons of non-European origin", and instead of saying "Swallows fly fast" it can be said "*All* swallows *are* fast fliers". In many instances when expressions significant for logical reasoning occur in ordinary language, a rather radical alteration of simple expressions is required to put their content into a proper logical form. For example, "Adults only" must be rendered as "*No* persons other than adults *are* persons who are admitted", "Six failed" must be rendered along the following lines: "*All* persons who failed *are* persons numbering six" (or as a singular proposition "The number of persons who failed *is* the number six"). For certain logical purposes some expressions which occur in ordinary language can be used without forcing them into the rigid forms of traditional logic. Thus "Churchill was a statesman" (rather than "Churchill *is* a person who was a statesman"), "Some decisions are bad" (rather than "*Some* decisions *are* bad decisions"), and "No one was injured" (rather than "*No* persons *are* persons who were injured") can be employed as premisses in inferences in the form which they have, provided that the context in which they occur assures that their meaning and logical structure are apparent.

2. Immediate Inference

In an immediate inference the conclusion is drawn from a single proposition as its *premiss*. This conclusion results either from affirmation or denial of the premiss; it can be either an affirmed or a denied proposition. An immediate inference is *valid* if the conclusion is drawn in accordance with the relevant rules of logic; otherwise it is *invalid*.

Provided that the terms in the premiss and the conclusion are the same, the following general rules apply to immediate inference:

(1) *If the premiss and the conclusion are either both affirmed or both denied, their quality must be the same; if one is affirmed and the other is denied, their quality must be different.*

(2) *If the premiss and the conclusion are both expressed as affirmed propositions, a term which is distributed in the conclusion must be distributed in the premiss.*

The four propositional forms, have various inferential relations to each other, which are set out below. As will be seen, certain logical consequences follow from affirmation or denial of these four forms. A convenient way to express affirmation of a proposition is to prefix it by the phrase "*It is the case that*" and a convenient way to express denial of a proposition is to prefix it by the phrase "*It is not the case that*". However, for the sake of economy of expression, the convention may be adopted that only the negating phrase will be employed. If a proposition is not preceded by the negating phrase, it is to be taken as affirmed.

The propositions in the form S *a* P and S *e* P having identical terms in the same order stand in *contrary opposition* to each other. The relation of contrariety yields the following immediate inferences which can be schematically presented as follows:

Premiss	Valid Conclusion
Affirmation of S *a* P	Denial of S *e* P
Affirmation of S *e* P	Denial of S *a* P
Denial of S *a* P	None
Denial of S *e* P	None

Examples

From "*All* felonies *are* crimes" it is valid to infer that "*It is not the case that* '*No* felonies *are* crimes'" (but not *vice versa*). From "*No* invitees *are* trespassers" it is valid to infer that "*It is not the case that* '*All* invitees *are* trespassers'" (but not *vice versa*).

It is invalid to infer, for instance, from "*It is not the case that* '*All* felonies *are* homicides'" either that "*No* felonies *are* homicides" or that "*It is not the case that* '*No* felonies *are* homicides'". The former proposition happens to be false, but its falsity is not warranted by the above mode of inference. This becomes obvious if, for instance, the proposition "*All* minors *are* adults" is denied. The contrary proposition "*No* minors *are* adults" happens to be true. From "*It is not the case that* '*No* felonies *are* crimes'" it is invalid to infer either that "*All* felonies *are* crimes" or that "*It is not the case that* '*All* felonies *are* crimes'". The former proposition happens to be true, but its truth is not warranted by the mode of inference here in question. If, for instance, the proposition "*No* minors *are* criminals" is denied, the contrary proposition "*All* minors *are* criminals" happens to be false.

The propositions in the form S *a* P and S *o* P on the one hand and S *e* P and S *i* P on the other having identical terms in the same order stand in *contradictory opposition* to each other. The relation of contradiction yields the following immediate inferences which can be schematically presented as follows:

Premiss	Valid Conclusion
Affirmation of S *a* P	Denial of S *o* P
Affirmation of S *e* P	Denial of S *i* P
Affirmation of S *i* P	Denial of S *e* P
Affirmation of S *o* P	Denial of S *a* P
Denial of S *a* P	Affirmation of S *o* P
Denial of S *e* P	Affirmation of S *i* P
Denial of S *i* P	Affirmation of S *e* P
Denial of S *o* P	Affirmation of S *a* P

Examples

From "*All* murders *are* felonies" it is valid to infer that "*It is not the case that* '*Some* murders *are not* felonies'" and *vice versa*. From "*No* invitees *are* trespassers" it is valid to infer that "*It is not the case that* '*Some invitees are* trespassers'" and *vice versa*. From "*It is not the case that* '*All* minors *are* criminals'" it is valid to infer that "*Some minors are not* criminals" and *vice versa*. From "*It is not the case that* '*No* murderers *are* sadists'" it is valid to infer that "*Some* murderers *are* sadists" and *vice versa*.

The propositions in the form S *a* P and S *i* P as well as S *e* P and S *o* P having identical terms in the same order stand in *implicative opposition* to each other. If the universal proposition is the implicans and the particular proposition is the implicate, the aspect of the opposition is *superaltern*; if the reverse is the case, the aspect of the opposition is *subaltern*. The relation of implication yields the following immediate inferences which can be schematically presented as follows:

Premiss	Valid Conclusion
Affirmation of S *a* P	Affirmation of S *i* P
Affirmation of S *e* P	Affirmation of S *o* P
Affirmation of S *i* P	None
Affirmation of S *o* P	None
Denial of S *a* P	None
Denial of S *e* P	None
Denial of S *i* P	Denial of S *a* P
Denial of S *o* P	Denial of S *e* P

Examples

From "*All* murders *are* felonies" it is valid to infer that "*Some* murders *are* felonies", but not *vice versa*. From "*No* invitees *are* trespassers" it is valid to infer that "*Some* invitees *are not* trespassers", but not *vice versa*. From "*It is not the case that* '*Some* invitees *are* trespassers'" it is valid to infer that "*It is not the case that* '*All* invitees *are* trespassers'", but not *vice versa*. From "*It is not the case that* '*Some* murders *are not* felonies'" it is valid to infer that "*It is not the case that* '*No* murders *are* felonies'", but not *vice versa*.

As to invalid inferences by sub- or superalternation, if, for instance, the proposition "*No* murders *are* felonies" is denied, the corresponding particular proposition "*Some* murders *are not* felonies" happens to be false, but its falsity is not warranted by the mode of inference here in question. If, for instance, the proposition "*No* minors *are* criminals" is denied, the corresponding particular proposition "*Some* minors *are not* criminals" happens to be true.

The propositions in the form S *i* P and S *o* P having identical terms in the same order stand in *subcontrary opposition* to each other. The relation of subcontrariety yields the following immediate inferences which can be schematically presented as follows:

Premiss	Valid Conclusion
Affirmation of S *i* P	None
Affirmation of S *o* P	None
Denial of S *i* P	Affirmation of S *o* P
Denial of S *o* P	Affirmation of S *i* P

Examples

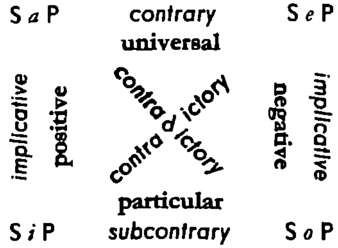
From "*It is not the case that* '*Some* minors *are* adults'" it is valid to infer that "*Some* minors *are not* adults", but not *vice versa.* From "*It is not the case that* '*Some* murders *are not* felonies'" it is valid to infer that "*Some* murders *are* felonies", but not *vice versa.*

It is invalid to infer, for instance, from "*Some* public servants *are* lawyers" that "*Some* public servants *are not* lawyers", even though the latter proposition happens to be true; for its truth is not warranted by the mode of inference here in question. If, for instance, "*Some* murders *are* crimes" is affirmed, the corresponding negative proposition "*Some* murders *are not* crimes" happens to be false.

The above exposition shows that the four kinds of opposition include the following logical consequences:

(1) *The contrary propositions cannot be both affirmed but they can be both denied.*
(2) *The contradictory propositions cannot be both affirmed or both denied.*
(3) *The implicative propositions can be both affirmed or denied.*
(4) *The subcontrary propositions can be both affirmed but they cannot be both denied.*

The relations between the four propositional forms can be diagrammatically represented in the so-called Square of Opposition:



It is to be noted that the word "opposition" in the context of immediate inference and in relation to the above Square has a special technical meaning importing that two propositions differ from each other by their quality or their quantity and not excluding their compatibility. The Square of Opposition holds only on the assumption that the "opposed" propositions contain terms which are referential, positive, and not transcendental. Should empty terms be admitted, some of the above inferences would become invalid. Thus, if it is assumed that the subject term in the universal propositions may be empty, the inference by subalternation would become invalid. Should negative terms be admitted, there would be sixteen different "opposed" propositional forms and a correspondingly more complex geometrical figure to represent them.

The assumption that all terms in the system of traditional logic presented here are referential implies that all propositions in this system have *existential import*. It may be argued that there is no need to assume existential import for all universal propositions and, accordingly propositions such as "*All* phlogiston deposits *are* deposits for which mining rights can be acquired" and "*No* nymphs *are* satyriasiacs" can be allowed even though there are no phlogiston deposits or nymphs at all. Perhaps there is no need to worry about examples from the realm of phantasy and they should be ignored for the purposes of logic whose scope should be limited only to the real world. However, even here states of affairs are encountered which have their counterparts in universal propositions apparently referring to non-existent entities. Consider, for example, criminal law provisions whose purpose is to ensure that there are no acts of murder, theft, etc. It is conceivable that some provisions of criminal law somewhere in the real world prove to be so efficient that certain crimes are never committed. It may happen in some societies that acts of sodomy are never committed, though there is the corresponding *possibility*, for which the criminal law applicable in these societies has provided prohibiting norms. In these conditions the proposition "*All* acts of sodomy *are* felonies" would have no existential import in the sense that its subject term does not refer to any *actual* occurrence.

The present Compendium proceeds from the view that "existence" for the purposes of logic means not only factual existence but also possible or conceivable existence. What the ontological nature of the entities is to which terms occurring in categoric propositions refer does not concern logic. In some instances they may refer to fictional entities, to mere constructs of thought, or what is only potentially the case. Systems of logic can be constructed, of course, which proceed from a different assumption so that terms interpreted as empty are admitted in categorical propositions. However, these systems are departures from traditional logic as it has been *traditionally* conceived. Traditional logic is admittedly not *entire logic* but only a part of logic as it exists today; it is a logic to which a rather limited scope may be assigned today.

On the assumption that all terms in the categoric propositions are referential, valid inferences can be made by *conversion* of two kinds. In the inference by conversion, the premiss and the conclusion are propositions having the same quality and the same terms whose order, however, is reversed.

Conversion yields the following immediate inferences which can be schematically presented as follows:

Premiss	Valid Conclusion
Affirmation of S *a* P	Affirmation of P *i* S
Affirmation of S *e* P	Affirmation of P *e* S
Affirmation of S *i* P	Affirmation of P *i* S
Affirmation of S *o* P	None

It is to be noted that from S *a* P it is not possible to infer validly P *a* S, for in this conclusion the term P is distributed whereas occurring in the premiss, it is undistributed, so that affirmation about the whole of a class is derived from a proposition which imports only a part of this class. P *a* S *might be* affirmed given S *a* P, *but only* on the basis of additional information — not merely from S *a* P. The conversion of S *a* P to P *i* S is called "*conversion by limitation*". The two other conversions are named "*simple conversion*". The proposition in the form S *o* P has no converse, since the subject term is undistributed in the premiss whereas it would be distributed as the predicate of the conclusion (P *e* S or P *o* S).

Examples

From "*All* murders *are* felonies" it is valid to infer that "*Some* felonies *are* murders". From "*No* invitees *are* trespassers" it is valid to infer that "*No* trespassers *are* invitees". From "*Some* minors *are* criminals" it is valid to infer that "*Some* criminals *are* minors".

The admission of negative terms gives rise to further immediate inferences. To express negative terms, the particle "*non-*" is employed as the negating factor. In the context of traditional logic, "*non*"- means "everything other than" or "anything apart from". For instance, "*non*-criminals" means everything other than criminals (that is to say, not only law-abiding people, new-born babies, etc., but also plants, stones, angels, triangles, etc.).

The immediate inference which arises when the predicate of a given premiss is negated, the subject of the premiss being retained as the subject of the conclusion, is called "*obversion*".

Schema of Obversion

Premiss	Valid Conclusion
Affirmation of S *a* P	Affirmation of S *e* *non*-P
Affirmation of S *e* P	Affirmation of S *a* *non*-P
Affirmation of S *i* P	Affirmation of S *o* *non*-P
Affirmation of S *o* P	Affirmation of S *i* *non*-P

The immediate inference which arises when the subject of a given premiss is negated, the predicate of the premiss being retained as the predicate of the conclusion is called "*inversion*".

Schema of Inversion

Premiss	Valid Conclusion
Affirmation of S *a* P	Affirmation of *non*-S *o* P
Affirmation of S *e* P	Affirmation of *non*-S *i* P
Affirmation of S *i* P	None
Affirmation of S *o* P	None

It is to be noted that in the first inference, P is undistributed in the premiss whereas it is distributed in the conclusion. This is nevertheless not a violation of any law of logic, because in the conclusion P is distributed not in relation to S but in relation to the negative term *non*-S, which obviously is a different term.

The immediate inference which arises when the premiss is obverted and the resulting proposition is converted is called "*partial contraposition*".

Schema of Partial Contraposition

Premiss	Valid Conclusion
Affirmation of S *a* P	Affirmation of *non*-P *e* S
Affirmation of S *e* P	Affirmation of *non*-P *i* S
Affirmation of S *i* P	None
Affirmation of S *o* P	Affirmation of *non*-P *i* S

The immediate inference which arises when the premiss is obverted, the resulting proposition is converted, and the proposition resulting therefrom is obverted again is called "*full contraposition*" (or "*obverted contraposition*").

Schema of Full Contraposition

Premiss	Valid Conclusion
Affirmation of S *a* P	Affirmation of *non*-P *a non*-S
Affirmation of S *e* P	Affirmation of *non*-P *o non*-S
Affirmation of S *i* P	None
Affirmation of S *o* P	Affirmation of *non*-P *o non*-S

Examples

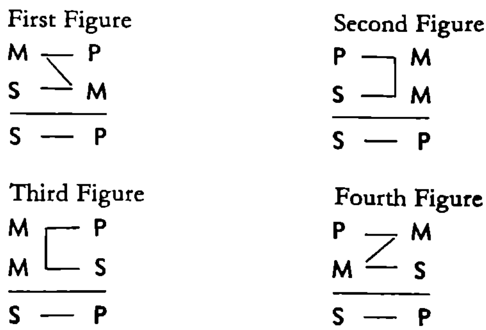
From "*All* murders *are* felonies" it is valid to infer that "*No* murders *are non*-felonies" by obversion. From "*No* trespassers *are* invitees" it is valid to infer that "*Some non*-trespassers *are* invitees" by inversion. From "*Some* contracts *are not* bilateral instruments" it is valid to infer that "*Some non*-bilateral instruments *are* contracts" by partial contraposition. From "*Some* minors *are not* criminals" it is valid to infer that "*Some non*-criminals *are not non*-minors" by full contraposition.

Further inferential possibilities lie in first converting and then obverting a proposition, in first inverting a proposition and then converting it, etc. By reference to the above schemata it is easy to determine which of the corresponding conclusions are valid and which not.

3. Syllogistic Inference

Syllogistic inference is a kind of mediate inference. In a mediate inference the conclusion is drawn from multiple propositions as premisses. This conclusion results from affirmation or denial of the premisses and is either affirmation or denial of a proposition. In a syllogism the conclusion, which is always an affirmed proposition, results only from affirmation of two or more premisses and depends on the terms contained in the premisses. It is to be noted that the word "syllogism" is used also in a wider sense to include other mediate inferences. Generally, this usage is not followed here. In the subsequent exposition only those syllogisms are considered whose terms are positive and referential, for only they fit neatly into the framework of traditional treatment of syllogisms.

A *simple syllogism* comprises three propositions: a *major premiss*, a *minor premiss*, and a *conclusion*, of which each must be in one of the four propositional forms and which contain no more than three terms. These three terms are: a *subject* (S), a *predicate* (P), and a *middle term* (M). The conclusion contains only S and P. The middle term occurs in each premiss only once: either as the subject of the premiss or as the predicate of the premiss. Depending on the position of the middle term, various combinations arise which determine the *figure* of the syllogism. In a syllogism, usually the major premiss (i.e. the premiss which contains the predicate of the conclusion) is stated before the minor premiss (i.e. the premiss which contains the subject of the conclusion). There are four figures of syllogism conforming to the following patterns:



The above diagram does not show what kind of proposition (the universal positive, the universal negative, the particular positive, or the particular negative) each premiss and the conclusion represents. Various placements of these propositional forms determine the *mood* of syllogism within each figure. Some combinations lead to invalid inferences because they violate one rule or several rules of syllogism; others lead to valid inferences because they comply with all the rules of syllogism.

The rules of simple syllogism are:

(1) *At least one premiss must be positive.*
(2) *The conclusion must be positive if both premisses are positive and the conclusion must be negative if one premiss is negative.*
(3) *The middle term must be distributed at least in one premiss.*
(4) *A term distributed in the conclusion must be distributed in the relevant premiss.*

These rules have the following important corollaries:

(a) At least one premiss must be universal.
(b) The conclusion must be particular if one premiss is particular.
(c) The minor premiss must be positive if the major premiss is particular.

There are *nineteen* conventionally accepted syllogisms which comply with the above rules. The valid inferences for each figure are set out in the schemata presented below. Since according to the definition of syllogism both the premisses and the conclusions are affirmed propositions, the schemata and the examples omit the indication of affirmation.

First Figure

Modus Barbara	*Modus Celarent*	*Modus Darii*	*Modus Ferio*
M *a* P	M *e* P	M *a* P	M *e* P
S *a* M	S *a* M	S *i* M	S *i* M
S *a* P	S *e* P	S *i* P	S *o* P

Examples

Modus Barbara:

All felonies *are* crimes
All murders *are* felonies
———
All murders *are* crimes

The following example represents an inference in which the minor premiss is a singular proposition. It is sometimes called "*Modus Barbara II*".

All murderers *are* criminals
John Axe *is* a murderer

John Axe *is* a criminal

Modus Celarent: *No* contracts *are* unilateral acts
All hire-purchase agreements *are* contracts

No hire-purchase agreements *are* unilateral acts

Modus Darii: *All* solicitors *are* lawyers
Some public servants *are* solicitors

Some public servants *are* lawyers

Modus Ferio: *No* torts *are* lawful acts
Some careless acts *are* torts

Some careless acts *are not* lawful acts

Second Figure

Modus Cesare	*Modus Camestres*	*Modus Festino*	*Modus Baroco*
P *e* M S *a* M ― S *e* P	P *a* M S *e* M ― S *e* P	P *e* M S *i* M ― S *o* P	P *a* M S *o* M ― S *o* P

Examples

Modus Cesare: *No* contracts *are* unilateral acts
All wills *are* unilateral acts

No wills *are* contracts

Modus Camestres: *All* business repairs *are* allowable deductions
No private expenses *are* allowable deductions

No private expenses *are* business repairs

Modus Festino: *No* torts *are* lawful acts
Some restraints of freedom *are* lawful acts

Some restraints of freedom *are not* torts

Modus Baroco: *All* unjustifiable homicides *are* crimes
Some killings *are not* crimes

Some killings *are not* unjustifiable homicides

Third Figure

Modus Darapti	*Modus Disamis*	*Modus Datisi*
M *a* P	M *i* P	M *a* P
M *a* S	M *a* S	M *i* S
S *i* P	S *i* P	S *i* P

Modus Felapton	*Modus Bocardo*	*Modus Ferison*
M *e* P	M *o* P	M *e* P
M *a* S	M *a* S	M *i* S
S *o* P	S *o* P	S *o* P

Examples

Modus Darapti: *All* felonies *are* crimes
All felonies *are* unlawful acts

Some unlawful acts *are* crimes

Modus Disamis: *Some* killings *are* crimes
All killings *are* matters of moral concern

Some matters of moral concern *are* crimes

Modus Datisi: *All* breaches of trust *are* acts creating liabilities
Some breaches of trust *are* excusable acts

Some excusable acts *are* acts creating liabilities

Modus Felapton: *No* equitable maxims *are* binding precedents
All equitable maxims *are* useful guides

Some useful guides *are not* binding precedents

Modus Bocardo: *Some* treaties *are not* bilateral acts
All treaties *are* legal instruments

Some legal instruments *are not* bilateral acts

Modus Ferison: *No* corporations *are* testators
Some corporations *are* owners of real property

Some owners of real property *are not* testators

Fourth Figure

Modus Bramantip	*Modus Camenes*	*Modus Dimaris*
P *a* M	P *a* M	P *i* M
M *a* S	M *e* S	M *a* S
S *i* P	S *e* P	S *i* P

Modus Fesapo	*Modus Fresison*
P *e* M	P *e* M
M *a* S	M *i* S
S *o* P	S *o* P

Examples

Modus Bramantip: *All* wilfully false statements to courts *are* perjuries
All perjuries *are* punishable acts
———
Some punishable acts *are* wilfully false statements to courts

Modus Camenes: *All* murders *are* felonies
No felonies *are* misdemeanours
———
No misdemeanours *are* murders

Modus Dimaris: *Some* legal expressions *are* ambiguities
All ambiguities *are* expressions of uncertain meaning
———
Some expressions of uncertain meaning *are* legal expressions

Modus Fesapo: *No* invitees *are* trespassers
All trespassers *are* tortfeasors
———
Some tortfeasors *are not* invitees

Modus Fresison: *No* perjuries *are* excusable acts
Some excusable acts *are* incorrect statements made to courts
———
Some incorrect statements made to courts *are not* perjuries

From the above exposition of valid syllogisms it appears that only the First Figure yields a universal positive proposition as a conclusion. Only the First Figure (in one instance), the Second Figure (in two instances) and the Fourth Figure (in one instance) yield universal negative propositions as conclusions. The Third Figure yields only particular propositions as conclusions.

Some logicians have regarded only the First Figure as "perfect" in the sense that conclusions here follow from the premisses in a transparent manner, whereas this is not the case with other figures. Therefore in the history of logic, *reduction* of the moods of other figures to the moods of the First Figure has played a role. This reduction is of two kinds: *direct* and *indirect*. Direct reduction is effected by transposition of premisses (stating the minor premiss first and treating it as the major premiss and stating the major premiss after it and treating it as the minor premiss), by conversion of premisses or of the conclusion, or by employing both methods. Thus *Modus Datisi* of the Third Figure can be reduced to *Modus Darii* of the First Figure by the conversion of its minor premiss (M *i* S is converted into S *i* M); *Modus Camenes* of the Fourth Figure can be reduced to *Modus Celarent* of the First Figure by transposing the premisses and by converting the conclusion; and *Modus Disamis* of the Third Figure can be reduced to *Modus Darii* of the First Figure by transposing the premisses, by converting the original major premiss, and by converting the conclusion.

Indirect reduction (*reductio per impossibile*) is performed by using a First Figure syllogism to show that the denial of the conclusion of a syllogism of this figure contradicts a premiss of another figure. This kind of reduction is needed in the cases where the premisses of a mood of syllogism to be reduced to a First Figure syllogism contain a universal positive proposition and a particular negative proposition. For if the former is converted, two particular propositions result, from which no conclusion is logically possible; the particular negative proposition cannot be converted under the relevant rule of distribution at all. Thus direct reduction is unfeasible with respect to *Modus Baroco* and *Modus Bocardo*. To illustrate how indirect reduction operates, a *Modus Baroco* syllogism is taken as an example:

All felonies *are* crimes
Some acts of killing *are not* crimes

Some acts of killing *are not* felonies

If the conclusion of this syllogism is denied, its contradictory must be affirmed, that is, "*All* acts of killing *are* felonies". Then a First Figure syllogism is formed in which this proposition appears as the minor premiss:

All felonies *are* crimes
All acts of killing *are* felonies
———————
All acts of killing *are* crimes

The conclusion of this syllogism is inconsistent with the minor premiss of the original syllogism by contradicting it ("*All* acts of killing *are* crimes" and "*Some* acts of killing *are not* crimes" are contradictory propositions). Hence "*Some* acts of killing *are not* felonies" must be affirmed, because if it is impossible to deny a proposition there is no other alternative (under the Principle of Excluded Middle) but to affirm this proposition.

It may be mentioned that, traditionally, reduction has been performed by not having recourse to obversion. Since obversion produces equivalent propositions, there is no need to discount this method. By the use of obversion of the premisses or the conclusion, the need for indirect reduction would disappear, because all syllogisms could be directly reduced to a First Figure syllogism.

The names of the moods of syllogism within each figure represent condensed instructions of how valid inferences ought to be made. The vowels in each of these names signify the quality and the quantity of the premisses and of the corresponding conclusion. Certain consonants in them signify the instructions of how the reduction is to be performed. For example, "*e*", "*a*", and "*o*" in "*Fesapo*" signify that in this syllogism the major premiss is a universal negative proposition, the minor premiss is a universal positive proposition, and the conclusion is a particular negative proposition. In "*Darii*" "*i*" signifies that the minor premiss and the conclusion are particular positive propositions and "*a*" signifies that the major premiss is a universal positive proposition.

The initial consonants of each name of the moods indicate to which moods of the First Figure a mood of another figure can be reduced. For example. "*C*" in "*Camenes*" indicates that *Modus Camenes* can be reduced to *Modus Celarent*. If "*m*" occurs in the name of a mood, this indicates that the premisses must be transposed (*muta*). If "*s*" occurs in it, this indicates that the proposition signified by the preceding vowel is converted simply (*simpliciter*), that is, the conversion is effected without changing the quantity of the proposition. If "*p*" occurs in it, this indicates that the conversion is by limitation (*per accidens*). The occurrence of "*c*" in the name of a mood indicates that indirect reduction is required (*conversio syllogismi*).

Because essential instructions for making valid inferences are contained in the names of the moods, mnemonic verses have been devised in which these names are combined with other words in Latin indicating the figure to which the named syllogisms belong. The following is one of the several versions of these mnemonic verses. The italicised words are the names of the moods:

Barbara Celarent Darii Ferio prioris;
Cesare Camestres Festino Baroco secundae;
Tertia *Darapti Disamis Datisi Felapton*
Bocardo Ferison habet; quart' insuper addit:
Bramantip Camenes Dimaris Fesapo Fresison.

It is to be noted that in this verse the names of five moods are missing which also constitute valid syllogisms. They are said to be syllogisms with "weakened" conclusions.

Modus Barbari (Figure I)	*Modus Celaront* (Figure I)	*Modus Cesaro* (Figure II)
M *a* P	M *e* P	P *e* M
S *a* M	S *a* M	S *a* M
S *i* P	S *o* P	S *o* P

Modus Camestros (Figure II)	*Modus Camenos* (Figure IV)
P *a* M	P *a* M
S *e* M	M *e* S
S *o* P	S *o* P

In ordinary as well as in learned discourse syllogistic inferences are seldom employed in the manner above presented. They usually occur in an abridged form in which either one of the premisses or the conclusion is suppressed. These inferential expressions are called "*enthymemes*". In an enthymeme the component propositions of syllogisms which are ***not stated*** are nevertheless ***tacitly present.*** The suppression of the component propositions is permissible only if it can be taken for granted that in the given argumentative situation the suppressed proposition is implicitly understood by the addressees of reasoning.

An enthymeme in which the major premiss is suppressed is called "***the first order enthymeme***".

Examples

. . . .	. . . .
All murders *are* felonies	*All* wills *are* unilateral acts
————————	————————
All murders *are* crimes	*No* wills *are* contracts

In the first example the major premiss "*All* felonies *are* crimes" is suppressed in a *Modus Barbara* syllogism, whereas in the second example the major premiss "*No* contracts *are* unilateral acts" is suppressed in a *Modus Cesare* syllogism.

An enthymeme in which the minor premiss is suppressed is called "***the second order enthymeme***".

Examples

No torts *are* lawful acts	*All* murders *are* crimes
. . . .	. . . .
————————	————————
No trespasses *are* lawful acts	*Some* homicides *are* crimes

In the first example the minor premiss "*All* trespasses *are* torts" is suppressed in a *Modus Celarent* syllogism, whereas in the second example the minor premiss "*All* murders *are* homicides" is suppressed in a *Modus Darapti* syllogism.

An enthymeme in which the conclusion is suppressed is called "***the third order enthymeme***".

Examples

All crimes *are* unlawful acts	*All* murders *are* felonies
All larcenies *are* crimes	*No* felonies *are* misdemeanours
————————	————————

In the first example the conclusion "*All* larcenies *are* unlawful acts" is suppressed in a *Modus Barbara* syllogism, whereas in the second example the conclusion "*No* misdemeanours *are* murders" is suppressed in a *Modus Camenes* syllogism.

It is to be noted an enthymemic inference is involved in legal reasoning when from a statement of facts (e. g. "Smith *is* a person who has entered land without being either an occupier or an invitee or a licencee") a legal consequence is derived (e. g. "Smith *is* a trespasser"). Here the major premiss stating a legal norm is suppressed (viz. "*All* persons entering land without being either occupiers, invitees, or licencees *are* trespassers").

Simple syllogisms, either complete or abridged, may occur in chains of reasoning in which one syllogism supports another. Such interconnected syllogisms constitute *complex syllogisms* and are called "*sorites*". The supporting syllogism is called "*prosyllogism*" and the supported syllogism is called "*episyllogism*".

Examples

Prosyllogism: *All* tortfeasors *are* law-breakers
All trespassers *are* tortfeasors

Episyllogism: *All* trespassers *are* law-breakers
Black *is* a trespasser

Black *is* a law-breaker

No matters of ethical concern *are* matters irrelevant to law reform
All major social interests *are* matters of ethical concern
Some interests protected by law *are* major social interests

Some interests protected by law *are not* matters irrelevant to law reform

The second example presents a sorites in which the component syllogisms are enthymemes. It has the form of the so-called *Goclenian sorites*, i. e. a sorites in which the first premiss contains the predicate of the conclusion and the last premiss contains the subject of the conclusion. In contrast, the so-called *Aristotelian sorites* is a sorites in which the first premiss contains the subject of the conclusion and the last premiss contains the predicate of the conclusion. The following is an example of Aristotelian sorites:

All judges *are* persons expected to have a sound knowledge of law
All persons expected to have a sound knowledge of law *are* persons liable to commit some errors in reasoning
All persons liable to commit some errors in reasoning *are* persons who may misinterpret law

---

All judges *are* persons who may misinterpret law

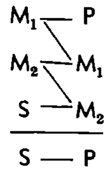
The Goclenian sorites must comply with the following rules:

(1) *If a negative proposition occurs in a Goclenian sorites, this proposition must be the first premiss.*
(2) *If a particular proposition occurs in a Goclenian sorites, this proposition must be the last premiss.*

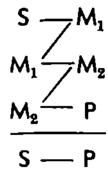
The Aristotelian sorites must comply with the following rules:

(1) *If a negative proposition occurs in an Aristotelian sorites, this proposition must be the last premiss.*
(2) *If a particular proposition occurs in an Aristotelian sorites, this proposition must be the first premiss.*

The Pattern of Goclenian Sorites



The Pattern of Aristotelian Sorites



There can be, of course, any number of middle terms in these sorites and other configurations of the premisses are also possible.

Any sorites must comply with the following rules:

(1) *No more than one negative proposition can occur in a sorites.*
(2) *Each middle term must be distributed at least once.*
(3) *The conclusion must be negative if a premiss is negative and one premiss must be negative if the conclusion is negative.*
(4) *A term distributed in the conclusion must be distributed in the premiss in which it occurs.*

These rules have the following important corollaries:

(a) No more than one particular proposition can occur in a sorites.
(b) The conclusion must be particular if a premiss is particular.

4. *Hypothetic and Disjunctive Inferences*

In the two foregoing sections those inferences were examined whose premisses were all categoric propositions. In the post-Aristotelian development of logic by megarians and stoics, a different kind of inference was elaborated whose premisses contained hypothetic or disjunctive propositions. This kind of inference which constitutes, in contrast to Aristotelian term-logic, the logic of propositions, has a special virtue in that the propositions of which the inferences are composed need not appear in the rigid four propositional forms. For the purposes of hypothetic and disjunctive inferences, propositions in the wording as they occur in common language are usually quite adequate. Therefore the inferences to be examined below involve less linguistic strain; they can usually be expressed in ordinary and natural prose.

It is to be noted that in the subsequent exposition of hypothetic and disjunctive inferences the indication of affirmation of the propositions is omitted. Any proposition in the context of these inferences which is simply stated is to be understood as being affirmed. Denial of a proposition in this context is expressed by the prefixed phrase "*it is not the case that*"; in the schemata of the inferences the expression "*not*" is used to signify denial.

The first premiss of a *simple hypothetic inference* consists in a complex affirmed proposition composed of the *antecedent* (proposition) and the *consequent* (proposition), connected with "*if . . . then . . .*". In the schemata of this inference the symbol p is employed for the antecedent and the symbol q for the consequent so that the first premiss is "*if* p *then* q". The second premiss of this inference is a categoric proposition which either affirms or denies a component proposition of the first one.

The rules of hypothetic inference are the following:

(1) *From affirmation of the antecedent it is valid to infer affirmation of the consequent (Modus Ponens).*
(2) *From denial of the consequent it is valid to infer denial of the antecedent (Modus Tollens).*

(3) ***Denial of the antecedent or affirmation of the consequent does not establish any conclusion.***

Modus Ponens	*Modus Tollens*
If p *then* q	*If* p *then* q
p	*not* q
q	*not* p

Examples

Modus Ponens: *If* Black is a trespasser *then* Black is a tortfeasor
Black is a trespasser

Black is a tortfeasor

Modus Tollens: *If* Black is a trespasser *then* Black is a tortfeasor
It is not the case that Black is a tortfeasor

It is not the case that Black is a trespasser

The following examples illustrate that no conclusion can be drawn from denying the antecedent or from affirming the consequent:

If there is no consideration *then* there is no contract
It is not the case that there is no consideration

? ? ?

The denial that there is no consideration does not establish any conclusion, for this denial leaves the possibilities open (1) that there is a contract because all essential contractual requirements have been fulfilled or (2) that there is no contract because some other essential contractual requirements apart from consideration have not been fulfilled.

If Brown is in Sydney *then* Brown is in Australia
Brown is in Australia

? ? ?

The affirmation that Brown is in Australia does not establish any conclusion to the effect that Brown is in Sydney or that Brown is not in Sydney, for the affirmation leaves it open whether (1) Brown in fact is in Sydney, or (2) Brown is in some part of Australia other than Sydney. Either state of affairs would be compatible with the premisses.

There are valid hypothetic inferences in which two or more premisses are hypothetic propositions. In contrast to the inferences considered above they constitute *complex hypothetic inferences* and may be exemplified by what is usually called "*hypothetic syllogism*". In a hypothetic syllogism the consequent of the first premiss must be identical with the antecedent of the second premiss; the conclusion is a hypothetic proposition whose antecedent is the antecedent of the first premiss and whose consequent is the consequent of the second premiss. The schema of this inference is the following:

If p *then* q
If q *then* r
———————
If p *then* r

Example

If this decision is legally unchallengeable *then* this decision is based on a valid statutory norm
If this decision is based on a valid statutory norm *then* the relevant statutory norm is based on a valid constitutional norm
———————
If this decision is legally unchallengeable *then* the relevant statutory norm is based on a valid constitutional norm

The first premiss of a *simple disjunctive inference* consists of two propositions (*disjuncts*) connected either with "... *or* ..." or with "*either* ... *or* ...". In the former case there is a *weak disjunction*; in the latter case there is a *strong disjunction*. In the schemata of this inference the symbol p is employed for the first disjunct and the symbol q for the second disjunct. The second premiss represents a proposition which either affirms or denies one of the disjuncts of the first premiss. All disjunctive inferences operate on the assumption that the disjuncts are exhaustive, i. e. that all relevant disjuncts have been stated. It is to be noted that "*either* ... *or* ..." is here understood to imply "*but not both*".

The rules of inference which apply where the first premiss is a weak disjunction are the following:

(1) *From denial of one disjunct it is valid to infer affirmation of the other disjunct (Modus Tollendo Ponens).*
(2) *Affirmation of one disjunct does not establish any conclusion.*

The inferences by *Modus Tollendo Ponens* where the first premiss is a weak disjunction can be presented in the following schemata:

p *or* q	p *or* q
not p	*not* q
q	p

Examples

This burglary was committed by North *or* this burglary was committed by South
It is not the case that this burglary was committed by North

This burglary was committed by South

This burglary was committed by North *or* this burglary was committed by South
It is not the case that this burglary was committed by South

This burglary was committed by North

The rules of inference which apply where the first premiss is a strong disjunction are the following:

(1) *From affirmation of one disjunct it is valid to infer denial of the other disjunct (Modus Ponendo Tollens).*
(2) *From denial of one disjunct it is valid to infer affirmation of the other disjunct (Modus Tollendo Ponens).*

The inferences where the first premiss is a strong disjunction can be presented in the following schemata:

Modus Ponendo Tollens

Either p *or* q	*Either* p *or* q
p	q
not q	*not* p

Modus Tollendo Ponens

Either p *or* q	*Either* p *or* q
not p	*not* q
q	p

Examples

Either Jones is an adult *or* Jones is a minor
Jones is an adult

It is not the case that Jones is a minor

Either Jones is an adult *or* Jones is a minor
It is not the case that Jones is an adult

Jones is a minor

Both weak disjunction and strong disjunction can occur in a complex form, in which there are more than two disjuncts in the first premiss: p *or* q *or* r *or* ...; *Either* p *or* q *or* r *or* ... In case of an inference which has a complex weak disjunction as the first premiss, denial of any of the disjuncts leads to affirmation of the remaining part of the disjunction. Affirmation of any of them does not establish any conclusion. In case of an inference which has a complex strong disjunction as the first premiss, affirmation of any of the disjuncts leads to denial of the remaining part of the disjunction and denial of any of the disjuncts leads to affirmation of the remaining part of the disjunction.

Disjunctive premisses can be expressed in a condensed form in which the disjuncts do not appear as propositions but as parts of propositions. For example, the proposition "This burglary was committed by North *or* this burglary was committed by South" can be abbreviated as "This burglary was committed by North *or* by South". The proposition "*Either* the accused ought to be imprisoned *or* the accused ought to be fined" can be abbreviated as "The accused ought to be *either* imprisoned *or* fined". Another way of abbreviating disjunctive as well as hypothetic premisses is to employ appropriate pronouns for certain expressions occurring in them. It is to be kept in mind that the abbreviated premisses in these cases are to be understood as being composed of propositions, namely of propositions expressed in an elliptic manner. Thus the above rules of inferences apply also if an inference contains abbreviated premisses. For example:

Either the accused ought to be imprisoned *or* fined
He ought to be fined

It is not the case that the accused ought to be imprisoned

Various complex inferences arise from combining hypothetic and disjunctive inferences. Only some configurations of the *dilemma*, which is the most important type of such complex inferences, will be considered here. There are four principal kinds of the dilemma: (1) the simple constructive dilemma, (2) the simple destructive dilemma, (3) the complex constructive dilemma, and (4) the complex destructive dilemma.

In the configurations of the dilemma here to be considered there are two premisses: one a conjunction of hypothetic propositions and the other a disjunctive proposition which either affirms disjunctively the antecedents of the hypothetic premiss or denies disjunctively its consequents. The disjunctive premiss can be either a weak disjunction or a strong disjunction. It is optional which of the premisses is stated first. In a simple dilemma, the conclusion is a categoric proposition whereas in a complex dilemma, the conclusion is a disjunctive proposition. In a constructive dilemma, the antecedents of the hypothetic premiss are disjunctively affirmed whereas in a destructive dilemma, the consequents of the hypothetic premiss are disjunctively denied. In a simple constructive dilemma, the hypothetic premiss has the same consequent for both antecedents whereas in a simple destructive dilemma, the hypothetic premiss has the same antecedent for both consequents. In a complex dilemma, be it constructive or destructive, both antecedents and both consequents of the hypothetic premiss are different propositions. In a constructive dilemma, be it simple or complex, the antecedents of the hypothetic propositions are different propositions. In a destructive dilemma, be it simple or complex, the consequents of the hypothetic premiss are different propositions.

The four kinds of the dilemma described above can be presented in the following schemata:

The Simple Constructive Dilemma:

If p *then* q *and if* r *then* q	*If* p *then* q *and if* r *then* q
p *or* r	*either* p *or* r
q	q

The Simple Destructive Dilemma:

If p *then* q *and if* p *then* r	*If* p *then* q *and if* p *then* r
not q *or not* r	*either not* q *or not* r
not p	*not* p

The Complex Constructive Dilemma:

If p *then* q *and if* r *then* s	*If* p *then* q *and if* r *then* s
p *or* r	*either* p *or* r
q *or* s	q *or* s

(Note that it is not correct to infer: *either* q *or* s)

The Complex Destructive Dilemma:

If p *then* q *and if* r *then* s	*If* p *then* q *and if* r *then* s
not q *or not* s	*either not* q *or not* s
not p *or not* r	*not* p *or not* r

(Note that it is not correct to infer: *either not* p *or not* r)

In the following illustrations complete as well as abbreviated premisses and different placement of premisses will be employed.

Examples

A Simple Constructive Dilemma:

Either restrictions on trade practices are imposed by law *or* law abstains from interfering with trade practices
If restrictions on trade practices are imposed by law *then* economic hardships must arise *and if* law abstains from interfering with trade practices *then* economic hardships must arise

Economic hardships must arise

A Simple Destructive Dilemma:

If an ideal State will be established *then* there will be general prosperity *and if* it will be established *then* State coercion will disappear
It is not the case that there will be general prosperity *or it is not the case that* State coercion will disappear

It is not the case that an ideal State will be established

A Complex Constructive Dilemma:

Either we retain *or* abolish death penalty
If we abolish death penalty *then* our criminal law will lose its deterrent effect *and if* we retain it *then* our criminal law will remain uncivilised

Our criminal law will lose its deterrent effect *or* it will remain uncivilised

A Complex Destructive Dilemma:

> *If* there is full freedom of expression *then* there will be obscene publications *and if* there is censorship *then* the quality of works of art will decline
>
> *It is not the case that* there will be obscene publications *or it is not the case that* the quality of works of art will decline
>
> ---
>
> *It is not the case that* there is full freedom of expression *or it is not the case that* there is censorship

Dilemmas are widely employed in polemics. Because of their rather complex logical structure, they lend themselves to abuse: invalid arguments here are not easy to detect for those not sufficiently familiar with the principles on which the dilemmas are based.

Chapter II: A System of Modern Logic

1. Preliminary Considerations

Modern logic, that is, the logic which is the principal object of study and development for contemporary logicians and which is widely applied in various branches of contemporary science, is a logic characterised by a thorough formalisation expressed in highly developed symbolism. It is therefore apt to call it "symbolic logic", even though use of symbols is not a specific feature which distinguishes it from traditional logic; for the latter, too, employs symbols. In a way, modern logic is contiguous to post-Aristotelian developments of hypothetic and disjunctive inferences. These have found an elaboration, generalisation, and refinement in propositional calculus, on which the whole structure of symbolic logic is ordinarily based. On the other hand, modern logic can be viewed as a generalisation of principles of mathematics, which generalisation raises certain mathematical notions to a level of abstraction which is no longer pertinent to mathematics but to all procedures of stringent reasoning.

Ordinary language is not capable of supplying words in common usage which would univocally convey the meanings intended by logicians when they deal with matters of symbolic logic. One of the main obstacles to the understanding of the principles and methods of symbolic logic lies precisely in this fact. Unless the student is *ab initio* put to notice and is constantly aware that words of ordinary language employed in the context of symbolic logic may carry a specific technical meaning, he will be constantly puzzled or confused about and may even rebel against what he finds in books of this logic. Thus he would find that the words "implication", "truth", and "and" employed by modern logicians are misleading when not considered in detachment from their ordinary meanings. Once the student has mastered the use of logical symbols, he is initiated into symbolic logic and should henceforth be able to follow its procedures without any feeling of consternation which ordinary words may produce when logical formulae are translated into prose.

Another main obstacle to the study of symbolic logic lies in the circumstance that there is not yet a uniform system of notation for this logic. There are various alternative systems, each having their advantages and disadvantages. Fortunately they are easily interchangeable, though this does not guarantee the facility of handling a notation to which the student has not been accustomed. A further difficulty arises for a novice in the study of symbolic logic from the fact that the words expressing certain key notions employed in its expositions vary. For example, what some logicians call "implication" (or "material implication" or "extensive implication") is called "conditional" by others, and what some call "disjunction" is called "alternation" by others.

The subsequent exposition tries to avoid as much as possible proliferation of terminology by shunning words not already in use in the literature of symbolic logic. However, it will prove necessary to make choices between alternative technical words in use. Occasionally, slightly modified technical words in use will be adopted in order to achieve a satisfactory terminological uniformity. The considerations determining these choices and adjustments include avoidance of ambiguities and disturbing associations in the present context and the least strain between ordinary language and the technical language of symbolic logic. The notation devised by Jan Łukasiewicz (Polish notation) will be selected, not just because of its subtlety and elegance but because for the present purposes it is simple and convenient requiring only few additional signs which are not ordinary letters of the English alphabet. By its constant but slightly varied use in different calculi an attempt will be made to bring out the unity of rational endeavour of all parts of symbolic logic.

The exposition of symbolic logic in this Compendium will start with what may be called "protological calculus". This calculus is a generalisation of the ideas of propositional calculus, which is usually chosen to incept treatments of modern logic. Protological calculus, being a system of uninterpreted signs, forms the basis for the whole structure of symbolic logic presented here. It may be regarded as being not a part of any logic but as a preliminary to logic. At the stage of protological calculus, the question as to what use its formulae may have is left open and no attempt is made to relate these formulae to "reality" by giving illustrations. In fact, this calculus can be viewed as a "play" with signs, which it is advisable to learn first as a game and then to proceed to make use of its principles. It proves that the direct applications of protological calculus are specific logical calculi, which in their turn find application in thought having material content.

One reason for this approach to the presentation of modern logic is didactic. Although what will be encountered in the exposition of protological calculus proves to be quite simple, perhaps in no way more difficult than chess, there are many questions which arise for the learner and require answers before he can assimilate its principles in his mind and acquire a facility of using them correctly. This process will be delayed or even frustrated if interpretations are given to the signs and their combinations which link them with any particular reality. This is so because confrontation of the mind with reality to which a formal system would apply gives rise to further questions diverting the attention from the problems of the formal system itself. It is therefore advisable to make a concentrated effort to learn this system in all its "purity" first. When this has been achieved, the time has arrived to proceed to the connecting of the formal system with reality. The other reason for the present approach is that protological calculus provides a unitary basis for the superstructure of modern logic: the calculi which belong to this superstructure prove to be special developments of the principles of protological calculus and to have common denominators in these principles.

The concepts of protological calculus will be here expressed in words which have a possibly "neutral" meaning in the sense that they are not intimately connected with any particular branch of learning. This is to avoid the impression that protological operations are mathematical operations or operations peculiar to any other special concern of the intellect.

2. *Protological Calculus*

Protological calculus employs the following signs:

(1) x, y, and z, or any of them with a numeral subscript (e.g. x_1, y_2, z_1, etc.), signifying *elements*.
(2) N signifying the *monadic operator*.
(3) C, A, K, E, D, I, J, and O signifying the *dyadic operators*.
(4) + and – signifying the *marks*.

In addition to these signs, also * and ^{0} will be employed here. What they signify will be stated in the end of this section.

Some of these signs singly or certain combinations of them constitute *units*. There is an expression of a unit if it is exactly

(a) a sign of an element, or
(b) an expression formed so that the sign of the monadic operator is immediately followed by the expression of one unit, or
(c) an expression formed so that the sign of one of the dyadic operators is immediately followed by the signs of two units.

According to the above rules of formation, for example, x, y, x_1, and x_2 express protological units (whereas a, c, N, and E do not); Nx, Nx_1 NNx, and $NNNx_1$ express units (whereas Cx, xN, x_1NN, and NyN do not); and Cxy, Kxx, $EAxyOzx_1$, and $DAKxyJly_1xy_2Cy_3z$ express units (whereas xCy, xxK, $AxyOzx_1$, and $DAxyJly_1xy_2y_3Cz$ do not).

Unless a unit appears only as a single element, it is a *compound.* For example, Nx, Cxy, and KNIzxy are expressions of compounds (whereas x, y, and x_1 are not).

Any protological expression can be tested as to whether or not it signifies a unit by underlining the expression in the following procedure:

I. Underline all signs of an element.

For example, in the following expression underline:

N I E N N N C x y O z A N x_1 z D N y N N x_2

II. Underline then all expressions in which one already underlined expression is preceded by any number of the signs of the monadic operator.

The underlining now continues:

N I E N N N C x y O z A N x_1 z D N y N N x_2

III. Underline then all expressions in which two already underlined expressions are preceded by exactly one dyadic operator.

The underlining now continues:

N I E N N N C x y O z A N x_1 z D N y N N x_2

IV. Underline then all expressions in which an already underlined expression is preceded by any number of the signs of the monadic operator.

The underlining now continues:

N I E N N N C x y O z A N x_1 z D N y N N x_2

V. Continue steps III and IV until the sign of the first operator is reached.

The whole expression signifies a unit if it turns out that all its parts are underlined by one continuous line. If it proves that this cannot be done, the expression is not a unit.

The above procedure can be expedited by underlining the signs of the monadic operator in the same steps in which the expressions of other units are underlined. The following represents such an expedited underlining procedure:

Wherever there is an expression of a unit, there is always Step 1. Whether or not other steps and which of them are required depends on the composition of the expressions.

Every unit in the present system is characterised by either plus or minus marks assigned to them according to rules stipulated below. Hence this system is a two-mark protologic. Three- and more-than-three-mark protological systems can be constructed. Because of their complexities, they will not be examined in this Compendium.

If a single element occurs as a unit, it has either a plus or a minus mark. If a unit is composed of more than one element, these elements have various mark-distributions, the number of which depends on the number of elements. The possible mark-distributions for two elements is 2^2, i. e. 4, for three elements 2^3, i. e. 8, and for n elements 2^n. In order to present a pattern of possible mark-distributions which can be conveniently surveyed and remembered, they are set out in a certain order, which is here called a "*guide-matrix*". The following table presents guide-matrices for one element, for two elements, for three elements, and for four elements:

Table I

x	x y	x y z	x y z x_1
+	+ +	+ + +	+ + + +
−	+ −	+ + −	+ + + −
	− +	+ − +	+ + − +
	− −	+ − −	+ + − −
		− + +	+ − + +
		− + −	+ − + −
		− − +	+ − − +
		− − −	+ − − −
			− + + +
			− + + −
			− + − +
			− + − −
			− − + +
			− − + −
			− − − +
			− − − −

In order to obtain mark-distributions in the guide-matrices which represent all possible combinations of the marks, proceed as follows:

I. Halve the first column and place the plus sign or signs into the first half and the minus sign or signs into the second half of the column.

II. If there is a second column, halve each half of the previous column and place the plus signs into the first resulting half and the minus signs into the second resulting half.

III. If there are further columns, halve each group of the plus signs and of the minus signs until the plus-minus-plus-minus . . . configuration is reached.

Each compound is characterised by a specific mark *constellation* depending on the mark-distribution of the guide-matrix of its elements and on the operator or operators by which it is formed as a unit. For a unit formed by a monadic operator and one element there are four possible constellations of marks. For units formed by a dyadic operator and two elements there are sixteen possible constellations of marks. This appears from the following tables:

Table II

x	1	2	3	4
+	+	+	−	−
−	+	−	+	−

Table III

x	y	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
+	+	+	+	+	+	+	+	+	+	−	−	−	−	−	−	−	−
+	−	+	+	+	+	−	−	−	−	+	+	+	+	−	−	−	−
−	+	+	+	−	−	+	+	−	−	+	+	−	−	+	+	−	−
−	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−

Each of the above constellations could be chosen to characterise a compound. However, for the present purposes only some of them will be selected, namely from Table II 3 to characterise the N-compound and from Table III 2, 3, 5, 7, 8, 9, 10, and 15 to characterise A-, D-, C-, E-, K-, J-, O-, and I-compounds respectively.

The compounds formed by the above operators can be presented by the following two tables:

Table IV

x	Nx
+	−
−	+

Table V

x	y	Cxy	Axy	Kxy	Exy	Dxy	Ixy	Jxy	Oxy
+	+	+	+	+	+	+	−	−	−
+	−	−	+	−		+	−	+	+
−	+	+	+	−	−	−	−	+	+
−	−	+	−	−	+	+	+	+	−

It is to be noted that the uniform sequence of the marks in the matrices as employed above (and as usually employed in works on modern logic) is only a didactic and mnemonic expedient. Other guide-matrix configurations and correspondingly different mark constellations are possible. The condition which they all must fulfil is that the distribution of marks exhausts all possible permutations of the marks. There is no need to avoid repetitions of the same horizontal sequence of the marks and there is no objection against reshuffling the horizontal sequences. Keeping this in mind, the following rules can be extracted from the employment of the tabular method by means of which protological compounds are characterised:

(1) *If a compound is formed by* N, *the mark of the compound is plus if the mark of the unit governed by this operator is minus and it is minus if the mark of such unit is plus.*

(2) *If a compound is formed by* C, *the mark of the compound is minus only if the mark of the first unit governed by this operator is plus and the mark of the second unit is minus; in every other case the mark of the compound is plus.*

(3) *If a compound is formed by* A, *the mark of the compound is minus only if the marks of both units governed by this operator are minus; in every other case the mark of the compound is plus.*

(4) *If a compound is formed by* K, *the mark of the compound is plus only if the marks of both units governed by this operator are plus; in every other case the mark of the compound is minus.*

(5) *If a compound is formed by* E, *the mark of the compound is plus if the marks of both units governed by this operator are the same and it is minus if they are different.*

The rules for D-, I-, J-, and O-compounds can be formulated in the same way from Table V.

All these rules can be formulated in an abbreviated way as follows:

(1) *For* N, *plus gives minus and minus gives plus.*
(2) *For* C, *only plus and minus give minus.*
(3) *For* A, *only two minuses give minus.*
(4) *For* K, *only two pluses give plus.*
(5) *For* E, *the same marks give plus and different marks give minus.*
(6) *For* D, *only minus and plus give minus.*
(7) *For* I, *only two minuses give plus.*
(8) *For* J, *only two pluses give minus.*
(9) *For* O, *the same marks give minus and different marks give plus.*

Application of Rule (1), according to which N converts the marks of the unit governed by it to its opposite in any compound formed by N, leads to the following results:

Table VI

x	y	Axy	NAxy	Kxy	NKxy	Exy	NExy
+	+	+	−	+	−	+	−
+	−	+	−	−	+	−	+
−	+	+	−	−	+	−	+
−	−	−	+	−	+	+	−

Comparison of this table with Table V shows that the constellation characterising NAxy is the same as the constellation characterising Ixy. This permits the latter to be replaced by the former. Likewise Jxy can be replaced by NKxy and Oxy by NExy. This makes it possible to reduce the number of operators. There is a further possibility of reducing the number of operators by expressing Dxy either as Cyx or as AxNy. This appears from the following table:

Table VII

x	y	Dxy	y	x	Cyx	x	Ny	AxNy
+	+	+	+	+	+	+	−	+
	−	+	−	+	+	+	+	+
−	+	−	+	−	−	−	−	−
−	−	+	−	−	+	−	+	+

The number of the operators can further be reduced by expressing Cxy as ANxy, Kxy as NANxNy, and Exy as NANANxyNAxNy. That each of these pairs of compounds has exactly the same mark constellation appears from Table VIII below. In this Table a convenient method of working out the ultimate constellation for each compound is employed which consists in placing the mark distributions of elements under the signs of each element of each compound and then working out the constellation of each compound.

Table VIII

x	y	C	x	y	A	N	x	y	K	x	y	N	A	N	x	N	y
+	+	+	+	+	+	−	+	+	+	+	+	+	−	−	+	−	+
+	−	−	+	−	−	−	+	−	−	+	−	−	+	−	+	+	−
−	+	+	−	+	+	+	−	+	−	−	+	−	+	+	−	−	+
−	−	+	−	−	+	+	−	−	−	−	−	−	+	+	−	+	−

x	y	E	x	y	N	A	N	A	N	x	y	N	A	x	N	y
+	+	+	+	+	+	−	−	+	−	+	+	−	+	+	−	+
+	−	−	+	−	−	+	+	−	−	+	−	−	+	+	+	−
−	+	−	−	+	−	+	−	+	+	−	+	+	−	−	−	+
−	−	+	−	−	+	−	−	+	+	−	−	−	+	−	+	−

In order to employ the above method, proceed as follows:

I. Place the marks of each guide-matrix column under the corresponding element signs in the compound whose ultimate mark constellation is sought.

II. Work out the constellation of each compound consisting of either one or two elements placing the appropriate marks under the operator signs.

III. Work out the constellation of each compound composed of compounds as their units placing the marks under the governing operator sign, moving backwards as in the underlining process.

IV. Continue this procedure until the sign of the first operator which governs the whole compound is reached.

It is helpful to cancel the columns under each element or operator as soon as the column under their governing operator sign has been completed.

An alternative way of reduction of the number of operators is the following:

Cxy can be expressed as NKxNy, Axy as NKNxNy, and **Exy** as KNKxNyNKNxy. That each of these pairs of compounds have exactly the same mark constellation can be demonstrated by the tabular method described above.

So far it has been shown that it is possible to express every compound by means of N and one appropriate dyadic operator. There are ways of even further reduction of the number of operators in expressing the

N-compound by means of an appropriate dyadic operator. These operators can be either I or J.

Table IX

x	x	I	x	x	J	x	x	N	x
+	+	−	+	+	−	+	+	−	+
+	+	−	+	+	−	+	+	−	+
−	−	+	−	−	+	−	−	+	−
−	−	+	−	−	+	−	−	+	−

This table shows that wherever two pluses occur in the units governed by either of these operators, the ultimate mark of the compound is minus and wherever two minuses occur in these units, the ultimate mark of the compound is plus. Hence the constellation characterising Ixx as well as Jxx corresponds exactly to the constellation resulting from application of N to x. This yields the following principle:

Any unit governed by N *can be expressed as the same unit repeated twice and governed either by* I *or* J.

It can be demonstrated by the tabular method that C-, A-, K-, and E-compounds can be expressed by means of either I or J as follows:

Cxy as NINxy, Kxy as INxNy, Axy as NIxy, and Exy as IINxyIxNy;
Cxy as JxNy, Axy as JNxNy, Kxy as NJxy, and Exy as NJJxNyJNxy.

Under the above principle, the compounds governed by either C, A, K, or E can be expressed by means of I as a single operator as follows:

Cxy as IIIxxyIIxxy
Axy as IIxyIxy
Kxy as IIxxIyy
Exy as IIIxxyIxIyy

By this ultimate reduction, extreme sign-economy and uniformity of protological calculus has been gained; however, there has been a loss in the simplicity of the expressions and in the ease of wielding them. This loss outweighs the gain, and therefore in protological operations (and in logical operations following their pattern) it has proved to be convenient to employ N together with certain dyadic operators.

There are compounds whose ultimate mark constellation proves to contain either only pluses, only minuses, or both marks for any mark distributions of their elements. The first ones may be called "*firm compounds*", the second ones "*loose compounds*", and the third ones "*pliant compounds*". To indicate that a compound is firm, an asterisk (*) may be placed before the first operator sign of its expression and to indicate that a compound is loose, a minute zero (^{0}) may be placed in the same position. The following tables provide illustrations:

Table X

x	*A	x	N	x	^{0}K	x	N	x	A	x	x
+	+	+	−	+	−	+	−	+	+	+	+
−	+	−	+	−	−	−	+	−	−	−	−

Table XI

x	y	*E	C	x	y	A	N	x	y	^{0}K	K	x	y	I	x	y
+	+	+	+	+	+	+	−	+	+	−	+	+	+	−	+	+
+	−	+	−	+	−	−	−	+	−		−	+	−	−	+	−
−	+	+	+	−	+	+	+	−	+	−	−	−	+	−	−	+
−	−	+	+	−	−	+	+	−	−	−	−	−	−	+	−	−

x	y	O	D	x	y	J	x	y
+	−	+	+	+	+	−	+	+
+	−	−	+	+	−	+	+	−
−	+	+	−	−	+	+	−	+
−	−	−	+	−	−	+	−	−

Table XII

x	y	z	*C	K	C	x	y	C	y	z	C	x	z	^{0}K	I	x	D	y	z	x
+	+	+	+	+	+	+	+	+	+	+	+	+	+	−	−	+	+	+	+	+
+	+	−	+	−	+	+	+	−	+	−	−	+	−	−	−	+	+	+	−	+
+	−	−	+	−	−	+	−	+	−	+	+	+	+	−	−	+	−	−	+	+
+	−	−	+	−	−	+	−	+	−	−	−	+	−	−	−	+	+	−	−	+
−	+	+	+	+	+	−	+	+	+	+	+	−	+	−	−	−	+	+	+	−
−	+	−	+	−	+	−	+	−	+	−	+	−	−	−	−	−	+	+	−	−
−	−	+	+	+	+	−	−	+	−	+	+	−	+	−	+	−	−	−	+	−
−	−	−	+	+	+	−	−	+	−	−	+	−	−	−	−	−	+	−	−	−

x	y	z	O	A	D	x	y	C	y	z	x
+	+	+	−	+	+	+	+	+	+	+	+
+	+	−	−	+	+	+	+	−	+	−	+
+	−	+	−	+	+	+	−	+	−	+	+
+	−	−	−	+	+	+	−	+	−	−	+
−	+	+	+	+	−	−	+	+	+	+	−
−	+	−	−	−	−	−	+	−	+	−	−
−	−	+	+	+	+	−	−	+	−	+	−
−	−	−	+	+	+	−	−	+	−	−	−

3. Propositional Calculus

Propositional calculus has the same structure as protological calculus. It may be viewed as a calculus in which protological signs and their combinations receive a special meaning and are thus made applicable to a kind of intellectual reality. The following signs are employed here for propositional calculus:

(1) p, q, r, s, t, or v, or any of them with a numeral subscript, signifying *propositional variables.*
(2) N signifying the *monadic propositional operator.*
(3) C, A, K, E, D, I, J, and O signifying the *dyadic propositional operators.*
(4) + and — signifying the *propositional values.*
(5) * and ^{0} signifying *properties of propositional compounds.*

The rules of the formation of propositional units correspond exactly to those of the formation of protological units. A unit in propositional calculus is called a "*well-formed propositional formula*" (here abbreviated as "WFOF"). Accordingly, any single propositional variable (p, q, etc.) is a WFOF and so is any propositional compound formed in accordance with rules governing the use of the operators (Np, Apq, CCpqKpr, etc.).

The propositional variables represent the propositions. For example, in the formula CKpqr, p may represent "*Bona fides* is a fundamental principle of international law", q may represent "*Pacta sunt servanda* is a fundamental principle of all treaty law", and r may represent "The Charter of the United Nations ought to be observed in good faith".

There are special names or locutions in ordinary language which are used for conveying the meaning of the propositional compounds and the operators employed for their formation. Some of these expressions are clumsy, artificial, and even misleading in their literal sense. However, they are employed as technical words in the context of logic; if this is borne in mind it should not be difficult (certainly not for lawyers, who often employ ordinary words in a special legal sense) to get accustomed to them and to avoid misconceptions which would arise if they are taken too literally.

The names adopted for the propositional compounds in this Compendium are the following:

N followed by one unit: "*negation*"
C followed by two units: "*conditional*"
A followed by two units: "*alternation*"
K followed by two units: "*conjunction*"
E followed by two units: "*equivalence*"
D followed by two units: "*comprehendal*"
I followed by two units: "*contralternation*"
J followed by two units: "*contrajunction*"
O followed by two units: "*contravalence*"

A C-compound is also called "extensive implication" (and often, but not aptly, "material implication"); an A-compound is also called "inclusive disjunction" or "weak disjunction"; an E-compound is also called "coimplication" or "biconditional"; a D-compound is also called "intensive implication"; an I-compound is also called "incompatibility"; and an O-compound is also called "exclusive disjunction" or "strong disjunction".

The operators forming the above named compounds may be called "*the operator of negation*", "*the operator of conditional*", "*the operator of alternation*", etc. In an abbreviated way they may be called "*the N-operator*", "*the C-operator*", etc.

The locutions which may be employed to express the propositional operators in ordinary language are the following:

"*It is not the case that* ..."	for N
"*If* ... *then* ..."	for C
"... *or* ..."	for A
"... *and* ..."	for K
"*If and only if* ... *then* ..."	for E
"*Only if* ... *then* ..."	for D
"*Neither* ... *nor* ..."	for I
"*Not both* ... *and* ..."	for J
"*Either* ... *or* ..."	for O

Note that "*it is not the case that*" may be abbreviated as "*not*". In legal parlance, "... *or* ..." is sometimes rendered as "... *and/or* ...". Note especially that logicians sometimes express "... *or* ..." as "*either* ... *or* ...", that is, they employ this locution not for contravalence but for alternation. This is linguistically permissible. Therefore it must be constantly borne in mind that the usage of "*either* ... *or* ..." in this Compendium is a technical usage and should be understood to signify "*either* ... *or* ... *(but not both)*".

The two propositional values (corresponding to the two protological marks) are "*true*" and "*false*". They are called "*the truth-values*" usually and are abbreviated as "T" and "F" respectively. However, it is not necessary to use these abbreviations when the tabular method is applied to propositional logic. It is simpler to employ the plus and minus signs wherever this method is applied in modern logic, keeping in mind that in its different calculi they may have different meanings.

The propositional compound whose ultimate value constellation contains only "true", whatever the values of the variables in the guide-matrices, is called "*tautology*" and is indicated by an asterisk (*) placed before the first operator sign of the compound. The propositional compound whose ultimate value constellation contains only "false" is here called "*dyslogy*" and is indicated by a minute zero (^{0}) placed in the same position. The compound whose ultimate value constellation contains both "true" and "false" is here called "*amphilogy*". The tautologous compounds correspond to the firm compounds, the dyslogous compounds to the loose compounds, and the amphilogous compounds to the pliant compounds in protological calculus. What is here called "dyslogy" is often called "self-contradiction".

It is to be noted that "*and*" in the context of propositional calculus is used so that the sequence of the propositions which it links is irrelevant. Thus for the purposes of this calculus, the expressions "Jones is a trespasser *and* Jones is a burglar" and "Jones is a burglar *and* Jones is a trespasser" are interchangeable. However, if "and" is used in ordinary language so that it allows only one sequence of propositions, for example, "Jones took arsenic and Jones died", it is not employed as a logical word and the expression in which it occurs is not a conjunction in the sense of propositional calculus.

It is also to be noted that in the application of propositional calculus it is irrelevant what the thought content of the propositions employed happens to be and whether there is any meaningful link between the logically connected thoughts at all. For instance, the statement "*If* there is political unrest in China *then* Roman law is not incorporated in English common law" is perfectly acceptable for operations under propositional calculus. "*If and only if* Australia is a federal State *then* Paris is not the capital of England" and "*It is not the case that* Australia is a federal State *or* Paris is not the capital of England *and* Australia is a federal State *or it is not the case that* Paris is not the capital of England" can be used as paradigmata of equivalent propositions, because they are characterised by the same value constellations. These examples show that the names of the propositional compounds and the logical words employed to link propositions with each other are used in a very abstract sense; they are used in a special formal sense. It is therefore appropriate to characterise the concepts of conditional, equivalence, etc. of propositional calculus as "minimum conditional", "minimum equivalence", etc. The task of supplying appropriate material content to propositions is not a task of logic and any nonsense in the above illustrations does not make them illogical in the logical sense of the word "illogical". This important task of rational discourse must be performed by activities other than logical operations. Logical reasoning operating on absurdities can (but need not) lead to absurdities, but this does not in any way reflect on the rationality of this reasoning; it reflects only on the rationality of the material content to which it may be applied.

Although impeccable logical operations, including valid inferences, can be conducted by meaningless or preposterous propositions or their connections, there is no need to do so except for the purpose of disclosing the nature of logical reasoning. The subsequent illustrations are therefore chosen to be such that the propositions and their connections are rational also under criteria other than logic.

The following examples illustrate the simple propositional compounds:

Np: *It is not the case that* Paul is married.

Cpq: *If* Brown is a trespasser *then* he is a tortfeasor.

Apq: This burglary was committed by Black *or* it was committed by White.

Kpq: Peter is a youth *and* he is a delinquent.

Epq: *If and only if* someone is a married man *then* he is a husband.

Dpq: *Only if* a person has committed murder *then* he can be sentenced to life imprisonment.

Ipq: *Neither* Green is a licencee *nor* he is an invitee.

Jpq: *Not both* this conduct is obligatory *and* it is prohibitory.
Opq: *Either* the defendant is an adult *or* he is a minor.

There are certain propositional compounds which are important as *laws of propositional calculus.* The logical requirement which each of them must satisfy is that it is a tautology. It can be easily demonstrated by the tabular method that each formula of the laws specified below is such that its ultimate truth-value constellation (appearing under its first operator) contains only "true" (signified by a +), whatever the values of the propositional variables may be.

For one variable, the laws of propositional calculus include:

The Law of Identity:	*Epp
The Law of Non-contradiction:	*JpNp
The Law of Excluded Middle:	*OpNp
The Law of Double Negation:	*EpNNp
The Laws of Autology:	*EpKpp, *EpApp
The Laws of Negation Elimination:	*ENpIpp, *ENpJpp
Consequentiae Mirabilis:	*CCNppp, *CDpNpp

It may be noted that under the laws of autology, any WFOF can be expressed as a compound of two such WFOFs. Under the laws of negation elimination any negated WFOF can be expressed as either a contralternation or a contrajunction of two of these WFOFs.

For two variables, the laws of propositional calculus include:

The Laws of Commutation:	*EApqAqp, *EKpqKqp, *EEpqEqp
The Laws of Transposition:	*ECpqCNqNp, *EDpqDNqNp
The Laws of Transformation:	*ECpqANpq, *EIpqNApq, *ENKpqANpNq, *EApqNKNpNq
The Laws of Absorption:	*EpApKpq, *EpKpApq, *EpCCpqKpq
The Laws of Adjunction:	*CpCqKpq, *CNpCNqIpq
The Laws of Simplification:	*CKpqp, *CINpqp
The Laws of Addition:	*CpApq, *CNpJpq
The Laws of *Modus Ponens:*	*CKCpqpq, *CKDpqqp
The Laws of *Modus Tollens:*	*CKCpqNqNp, *CKDpqNpNq
The Laws of *Modus Ponendo Tollens:*	*CKOpqpNq, *CKOpqqNp
The Laws of *Modus Tollendo Ponens:*	*CKApqNpq, *CKOpqNpq

It may be noted that under the laws of commutation, all compounds formed by the operators other than *C* and *D* are such that the sequence of the WFOFs which these other operators (*A*, *K*, *E*, *I*, *J*, and *O*) govern can be reversed. Under the laws of transformation, any compound can be expressed by means of an operator or operators which are different from the operator employed in the premiss either with or without reversion of the sequence of the WFOFs which they govern. **EApqNKNpNq* and **EKpqNANpNq* are called "*De Morgan Laws*" and **ECpqANpq* and **EDpqApNq* are called "*duality laws*". Under the laws of simplification, a WFOF can be concluded from certain compounds in which the WFOF occurs.

For three variables, the laws of propositional calculus include:

The Laws of Association:	**EKKpqrKpKqr*, **EAApqrApAqr*
The Laws of Permutation:	**ECpCqrCqCpr*, **EDDpqrDDprq*
The Laws of Distribution:	**EKpAqrAKpqKpr*, **EApKqrKApqApr*
The Laws of Importation:	**ECpCqrCKpqr*, **ECpDqrCKprq*
The Laws of Exportation:	**ECKpqrCpCqr*, **ECKpqrCpDrq*
The Laws of Hypothetic Syllogism:	**CKCpqCqrCpr*, **CKDpqDrpDrq*
The Laws of Simple Constructive Dilemmas:	**CKKCpqCrqAprq*, **CKKCpqCrqOprq*
The Laws of Simple Destructive Dilemmas:	**CKKCpqCprANqNrNp*, **CKKCpqCprONqNrNp*

It may be noted that under the laws of distribution, a repeated WFOF or repeated WFOFs can be eliminated as a result of appropriate placement of operators. Under the laws of association, permutation, importation, and exportation, reshuffling of WFOFs can be effected.

For four variables, the laws of propositional calculus include:

The Laws of Complex Constructive Dilemmas:	**CKKCpqCrsAprAqs*, **CKKCpqCrsOprAqs*
The Laws of Complex Destructive Dilemmas:	**CKKCpqCrsANqNsANpNr*, **CKKCpqCrsONqNsANpNr*

It may be noted that under commutative laws the alternational premisses can be expressed first in the dilemmas and the conditional premisses second, which is often done in the application of dilemmas. Under transformation laws, the compounds *ANqNs* and *ANpNr* can be expressed as *Jqs* and *Jpr* respectively.

The laws of propositional calculus find application in the inferences of this calculus. In a statement form, a *valid* propositional inference appears as a tautologous propositional formula whose first operator sign is C. The second WFOF governed by this sign can be *detached* from the total formula as the conclusion of the inference if the first WFOF (representing the premiss or premisses of the inference) is posited. The conclusion of a valid inference is *logically necessary*. A propositional inference is *invalid* if it appears either as a dyslogous or as an amphilogous formula in its statement form. In the case of dyslogy, the conclusion is *logically repugnant;* in the case of amphilogy it is *logically contingent.*

It is to be noted that if the premiss or conjunction of the premisses has only "false" (represented by the minus sign in the logical computation) in its value constellation, any conclusion can be drawn from it. For a C-compound is true whenever the first WFOF in it has "false" as its truth-value, be the truth-value of the second WFOF "true" or "false". This principle is expressed by the Latin maxim "*ex falso quod libet*". Hence logical falsity is obnoxious to the rational endeavour of inference not because it would make the conclusion impossible but because it makes reasoning licentious.

In addition to the laws of propositional calculus, there are the following operational rules which find application in propositional inferences:

The Rule of Substitution: *In any tautologous formula, any WFOF can be uniformly substituted for any variable there occurring.*

The Rule of Replacement: *In any formula, any WFOF can be replaced with any WFOF having the same value constellation.*

The tenability of these rules becomes manifest in the application of the tabular method which shows that the formulae emerging from these operations have exactly the same value constellation as the formulae on which these operations are performed.

It is feasible to express propositional inferences in ordinary language by using appropriate logical words. For example, a simple *Modus Ponens* inference can be expressed under the law *CKCpqpq as follows:

If, *if* Black is in Melbourne *then* Black is in Australia *and* Black is in Melbourne, *then* Black is in Australia.

It appears that even such a simple inference can be expressed in ordinary language only in a rather clumsy manner. Complex inferences under the laws of propositional calculus when expressed in ordinary language turn out to be forbiddingly involved and virtually unintelligible. This shows that the logical words which ordinary language may provide are crutches to be thrown away when the ability to walk freely in the field of symbolic logic has been acquired.

In order to make convenient use of the inferential possibilities offered by propositional calculus, the following procedure may be adopted:

I. Translate into symbols the premisses in ordinary language after they have been identified and expressed in proper logical form.
II. Select the appropriate rule of inference.
III. By the aid of this rule or these rules, reach the conclusion in the symbolic form.
IV. Translate the conclusion expressed in the symbolic form back into ordinary language.

Suppose that the premisses for an inference are:

If this statute is constitutional *then* this statute is legally valid.
If this statute is legally valid *then* the regulations issued in accordance with it are legally unchallengeable.

These premisses can be translated into symbols as *Cpq* and *Cqr* respectively. The appropriate rule of inference is that of the law of "hypothetic syllogism" (in its conditional version), namely **CKCpqCqrCpr*. Hence the conclusion is *Cpr*, whose ordinary language correspondent is:

If this statute is constitutional *then* the regulations issued in accordance with it are legally unchallengeable.

In statement form, the above inference is **CKCpqCqrCpr*. Presented in argument form, it appears as follows:

$$
\begin{array}{ll}
Cpq & \text{(first premiss)} \\
\underline{Cqr} & \text{(second premiss)} \\
\therefore Cpr & \text{(conclusion by hypothetic syllogism)}
\end{array}
$$

Although truth tables can establish the validity of any propositional inference, they may prove to be complex and cumbersome, and where a great number of entries must be made, the scope of "clerical error" is considerable. For instance, a rather elementary inference (consisting of merely three transformations) under the formula

**ECKNKpqNArsCtvANKANpNqKNrNsANtv*

would require a truth table of 64 rows and 31 columns. To avoid the unwieldiness of the tabular method, other methods have been devised to establish either the validity or invalidity of inferences. Among these is the establishment of the validity of a given inference by *direct proof*, *conditional proof*, or *indirect proof*. The method here involved is, briefly, to select from the foregoing laws of propositional calculus certain elementary ones (each of which can be easily proved by the tabular method) and to set out any given inference in argument form, commencing with the premisses, then stating the conclusion, and finally presenting the proof of validity in which proof each step is justified by reference to a premiss or premisses or a previous step or steps, and by reference to one of the given laws of propositional calculus.

Direct Proof

Name of Law	Law Expressed in Ordinary Language	Law in Argument Form
Modus Ponens (M. P.)	*If* p *then* q; p; *therefore* q	Cpq p / .·.q
Modus Tollens (M. T.)	*If* p *then* q; *not*-q *therefore not*-p	Cpq Nq / .·.Np
Hypothetic Syllogism (H. S.)	*If* p *then* q; *if* q *then* r; *therefore if* p *then* r	Cpq Cqr / .·.Cpr
Modus Tollendo Ponens (M. T. P.)	p *or* q; *not*-p; *therefore* q	Apq Np / .·.q
Complex Constructive Dilemma (C. C. D.)	*If* p *then* q; *if* r *then* s; p *or* r; *therefore* q *or* s	KCpqCrs Apr / .·.Aqs
Complex Destructive Dilemma (C. D. D.)	*If* p *then* q; *if* r *then* s; *not*-q *or not*-s; *therefore not*-p *or not*-r	KCpqCrs ANqNs / .·.ANpNr
Simplification (Simp.)	p *and* q; *therefore* p	Kpq / .·.p
Adjunction (Adj.)	p; q; *therefore* p *and* q	p q / .·.Kpq
Addition (Add.)	p; *therefore* p *or* q	p / .·.Apq

In addition to the above forms of inference, the following equivalences are frequently used for the purposes of proof so that wherever a formula in Column A appears, it can be replaced by the equivalent formula stated in Column B, and wherever a formula in Column B appears, it can be replaced by the equivalent formula stated in Column A.

Name of Equivalence	Column A	Column B
De Morgan	*NKpq*	*ANpNq*
(De M.)	*NApq*	*KNpNq*
Duality for Equivalence	*Epq*	*KCpqCqp*
(Equiv.)	*Epq*	*AKpqKNpNq*
Duality for Conditional	*Cpq*	*ANpq*
(Cond.)		
Commutation	*Kpq*	*Kqp*
(Comm.)	*Apq*	*Aqp*
Transposition	*Cpq*	*CNqNp*
(Trans.)		
Exportation	*CKpqr*	*CpCqr*
(Exp.)		
Double Negation	*p*	*NNp*
(D. N.)		
Autology	*p*	*Kpp*
(Aut.)	*p*	*App*
Association	*KpKqr*	*KKpqr*
(Assoc.)	*ApAqr*	*AApqr*
Distribution	*KpAqr*	*AKpqKpr*
(Dist.)	*ApKqr*	*KApqApr*

It is to be noted that the Rule of Substitution and the Rule of Replacement apply to both the above forms of inference and the above equivalences, so that where any propositional variable occurs above, the inference remains valid or the equivalence remains tautological where any WFOF is substituted consistently for such variable throughout such inference or equivalence. For example, *NKCpqEqNp* is equivalent, under a De Morgan Law, to *ANCpqNEqNp*; again the following is a valid form of *Modus Tollens* inference:

CNEpKqrCqNp
NCqNp / ∴ *NNEpKqr*

The facility of recognising the elementary forms of valid inference and the elementary equivalences is acquired by constant practice. All the above now form part of the basic equipment in dealing with complex inferences.

A simple example of direct proof is given below to illustrate the method involved. Suppose the following inference is required to be proven as valid:

> There is consideration or there is no contract. If either party is a volunteer, there is no consideration. Hence, if there is a contract, neither party can be a volunteer.

The first step is to put the inference into symbols. Using *p* for "there is consideration", *q* for "there is a contract" and *r* for "either party is a volunteer", the inference would be symbolised as follows:

1. *ApNq*
2. *CrNp* / ∴ *CqNr*

The proof would then proceed as follows:

Firstly, the first premiss is equivalent to "*if* q *then* p" by virtue of the equivalence by commutation and duality for conditional. This would be written: 3. *ANqp* 1, Comm. and 4. *Cqp* 3, Cond. The second premiss is equivalent to "*if not-not-*p *then not-r*" by virtue of the equivalence by transposition. This would be written: 5. *CNNpNr* 2, Trans. In this last step, "*not-not-p*" is equivalent to *p*, by virtue of the equivalence by double negation. Thus *NNp* can be replaced by *p*. This would be written: 6. *CpNr* 5, D. N. Now, taking steps 4 ("*if* q *then* p") and 6 ("*if* p *then not-r*"), the conclusion "*if* q *then not-r*" is derivable by means of the elementary inference of hypothetic syllogism. This would be written: 7. *CqNr* 4, 6, H. S., which is the conclusion required to be proved (*QED*), so that the proof is now complete.

The above detailed explanation was to indicate how each step was derived. In practice, after stating the propositions represented by each symbol, the inference and proof would appear simply as follows:

	1. *ApNq*		
	2. *CrNp*	/ ∴ *CqNr*	
Proof:	3. *ANqp*	1, Comm.	
	4. *Cqp*	3, Cond.	
	5. *CNNpNr*	2, Trans.	
	6. *CpNr*	5, D. N.	
	7. *CqNr*	4, 6, H. S.	*QED*

It is to be noted that each step in the above proof is validated by one or more premisses or one or more previous steps, by virtue of an elementary inference or an elementary equivalence. The proof required five simple steps; application of the tabular method would have required eight rows and fourteen columns.

Conditional Proof (abbreviated as C. P.)

Some valid complex inferences which may not lend themselves easily to Direct Proof, can be proved by means of the method of Conditional Proof. This method can be briefly described as follows: Any proposition, simple or complex, may be assumed and used together with any premiss or premisses or any derived steps. The steps involving conditional proof are usually bracketed and when a desired result is ultimately obtained, the bracket is closed, and the next step is expressed as a conditional in which the assumed proposition is the antecedent and the desired result is the consequent. This method is usually helpful where the conclusion of the inference to be proved is a conditional proposition, whereupon the antecedent of the conclusion is assumed as an extra premiss. The following illustration may clarify this procedure. Suppose the following inference is required to be proven as valid:

> If the defence of diminished responsibility is available or the accused can establish provocation, the accused will not need to establish insanity and the charge of murder will be reduced to manslaughter. If the accused cannot establish provocation or his evidence as to insanity is not accepted, then he is guilty of murder. Therefore if the accused will need to establish insanity, then he is guilty of murder.

Using p for "the defence of diminished responsibility is available", q for "the accused can establish provocation", r for "the accused will need to establish insanity, s for "the charge of murder will be reduced to manslaughter", t for "the accused's evidence as to insanity is accepted", and v for "the accused is guilty of murder", the inference and the proof are symbolised as follows:

	1. $CApqKNrs$	
	2. $CANqNtv$	/ ∴ Crv
Proof:	3. r	C. P.
	4. $ArNs$	3, Add.
	5. $NKNrs$	4, De M.
	6. $NApq$	5, 1, M. T.
	7. $KNpNq$	6, De M.
	8. $KNqNp$	7, Comm.
	9. Nq	8, Simp.
	10. $ANqNt$	9, Add.
	11. v	2, 10, M. P.
	12. Crv	3–11, C. P. *QED*

Note how the above differs from ordinary direct proof:

(1) The brackets around Steps 3 to 11, i. e. the steps involving conditional proof.

(2) Step 3, in which the proposition r is assumed; r is the antecedent of the conclusion Crv.

(3) After v is derived in Step 11 (by ordinary direct proof methods in Steps 4 to 11), the bracket is closed as the desired result has been obtained.

(4) Step 12, which is a conditional in which the first step in the conditional proof is the antecedent and the last step in the conditional proof is the consequent.

Note here also that although the proof required ten steps, application of the tabular method would have required twenty columns and sixty four rows. Finally note the following: *No step outside the bracketed steps may refer to any step within the bracketed steps as justification.*

Indirect Proof (abbreviated as I. P.)

To show that a conclusion follows validly from certain premisses, it is permissible to assume the denial of the conclusion as an additional premiss and to derive from all the premisses including this assumed denial any dyslogous (self-contradictory) formula. As soon as this dyslogy has been derived, the conclusion has been established. The rationale for this method is that, by definition, an argument is valid provided that a false conclusion does not follow from true premisses. If the premisses are inconsistent with each other (as is shown by deriving a dyslogy from the premisses) no substitution of truth values can produce true premisses. Since such premisses can never be true, no false conclusion follows from true premisses. Thus, the argument must be valid. It may be here remarked that this rationale also lies behind the statement that any argument with inconsistent premisses is always valid. It is therefore important to determine whether the given premisses in an inference are inconsistent. This can be done by the tabular method by conjoining all the premisses; if a dyslogy is derived, the premisses are inconsistent. The indirect proof method is similar to the conditional proof method and could even be categorised as a special application of the conditional proof method, since in conditional proof, any assumption may be made, while in indirect proof, a particular assumption is made immediately at the beginning of the proof, namely the contradictory of the conclusion to be proved. Suppose it is desired to prove the validity of the following:

> If Dherosia is to be treated as having statehood, then Dherosia is a party to the dispute before the Security Council. If Dherosia is to be treated as having statehood and is a party to the dispute before the Security Council then Dherosia ought to have been invited to participate in the discussions. If Dherosia is not to be treated as having statehood and ought not to have been invited to participate in the discussions, then the Security Council was not in breach of its obligations under the Charter. Hence, the claim that the Security Council was in breach of its obligations under the Charter implies that Dherosia ought to have been invited to participate in the discussions.

Using p for "Dherosia is to be treated as having statehood", q for "Dherosia is a party to the dispute before the Security Council", r for "Dherosia ought to have been invited to participate in the discussions" and s for "the Security Council was in breach of its obligations under the Charter", the inference and the proof would proceed:

	1. *Cpq*	
	2. *CKpqr*	
	3. *CKNpNrNs*	/∴*Csr*
Proof:	4. *NCsr*	I. P. (Indirect Proof)
	5. *NANsr*	4, Cond.
	6. *KsNr*	5, De M.
	7. *s*	6, Simp.
	8. *NKNpNr*	3, 7, M. T.
	9. *Apr*	8, De M.
	10. *CKqpr*	2, Comm.
	11. *CqCpr*	10, Exp.
	12. *CpCpr*	1, 11, H. S.
	13. *CKppr*	12, Exp.
	14. *Cpr*	13, Aut.
	15. *Nr*	6, Comm. and Simp.
	16. *Np*	14, 15, M. T.
	17. *p*	9, 15, M. T. P.
	18. *KpNp*	16, 17, Adj. *QED*

Step 18 is a dyslogy and nothing further is necessary to establish the conclusion. The astute reader will note that the above inference could also have been proved by conditional proof (by the assumption of *s*) or even by direct proof, though the latter would be more involved. Any of the three modes of proof may be utilised in any particular inference, and it will vary from inference to inference which of the three provides the most convenient proof.

Some comments are warranted about the foregoing methods. Firstly, whereas the tabular method is automatic, the various proof methods are not. The determination of the starting point and the order of the steps of proof often require considerable insight and ingenuity. Nevertheless the methods of proof are simpler than the tabular method which may involve hundreds or even thousands of entries. Secondly, these methods only establish validity; they do not establish invalidity. The failure to prove an inference valid does not establish that the inference is invalid; it may happen that the reasoner has not displayed sufficient ingenuity to construct a proof. However, where it has not been possible to construct a formal proof and invalidity is suspected, such invalidity can be tested by the tabular method or by the "short-cut" method of assigning truth-values (as set out in Appendix B).

4. Predicational Calculus

In propositional calculus, the propositional variables p, q, etc. are treated as symbols for unanalysed propositons. However, as it appears from the exposition of traditional logic in Chapter I, propositions have a logically significant internal structure relevant to immediate inferences and syllogistic inferences. This structure is significant also in predicational calculus of modern logic.

Predicational calculus can be viewed as an articulation of propositional calculus, which provides a basic formal framework for it. Thus the propositional operators C, A, K, E, etc. are employed in predicational calculus and use is made of the truth-values "true" and "false". Predicational calculus as presented in this Compendium has the following principal signs:

(1) F, G, and H, or (where convenient) any italicised capital letter, signifying *predicators.*
(2) h, k, and l, or any of them with a numeral subscript, signifying *hypotact-constants* (usually called "individual constants").
(3) x, y, and z, or any of them with a numeral subscript, signifying *hypotact-variables* (usually called "individual variables").
(4) Π signifying *the universal quantifier* (or *the universaliser*) and Σ signifying *the particular quantifier* (or *the particulariser*).

In addition, some further signs will be introduced below, where a need arises for them.

The hypotact-constants and the hypotact-variables constitute two kinds of thought-formations on which a predicator has a bearing as a governing factor. The name common to a hypotact-constant and a hypotact-variable is "*hypotact*". The predicators are divided into *monadic predicators* and *polyadic predicators.* The latter are divided into *dyadic, triadic,* etc. *predicators.* This division is based on the number of hypotacts which a predicator governs. Only the logical structures formed by monadic and dyadic predicators will be considered in this Compendium.

The specific formulae of predicational calculus are *predicational formulae* and *quantification formulae.* The thought-formations for which the former stand may be called "*predications*". The thought-formations for which the latter stand may be called "*quantification indications*".

A predicational formula is a *well-formed predicational formula*, in short, a WFEF, if

(*a*) *a predicator sign is immediately followed by one or more hypotact signs* (e. g. Fh, Gx, Rxy, $Qxyz$), or
(*b*) *exactly one WFEF is immediately preceded by an* N (e. g. NFx, $NNGx$, $NRxy$), or
(*c*) *exactly two WFEFs are immediately preceded by the sign of one dyadic propositional operator* (e. g. $CFxGx$, $AFhRxy$, $KRxyNQkz$), or
(*d*) *an appropriate quantification formula precedes a WFEF which contains at least one hypotact-variable sign occurring in the former and the same contains no such sign which does not occur in the latter* (e. g. ΠxFx, $\Pi x\Sigma yRxy$, $\Sigma x\Pi yAFxRxy$).

A quantification formula is appropriate in relation to a well-formed predicational formula if

(*a*) *either one* Π *or one* Σ *immediately precedes at least one hypotact-variable sign* (e. g. Πx, Σxy, $\Pi x\Sigma y$, $\Sigma x\Pi yz$), and
(*b*) *it contains only such hypotact-variable signs which are contained in the WFEF to which it relates*, and
(*c*) *it contains no such hypotact-variable sign which has already been quantified in the WFEF to which it relates.*

According to the above stated rules, for example, ΠxFx, $\Pi xRxy$, $\Pi xyQxy$, $\Sigma xyCRxyKFxGy$, and $\Pi x\Sigma yCRyzFx$ are WFEFs whereas, for example, $Fx\Pi x$, $x\Sigma yQxy$, $\Pi xy\Sigma xQy$, $ARxyKF\Sigma xyGy$, and $ERRx\Pi yy\Sigma Fx$ are not.

A quantifier applies only to the next succeeding WFEF. It is to be noted that the occurrence of a sign of a hypotact-constant or a sign of a hypotact-variable not contained in the quantification formula does not break the application of the quantifier. For example, in the formula $\Pi xCKRxyFhQzx$ the quantifier applies to x not only in Rxy but also in Qzx. It is also to be noted that alternative placements of quantification formulae are possible. Such different placements may affect the meaning of the predications for which the formulae stand.

Variables to which a quantifier applies are called "*bound variables*" whereas variables to which no quantifier applies are called "*free variables*". Predicational formulae in which no free variables occur are called "*closed formulae*" whereas predicational formulae in which at least one free variable occurs are called "*open formulae*". It is to be noted that quantifiers are not applied to hypotact-constants, of course.

The actual use which can be made of the formalistic devices and of the specific concepts of predicational calculus will be first examined in connection with monadic predicators. For this purpose various linguistic utterances will be provided by way of illustration.

The formulae such as Fh, Gk, Hl, etc. stand for singular propositions such as "London is a city", "Your argument is unsound", "This plea fails", "The testimony of the witness Brown seems to be most unreliable", and the like. These propositions can be analysed into hypotact locutions ("London", "your argument", "this plea", "the testimony of the witness Brown") and into predicator locutions ("is a city", "is unsound", "fails", and "seems to be most unreliable"). The above illustrations show that h, k, etc. stand for concepts which designate something unique (that is, their designatum is an individual or concrete entity) whereas, F, G, etc. stand for a property possessed by such entities. This property can be rendered in various ways: by locutions containing a verb and a noun, a single verb, a verb and an adjective, and a verb combined with various other parts of speech. It is to be noted that in the symbolic expression, the English grammatical order of the hypotact and predicator locutions is reversed.

The formulae such as Fx, Gy, Hz, etc. stand for propositional schemata such as ". . . is a trustee", ". . . does not apply", ". . . cannot be performed", and the like. These illustrations indicate that the signs of hypotact-variables stand for a mere logical vacancy, a place where a concept of which a property is predicated may appear. The above formulae, being instances of simple open formulae, do not represent any propositions but only "dummies" of propositions which can be developed into propositions if the vacancies in them indicated by the leaders are filled with appropriate concepts. Thus the open predicational formulae have no truth-value and they cannot be treated truth-functionally. These schemata can be developed into formulae representing propositions if their hypotact-variables are quantified.

The quantification formulae Πx and Σx can be rendered in ordinary language as "For all x" (or "For whatever x") and "For some x" (or "There is an x such that") respectively. In this Compendium only the first alternatives will be employed. They represent elliptic propositions which can be spelt out as follows: "For all x, what is stated hereafter about x holds" and "For some x, what is stated hereafter about x holds".

Thus they can be treated truth-functionally and, accordingly, be negated. The negation of quantification formulae yields the following equivalences:

$$E\Pi xNFxN\Sigma xFx$$
$$E\Sigma xNFxN\Pi xFx$$
$$E\Pi xFxN\Sigma xNFx$$
$$E\Sigma xFxN\Pi xNFx$$

These equivalences show that (1) it is not necessary to negate the quantification formulae, because for each negated quantification formula an expression is available in which the formula governed by a quantifier is negated instead; (2) the number of the quantifiers can be reduced to one, because for each formula in which a quantifier appears another equivalent formula is available in which only the other quantifier appears. Such a reduction is an important theoretical possibility. However, in actual application of predicational calculus employment of both quantifiers proves to be convenient. It is to be noted that the equivalences set out above are often used in direct proof methods employing *quantification negation* (abbreviated as "Q. N").

The quantified formulae such as ΠxFx, ΣxGx, $\Pi xNHx$, and $\Sigma xNIx$ stand for propositions such as "For all x, x is something", "For some x, x is a trespasser", "For all x, *it is not the case that* x is a criminal" and "For some x, *it is not the case that* x is nothing whatsoever". The illustrations which can be provided for ΣxFx and for $\Pi xNIx$ are rather artificial and only of theoretical interest. However, in complex predicational formulae arising from the use of propositional operators both kinds of quantification find significant application. This will appear from the following illustrations:

(1) $\Pi xCFxGx$: "For all x, *if* x is a trespasser *then* x is a tortfeasor".

(2) $\Pi xCFxNHx$: "For all x, *if* x is a trespasser *then it is not the case that* x is an invitee".

(3) $\Sigma xKIxJx$: "For some x, x is a lawyer *and* x is a public servant".

(4) $\Sigma xKIxNJx$: "For some x, x is a lawyer *and it is not the case that* x is a public servant".

These illustrations suggest that (1), (2), (3), and (4) are similar to the four propositional forms of traditional logic. Thus the illustration for (1) can be expressed as "*All* trespassers *are* tortfeasors", for (2) as "*No* trespassers *are* invitees", for (3) as "*Some* lawyers *are* public servants", and for (4) as "*Some* lawyers *are not* public servants" (i.e. as S *a* P, S *e* P, S *i* P, and S *o* P respectively).

Translation of the four propositional forms of traditional logic into the above predicational formulae is, however, possible only if the universal propositions are conceived as not having existential import. The assumption adopted in this Compendium according to which all terms in the propositions of traditional logic are referential (or instantiated) excludes the formulae (1) and (2) as appropriate correspondents of S *a* P and S *e* P. Their admissible translations would be $KK\Sigma xFx\Sigma xGx\Pi xCFxGx$ and $KK\Sigma xFx\Sigma xGx\Pi xCFxNGx$ respectively.

The second formula provides an occasion to show that the placement of quantification formulae affects the meaning of the thought-formations for which they stand. If an S *e* P proposition were rendered by the formula $K\Sigma xKFxGx\Pi xCFxNGx$ and if Fx were to stand for "is guilty of this crime" and Gx were to stand for "is innocent of this crime", the first part of the formula would stand for "For some x, x is guilty of this crime *and* x is innocent of this crime". Σx placed immediately before Fx and repeated immediately before Gx produces an expression ($K\Sigma xFx\Sigma xGx$) which would stand for "For some x, x is guilty of this crime *and* for some x, x is innocent of this crime" which is a different (and not an absurd) proposition.

Any predicational formula which contains a sign of a hypotact-constant and shares the predicator sign with another predicational formula which contains a sign of a hypotact-variable is a *substitution instance* for such a formula. Thus Fb, Fk, etc. are substitution instances for Fx. The following rules state the logical links between the predicational formulae containing signs of hypotact-constants and the predicational formulae having the same predicator signs but containing signs of hypotact-variables:

Universal Instantiation

From any predicational formula having a sign of a hypotact-variable quantified by the universaliser, which is or can be placed before the first operator-sign, it is valid to infer any substitution instance of it.

Particular Generalisation

From any predicational formula containing a sign of a hypotact-constant, it is valid to infer any predicational formula which has the same predicator sign *and whose sign of the hypotact-variable governed by it is quantified by the particulariser.*

Universal Generalisation

From any predicational formula containing a sign of an arbitrarily selected hypotact-constant, it is valid to infer any predicational formula which has the same predicator sign governing the sign of a hypotact-variable quantified by the universaliser.

Particular Instantiation

From any predicational formula whose hypotact-variable is quantified by the particulariser, it is valid to infer any substitution instance containing the sign of an arbitrary hypotact-constant which has no prior occurrence in the same logical expression.

Given "For all x, *if* x is a trespasser *then* x is a tortfeasor" as the premiss, it follows by way of universal instantiation that "*If* Black is a trespasser *then* Black is a tortfeasor". The form of this inference is

$$C\Pi xCFxGxCFbGb$$

Given "The Charter of the United Nations is a treaty" as the premiss, it follows by way of particular generalisation that "For some x, x is a treaty". The form of this inference is

$$CHk\Sigma xHx$$

Given "*If* the will made by the late John Smith is not signed by the testator *then* it is null and void" as the premiss, it follows by way of universal generalisation, provided that "the will made by the late John Smith" is an arbitrarily selected occurrence, that "For all x, *if* x is a will not signed by the testator *then* x is null and void". The form of this inference is

$$CC I\varkappa J\varkappa\Pi xCIxJx$$

(where $\varkappa$ is introduced as a new symbol to signify any arbitrarily selected hypotact-constant)

Given "For some x, x is an association *and* x is a legal personality" as the premiss, it follows by way of particular instantiation that there is an entity which has "association" and "legal personality" among its properties, provided that this entity is indeterminate and is not previously referred to in the context of the logical operations in which the conclusion is inferred.

The form of this inference is

$$C\Sigma xKLxMxKL\tau M\tau$$

(where τ is introduced as a new symbol to signify an arbitrary hypotact-constant having no prior occurrence in the same logical expression)

There are thought-formations which require for their adequate logical expression in predicational calculus more than two signs of hypotact-constants or hypotact-variables. For example, "Negotiations precede the conclusion of all peace treaties" and "Paris is south of London" are propositions which are to be rendered each as a combination of a predicator sign with two hypotact signs. These propositions can be conceived as dyadic predications whose symbolic expression is $C\Pi xyKFxGy\Sigma x\Pi yPxy$ and Shk respectively. Thought-formations like these are usually called "*relations*". Their predicators are called "*relators*" and their hypotacts "*terms*". The first hypotact in a dyadic relation may be called "*the foreterm*" and the second hypotact "*the afterterm*". The relations which have the same relators (whether or not they have also a common term) may be called "*equipredicative relations*". It may be noted that dyadic relational propositions would find more natural symbolic expressions in English if the relator were placed between the two hypotacts. However, this is not done in the present Compendium because the notation here adopted would produce in that case too unwieldy formulae which would also require use of brackets.

Relations have various logically significant properties of their own, the most important of which will be discussed below. The illustrations which will be provided first will contain only hypotact-constants, because the absence of quantification makes it possible to grasp more easily the logical ideas involved.

Consider relations such as (1) "Jane *is the wife of* Jack" and (2) "Jack *is the husband of* Jane". They are examples of *converse* relations: having different relators but the same terms in reversed sequence, they refer to the same state of affairs. If the original relation is expressed in symbols as Rhk, its converse relation may be expressed as $\breve{R}kh$, where $\smile$ on the top of R indicates that the relator for which the $\breve{R}$ sign stands belongs to a relation which is converse to the relation constituted by the relator for which the R sign stands.

A relation is converse with respect to another relation if this other relation is such that it refers to the same state of affairs but by a different relator and by reversed sequence of the terms.

Consider relations such as (1) "Black *is a partner of* White", (2) "Mason *is wealthier than* Taylor", and (3) "Edwards *is an agent of* Richards". The first relation is such that its terms can be reversed without changing the relator, the resulting relation still referring to the same state of affairs. The second relation is such that this reversion produces a relation which does not hold if the original relation holds. The third relation is such that depending on circumstances (in the given instance on specific legal arrangements between Edwards and Richards), this reversion produces a relation which on some occasions holds and on other occasions does not hold. The above examples illustrate *symmetric*, *asymmetric*, and *parasymmetric* (usually but not aptly called "non-symmetric) relations respectively.

A relation is symmetric where the equipredicative relation with reversed sequence of its terms always holds. A relation is asymmetric if the other equipredicative relation with reversed sequence of its terms never holds. A relation is parasymmetric if, depending on circumstances, the other equipredicative relation with reversed sequence of its terms either holds or does not hold.

Consider relations such as (1) "Cooper *has the same nationality as* Smith", (2) "Blackacre *is larger than* Whiteacre", and (3) "Green *has contractual relations to* Brown". The first relation is such that each term of it relates to itself in the same way as it does to the other term (for obviously Cooper has the same nationality as Cooper and Smith has the same nationality as Smith), whereas the second relation is such that this is quite out of the question. The third relation is such that depending on circumstances either the former or the latter is the case. The above three relations exemplify *reflexive*, *irreflexive*, and *parareflexive* (usually but not aptly called "non-reflexive") relations respectively.

A relation is reflexive if the equipredicative relations connecting the foreterm with itself and the afterterm with itself always hold. A relation is irreflexive if the equipredicative relations connecting the foreterm with itself and the afterterm with itself never hold. A relation is parareflexive if, depending on circumstances, the equipredicative relations connecting the foreterm with itself and the afterterm with itself either hold or do not hold.

Consider relations such as (1) "White *is a relative of* Black" and "Black *is a relative of* Grey", (2) "Paul *is the father of* Peter" and "Peter *is the father of* John", and (3) "Hill *is a representative of* Mill" and "Mill *is a representative of* Till". As to (1), it must hold that "White *is a relative of* Grey". As to (2), it cannot hold that "Paul *is the father of* John" (it holds that "Paul *is the grandfather of* John"). As to (3), it depends on circumstances (namely on specific legal arrangements between the persons involved) whether or not "Hill *is a representative of* Till". The above three relations exemplify *transitive*, *intransitive*, and *paratransitive* (usually but not aptly called "non-transitive") relations.

> A relation is transitive if in conjunction with another equipredicative relation sharing one differently placed term with it a further equipredicative relation holds whose terms are the same and occupy the same positions as in the first and second relations. If such a further equipredicative relation never holds, the relation is intransitive. If, depending on circumstances, such a further equipredicative relation either holds or does not hold, the relation is paratransitive.

Of the interconnections between the above discussed kinds of relations, the following may be mentioned:

(1) A symmetric relation may or may not be reflexive and it may or may not be transitive.
(2) A transitive relation may or may not be symmetric and it may or may not be reflexive.
(3) An intransitive relation may or may not be symmetric and it is not reflexive.
(4) Any reflexive relation is symmetric and transitive.
(5) Any asymmetric relation is irreflexive.
(6) Any transitive and irreflexive relation is asymmetric.

Any relational formula containing only hypotact-constant signs represents a proposition (as does any monadic predicational formula containing only such a sign or such signs). Therefore such relational formulae can be linked with the signs of propositional operators and they can be used as formulae of premisses of propositional inferences. For example, from "*If* Peter *was born before* Paul *then* Paul *is younger than* Peter" and "Peter *was born before* Paul", it follows by *Modus Ponens* that "Paul *is younger than* Peter". The form of this inference is *CKCBhkYkhBhkYkh*.

Besides, relations have their own inferential potentialities, of which the most important is that resulting from transitivity.

Since relations are predications, they are governed by the rules of quantification if their terms are hypotact-variables. In order to have a closed relational formula, all variable signs must be governed in it by an appropriate quantification formula; otherwise the relational formula is not a WFEF or is an open formula (having one or several free variables). Quantification formulae characteristic of relational formulae are Σxy, Πxy, $\Sigma x\Pi y$, Σxyz, $\Pi x\Sigma yz$, etc.

It is possible to give symbolic expression to the generalisations of the above discussed principles of relationality as follows:

Symmetry:	$\Pi xy\mathrm{E}RxyRyx$
Asymmetry:	$\Pi xy\mathrm{C}Rxy\mathrm{N}Ryx$
Parasymmetry:	$\mathrm{K}\Sigma xy\mathrm{K}RxyRyx\Sigma xy\mathrm{K}Rxy\mathrm{N}Ryx$
Reflexivity:	$\Pi xRxx$
Irreflexivity:	$\Pi x\mathrm{N}Rxx$
Parareflexivity:	$\mathrm{K}\Sigma xRxx\Sigma x\mathrm{N}Rxx$
Transitivity:	$\Pi xyz\mathrm{CK}RxyRyzRxz$
Intransitivity:	$\Pi xyz\mathrm{CK}RxyRyz\mathrm{N}Rxz$
Paratransitivity:	$\mathrm{K}\Sigma xyz\mathrm{CK}RxyRyzRxz\Sigma xyz\mathrm{CK}RxyRyz$-$\mathrm{N}Rxz$
Conversity:	$\Pi xy\mathrm{E}Rxy\breve{R}yx$
Inconversity:	$\Pi xy\mathrm{E}Rxy\mathrm{N}\breve{R}yx$
Paraconversity:	$\mathrm{K}\Sigma xy\mathrm{K}Rxy\breve{R}yx\Sigma xy\mathrm{K}Rxy\mathrm{N}\breve{R}yx$

Since the hypotact-variable signs are signs which stand for a logical vacancy, the formula $\Pi xySxy$ would represent, for example, "For all x and all y, x is south of y", which leaves completely open what x and what y may mean, and the formula $\Pi x\Sigma yOxy$ would represent, for example, "For all x and some y, x ought to do y". Such indefinite expressions are of little practical value. It is therefore requisite to limit the range of application of the signs of the terms in relational formulae. One way of doing this is by prefixing the relational formulae containing hypotact-variables by appropriate limitative formulae, for example by the formula $\Pi xy\mathrm{K}FxGy$.

Accordingly, a formula such as $\Pi xy\mathrm{CK}FxGySxy$ would represent, for example, "For all x and all y, *if* x is an Italian city *and* y is a German city *then* x is south of y" (i.e. "All Italian cities are south of all German cities"). A formula such as $\Pi xCFx\Sigma yCGyOxy$ would represent, for example. "For all x, *if* x is an owner of a motor vehicle *then* for some y, *if* y is a registration fee *then* x ought to pay y" (i.e. "Every motor vehicle owner ought to pay a registration fee").

The use of the quantifiers in connection with relations can further be illustrated by an attempt to give a special interpretation to the four propositional forms of traditional logic. For this purpose, the following assignments are made: x to the predicate (in the sense of traditional logic), y to the subject, and P to the relator "*is predicated of*". The four propositional forms can now be expressed by the following relational formulae:

(1) S *a* P: $C\Pi xyKFxGy\Sigma x\Pi yPxy$
(2) S *e* P: $\Pi xyCKFxGyNPxy$
(3) S *i* P: $C\Pi xyKFxGy\Sigma xyPxy$
(4) S *o* P: $C\Pi xyKFxGy\Pi x\Sigma yNPxy$

These formulae can be illustrated by the following examples:

(1) *If* for all x and all y, x is a tortfeasor *and* y is a trespasser *then* for some x and all y, x is predicated of y.
(2) For all x and all y, *if* x is an invitee *and* y is a trespasser *then it is not the case that* x is predicated of y.
(3) *If for* all x and all y, x is a lawyer *and* y is a public servant *then* for some x and some y, x is predicated of y.
(4) *If* for all x and all y, x is a lawyer *and* y is a public servant *then* for all x and some y, *it is not the case that* x is predicated of y.

This way of expressing the traditional four propositional forms makes it possible to construct relational inferences by recourse to the principle of transitivity. If the symbol z is assigned to the middle term, *Modus Barbara* can be expressed as follows:

$$C\Pi xyzKKFxGyHzCK\Sigma x\Pi yPxy\Sigma y\Pi zPyz\Sigma x\Pi zPxz$$

It is to be noted that for making use of the principle of transitivity in order to express syllogistic inferences by means of relations, it is necessary to reduce all valid moods of these inferences to the moods of the First Figure. The relevant procedure was discussed in Ch. I, § 3 and will be further discussed in § 5 of the present chapter.

Some examples will now be given to show how direct, conditional, or indirect proof methods may be used in predicational calculus. The additional abbreviations here used are U. I. for "Universal Instantiation", P. G. for "Particular Generalisation", U. G. for "Universal Generalisation", P. I. for "Particular Instantiation", and Q. N. for "Quantification Negation".

I. Suppose that it is desired to prove the validity of the following argument:

> All barristers and judges are qualified legally and appreciate the the usefulness of logical training for legal reasoning. Therefore all barristers appreciate the usefulness of logical training for legal reasoning.

Using the symbols

Bx to represent "x is a barrister",
Jx to represent "x is a judge",
Qx to represent "x is qualified legally",
Ux to represent "x appreciates the usefulness of logical training for legal reasoning",

the argument and the proof therefor may be symbolised as follows:

1. $\Pi xCABxJxKQxUx$ / ∴ $\Pi xCBxUx$

2. Bx	C. P.	
3. $ABxJx$	2, Add.	
4. $CABxJxKQxUx$	1, U. I.	
5. $KQxUx$	3, 4, M. P.	
6. $KUxQx$	5, Comm.	
7. Ux	6, Simp.	
8. $CBxUx$	2–7, C. P.	
9. $\Pi xCBxUx$	8, U. G.	*QED*

II. Suppose that it is desired to prove the validity of the following argument:

> All business expenses are allowable taxation deductions. Some repairs are business expenses. Therefore some repairs are allowable taxation deductions.

Using the symbols

Bx to represent "*x* is a business expense",
Tx to represent "*x* is an allowable taxation deduction",
Rx to represent "*x* is a repair",

the argument and the proof therefor may be symbolised as follows:

1. $\Pi xCBxTx$		
2. $\Sigma xKRxBx$	/ ∴ $\Sigma xKRxTx$	
3. $KR\tau B\tau$	2, P. I.	
4. $KB\tau R\tau$	3, Comm.	
5. $B\tau$	4, Simp.	
6. $CB\tau T\tau$	1, U. I.	
7. $T\tau$	5, 6, M. P.	
8. $R\tau$	3, Simp.	
9. $KR\tau T\tau$	8, 7, Adj.	
10. $\Sigma xKRxTx$	9, P. G.	*QED*

III. Suppose that it is desired to prove the validity of the following argument:

Anybody who shot the deceased would have been detected. Anybody who would have detected the accused would have recognised him. Anybody who would have recognised the accused would have notified the police. The police were not notified. Therefore the accused did not shoot the deceased.

Using the symbols

k to represent "the accused",
Sx to represent "*x* shot the deceased",
Dxy to represent "*x* detected *y*",
Rxy to represent "*x* recognised *y*",
Px to represent "*x* notified the police",

the argument and the proof therefor may be symbolised as follows:

1. $\Pi xCSx\Sigma yDyx$	
2. $\Pi xCDxkRxk$	
3. $\Pi xCRxkPx$	
4. $N\Sigma xPx$	/ ∴ NSk

5. *CRlkPl*	3, U. I.
6. *ΠxNPx*	4, Q. N.
7. *NPl*	6, U. I.
8. *NRlk*	5, 7, M. T.
9. *CDlkRlk*	2, U. I.
10. *NDlk*	8, 9, M. T.
11. *ΠyNDyk*	10, U. G.
12. *CSkΣyDyk*	1, U. I.
13. *NΣyDyk*	11, Q. N.
14. *NSk*	12, 13, M. T. *QED*

IV. Suppose that it is desired to prove the validity of the following argument:

> Siblings are not permitted to marry. Tom is a sibling of Kate. Harry is a sibling of Tom. Therefore Harry and Kate are not permitted to marry.

Using the symbols

h to represent "Harry",
k to represent "Kate",
t to represent "Tom",
Mxy to represent "*x* is permitted to marry *y*",
Sxy to represent "*x* is a sibling of *y*",

and assuming that "is a sibling of" is a relator which constitutes transitive and symmetric relations, the argument and the proof therefor may be symbolised as follows:

	1. *ΠxyCSxyNMxy*	
	2. *Stk*	
	3. *Sht*	
Transitivity:	(4. *ΠxyzCKSxySyzSxz*)	
Symmetry:	(5. *ΠxyESxySyx*)	/ ∴ *KNMhkNMkh*
	6. *CShkNMhk*	1, U. I.
	7. *CKShtStkShk*	4, U. I.
	8. *KShtStk*	3, 2, Adj.
	9. *Shk*	7, 8, M. P.
	10. *EShkSkh*	5, U. I.
	11. *KCShkSkhCSkhShk*	10, Equiv.
	12. *CShkSkh*	11, Simp.

13. *Skb*	12, 9, M. P.
14. C*Skb*N*Mkb*	1, U. I.
15. N*Mkb*	14, 13, M. P.
16. N*Mbk*	6, 9, M. P.
17. KN*Mbk*N*Mkb*	16, 15, Adj. *QED*

5. Extensional Calculus

Besides the method discussed in connection with predicational calculus, there is a further way of logical treatment of properties which lies in making use of the concept of classes. A notable advantage of this method of organising thought is that it lends itself to diagrammatic representations offering a visual aid for apprehending logical connections. A class can be conceived of as an extension determined by a predicator, in other words, it can be conceived of as an entity range characterised by a certain property or by certain properties. In this Compendium only those classes which are determined by monadic predicators will be considered.

The link between classes and predications is that a class results from a predication by substituting for its predicator the indication of the range of entities for which the predication holds. Supposing that the formula *Fk* stands for the predication "Paul is a minor", *F* stands for the property "minor" characterising the range of entities of which Paul is a member. If the symbol a is assigned to the range of entities called "minors", *Fk* can be rendered as *Rᵋk*a, where the symbol *Rᵋ* (which may be called "the epsilon relator") stands for "*is a member of*". Thus the predication "Paul is a minor" can be rendered as "Paul *is a member of* the class 'minors'".

Suppose that the monadic predicational formula *Π*x*CFxGx* stands for the predication "For all *x*, *if x* is a trespasser *then x* is a tortfeasor". In this formula, *F* stands for the property "trespasser" characterising the range of entities of which *x* is a member and *G* stands for the property "tortfeasor" characterising the range of entities of which *x* is a member. If the symbol **a** is assigned to the class "trespassers" and the symbol **e** is assigned to the class "tortfeasors", *ΠxCFxGx* can be rendered by the dyadic predicational formula *ΠxCRᵋx***a***Rᵋx***e**, i.e. by a relational formula. Apart from the logical connections existing between relations (interpreted as propositions) which have for their terms class members and classes, there are parallel logical connections between the relevant classes. Thus the formula **Cae** represents extensional inclusion and signifies, for example, that the class "trespassers" is included in the class "tortfeasors".

There is a certain superficial similarity but also an important logical difference between class membership and extensional inclusion relations. This difference is that extensional inclusion is a transitive relation whereas class membership is not. For example, from "The class 'murders' *is included in* the class 'crimes'" and "The class 'crimes' *is included in* the class 'illegal acts'", it follows that "The class 'murders' *is included in* the class 'illegal acts'". In contrast, from "Jack the Ripper *is a member of* the class 'murderers'" and "the class 'murderers' *is a a member of* the class 'legally significant thought-formations'", it does not follow that "Jack the Ripper *is a member of* the class 'legally significant thought-formations'", for Jack is an entity which existed as a human being and not as a thought-formation.

It is to be noted that classes, too, can be members of classes, as it appears from the above illustration. This is so because, by their external aspect, classes are "conceptual singularities" and can therefore be treated as individual entities for certain purposes. It is also to be noted that there are classes which have only one member, for example, "the class consisting of the range of entities identical with Jack the Ripper". Accordingly, in appropriate contexts, it is possible to have extensional inclusion relations in which such classes occur. Thus from "The class 'Jack the Ripper' *is included in* the class 'murderers'" and "The class 'murderers' *is included in* the class 'criminals'", it follows under the principle of transitivity that "The class 'Jack the Ripper' is *included in* the class 'criminals'".

Although there are essential differences between propositions and classes, logical connections between classes are parallel to those between propositions. Therefore an extensional calculus can be established which employs operator signs similar to the signs of the propositional operators and a method of logical computation can be provided which is similar to the one applied to propositional units. The difference between the symbols used in extensional calculus and the symbols used in propositional calculus is here expressed by employing a different type style for the formulae containing expressions of class units and also by employing vowel letters for signifying class variables (in contrast to consonant letters employed for signifying propositional variables).

The following signs are employed here for extensional calculus:

(1) **a**, **e**, **i**, **o**, and **u**, or any of them with a numeral subscript, signifying *class variables.*

(2) **N** signifying the *monadic extensional operator.*

(3) **C**, **A**, **K**, **E**, **D**, **I**, **J**, and **O** signifying the *dyadic extensional operators.*

(4) + and — signifying the *extensional values.*

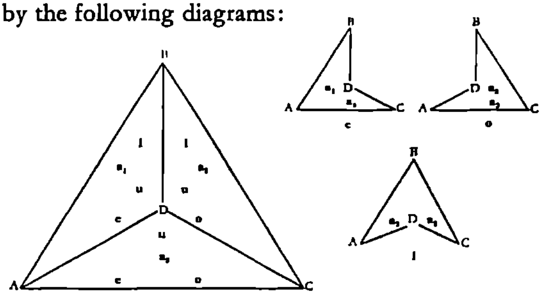
(5) * and ^{0} signifying *extensional tautology* and *extensional dyslogy* respectively.

The rules of the formation of extensional units correspond exactly to those of the formation of protological units. A unit in extensional calculus may be called a "*well-formed class formula*" (abbreviated here as "WFAF"). Accordingly, a single class variable (a, e, etc.) is a WFAF, and so is any extensional compound formed in accordance with the rules of the use of the operators.

As in propositional calculus any WFOF represents a proposition so in extensional calculus every WFAF represents an extensional state of affairs. Thus any class variable represents an extensional state of affairs and so does any extensional compound. The two extensional values are "*existent*" and "*non-existent*". In the tabular method applied to extensional states of affairs, plus (+) and minus (—) signs are employed for these values respectively bearing in mind that these signs have a special meaning in extensional calculus.

The extensional compounds whose ultimate value constellation contains only "existent", whatever the values of the variables in the guide-matrices, is here called "*extensional tautology*" (or simply "tautology") and is indicated by an asterisk placed before the first operator sign of the compound. Those compounds whose ultimate value constellation contains only "non-existent" is here called "*extensional dyslogy*" (or simply "dyslogy") and is indicated by a minute zero placed before the first operator sign of the compound. The compounds whose ultimate value constellation contains both "existent" and "non-existent" is here called "*extensional amphilogy*" (or simply "amphilogy").

Various extensional units and logical connections between them can be represented by the following diagrams:



In the above diagrams, Figure ABD represents the class $\mathbf{a_1}$, Figure DBC the class $\mathbf{a_2}$, and Figure ADC the class $\mathbf{a_3}$; Figure ABDC represents the class **e**, Figure ABCD the class **i**, and Figure ADBC the class **o**. The total triangle ABC represents the class **u**, which is the universal class (or the extensional universe) here.

The diagrams help to apprehend the following:

(1) $\mathbf{a_1}$ together with **o**, $\mathbf{a_2}$ together with **e**, and $\mathbf{a_3}$ together with **i** exhaust the extensional universe. This means that they are complementary to each other within **u**. *Extensional complementarity* thus diagrammatically represented can be expressed in symbols as $\mathbf{Na_1}$, $\mathbf{Na_2}$, $\mathbf{Na_3}$, **Ne**, **Ni**, and **No**.

(2) $\mathbf{a_1}$ and $\mathbf{a_3}$ are included in **e**, $\mathbf{a_1}$ and $\mathbf{a_2}$ are included in **i**, $\mathbf{a_2}$ and $\mathbf{a_3}$ are included in **o**, and each of these classes is included in **u**. *Extensional inclusion* thus diagrammatically represented can be expressed in symbols as $\mathbf{Ca_1e}$, $\mathbf{Ca_3e}$, $\mathbf{Ca_1i}$, $\mathbf{Ca_2i}$, $\mathbf{Ca_2o}$, $\mathbf{Ca_3o}$, **Ceu**, **Ciu**, **Cou**, $\mathbf{Ca_1u}$, $\mathbf{Ca_2u}$, and $\mathbf{Ca_3u}$.

(3) **e** results as an extensional "sum" from the union of $\mathbf{a_1}$ and $\mathbf{a_3}$, **i** results as the same from the union of $\mathbf{a_1}$ and $\mathbf{a_2}$, **o** results as the same from the union of $\mathbf{a_2}$ and $\mathbf{a_3}$, and **u** results as the same from the union of **e** and **i**, **i** and **o**, or **o** and **e**. *Extensional union* thus diagrammatically represented can be expressed in symbols as $\mathbf{Aa_1a_3}$, $\mathbf{Aa_1a_2}$, $\mathbf{Aa_2a_3}$, **Aei**, **Aio**, and **Aoe**.

(4) $\mathbf{a_1}$ results as an extensional "product" from the intersection of **e** and **i**, $\mathbf{a_2}$ results as the same from the intersection of **i** and **o**, and $\mathbf{a_3}$ results as the same from the intersection of **e** and **o**. *Extensional intersection* thus diagrammatically represented can be expressed in symbols as **Kei**, **Kio**, and **Keo**.

(5) $\mathbf{a_1}$ is coextensive with **Kei**, $\mathbf{a_2}$ is coextensive with **Kio**, and $\mathbf{a_3}$ is coextensive with **Keo**. *Extensional parity* (usually called "class identity") thus diagrammatically represented can be expressed in symbols as $\mathbf{Ea_1Kei}$, $\mathbf{Ea_2Kio}$, and $\mathbf{Ea_3Keo}$.

(6) **e** encloses $\mathbf{a_1}$ and $\mathbf{a_3}$, **i** encloses $\mathbf{a_1}$ and $\mathbf{a_2}$, **o** encloses $\mathbf{a_2}$ and $\mathbf{a_3}$, and **u** encloses each of these classes. *Extensional enclosure* thus diagrammatically represented can be expressed in symbols as $\mathbf{Dea_1}$, $\mathbf{Dea_3}$, $\mathbf{Dia_1}$, $\mathbf{Dia_2}$, $\mathbf{Doa_2}$, $\mathbf{Doa_3}$, **Due** etc., and $\mathbf{Dua_1}$ etc.

(7) $\mathbf{a_1}$ results as an extensional "remainder" from the disunion of $\mathbf{a_2}$ and $\mathbf{a_3}$, $\mathbf{a_2}$ results as the same from the disunion of $\mathbf{a_1}$ and $\mathbf{a_3}$, and $\mathbf{a_3}$ results as the same from the disunion of $\mathbf{a_1}$ and $\mathbf{a_2}$. *Extensional disunion* thus diagrammatically represented can be expressed in symbols as $\mathbf{Ia_2a_3}$, $\mathbf{Ia_1a_3}$, and $\mathbf{Ia_1a_2}$.

(8) $\mathbf{a_1}$, $\mathbf{a_2}$, and $\mathbf{a_3}$ are "dissected" in the sense that they are separate classes and that any two of them do not exhaust the extensional universe (**u**). *Extensional dissection* (usually called "class exclusion") thus diagrammatically represented can be expressed in symbols as $\mathbf{Ja_1a_2}$, $\mathbf{Ja_1a_3}$, and $\mathbf{Ja_2a_3}$.

(9) $\mathbf{a_1}$ and **o**, $\mathbf{a_2}$ and **e**, and $\mathbf{a_3}$ and **i** are complementary classes. Any of them is "disparate" from its complementary class in the sense that they are separate classes and that together they exhaust the extensional universe (**u**). *Extensional disparity* thus diagrammatically represented can be expressed in symbols as $\mathbf{Oa_1o}$, $\mathbf{Oa_2e}$, and $\mathbf{Oa_3i}$.

The above presented extensional compounds have logical properties which can be expressed by means of a tabular method based on principles of protological calculus as follows:

a	Na
+	−
−	+

a	e	Cae	Aae	Kae	Eae	Dae	Iae	Jae	Oae
+	+	+	+	+	+	+	−	−	−
+	−	−	+	−	−	+	−	+	+
−	+	+	+	−	−	−	−	+	+
−	−	+	−	−	+	+	+	+	−

Because propositional calculus and extensional calculus have the same structure and hence there is a parallelism between propositional compounds and extensional compounds, it is possible to express the laws of logic for extensional calculus which correspond to those of propositional calculus. Accordingly, there are extensional laws of commutation, distribution, duality, etc. There is no need to specify them here anew. To express them by formulae, all that is necessary to do is to substitute the signs of extensional calculus for the corresponding signs of propositional calculus. It is to be noted that there is a class whose scope is absolutely universal (enclosing everything whatsoever) and there is a class which is absolutely void. The latter arises under principles of extensional calculus, for dyslogous class compounds (e. g. **KaNa**) represent a class which has no members whatsoever.

Given an extensional universe and its constitution which specifies the relations of its classes to each other, it is possible to draw logical consequences from extensional states of affairs. This procedure is parallel to the procedure of propositional inference, and it may be called "*extensional derivation*".

It is possible to express the four propositional forms of traditional logic as propositions about *definite* classes, i. e. classes which are neither empty nor have everything whatsoever as their members. This qualification corresponds to assumptions on which the system of traditional logic presented in this Compendium is based. To express that a class is a definite class, a dot is placed here on the top of any letter signifying such a class. To express the relations between the relevant classes, the symbol *C* will be employed in the pertinent formulae to signify the relator "*inheres in*". The four propositional forms can thus be expressed as follows:

S *a* P: *C*ȧė; S *e* P: *C*ȧNė; S *i* P: *NC*ȧNė; S *o* P: *NC*ȧė

They can be read as "ȧ inheres in ė", "ȧ inheres in the complement of ė", "It is not the case that ȧ inheres in the complement of ė", and "It is not the case that ȧ inheres in ė", respectively.

The following principles are of specific significance for the logical treatment of definite classes:

Subalternation (Subalt.):	*CC*ȧė*NC*ȧNė
Full Contraposition (F. Contrap.):	*EC*ȧė*C*NėNȧ
Inversion (Inv.):	*CC*ȧė*NC*Nȧė
Transitivity (Transit.):	*CKC*iė*C*u̇i*C*u̇ė
Double Complementarity (D. C.):	ENNȧȧ

Note that in the present context the letter "i", which already has a dot, is treated as a sign of a definite class.

Apart from these, other relevant principles of the calculi of modern logic find application for this treatment and the rule of substitution in the following formulation:

In any formula representing a law of logic, any WFAF can be uniformly substituted for any class variable.

The formula for the principle of transitivity represents *Modus Barbara* of traditional logic, i standing for the middle term, ė for the predicate term, and u̇ for the subject term. The formula for this principle validates also *Modus Celarent*, which can be derived from *Modus Barbara* by replacing the expression ė with the expression Nė. Thus the following formula represents *Modus Celarent*:

CKCiNėCu̇iCu̇Nė

Modus Darii, expressed by the formula CKCiėNCu̇NiNCu̇Nė, can be validated in the procedure of indirect proof (I. P.) as follows:

Premisses:	1. Ciė		
	2. NCu̇Ni	/ ∴ NCu̇Nė	
Proof:	3. Cu̇Nė	I. P.	
	4. CNėNi	1, F. Contrap.	
	5. Cu̇Ni	3, 4, Transit.	
	6. KNCu̇NiCu̇Ni	2, 5, Adj.	*QED*

By reaching the last formula, the aim of the proof has been reached, because it represents a dyslogy. Since a proposition which is contradictory to the proposition representing the conclusion of *Modus Darii* proves to be logically repugnant, the above method of *reductio ad absurdum* has established the validity of the inference in question.

The formula for *Modus Ferio* can be derived from the formula for *Modus Darii* by substituting Nė for ė. After the elimination of double complementarity which arises through this substitution, the formula for *Modus Ferio* is the following:

CKCiNėNCu̇NiNCu̇ė.

As was shown in the treatment of the syllogistic inferences of traditional logic, all moods of the other syllogistic figures can be reduced to the moods of the first figure. By employing the appropriate reductions, they can therefore be shown to be valid also when expressed as formulae representing propositions about relations between definite classes.

How the methods of proof presented above in connection with propositional and predicational calculi can be utilised to demonstrate validity of further syllogistic inferences expressed in the form discussed in the present section may be illustrated by a *Modus Baroco* syllogism:

All unjustifiable homicides *are* crimes	(*C*ėi)
Some killings *are not* crimes	(*NC*ùi)
Some killings *are not* unjustifiable homicides	(*NC*ùė)

In the statement form, this argument appears as *CKC*ėi*NC*ùi*NC*ùė.

By direct proof, it can be shown to be valid as follows:

1. *C*ėi	
2. *NC*ùi	/ ∴ *NC*ùė
3. *C*NiNė	1, F. Contrap.
4. *NNC*ùNi	2, Subalt.
5. *C*ùNi	4, D. N.
6. *KC*NiNė*C*ùNi	3, 5, Adj.
7. *C*ùNė	6, Transit.
8. *NNC*ùNė	7, D. N.
9. *NC*ùė	8, Subalt. *QED*

By indirect proof (I. P.), the argument can be shown to be valid as follows:

1. *C*ėi	
2. *NC*ùi	/ ∴ *NC*ùė
3. *C*ùė	I. P.
4. *KC*ėi*C*ùė	1, 3, Adj.
5. *C*ùi	4, Transit.
6. *KC*ùi*NC*ùi	5, 2, Adj. *QED*

Step 6 represents a dyslogous formula. By having reached such a formula, the aim of indirect proof has been attained.

Chapter III: Modern Logic in the Legal Universe of Discourse

1. *Logical Structure of the Legal Norm*

A legal norm is a thought-formation directed to a person or persons and containing a legally authoritative stipulation concerning an instance or instances of behaviour. "Every partner may take part in the management of the partnership business" is a legal provision representing a rather simple legal norm. In order to subject it to logical treatment, a slightly modified (and unavoidably unnatural) expression of it is required: "Every partner may carry out taking part in the management of the partnership business." Now the following components can be distinguished in this norm: the concepts of *the norm-subject*, *the norm-object*, and *the norm-nexus*.

The norm-subject is any entity whose behaviour a legal norm regulates (e. g. "every partner"). The norm-object is any instance of behaviour regulated by a legal norm (e. g. "taking part in the management of the partnership business"). The norm-nexus links the norm-subject and the norm-object into a norm-unity (e. g. "may carry out"). One of the essential elements of the concept of the norm-nexus is *the performatory factor* appearing either as "*carry out*" or "*refrain from*", by which reference is made to legally relevant behaviour.

There are different kinds of the norm-nexus and different ways of expressing each of them. In English legal provisions, the modal verb "shall" is often used for saying that a person has a duty to do something (e. g. "The defendant shall file a Notice of Grounds of Defence within ten days after service of summons upon him", "The Commissioner shall deliver to the parties copies of any statement he submits to the Court"). For saying that a person has a right to do something, the modal verb "may" is often used (e. g. "The Institute may recover any sum of money payable to it", "The deliverer may treat demand of delivery as ineffectual unless made at a reasonable hour"). It is to be noted that legal provisions are often expressed in passive voice; however, these lend themselves easily to conversion into active voice, which is more amenable to logical treatment.

Whatever the linguistic expression of the norm-nexus, it is to be understood that in the context of law these expressions are not used to signify a thought of a fact but of a fiat. It is to be noted that the same expressions can be employed also for speaking *about* law. In this case, the expressions are in normative *meta-language*, which is used, for example, in a juristic account of contents of a statute and in juristic reasoning relating to law. In contrast, a statute itself must always be understood as being in normative *object-language*, i. e. in the language of law itself, in which expressions of the norm-nexus are employed as having *normative force* or *normative import.*

It seems that thought-formations expressed in normative object language require logical values other than truth-values for their logical treatment. This, however, would lead to an area of logical theory which has not yet been sufficiently tested and possibly to some misadventures of exposition. To avoid intellectual hazards here involved, norms will be treated as propositions, namely as propositions having normative import, in the subsequent discussions. Since legal provisions usually occur in indicative mood (or where they occur in imperative mood they can be translated into indicative mood), this way of handling them is feasible. It is to be understood, however, that "truth" and "falsity" as applied to a norm are words which do not have their ordinary epistemological sense. They do not mean truth and falsity by virtue of actual facts but by virtue of legal fiats. Such an extension of the meaning of these words is plausible in logical contexts because in logic "truth" and "falsity" are employed in a special logical sense.

In order to lend a stronger emphasis to the duty to be performed, the modal verb "must" is often employed in expressions of the norm-nexus. The difference in meaning between "shall" and "must" is irrelevant for logical purposes. In the subsequent exposition the expression "ought to" will be employed to cover both and to contrast them with "may". It is to be noted that in expressions of legal provisions "may not" occurs in the sense of "shall not".

For logical treatment, legal norms can be conceived as dyadic relations in which the concepts of a norm-subject and a norm-object appear as two terms and the concept of the norm-nexus appears as the relator linking these terms. There are *four varieties of the norm-nexus*, which by means of modal verbs and of the corresponding symbols can be expressed as follows:

"... *ought to carry out* ..." (O^c)

"... *ought to refrain from* ..." (O^r)

"... *may carry out* ..." (M^c)

"... *may refrain from* ..." (M^r)

These relators belong to relations which can be treated as propositions and thus the principles and methods of propositional calculus can be brought to bear on them. The relations in question are asymmetric, irreflexive, and intransitive.

The four varieties of the norm-nexus can also be expressed by means of the words "duty" and "right" together with the expression of the performatory factor. Thus "*ought to carry out*" can be expressed as "*has the duty to carry out*", "*ought to refrain from*" as "*has the duty to refrain from*", "*may carry out*" as "*has the right to carry out*", and "*may refrain from*" as "*has the right to refrain from*". These expressions can be symbolised as D^c, D^r, R^c, and R^r respectively. The corresponding relations have, of course, the same logical properties as their counterpart ought- and may-relations.

A legal norm in which both the subject and the object are signified generally may be called a "*general-abstract norm*". For example, "Persons whose annual income is over $ 1000 ought to carry out submitting tax returns". The logical form of this norm is $\Pi x \Sigma y CKFxGyO^cxy$. General-abstract norms whose logical form is such that either both terms are quantified by the universaliser, or both terms are quantified by the particulariser, or the foreterm is quantified by the particulariser and the afterterm is quantified by the universaliser are also possible.

A legal norm in which the subject is signified generally and the object is signified individually may be called a "*general-concrete norm*". For example, "Any person who captures Billy the Kid may carry out receiving the reward of $ 5000". The logical form of this norm is $\Pi xCFxM^cxh$. General-concrete norms whose logical form is such that the foreterm is quantified by the particulariser are also possible.

A legal norm in which the subject is signified individually and the object is signified generally may be called a "*singular-abstract norm*". For example, "The innkeeper Jane Green may refrain from selling liquor to drunken men". The logical form of this norm is $\Pi yCGyM^rky$. Singular-abstract norms whose logical form is such that the afterterm is quantified by the universaliser are also possible.

A legal norm in which both the subject and the object are signified individually may be called a "*singular-concrete norm*". For example, "Robert Wretch may carry out leaving the Long Bay Prison on December 24, 1969 from 10 a. m. to 10 p. m.". The logical form of this norm is M^ckl.

Besides the above considered ways, there is a further way of expressing legal norms for the purposes of logical treatment by employing deontic adjectives. An advantage of this method is that the normative states of affairs thus expressed lend themselves to a convenient diagrammatic representation because an extensional interpretation can be given to them. By means of deontic adjectives, the meaning of the four varieties of the norm-nexus can be expressed as follows:

"... *is obligatory conduct-to-be-carried-out*"
"... *is obligatory conduct-to-be-refrained-from*"
"... *is licensory conduct-to-be-carried-out*"
"... *is licensory conduct-to-be-refrained-from*"

In the following exposition, the norm-subject and the norm-object — be they signified generally or individually — are considered as constant and "bracketed out" for the purposes of analysis, which is concentrated on the concepts of conduct-to-be-carried-out and conduct-to-be-refrained-from (corresponding to the two performatory factors) and on the deontic concepts qualifying them ("obligatory", "licensory", and others later to be introduced). The norms will be framed as monadic predications whose hypotacts stand for a person's behaviour (e. g. "Black's repairing the roof of Sydney Town Hall", "Every motorist's not exceeding the speed limits indicated by the road signs").

In the deontic universe, the denotation of the concept "conduct" covers the entire relevant extensional universe. The letter **u** is assigned to it here and treated as a *deontic class symbol.* The class "conduct" encloses the classes "conduct-to-be-carried-out" and "conduct-to-be-refrained-from" symbolised as **a** and **o** respectively, both of which are treated as further such symbols. The logical connections between **u**, **a**, and **o** are **Dua**, **Duo**, **Aao**, and **EuAao**. Their relations can be diagrammatically represented as follows:

Diagram I

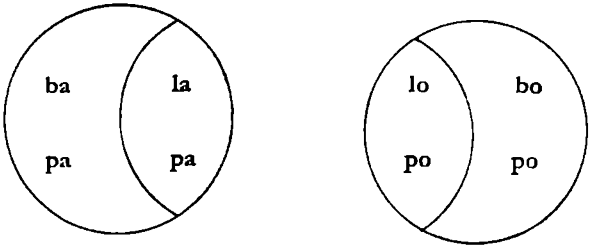
u	**u**	**u**
a	**a**	
	o	**o**

In order to form further classes within the deontic universe, *deontic functors* are employed, of which the following are introduced at this stage: "*obligatory*" (**b**), "*permissory*" (**p**), and "*licensory*" (**l**). These are connected with the concepts of the above specified deontic classes (**u**, **a**, and **o**), which may be called "*deontic hypotacts*" by placing their symbols immediately before the symbols of the latter. Thus **bu** ("obligatory conduct"), **pa** ("permissory conduct-to-be-carried-out"), **lo** ("licensory conduct-to-be-refrained-from"), etc. are formed representing *deontic modalities*.

It is possible to conceive of legal systems in which the modalities **bu** and **lu** are complementary classes within the given legal universe. Such systems are *closed legal systems*, because in them a legal norm corresponds to some or other deontic modality and the whole legal universe is filled with such deontic modalities. The principle which assures the normative plenitude of a closed legal system is "*the sealing legal principle*", according to which any instance of conduct is either obligatory or licensory (**Obulu**). The alternative formulations of the same principle are: Any instance of conduct is either obligatory conduct-to-be-carried-out or permissory conduct-to-be-refrained-from (**Obapo**) or any instance of conduct is either obligatory conduct-to-be-refrained-from or permissory conduct-to-be-carried-out (**Obopa**). For the closed legal systems, the sealing principle is usually framed as *the residual negative legal principle* in the words "Whatever is legally not prohibited is legally permitted". It could also be framed in the words "Whatever is legally not permitted is legally prohibited". It is to be noted that formulation of these principles so that "not" appears before "legally" is unfeasible. This becomes clear if it is said that "Whatever is not legally permitted is legally prohibited", which statement is quite absurd, because legal experience shows that absence of a legal permission does not necessarily mean that there is a legal prohibition. However, "Whatever is not legally prohibited is legally permitted" is a plausible statement and could pass as a loose formulation of the first version of the residual negative legal principle. It is further to be noted that this first version is of greater practical significance, for in any developed legal system all prohibitions are formulated in legal provisions whereas many permissions are not.

The relations between **ba**, **la**, and **pa** on the one hand and **bo**, **lo**, and **po** on the other can be diagrammatically represented as follows:

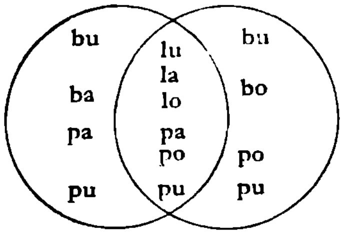
Diagram II



From this diagram it can be seen that **ba** and **la** are complementary classes within the total class **pa,** and **lo** and **bo** are complementary classes within the total class **po.**

The whole deontic system representing a closed legal system can be expressed by the following diagram:

Diagram III



This diagram shows that a closed legal system has the following logical features:

(1) **Dpubu, Dpulu, Dpupa, Dpupo, Dpuba, Dpubo, Dpula, Dpulo; Dpaba, Dpala, Dpalo, Dpalu; Dpobo, Dpolo, Dpola, Dpolu.**

(2) **Apapo.**

(3) **Jbabo, Jlaba, Jloba, Jluba, Jlabo, Jlobo, Jlubo, Jlubu.**

(4) **Cbupu, Clupu, Cpapu, Cpopu, Cbapu, Cbopu, Clapu, Clopu; Cbapa, Clapa, Clopa, Clupa; Cbopo, Clopo, Clapo, Clupo.**

(5) **Obapo, Obopa, Olabu, Olobu, Olubu.**

(6) **Elalo, Elalu, Elolu.**

(7) **ElaKpapo, EloKpapo, EluKpapo.**

(8) **EpuApapo.**

It is possible to conceive of legal systems which are not governed by the sealing principle. This means that permissory conduct does not exhaust the legal universe but only a part of it. The remaining part is occupied by *neutral conduct* (**nu**), which has extensional parity with *neutral conduct-to-be-carried-out* (**na**) and with *neutral conduct-to-be-refrained-from* (**no**). A legal system which is thus constituted may be called an "*open legal system*", because the modality which is complementary to "obligatory conduct-to-be-carried-out" includes not only that for which a norm is present but also that for which a norm is absent, that is, neutral conduct (the same holds for "obligatory conduct-to-be-refrained-from"). **pa** together with **na** constitute what may be called "*allowable conduct-to-be-carried-out*" (**wa**) and **po** together with **no** constitute what may be called "*allowable conduct-to-be-refrained-from*" (**wo**). Their extensional union represents *allowable conduct* (**wu**), which is the modality representing the total class of legally significant conduct. It is to be noted that "legal significance" is not to be understood to mean what is legally regulated but what is of legal concern, i.e. pertinent to legal discourse.

An open legal system has the following specific logical features:

Enano, Enanu, Enonu, Onupu, Obawo, Obowa, Jnubu, Jnulu, EwuAwawo.

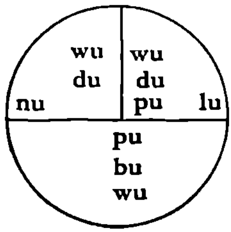
Licensory conduct and neutral conduct have in common that both of them are neither obligatory conduct-to-be-carried-out nor obligatory conduct-to-be-refrained-from. The modality representing this deontic state of affairs is *indifferent conduct* (**du**), its corresponding submodalities being *indifferent conduct-to-be-carried-out* (**da**) and *indifferent conduct-to-be-refrained-from* (**do**). Obligatory conduct-to-be-carried-out and obligatory conduct-to-be-refrained-from have in common that both are submodalities of *obligatory conduct* (**bu**).

There are the following principal intermodal relations between **du** and other deontic modalities:

Eduda, Edudo, Odubu, Jduba, Jdubo, Ddulu, Ddunu, Edulbabo.

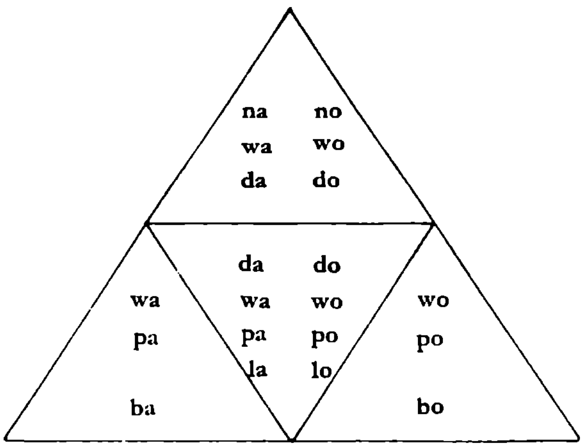
The following diagram represents a deontic system containing all modalities formed by the above introduced deontic functors and by the deontic hypotact **u** only:

Diagram IV



In this diagram the submodalities of **bu** (which are legally most important) could not be represented because only **u** was employed in it as a deontic hypotact. The whole deontic system adequately articulated representing an open legal system can be expressed by the following diagram, in which **a** and **o** are employed as deontic hypotacts:

Diagram V



The number of the deontic concepts encountered so far can be reduced in view of extensional parities which exist in the deontic systems. Thus in the deontic system representing the structure of a closed legal system the following eliminations can be effected: **u** because **EuAao**; **lo** because **Elola**; **la** because **ElaKpapo**; **pa** because **EpaNbo**; **po** because

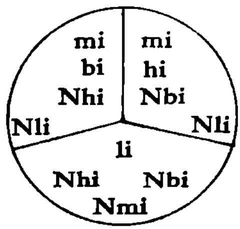
EpoNba. After these eliminations it proves that this structure can be expressed by means of one deontic functor (**b**) and of two deontic hypotacts (**a** and **o**), and of appropriate extensional operators. The principal deontic modalities of this system are: **ba, bo, Nba, Nbo,** and **Ibabo.** All intermodal relations in this system can be derived from the axiom **Jbabo**.

In the deontic system representing the structure of an open legal system the following eliminations can be effected: **u** because **EuAao**; **do** because **Edoda**; **da** because **EdaKwawo**; **wa** because **EwaNbo**; **wo** because **EwoNba**; **pa** because **EpaInabo**; **po** because **EpoInaba**; **no** because **Enona**; **lo** because **Elola**; **la** because **ElaIAnababo**. After these eliminations, this structure can be expressed by means of two deontic functors (**b** and **n**), of two deontic hypotacts, (**a** and **o**), and of appropriate extensional operators. The principal deontic modalities of this system are: **ba, bo, na, Nba, Nbo, Nna,** and **IAnababo**. All intermodal relations in this system can be derived from the axioms **Jbabo, Jbana**, and **Jbona**.

By employing "*prohibitory*" as a further deontic functor and "*incidence*" as the only deontic hypotact, it is possible to construct one-hypotact deontic systems representing the structure of any closed and any open legal system in an adequately articulated manner. If the symbol **h** is assigned to "prohibitory" and the symbol **i** to "incidence", the formulae **bi** and **hi** can be constructed standing for "*obligatory incidence*" and "*prohibitory incidence*" respectively.

In the one-hypotact deontic system representing a closed legal system, the extensional disunion of **bi** and **hi** constitutes *licensory incidence*. The extensional disparity of both constitutes *mandatory incidence*. This system can be expressed by the following diagram:

Diagram VI



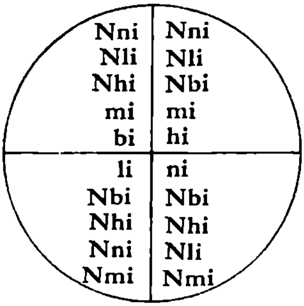
The following formulae express the principal deontic states of affairs in the above system:

Jbihi, Jbili, Jhili, CbiNhi, CbiNli, ChiNbi, ChiNli, CliNbi, CliNhi, Cbimi, Chimi, EliNmi, EliIbihi, EmiNli, EmiAbihi.

Because of the extensional parities stated above, the concepts of mandatory conduct and licensory conduct can be eliminated here. All intermodal relations in this system can be derived from the axiom **Jbihi**.

In the one-hypotact deontic system representing an open legal system, "*neutral incidence*" appears as a specific modality. The formula expressing it is **ni**. This system can be expressed by the following diagram:

Diagram VII



The following formulae express the principal deontic states of affairs in the above system:

Jbihi, Jbili, Jhili, Jmili, Jbini, Jhini, Jnili, Jnimi, Cbimi, Chimi, CbiNhi, CbiNli, CbiNni, ChiNbi, ChiNli, ChiNni, CliNbi, CliNhi, CliNni, CliNmi, CniNbi, CniNhi, CniNli, CniNmi, EliIAnibihi, EmiIniIAnibihi.

Because of the extensional parities stated above, the concepts of mandatory incidence and licensory incidence can be eliminated here. All intermodal relations in this system can be derived from the axioms **Jbihi, Jbini,** and **Jhini**.

The advantage of the one-hypotact deontic systems is that the norms corresponding to the appropriate deontic states of affairs can be given relatively simple symbolic expressions. Thus the singular-concrete norm corresponding to **bi** can be expressed simply as *Bhk* (to read: "*b is* *obligatory for k*"), the singular-concrete norm corresponding to **hi** as *Hhk* (to read: "*h is prohibitory for k*"), the singular-concrete norm corresponding to **li** as *Lhk* (to read: "*h is licensory for k*"), the singular-concrete norm corresponding to **Nbi** as *NBhk* (to read: "*It is not the case that h is obligatory for k*"), and the singular-concrete norm corresponding to **Nhi** as *NHhk* (to read: "*It is not the case that h is prohibitory for k*"). It is to be noted that *NBhk* and *NHhk* represent univocally norms only in a closed legal system; applied to an open legal system, they represent a state of affairs which encompasses also absence of law.

The disadvantage of the one-hypotact deontic systems is that the appropriate deontic modalities cannot find expression in the corresponding norms by means of the concepts "ought to" and "may" or "duty" and "right". The use of these current legal concepts requires the concepts of the performatory factors "carry out" and "refrain from", which have no correspondents in the norms whose symbolic expression is *Bhk*, *Hhk*, etc.

The above expounded systems provide an intellectual framework for thoughts *about* normative systems as well as for organising thoughts contained *in* normative systems. Apart from legal systems, deontic concepts are significant also for moral, aesthetic, and other normative systems. It may be mentioned that deontic modalities have counterparts in ontic modalities (e. g. "necessary event", "impossible event", and "contingent event"). It may also be mentioned that ontic and deontic modal concepts can be combined, which gives rise to modalities such as "necessarily obligatory incidence", "contingently licensory incidence", etc.

2. *Logical Expression of Some Legal Structures*

Legal orders are among those normative systems whose structure is characterised by hierarchic relations between their norms. Thus any legal order contains "higher" and "lower" norms in the sense that a norm, to be considered valid, must satisfy the conditions of validity laid down by a higher valid norm. The latter in its turn must satisfy the conditions of validity laid down by a still higher valid norm, etc., until the ultimate validating norm of the system is reached. The status of this norm as a norm of the given legal order is not determined by another validating norm but by the desideratum of a unitary intellectual organisation of the contents of the given legal system leading to its construction from the data of relevant legal experience by way of postulation.

The number of the levels of validity and their interrelations differ in different legal systems; thus there are differences between unitary States' legal orders and federal States' legal orders. To offer a rough schema by way of illustration for the present purposes, it may be said that under the ultimate validating norm, the norms issued by the constitutive authority of the State S are of the highest level of validity of its legal order, the norms issued by its principal legislative authority are of the next-lower level, the norms issued by its subordinate legislative authorities are of the next-next-lower level, and the norms issued by its administrative authorities or by private persons or by its judicial authorities finally applying the last category of norms are of the lowest level. Accordingly, there are norms of levels I, II, III, and IV with respect to their validity.

It may be mentioned that the division of norms into general-abstract, general-concrete, singular-abstract, and singular-concrete norms is not necessarily linked with the validity levels of the norms. However, the norms issued by the constitutive or legislative authorities are, as a rule, general norms. Whether a norm results from custom, statute, treaty, or judicial precedent is irrelevant to the determination of its validity level. Every normative system has a *constitution in the material sense*; this may be embodied in a constitutional instrument or instruments only, in customary law expressions only, or in customary law expressions and in a constitutional instrument or instruments.

The constitution in the material sense contains norms which lay down the validity-conditions for all subordinate norms of the relevant legal system; that is, it contains the norms of Level-I. The principal role of these is to set up, to specify the functions, and to determine the competence of the organs which create norms of Level-II.

The logical relations between norms of different validity-levels can be stated in the form of transitive relations in which these norms appear as terms. In order to give symbolic expression to such relations here, the assignment of the following symbols is made: n^1 to any norm of Level-I, n^2 to any norm of Level-II, n^3 to any norm of Level-III, and n^4 to any norm of Level-IV; the symbol V is assigned to the relator "*derives its validity from*", which links any two of them as terms into a relation.

The symbols n^1, n^2, etc. stand for hypotact-variables; however, it is to be noted that in contrast to x, y, etc. (employed in this Compendium as hypotact-variable signs having unlimited range of application), they are hypotact-variable signs *having a limited range of application*. What this range is in each case in which they are employed follows from a statement by which their universe of reference is determined. In the present case, the range of application of the symbols in question is the legal norms of a hypothetical legal order whose hierarchic structure corresponds to the above rough schema of the validity-levels. After these preliminaries, the validity relations holding in the case in which norms of all four levels are involved can be expressed by the following formula:

$$KK\Pi n^4 \Sigma n^3 V n^4 n^3 \Pi n^3 \Sigma n^2 V n^3 n^2 \Pi n^2 \Sigma n^1 V n^2 n^1$$

Under the principle of transitivity, the following formulae can be concluded from the above formula:

$$\Pi n^4 \Sigma n^2 V n^4 n^2$$
$$\Pi n^3 \Sigma n^1 V n^3 n^1$$
$$\Pi n^4 \Sigma n^1 V n^4 n^1$$

These formulae express the legal state of affairs that all norms of any lower level of validity derive their validity from some norm of a higher validity level and, finally (for practical purposes), from some norm of the highest level of validity in the given legal system (i.e. from the constitution). This means, for example, that any judicial decision must not only be in accordance with relevant regulations (provided that there are such applicable to the given judicial decision) or statutes (with the same proviso) but it must also be constitutional.

There are legal structures in which norms are linked with each other so that one norm stipulates what ought to be done in case another norm is infringed. The former may be called a "*sanctioning norm*" and symbolised as n^ν; the latter may be called a "*sanctioned norm*" and symbolised as n^σ. The connection between a sanctioned norm and its corresponding sanctioning norm can be expressed by propositional operators applying to relations formed by the relators "*is infringed by*" (I) and "*ought to be applied by*" (A). The foreterm of the first relation so formed is "sanctioned norm" and its afterterm is "the subject of the sanctioned norm" (s^σ); the foreterm of the second relation so formed is "sanctioning norm" and its afterterm is "subject of the sanctioning norm" (s^ν). The logical link between the sanctioned and the sanctioning norms in a hypothetical legal order can now be expressed as follows:

$$C\Pi n^\sigma s^\sigma I n^\sigma s^\sigma \Sigma n^\nu s^\nu A n^\nu s^\nu,$$

where the symbols for the terms appear as hypotact-variables with a limited range of application.

The above formula expresses the legal state of affairs that in a given legal order infringement of any sanctioned norm by its subject is a sufficient condition for the duty to apply a sanctioning norm by a subject of the latter.

Under any legal system there arise various legally significant relations between different norm-subjects. In order to express some of these relations, the following relators and their corresponding symbols may be employed: (1) "*has a duty towards*" ($\overset{+}{D}$), signifying the presence of a duty; (2) "*has not a duty towards*" ($\bar{D}$), signifying the absence of a duty; (3) "*has a right against*" ($\overset{+}{R}$), signifying the presence of a right; and "*has not a right against*" ($\bar{R}$), signifying the absence of a right. Thus there are relations and their symbolic expressions such as:

Black *has a duty towards* White: $\overset{+}{D}hk$
Black *has not a duty towards* White: $\bar{D}hk$
Black *has a right against* White: $\overset{+}{R}hk$
Black *has not a right against* White: $\bar{R}hk$

The application of such formulae may be illustrated by the following legally significant situations:

There is a treaty of mutual assistance between two States (*h* and *k*). The treaty imports norms which stipulate that either State has a duty to give military assistance to the other State in case of an armed attack by a third State. It also imports norms which stipulate that either State has a right to have military assistance given to it by the other State. In this situation there is, firstly, a conjunction of symmetric relations, which can be symbolised as follows:

$$KE\overset{+}{D}hk\overset{+}{D}khE\overset{+}{R}hk\overset{+}{R}kh$$

Secondly, in the same situation there is a conjunction of converse relations (in which the duty-relation specified above has for its converse the right-relation specified above and *vice versa*), which can be symbolised as follows:

$$KE\overset{+}{D}hk\overset{+}{R}khE\overset{+}{R}hk\overset{+}{D}kh$$

A State (*h*) has acquired the status of a neutralised State by pledging to another State (*k*) that it will not enter into any military alliance.

Hence with respect to this matter, the promisor State has a duty towards the promisee State but the latter does not have the same duty towards the former. The promisee State has a right against the promisor State in this matter but the latter does not have the same right against the former. This situation can be symbolised as follows:

$$KK\overset{+}{D}bk\bar{D}kbK\overset{+}{R}kb\bar{R}bk$$

From this formula it follows that also the formula $K\overset{+}{D}bk\overset{+}{R}kb$ holds.

There is no treaty of mutual assistance between two States. Moreover there is no other norm of international law which would impose a duty on one State to assist the other or would confer a right on one State to have this assistance from the other. This situation can be symbolised as follows:

$$KK\bar{D}bk\bar{D}kbK\bar{R}bk\bar{R}kb$$

From this formula it follows that also the formula $KK\bar{D}bk\bar{R}kbK\bar{D}kb\bar{R}bk$ holds. In the above situation there are symmetric absence-of-duty-relations and symmetric absence-of-right-relations as well as absence-of-duty-relations with converse absence-of-right-relations.

In discharge of a debt, White (*b*) has entered into an agreement with Grey (*k*) according to which he has a duty to pay Black (*l*) $1000 annually until Black comes of age. In this situation White has a duty towards Grey with respect to this payment and Grey has a right against White with respect to this payment, but neither has such a duty towards Black. As to Black, he has no such right against either. The whole situation can be symbolised as follows:

$$KKKKK\overset{+}{D}bk\overset{+}{R}kb\bar{D}bl\bar{D}kl\bar{R}lb\bar{R}lk$$

In the above illustrations various right- or duty-relations between individual norm-subjects were considered and logical expressions of these relations were supplied. In order to express those relations between norm-subjects of which at least one is signified generally, appropriate quantification formulae must be employed. These will appear in the symbolic expressions of the situations described below, in which the hypotact-variables will be again ones with a limited range of application.

There is a law which prohibits defamation. It imposes a duty on any person (r_1) to abstain from defaming any other person (r_2). At the same time it confers a right on any person not to be defamed by any other person. This situation can be symbolised as follows:

$$\Pi r_1 r_2 K \overset{+}{D} r_1 r_2 \overset{+}{R} r_2 r_1$$

There is a law which prohibits murder. It imposes a duty on any person (r) to abstain from murdering any other person. Supposing that the proper construction of the relevant provision is that this duty is not owed to any potential victim but to the State (b) and that the State has also a right towards any person not to have murder committed, this situation can be symbolised as follows:

$$\Pi r K \overset{+}{D} r b \overset{+}{R} b r$$

The constitution of a State contains a norm according to which any citizen of this State (r_1) has a right to be employed. It contains, however, no provision as to the implementation of this right and thus does not impose any duty on anyone else (r_2) to give employment. Hence the right conferred is an instance of *ius nudum*. It may be argued that the relevant provision of the constitution does not import a legal right at all. The tenability of this contention (resting on extra-logical grounds) depends on how the concept "right" is defined. It is conceivable that it is defined so that the relation $\Pi r_1 \Sigma r_2 R r_1 r_2$ holds independently of a corresponding presence-of-duty relation. Assuming that there, is no corresponding duty imposed on anyone, the whole situation can be symbolised as follows:

$$K \Pi r_1 \Sigma r_2 \overset{+}{R} r_1 r_2 \Pi r_2 r_1 \bar{D} r_2 r_1$$

There is no law which would impose a duty on anyone (r_1) to take one's children to the zoo nor is there any law which would confer a right on anyone else (r_2) to have this done. This situation is characterised by absence of the duty in question as well as by absence of the corresponding right and can be symbolised as follows:

$$\Pi r_1 r_2 K \bar{D} r_1 r_2 \bar{R} r_2 r_1$$

3. Logical Nature of Some Defects of Legal Regulation

Defects of legal regulation include inconsistencies in law and gaps in law. Their logical nature can be apprehended by recourse to principles and methods discussed above. There are inconsistencies in law also in the sense of impracticability of observing different legal norms at the same time, even though they are not logically inconsistent and there are gaps in law in the sense that law fails to provide a reasonable or desirable solution to a legal problem. Such inconsistencies and gaps are important shortcomings of legal regulation and they call for appropriate remedies. However, because of their extra-logical character they are not matters to which attention will be directed here.

Any deontic modality in the field of law represents a legal situation resulting from a legal norm. Certain logical relations between legal modalities reflect antinomies in law and certain others do not. Two legal modalities are logically *adverse* to each other if their relationship is either extensional dissection or extensional disparity. In the former case, extensional intersection of the relevant deontic principle with extensional intersection of the adverse deontic modalities produces dyslogy, as will appear from the following table. As regards the latter, extensional intersection of a deontic modality with its complementary deontic modality always produces dyslogy.

^{0}KJbihiKbihi
− − + + + + +
− + + − − + −
− + − + − − +
− + − − − − −

^{0}KbiNbi
(Obvious dyslogy)

In a closed legal system, the following legal modalities are adverse to each other and in the following way:

(1) **Jbihi**, (2) **Jbili**, (3) **Jhili**,
(4) **ObiNbi**, (5) **OhiNhi**, (6) **Olimi**.

In an open legal system, the following legal modalities are adverse to each other and in the following way:

(1) **Jbihi**, (2) **Jbili**, (3) **Jhili**,
(4) **JbiAhili**, (5) **JhiAbili**, (6) **Jlimi**.

There are further modal adversities in the open legal system; thus "obligatory incidence" is adverse to "allowable incidence" and "permissory incidence" is adverse to "neutral incidence". However, these modalities are not relevant to the problem of antinomies in law, because no legal norm corresponds to neutral incidence and a legal norm may or may not correspond to allowable incidence.

In order to express legal norms in logical language for the present purposes, they will be conceived as relations constituted by the following relators: "*is obligatory for*" (*B*), "*is probibitory for*" (*H*), and "*is licensory for*" (*L*). Supposing that only singular-concrete norms are involved, the following norms are antinomic and in the following way in a closed legal system:

(1) *JBbkHbk*, (2) *JBbkLbk*, (3) *JHbkLbk*,
(4) *OBbkNBbk*, (5) *OHbkNHbk*, (6) *OLbkNLbk*,

where *NBbk* and *NHbk* represent permitting norms corresponding to the modalities "permissory incidence" and where *NLbk* represents a prescribing norm corresponding to the modality "mandatory incidence".

Supposing again that only singular-concrete norms are involved, the following norms are antinomic and in the following way in an open legal system:

(1) *JBbkHbk*, (2) *JBbkLbk*, (3) *JHbkLbk*,
(4) *JBbkAHbkLbk*, (5) *JHbkABbkLbk*, (6) *JLbkABbkHbk*,

where *AHbkLbk* and *ABbkLbk* represent permitting norms and *ABbkHbk* represents a prescribing norm in an open legal system.

As regards antinomies between general-abstract norms, general-concrete norms, and singular-abstract norms, they have the same logical nature in both kinds of legal systems respectively, provided that the quantification formulae governing the hypotact signs in their logical expressions are exactly the same for each instance of the conflicting norms. For example, the following norms are antinomic:

ΠcsBcs and *ΠcsHcs*,
ΣsHbs and *ΣsNHbs*,
ΠcBck and *ΠcAHckLck*

where *c* and *s* represent hypotact-variables with a limited range of application standing for a norm-object and a norm-subject respectively.

The existence of two antinomic norms in a legal system imports dyslogy under the relevant normative principle, which can be shown by the tabular method in the same way as above in connection with adverse legal modalities. For example, the following formulae representing antinomic legal norms can be shown to be dyslogous:

KJBbkHbkKBbkHbk
ΠcsKJBcsLcsKBcsLcs

In the above exposition of antinomies in law it was presumed that in each case of the antinomic norms only the relators employed for their formation were different whereas the hypotacts standing for their objects or their subjects were exactly the same entities or exactly the same ranges of entities. Conflicts can occur also between a general norm and a singular norm, between an abstract norm and a concrete norm, and between norms of different levels of generality or abstractness. These conflicts do not exhibit their antinomic character directly, because the logical expressions of the conflicting norms here prove to be not dyslogous but amphilogous. For example, the formula

$$KK\Pi csJBcsHcsJBbkHbk\Pi csKBcsHbk$$

can be shown to represent an amphilogy. However, there is an antinomy between a norm validly inferred from a general or abstract norm and a given norm whose object and subject are represented by the same hypotacts as those of the inferred norm but whose relator is such that it would produce an antinomy. For example, *Bbk* can be validly inferred from $\Pi csBcs$ under the principle of universal instantiation. Given a norm whose symbolic expression is *Hbk*, it is obvious that it is antinomic to the inferred norm *Bbk*.

Examples of Antinomies

(1) "Entering Blackacre is licensory for Jones" and "Entering Blackacre is prohibitory for Jones".

(2) "Paying no less than $ 50 weekly to each employee is licensory for all employers" and "Paying no less than $ 50 weekly to each employee is obligatory for all employers".

(3) "Submission of tax returns is obligatory for all persons resident in this State whose annual income is over $ 1000". From this norm it can be inferred that "Submission of tax returns is obligatory for all foreign diplomats resident in this State". There is, however, a norm according to which "It is not the case that the submission of tax returns is obligatory for all foreign diplomats resident in this State". (It is here assumed that the annual income of any foreign diplomat in the State in question would be over $ 1000.)

(4) "Exceeding 35 MPH speed limit is prohibitory for all motorists on the streets of Sydney". From this norm it can be inferred that "Exceeding 35 MPH speed limit is prohibitory for the police officer Smith chasing a speeding motorist in George Street, Sydney,

at 1 a. m. on 1.10.1969". There is, however, a norm according to which "Exceeding 35 MPH speed limit is licensory for all police officers chasing a speeding motorist on the streets of Sydney". From this norm it can be inferred that "Exceeding 35 MPH speed limit is licensory for the police officer Smith chasing a speeding motorist in George Street, Sydney, at 1 a. m. on 1.10.1969".

Antinomies in law arise only if the norms which are inconsistent with each other belong to the same legal system. If they belong to different legal systems or to a system of positive law on the one hand and to a system of natural law on the other, there are what may be called "conflicts between foreign laws" and "conflicts between laws and standards of good law" respectively. Such conflicts may be relevant in a context of private international law or in a context of law reform.

Whereas the logical effect of the antinomies in law is dyslogy, the legal effect of them may differ in different legal systems. It is possible that a legal system contains a principle according to which the antinomic norms whose inconsistency cannot be removed by any methods provided in this legal system are to be treated as being both legally invalid. But it is also conceivable that both are treated as legally valid. This may give rise to situations in which one norm imposes a duty on a person to carry out an instance of conduct and another norm imposes a duty on the same person to refrain from it. If a mandatory penalty is provided for the violation of either norm, the person is liable to be punished in any event. This may be wicked or absurd, but it is plausible and practicable. If the relevant legal system does not offer any means to remove the antinomies in certain cases or if these means are not resorted to, the above harassing situations must actually occur.

For removing antinomies in law, the following principles are frequently resorted to:

(1) *Lex superior derogat legi inferiori.*
(2) *Lex posterior derogat legi priori.*
(3) *Lex specialis derogat legi generali.*

These principles are not capable of removing all antinomies, because it may happen that norms of the highest validity level enacted simultaneously and being of the same level of generality or speciality are inconsistent with each other. Moreover, if these principles are conceived as being on the same validity level, their role in the removal of antinomies is rather limited. It is therefore important for any legal system to establish their order of rank, for example, by assigning the highest rank to Principle (1), the rank immediately subordinate to it to Principle (2), and the lowest rank to Principle (3). Antinomies are often removed by the exercise of the power given to courts or to administrative authorities to interpret extensively or restrictively inconsistent norms so that the duties which they impose or the rights which they confer are construed as being different and their objects or subjects are construed as being different.

Gaps in law are a special case of absence of law. Absence of law may arise from the fact that the law-maker has never proposed to provide legal norms for an area of conduct because this area is already adequately regulated by norms of morals or *mores*, or because there is no need for any normative regulation at all in such an area. But absence of law may also arise from a shortcoming of legislative activity: it may happen that the law-maker should have tried or has actually tried to provide a legal norm to govern an instance of conduct but has failed to do so either because of oversight, incompetence, or some other factor.

Genuine gaps in law are present where expressions of law indicate that a legal norm necessary for the administration of law has been purported but it has not been actually provided. From the legal point of view, such gaps are something more than absence of norms declared to be desirable or which are actually desired by those who make or apply law; they are something more than even absence of norms which are required for the strongest extra-legal reasons. Genuine gaps can arise only in those legal systems which do not contain the "sealing" principle, such as the one according to which any instance of conduct is either obligatory or licensory, that is, they can arise only in an open legal system. In a closed legal system absence of law cannot arise at all, for any instance of conduct whatsoever finds some legal qualification in it because the "sealing" principle assures its juridic plenitude. Supposing that an irremovable antinomy arises in a closed legal system, it does not open a gap in this legal system; for if there is no provision at all as to how to treat the antinomy, both antinomic norms remain legally valid and entail legal consequences however awkward these may be.

The following situations illustrate existence of genuine gaps in law:

In a bilateral treaty there is a norm according to which "The boundary between the territories of the High Contracting Parties in the Eastern

Alps is the line established by local custom". There is no relevant local custom nor is there any norm under which it would be obligatory for the Parties or any other legal authority of the relevant legal system to provide the requisite missing norm.

There is a statute which provides that "The employers of this industry shall pay the wages to their skilled workers which are no less than those specified in the Fourth Schedule appended to the present statute". There is no Fourth Schedule nor is there any norm under which it would be obligatory for the competent law-making authority to supply the requisite norms.

The logical nature of a genuine gap in law appears in the following formula, in which the symbol *p* is assigned to the proposition "The referring norm exists" and the symbol *q* to the proposition "The norm referred to exists" (i. e. the norm which the former norm assumes to be present but which proves to be absent):

$^{0}KCpqKpNq$
− + + + − + − +
− − + − + + + −
− + − + − − − +
− + − − − − + −

As the above table shows, the normative situation here in question involves dyslogy.

4. *Logical Aspects of Some Defects in Legal Expression*

The process in which law is made or applied is often disturbed by unsound organisation of thought. This gives rise to uncertainties in law which uncertainties are manifested in defects of legal expression. These defects may be grouped as ambiguities in law and vaguenesses in law.

A legal expression is ambiguous if it has more than one legally significant meaning; it is vague if its meaning is indefinite or obscure. Ambiguity and vagueness can occur separately but also in conjunction. They occur in conjunction when a linguistic expression has more than one meaning and some or all of its meanings are indefinite or obscure. It is important that those who make or apply law avoid ambiguous or vague legal expressions and when confronted with them seek to remove these defects.

What may be ambiguous or vague in an isolated legal expression may not be so if be whole context in which it occurs is taken into consideration. If this is not sufficient for the removal of ambiguities or vaguenesses in law, further interpretative efforts are required in which recourse is made to various canons of legal interpretation. These canons themselves do not provide a perfect remedy for establishing the meanings of legal expression, for they do not represent a system of axioms from which stringent reasoning can deduce compelling conclusions but only guidelines for non-stringent reasoning which help the reasoner to arrive at conclusions which are cogent as being merely convincing to competent and reasonable men. Moreover it may happen that the relevant canons of interpretation themselves are ambiguous or vague or stand in antinomic relations to each other. Thus the decisive remedy for the removal of ambiguities or vaguenesses in law lies in the competence and capability of the authorities which make or apply law to stipulate what a legal expression ought to mean for a category of legal cases or for an instant case.

Although ambiguities and vaguenesses in law are largely a problem for the theory of non-stringent reasoning, logic is relevant to the treatment of both. Non-stringent reasoning involves steps of logical reasoning in its total course. These steps enhance the lucidity and intellectual restraint of that kind of reasoning. Moreover when non-stringent reasoning has achieved its goal in a statement to which insightful assent is sought, its soundness is tested by examining the merits and demerits of its corollaries, which are formulated by applying the principles and methods of logic. If these corollaries prove to be objectionable, there may be something wrong with the formulation of the statement or it may be materially unsound. Logic can assist in the achievement of satisfactory formulations of statements of law by providing formulae whose logical features are transparent. By the aid of these formulae self-contradictions of thought-formations having legal significance, their redundancies, and circularities can be exposed. Logic provides also a form, of course, in which statements of law can be refuted on the grounds of unsoundness of their corollaries. This refutation follows the pattern of *Modus Ponens* or *Modus Tollens*:

CKCpqpq: *If* at least one corollary of this statement of law is unsound *then* this statement of law is unsound
At least one corollary of this statement of law is unsound

This statement of law is unsound

CKCpqNqNp: *If* this statement of law is sound *then* all its corollaries are sound
It is not the case that all its corollaries are sound

It is not the case that this statement of law is sound

Ambiguities or vaguenesses in law can relate to single legal words, to single legal phrases, to single legal sentences, or to larger units of legal expression. If these defects are not averted and properly attended to, they give rise to uncertainties in the application of law and to fallacies in legal reasoning.

It is notable that even in the technical legal language such an important legal word as "right" is ambiguous. For example, occurring in the expression of the legal provision according to which all employers of an industry have a right to pay no less than $ 50 weekly to its skilled workers, this word may mean "right or duty" (whose corresponding deontic concept is "permissory"). But in the same context it may also mean "right but not duty" (whose corresponding deontic concept is "licensory"). The symbolic expression of this provision could accordingly be either *IIcsNHcs* or *IIcsLcs*.

A complex ambiguity in law can be found in a norm of a federal constitution which stipulates that "Commerce and traffic between the member States shall be absolutely free". "Free" in this context can mean free *from* any legal restraint, that is, neither the member States nor the Federation is entitled to impose any restrictions on the commerce and traffic between the member States. But "free" in the same context can also mean free *within* certain legal restraints which assure that freedom in question does not amount to licentiousness. The ambiguity of the word "free" is compounded by the adverb preceding it. For "absolutely" can mean "entirely" (or "completely"), which simply stresses what is conveyed by the word "free". But "absolutely" can also mean that there are no restraints on the freedom in question at all, that is, freedom here would entail that the norm in question overrides any other norm of the same constitution which may impose restrictions on what otherwise would be permissible to do. The norm thus interpreted would override, for example, the norm according to which the Federal Parliament may enact laws which are required for the defence of the country. Depending on the possible interpretations of the words "free" and "absolutely", the above sentence in the Federal constitution in question can mean at least four different norms. What may be inferred from one of them may not be consistent with what may be inferred from the others.

A special kind of ambiguity in law results from the fact that the words "if ... then ..." and "... or ..." are used loosely in legal language. "If ... then ..." may signify here the operator of conditional but also the operator of comprehendal, and even the operator of equivalence. Accordingly, the relevant propositional compounds may be either *Cpq*, *Dpq*, or *Epq*. As used in legal language, "... or ..." may signify the operator of alternation but also the operator of contravalence. Accordingly, the relevant propositional compounds may be either *Apq* or *Opq*. The above mentioned ambiguities can be illustrated by the following examples:

In a law of naturalisation there is a provision expressed as follows: "If an alien has been a resident of the country for five years, he may apply for citizenship." This sentence can mean that the residence of five years in the country is a *sufficient* condition for the application. In this case the relevant logical words are "*if* ... *then* ..." and the relevant propositional compound is to be symbolised as *Cpq*. This sentence can also mean that such a residence is a *necessary* condition for the application. In this case the relevant logical words are "*only if* ... *then* ..." and the relevant propositional compound is to be symbolised as *Dpq*. Further, this provision can mean that such a residence is both a *sufficient* and a *necessary* condition for the application. In this case the relevant logical words are "*if and only if* ... *then* ..." and the relevant propositional compound is to be symbolised as *Epq* (or alternatively as *KCpqDpq*). Should the law contain additional requirements (e. g. the requirement of the release from the previous citizenship or the requirement of not being convicted of any felony), the appropriate propositional compound is that of comprehendal. In case there are no such additional requirements but there are also other ways of obtaining the right to apply for citizenship (e. g. by entering the military service of the country of residence), the appropriate propositional compound is that of conditional. In case there are no additional requirements and there are no alternative ways of obtaining the right to apply for citizenship, the appropriate propositional compound is that of equivalence.

In a will there is a provision saying that the testator leaves $ 10,000 to his nephew, a graduate in law, if he graduates also in economics or if he becomes a full-time university lecturer; otherwise the sum will go to charity. The intention of the testator that can be gathered from the expression of this provision might have been that the sum will be given to the nephew in case he *either* graduates in economics *or* becomes a full-time university lecturer (*Opq*), for the testator might have thought that in case the nephew graduates in economics *and* becomes a full-time university lecturer (*Kpq*), the future of the nephew will be financially so secure that he would not need any assistance, and thus the charity should benefit from the will. On the other hand, the testator's intention might have been that the nephew should receive the money also in case he graduates in economics *and* becomes a full-time university lecturer. The money should go to charity only in case neither of the alternatives is realised (*Apq*).

A further kind of ambiguity in law occurs where it is not certain to which part of a sentence of legal expression a word or a phrase contained in it refers. This kind of ambiguity, called "*amphiboly*" (or "*syntactic ambiguity*") can be illustrated by the following example:

A statute stipulates: "Non-industrial buildings are buildings which are not those of factories, mills, or other premises of similar character used mainly for industrial purposes." The ambiguity here arises from the fact that the phrase "used mainly for industrial purposes" can be read to govern only "other premises of similar character" but it can also be read to govern "factories [or] mills", because there are such factories or mills which are not used mainly for industrial purposes but, for example, mainly for the purpose of training apprentices in skilled work required in industry.

One way to expose the logical character of the amphiboly encountered in this illustration is by means of the circuit-diagram method of sentence analysis, recently elaborated by Layman E. Allen. The procedure is to label each material part of the given sentence, for example, by arabic numerals. The diagram is an arrow which divides wherever a conjunction or disjunction occurs into as many paths as there are conjuncts or disjuncts and as soon as the conjunction or disjunction is concluded, the paths merge into one again, concluding ultimately with the head of the arrow followed by the letter representing the consequent. Applying this method to the above legal expression, the labelling could appear as follows:

1 2
(Non-industrial buildings) are (buildings which are not those of)
3 4 5
(factories), (mills), (or other premises of similar character)
6
(used mainly for industrial purposes).

The two possible interpretations suggested above would appear respectively as follows:

Diagram I

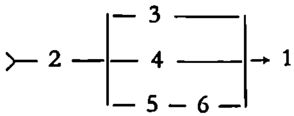


Diagram II

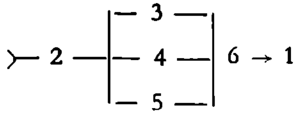


Diagram I represents the interpretation that "used mainly for industrial purposes" qualifies only "other premises of a similar character", while Diagram II represents the interpretation that it qualifies "factories" and "mills" as well. Both interpretations could also be represented by means of predicational calculus as follows:

Diagram I: $\Pi xCBxCAAFxMxKOxUxKBxNIx$
Diagram II: $\Pi xCBxCKAAFxMxOxUxKBxNIx$

In the above formulae *Bx* represents "*x* is a building", *Fx* represents "*x* is a factory", *Mx* represents "*x* is a mill", *Ox* represents "*x* is other premises of similar character", *Ux* represents "*x* is used mainly for industrial purposes", and *Ix* represents "*x* is industrial".

It is to be noted that the circuit diagram method does not cope with all varieties of syntactic ambiguity but only with those involving conjunctions or disjunctions. The method known as "isomer diagrams" assists in the treatment of syntactic ambiguities which cannot be dealt with by the method expounded above.

Consider, for example, the advertisement, "For Sale: Perfect gentleman's bicycle". The two interpretations appropriate for this advertisement could be shown thus:

(For Sale): (perfect) (gentleman's bicycle).
(For Sale): (perfect gentleman's) (bicycle).

Both interpretations could be represented by means of predicational calculus as follows:

$$KKPbGbSb$$
$$KFbSb$$

In the above formulae *b* represents "this bicycle", *Pb* represents "this bicycle is perfect", *Gb* represents "this bicycle is a gentleman's", *Sb* represents "this bicycle is for sale", and *Fb* represents "this bicycle is a perfect gentleman's".

Consider also a regulation stipulating that "a restraint is permissible if the removal of the restraint would cause a reduction in the profit of the business which is substantial". The problem which here arises is whether "which is substantial" qualifies "reduction" (Interpretation I), "profit" (Interpretation II), or "business" (Interpretation III).

I: ... (reduction which is substantial) (in the profit) (of the business)
II: ... (reduction) (in the profit which is substantial) (of the business)
III: ... (reduction) (in the profit) (of the business which is substantial)

The difference between these three interpretations could also be expressed in predicational calculus, omitting those respects in which the formulae are the same, as follows:

I: ... $\Pi xyzKKKRxPyBzSx$ ...
II: ... $\Pi xyzKKKRxPyBzSy$ ...
III: ... $\Pi xyzKKKRxPyBzSz$ ...

In the above formulae *R*... represents "... is a reduction", *P*... represents "... is a profit", *B*... represents "... is a business", and *S*... represents "... is substantial".

The logical treatment of ambiguities consists in providing an appropriate logical expression for each meaning of an ambiguous locution occurring patently or latently in ordinary linguistic usage. For subjecting ambiguities to this treatment, it is necessary to paraphrase their linguistic embodiments in order to state the various meanings which they embrace and to express such variations in the precise language of logic.

Both the circuit-diagram method and the isomer diagram method are useful in assisting the draftsmen in drafting a legal instrument by clarifying its intended structure and in elimination of any possible syntactic ambiguity. They are also useful in assisting counsel as well the judge in the interpretation of legal instruments by showing the possible choices of available interpretation. However, it must be noted that, even though these techniques make explicit any syntactic ambiguity involved, they do not assist in the resolution of the ambiguity materially. To determine which alternative to choose is an extra-logical matter.

In all instances of vagueness of law, the range of entities to which a notion applies is not definite. In most cases the vague notion has a "core of certainty" in the sense that it applies with certainty to some entities but it also has a "penumbra of doubt", that is, an area of uncertainty of its application. For example, the notion "vehicle" certainly applies to those motorcars, trams, or bicycles which are in running order. But it is not certain whether it also applies to a motorcar whose engine has been removed, to prams, or to supermarket trolleys. The notion "aggression" undoubtedly applies to an unprovoked armed attack by a strong military power against a small State which has no regular army, navy, or air force. But it is not certain whether it also applies to an armed attack by a strong military power against a small hostile State in whose territory another strong military power has established bases for launching nuclear missiles. The notion "judicial organ" undoubtedly applies to the law courts, but it is not certain whether it also applies to tribunals of arbitration or to boards of review of administrative acts. Vagueness in law is encountered also in the following familiar situation: In a traffic intersection a red flicking light appears showing the words "Don't walk". It may be regarded as certain that the instruction is to the effect that when a pedestrian has not yet left the footpath, he is forbidden to start crossing the street in a leisurely manner. But it is not certain whether he ought to turn back, to stop, or to proceed when he has covered one third of the width of the street and the flicking light appears. A whole group of instances of vagueness is constituted by the legal standards such as good faith, justice, expediency, and reasonableness. Each of them has a core of certainty but also an extensive penumbra of doubt. For example, it is certain that it is reasonable to respect the rules of logic but uncertain whether and to what extent it is reasonable to sacrifice legality to justice if in a particular case considerations of justice and requirements of expediency also conflict with each other.

In some instances the given situation in which vague notions occur in law provides indication how to deal with doubtful cases. For example, it appears to be a better course of action either to turn back quickly or to cross the street when the red flicking light showing the words "Don't walk" is still showing rather than to stop in the street between the footpaths. In some instances of vagueness in law, the given situation in which a vague notion is legally relevant may not provide any indication how to deal with doubtful matters resulting from the occurrence of such a notion. Therefore subsequent acts of legislation or judicial decisions may be required to remove or reduce vagueness in law by appropriate definitions. In some further instances of vagueness in law, for example, in connection with the notion of "aggression" or in connection with legal standards such as "reasonableness", "justice", etc., satisfactory definitions which would remove the area of doubt may be virtually impossible to achieve because of the unforeseeability of aggressive situations and the correlative situations of legitimate self-defence or because of the intractability of what ought to be considered reasonable, just, etc. in situations which have certain prominently unique features. Vagueness affecting such notions cannot be removed in advance for all future cases but it can be remedied for the instant case by authoritative clarifying pronouncements.

Ambiguities or vaguenesses in law may be either patent or latent. If they are patent, the legal decision-maker must make a choice between alternative meanings of legal expressions and must make up his mind about the proper scope of a vague notion before he can affirm or deny a legal proposition and thus perform logical operations for drawing legal conclusions under relevant rules of logic. If ambiguities or vaguenesses in law are latent, the legal decision-maker may be deceived about their presence and may thus abstain from making the requisite choice between alternative meanings of legal expressions and from seeking the requisite definite meaning for vague notions. Thus he may have no sufficient reasons to affirm or to deny a legal proposition and thus the legal conclusions which he draws may be logically impeccable but prove to be materially unsound. Ambiguities or vaguenesses of the expressions of thought-formations on which logical operations are performed are no obstacles for these operations just as falsity of propositions or absurdity of terms is not. All these defects or shortcomings of reasoning are extra-logical matters which can be attended to by the aid of the principles and methods of logic but which nevertheless are not removable by logic alone.

Finally it is to be noted that uncertainties are "constitutionally" present in law owing to the fact that a great deal of law is not ready-made for direct application but is meant to be in a constant state of making, through organs exercising their discretion in determining the extent of application of a legal provision in a concrete instance or in a range of instances. The activity of these organs must, of course, take place within the framework of the given law, that is, within its logical possibilities; but as long as they remain within these they can act without formal inhibitions.

5. Logical Aspects of Some Defects in Legal Reasoning

Defects in legal reasoning may manifest themselves as what are called "fallacies". A fallacy occurs when an inference appears to be valid but proves not to be so. All fallacies contain an error or several errors in reasoning. Some of these result from non-compliance with principles of logic, some from other sources of unsound reasoning. The former are *logical fallacies*, the latter *extra-logical fallacies.* Although in the area of legal thought (especially in forensic argumentation) extra-logical fallacies are of considerable importance (being frequently, and sometimes most efficiently, employed), they will not be an object of concern in this Compendium. A proper place for their treatment is in works on theory of argumentation. However, a few instances of them will be mentioned below in order to show the specific nature of logical fallacies by contrast.

Examples of Extra-logical Fallacies

Counsel argues that a case ought to be decided in a certain way because one of the greatest judicial authorities has said that this is the right way to decide it.
The type of fallacy here involved is argumentum ad verecundiam *(appeal to authority). It is unsound unless the view in question has been expressed in a manner which makes it an authoritative statement of the law in question.*

A plaintiff sues a university for unlawful dismissal from the chair of philosophy. The defendant's counsel argues that the plaintiff is a person of bad moral character because he has published views which are morally abhorrent to the vast majority of the community.
The type of fallacy here involved is argumentum ad hominem. *It is unsound unless in the given community the publication of the alleged objectionable views is inconsistent with the terms of the plaintiff's employment.*

A youth is charged with having murdered his parents. He admits the facts which make his act legally a murder. However, he pleads before the jury that his parents had treated him in an abominable manner, that he is now an orphan, and that the girl whom he wishes to marry is pregnant by him.
The type of fallacy here involved is argumentum ad misericordiam *(appeal to pity). It is unsound as a legal justification of the defendant's act (but may be relevant to achieving a mitigation of his punishment).*

There are as many logical fallacies as there are possible violations of the rules of logic or diversions from basic assumptions on which logic operates. Often extra-logical and logical fallacies occur in combination. In some instances a logical fallacy is committed but no person of average intelligence is likely to be deceived. For example, if it is argued that no politicians are reliable, because most politicians are liars and no liars are reliable, not only experts in logic can discover the unsoundness of the argument but anyone endowed with common sense. It is to be noted, however, that the common man usually rejects such arguments not by logical considerations but on grounds of his experience of life. He may be deceived even by crudest logical errors if these are concealed in a subtle manner.

Logical fallacies arise, firstly, because certain basic assumptions of logical reasoning are not observed. One of these is that in logical inferences the terms of the premisses must retain the same meaning. If they do not, the *fallacy of equivocation* occurs. Words like "night-time" (because it has a different meaning in law and in extra-legal contexts), "marriage" (because its essential requirements differ in Christian countries and in Moslem countries), and "possession" (because its meaning is different in criminal law and in property law) are among those whose equivocations are of legal significance. When equivocal terms are employed in a simple syllogism, the fallacy of *quaternio terminorum* is likely to occur (i. e. instead of three and only three terms which a valid simple syllogism must have it turns out that, by a shift of meaning, four or more terms are employed). Another of these assumptions is that the relations which hold between a class and its members are not the same as the relations which hold between a collective entity (e. g. a jurisprudence class) and its component parts (e. g. the students of a jurisprudence class). Thus it would be fallacious to argue that because the International Court of Justice has delivered an objectionable judgment its individual judges, too, have delivered objectionable judgments; or to argue that because every shareholder of a company is solvent the company, too, is solvent. The former fallacy is the *fallacy of division*, the latter the *fallacy of composition.* A further basic assumption of logic is that the conclusion which is drawn by an inference must not be any other than what one claims to prove. For example:

> The Government of State A sends military aircraft over the territory of State B for the purpose of gaining military information. The Government of State B protests. In its reply, the Government of State A tries to justify its action by pointing out that State B has engaged in extensive espionage in the territory of State A.
> *The fallacy here committed is that of* irrelevant conclusion (ignoratio elenchi), *because the Government of State A may succeed only in proving that State B has committed an international wrong but not that its own action is legally justified. In other words, there is no logical force in a mere* tu quoque *("you too", or shorter: "U-2") argument.*

Secondly, a source of logical fallacies is non-compliance with rules of immediate inference. For example, if it is affirmed that all criminals are corrigible because it must be denied that no criminals are corrigible, the conclusion is fallacious; for logically only the conclusion that some criminals are corrigible follows from this denial. It would also be logically fallacious to argue that some delinquents are not youths because some youths are not delinquents; for a proposition in the form of S *o* P has no converse.

Thirdly, a source of logical fallacies is non-compliance with rules of simple syllogism.

Examples

All criminals *are* lawbreakers
All trespassers *are* lawbreakers
———
All trespassers *are* criminals

Some lawyers *are* married men
Some monks *are* lawyers
———
Some monks *are* married men

In these examples the fallacy of undistributed middle *is committed; for the middle term must be distributed in at least one premiss of a valid syllogism.*

All treaties *are* legal instruments
No wills *are* treaties

No wills *are* legal instruments

All murders *are* felonies
All murders *are* homicides

All homicides *are* felonies

In these examples the fallacy of illicit process *is committed; for no term can be distributed in the conclusion of a syllogism if it is undistributed in the premiss. In the first example, the fallacy is that of the* illicit major, *because in the major premiss the term "legal instruments" is undistributed but occurs as distributed in the conclusion. In the second example, the fallacy is that of the* illicit minor, *because in the minor premiss the term "homicides" is undistributed but occurs as distributed in the conclusion.*

In the following example the fallacy of exclusive premisses *is committed; for from two negative premisses no syllogistic conclusion follows:*

No valid contracts *are* acts in which
offer and acceptance are absent
No conspiracies *are* valid contracts

No conspiracies *are* acts in which
offer and acceptance are absent

Fourthly, a source of logical fallacies is non-compliance with rules of hypothetic or disjunctive inference.

Examples

If this law is adopted *then* the economy of the country will decline
It is not the case that this law is adopted

It is not the case that the economy of the country will decline

In this example the fallacy of unwarranted denial *is committed; for denial of the antecedent does not lead to denial of the consequent. This is clear if it is considered that the economy of the country may decline also for reasons other than the adoption of a law which is objectionable from the economic point of view. The symbolic expression of the above fallacy is* CKCpqNpNq, *which is not a tautologous propositional formula and therefore cannot represent a valid inference.*

If Black was in London *then* he was in England
Black was in England
____
Black was in London

In this example the fallacy of unwarranted affirmation *is committed; for affirmation of the consequent does not lead to affirmation of the antecedent. This is clear if it is considered that Black could have been in Manchester, Oxford, or elsewhere in England, but not in London. The symbolic expression of this fallacy is* CKCpqqp, *which is not a tautologous formula.*

This burglary was committed by Black *or* it was committed by White
This burglary was committed by Black
____
It is not the case that this burglary was committed by White

In this example the fallacy of misconceived disjunction *is committed; for in weak disjunction affirmation of one disjunct does not lead to denial of the other disjunct. The above conclusion would be warranted if the premiss had been a strong disjunction. The symbolic expression of this fallacy is* CKApqpNq, *which is not a tautologous formula.*

A peculiar logical fallacy is encountered in attempts to rebut a dilemma by constructing another dilemma the conclusion of which is inconsistent with the conclusion of the disputed dilemma. To illustrate such a specious rebuttal, a slightly modified classical paradigm will be presented in which a young man anxious to become a politician was cautioned as follows:

If you tell the truth *then* men will hate you *and if* you tell lies *then* gods will hate you.
You tell the truth *or* you tell lies
____
Men will hate you *or* gods will hate you

The symbolic expression of this dilemma is *CKKCpqCrsAprAqs.

The same man was reassured by the following counter-dilemma:

If you tell lies *then it is not the case that* men will hate you *and if* you tell the truth *then it is not the case that* gods will hate you
You tell the truth *or* you tell lies
____
It is not the case that men will hate you *or it is not the case that* gods will hate you

The symbolic expression of this dilemma is *CKKCrNqCpNsAprANqNs.

*This rebuttal is fallacious, for though the disjuncts in the conclusions (*q and Nq, s and Ns*) of the above dilemmas are inconsistent with each other, the disjunctive conclusions (*Aqs and ANqNs*) are not so. That men will hate a person or gods will hate him is consistent with (1) men will hate a person and gods will not hate him, (2) men will not hate a person and gods will hate him. A successful rebuttal of the conclusion of the original dilemma would have required affirmation that neither men nor gods will hate the would-be politician (*Iqs, i. e. NAqs*), which the counter-dilemma failed to establish.*

Among logical fallacies the *fallacy of non sequitur* is sometimes mentioned. It occurs when a conclusion is drawn from premisses which are consistent with it but does not follow from them. Strictly speaking, there need not be any fallacy involved in this procedure, because the reasoning may represent an enthymeme with a suppressed premiss or suppressed premisses. The reproach of *non sequitur* can, however, be properly made if a requisite premiss cannot be supplied by the reasoner or if it cannot be expected that his audience would readily identify it. But all this is an extra-logical matter; hence *non sequitur* of this kind cannot be regarded to be a logical fallacy. On the other hand, "*non sequitur*" may be employed as a covering term for all logically fallacious inferences where a conclusion is consistent with the premisses but does not follow from them under the rules of any valid inference.

Defects in legal reasoning are also encountered if this reasoning leads to paradoxical results. Paradoxes arise in the course of logical reasoning when reasoning, though logically impeccable, leads to absurdities. Such paradoxes had already been discovered and widely discussed in classical antiquity; some of them continue to engage the minds of logicians.

Examples

In cross-examination counsel tries to discredit the reliability of a witness, and contends that he is a person who only tells lies. The witness replies: "Well, I am lying that I am telling a lie". He observes that if it is true that he is lying then he is not telling a lie but the truth. If it is false that he is lying then he is telling the truth. Counsel insists that if it is true that the witness is lying then the witness is lying and if it is false that the witness is lying then the witness is not telling the truth but a lie.

The symbolic expression of the argument of the witness is

*CKCpNppNp and *CKCNpNpNpNp

whereas the symbolic expression of the counsel's argument is

*CKCpppp and *CKCNppNpp,

which all represent valid inferences. Hence both conclusions, though inconsistent, are logically necessary. The source of the Liar's Paradox *(here presented in a modern setting) lies not in any principle of logic but in the statement of the witness that he is lying that he is telling a lie. This statement is "impredicative" (in that it includes itself within its scope) and in its impredicativeness it imports self-contradiction. Therefore it is materially unsound and as such productive of mutually inconsistent conclusions (*ex falso quod libet!*).*

Protagoras agrees to teach law to Eulathus on the condition that the latter pays the tuition fee when he has won his first case. After the end of the course, Eulathus abstains from going into legal practice. Protagoras brings suit against Eulathus for the tuition fee. In the court Protagoras presents his plea in the form of the following simple constructive dilemma:

If the defendant loses this case *then* he has the duty to pay the tuition fee *and if* he wins this case *then* he has the duty to pay the tuition fee

Either the defendant loses *or* wins this case

The defendant has the duty to pay the tuition fee

The symbolic expression of this argument is *CKKCpqCrqOprq.

Protagoras is contending that if Eulathus loses the case then Eulathus must pay the tuition fee by the judgment of the court and if Eulathus wins the case then Eulathus must pay it under the terms of the agreement.

Eulathus attempts to rebut the dilemma of Protagoras by the following simple constructive dilemma:

If the defendant loses this case *then* he has not the duty to pay the tuition fee *and if* he wins this case *then* he has not the duty to pay the tuition fee

Either the defendant loses *or* wins this case

The defendant has not the duty to pay the tuition fee

The symbolic expression of this argument is *CKKCpNqCrNqOprNq.

Eulathus is contending that if he loses this case he shall not have to pay the tuition fee under the terms of the agreement and if he wins this case he shall not have to pay it by the judgment of the court.

The attempted rebuttal of the dilemma of Protagoras by Eulathus is only a rhetorical but not a logical refutation; for even though the conclusions arrived at are inconsistent with each other, they are derived from different premisses, for the first premiss of each dilemma represents a different propositional compound. The inconsistency of the conclusions brings out that the agreement which gave rise to the litigation is unsound. It is not the task of logic to remedy the situation resulting from the defective agreement. The court is faced with the problem what to do about the agreement productive of a paradox.

Someone passes the remark: "All solicitors in this building are crooks." In actual fact there are no solicitors at all in the building in question; therefore the proposition can be claimed to be false. Under the principle of Excluded Middle, the contradictory proposition "Some solicitors in this building are not crooks" can be claimed to be true. Since the latter proposition is one in the form of S *o* P, it has existential import and hence means that there is at least one solicitor in the building in question, namely one who is not a crook. This is inconsistent with the basis of denial of the original proposition.

The source of the Paradox of Existential Import *is that the proposition "All solicitors in this building are crooks" is treated as false because there are no solicitors in the building in question at all. Since it is an assumption of the system of traditional logic as expounded in this Compendium that all categoric propositions have existential import, the above proposition is to be taken to imply that there is at least one solicitor in the building. Hence either this proposition is an inappropriate vehicle for conveying the relevant thought or its denial on the above basis is inappropriate. If one desires to convey a proposition lacking existential import, this proposition should be hypothetic, for example, "If there are solicitors in this building then all of them are crooks".*

A scoffer of modern logic in the service of law says that from the conditional "If it is not the case that this innocent man ought to be hanged for the murder he has not committed then this innocent man ought to be hanged for the murder he has not committed" it follows under the formula **CCNppp* that "This innocent man ought to be hanged for the murder which he has not committed". This logically necessary conclusion is, however, absurd.

The source of the Paradox of Consequentia Mirabilis *here presented is not logic but its absurd premiss. The logical "miracle" worked in it is another instance showing that logic does not provide adequate protection against consequences of materially unsound thought-formations. Once absurdity appears in the course of logical reasoning, its source can be traced by the aid of logic. The removal of absurdity is effected by extra-logical ways of reasoning.*

A novice in the study of propositional calculus argues that from the premiss "X wilfully and maliciously split the head of Y by an axe and Y died", it follows under a law of commutation that "Y died and X wilfully and maliciously split the head of Y by an axe".

The Paradox of Conjunction *here presented involves that if the premiss is true, X is liable for murder, whereas if the conclusion is true, X has not committed murder but only maltreated a corpse. This is odd indeed; however, the paradox is not produced by logic. The oddity arises due to the ambiguity of the word "and", which is not employed in the premiss in the sense that it can be regarded as standing for the* K*-operator; it is used in the sense of "and thereafter", which is not a logical operator.*

The above examination of paradoxes suggests that there are no *logical* paradoxes but only paradoxes resulting from the circumstance that propositions subjected to logical operations have sometimes a content which makes them liable to lead to odd results. The absurdities which they explicitly or implicitly contain require attention, but an appropriate handling of them falls beyond the pale of logic. Logic is not responsible for them but has a responsibility to expose them, to trace their source, and to show ways in which they can be avoided.

6. Logical Aspects of Some Specific Juristic Arguments

There are juristic arguments which purport to establish legally binding conclusions but which prove to be logically invalid inferences. They are encountered in *argumentum a contrario*, *argumentum a fortiori*, and *argumentum a simile,* all of which play an important role in legal reasoning.

Argumentum a contrario proceeds from the idea that given a proposition about a legal state of affairs as an antecedent of a hypothetic proposition whose consequent is a proposition about the relevant legal consequence and given that the antecedent is to be denied, the consequent is also to be denied. For example:

If this act is murder *then* this act ought to be punished by life imprisonment
It is not the case that this act is murder

It is not the case that this act ought to be punished by life imprisonment

That this argument is unsound may be gathered from the fact that there are or may be offences (e. g. high treason) which are also punishable by life imprisonment; hence denial that an act is murder does not exclude affirmation that it ought to be punished with the same severity as murder. The logical form of an *argumentum a contrario* is represented in the formula *CKCpqNpNq*. The following table shows that it is amphilogous; hence a purported inference under it leads only to a logically contingent conclusion:

Table I

CKCpqNpNq
+ – + + + – + – +
+ – – + – – + + –
– + + – + + – – +
+ + + – – + – + –

From the amphilogy of the formula it follows that it depends on circumstances whether or not the conclusion of the inference is in fact warranted.

Argumentum a contrario would represent a valid inference if the hypothetic premiss from which the conclusion is drawn could be considered to have either the form "*Only if* p *then* q" (**CKDpqNpNq*) or the form "*If and only if* p *then* q" (**CKEpqNpNq*). Both formulae are tautologous and as such they represent inferences which lead to logically necessary conclusions. This appears from the following table:

Table II

**CKDpqNpNq*	**CKEpqNpNq*
+ – + + + – + – +	+ – + + + – + – +
+ – + + – – + + –	+ – – + – – + + –
+ – – – + + – – +	+ – – – + + – – +
+ + + – – + – + –	+ + + – – + – + –

Examples

A statute stipulates that registered companies have the right to own land. From this provision it does not follow that unregistered companies do not have the same right. However, in a legal dispute in which the ownership of land by an unregistered company is at issue, the court may find (by recourse to appropriate canons of interpretation) that for the instant case the relevant provision of law must be construed as follows: "*Only if* this company is registered *then* this company has the right to own land". It may be noted that under a law of transformation, this proposition is equivalent to "This company is registered *or it is not the case that* this company has the right to own land". Given either of these premisses and the premiss "*It is not the case that* this company is registered", it follows as a logically necessary conclusion that "*It is not the case that* this company has the right to own land".

Under the Crimes Act of a country, death penalty is imposed *if and only if* a person has committed murder. Black attempts to murder White but actually only inflicts bodily harm on him which is so grievous that the victim is reduced to a "human vegetable" for the rest of his life. Despite the heinousness of the crime the court rejects imposition of death penalty arguing that "*If and only if* Black has committed murder *then* death penalty ought to be imposed on Black" and "*It is not the case that* Black has committed murder"; *consequently*, "*It is not the case that* death penalty ought to be imposed on Black".

The Crimes Act of a country imposes penalties on various acts of socially harmful behaviour. In a recent amendment of the Act, soliciting in public places was made a punishable offence. Smart publishes a book in which addresses, telephone numbers, and descriptions of prostitutes as well as their special services are printed and offers it for sale. In relying on the principle "*Nullum crimen nulla poena sine lege*", counsel for Smart argues before the court of first instance that his client's behaviour does not constitute a criminal offence. This reasoning, which follows the pattern of *Modus Tollens* of comprehendal (**CKDpqNpNq*) is accepted by the court. On appeal the decision is reversed. The Court of Appeal finds that in the relevant legal system not all offences are included in the Crimes Act but the courts have a residual power to find that a morally outrageous and socially harmful act constitutes a punishable offence. The reasoning of the Court of Appeal follows the pattern which can be symbolised as **CKKCrqEprpq*.

Both courts inferred correctly from the logical point of view. However, each assumed a different legal point of view.

Argumentum a fortiori proceeds from the idea that if there is a weaker reason to affirm something, then given a stronger reason, the same can be affirmed. The inference by *argumentum a fortiori* is logically valid if the relevant reasons are logical reasons. Thus the inferences by subalternation, according to which S *i* P can be concluded from S *a* P and S *o* P can be concluded from S *e* P, may be regarded as instances of *argumentum a fortiori* (e.g. "*Some* murders *are* felonies" can be concluded from "*All* murders *are* felonies" *a fortiori*).

The relevant reasons in case of the specific juristic *argumentum a fortiori* are, however, extra-logical reasons. Thus the form of the purported inference is here *CKCpqrq*, which represents an amphilogous formula incapable of providing a logically necessary conclusion.

Table III

C	*K*	*C*	*p*	*q*	*r*	*q*
+	+	+	+	+	+	+
+	−	+	+	+	−	+
+	−	−	+	−	+	−
+	−	−	+	−	−	−
+	+	+	−	+	+	+
+	−	+	−	+	−	+
−	+	+	−	−	+	−
+	−	+	−	−	−	−

For example, it is argued that if it is prohibited to take dogs on public transport then it is also prohibited to take (for instance) wolves on public transport. Such an inference is invalid from the logical point of view, because wolves are not dogs, even though there is a reason to believe that they are greater potential nuisance than dogs (even if a wolf is domesticated and has proved to be well-behaved). The invalid inference of the above example follows the pattern *CKCpqrq* and can be expressed in ordinary language as follows:

If this animal is a dog *then* this animal is not admitted on public transport

This animal is a wolf

This animal is not admitted on public transport

There are two kinds of juristic *a fortiori* arguments: (1) *argumentum a minori ad maius* and (2) *argumentum a maiori ad minus*. By employing (1) it is argued that if someone ought to refrain from an instance of conduct which is of *lesser* significance than some other instance of conduct then likewise he ought to refrain from the latter. For example, if all persons ought to refrain from walking on a lawn then likewise they ought to refrain from riding a horse on this lawn. By employing (2) it is argued that if someone may carry out an instance of conduct which is of *greater* significance than some other instance of conduct then likewise he may carry out the latter. For example, if all persons may carry out riding a horse in a park then likewise they may carry out walking in this park.

Any *a fortiori* argument as a specific juristic inference of both kinds can be converted into a logically valid inference if there is a sufficient reason for supplying a premiss by virtue of which given stronger legal reasons relevant to a case the same legal consequence is to follow as is provided when weaker legal reasons are given. For example, supposing that the italicised clause and its link ("*or*") with the preceding clause are legally well founded, the following inference is logically valid:

If this creature is a dog *or this creature is of greater potential nuisance than a dog* then it is prohibited to take this creature on public transport
This creature is of greater potential nuisance than a dog

It is prohibited to take this creature on public transport

The form of this inference is *CKCApqrqr*. That it leads to a logically necessary conclusion appears from the following table:

Table IV

*C	K	C	A	p	q	r	q	r
+	+	+	+	+	+	+	+	+
+	−	−	+	+	+	−	+	−
+	−	+	+	+	−	+	−	+
+	−	−	+	+	−	−	−	−
+	+	+	+	−	+	+	+	+
+	−	−	+	−	+	−	+	−
+	−	+	−	−	−	+	−	+
+	−	+	−	−	−	−	−	−

Argumentum a simile proceeds from the idea that if a certain legal consequence is attached to certain legally relevant facts, one is entitled to attach the same legal consequence to essentially similar legally relevant facts. For example:

If the set of facts F is given *then* this act ought to be treated as larceny
A set of facts essentially similar to F is given

This act ought to be treated as larceny

This purported inference is logically invalid because it has the pattern *CKCpqrq*, which formula represents an amphilogy, as appears from Table III above.

Suppose that the set of facts F in the above illustration is: taking unlawfully and removing a thing with intent to deprive the right owner of the same. A person unlawfully takes electricity with intent to deprive the Electricity Commission of it. Although electricity is not a thing, taking it unlawfully is essentially similar to taking and removing a thing unlawfully (the economic and social consequences of both acts being virtually the same). Therefore it may be argued that this act deserves the same punishment as larceny in the strict sense. However sound this conclusion may be from the viewpoint of morals or social policy, it is unsound from the logical viewpoint. It can be logically justified only if there is a sufficient reason for supplying a premiss by virtue of which the same legal consequence is to follow for the unprovided case as it is to follow for the provided case. For example, assuming that the italicised clause and its link ("*or*") with the preceding clause are legally well founded, the following inference is logically valid:

If this act is taking a thing unlawfully ... *or this act is found by a competent judicial authority to be essentially similar to taking a thing unlawfully then* this act ought to be treated as larceny

This act is found by a competent judicial authority to be essentially similar to taking a thing unlawfully ...

This act ought to be treated as larceny

The formula expressing the above inference is the same as the one expressing the inference by which *argumentum a fortiori* is converted into a valid inference, viz. **CKCApqrqr*. That the conclusion is logically necessary appears from Table IV above, which shows that this formula represents a tautology.

Argumentum a contrario, *argumentum a fortiori*, and *argumentum a simile* are all instances of *modus deficiens* and from their expressly stated premisses no logically necessary conclusions follow. If the premisses are supplied which would convert them into logically valid inferences, these are, strictly speaking, not instances of the arguments in question. Provided that the requisite additional premisses, even though not stated, are sure to be understood and accepted in the given legal community, the above three specific juristic arguments may be regarded as representing abridged (enthymemic) inferences with suppressed but readily available premisses.

Conclusion

In the foregoing summary treatment of logic in the service of law it became apparent that in legal thought logic has certain uses. Various familiar patterns of legal thought fit neatly into logical patterns, and thus principles and methods of logic can be taught and learnt by means of legal illustrations. The above exposition of a system of traditional logic and of a system of modern logic indicates that logic is concerned with consistency of thought in abstraction of its material content; in other words, it is concerned with drawing conclusions in a stringent manner. In order to provide principles and methods by recourse to which stringent reasoning can take place, logic categorises and articulates thought-formations on which it operates.

The size of the present book was reduced to a minimum not only by avoiding discussion of matters belonging to neighbouring disciplines such as semantics, theory of argumentation, formal ontology, and scientific method but also by avoiding various details of systems of logic which might prove to be of juristic interest. To consider three- or more-than-three-valued systems of logic would have been unmanageable within the scope of the present work, though it is arguable that formal treatment of the "multidimensionality" of law provides occasions for applications of these systems.

The whole of this Compendium can be viewed as making a case for the submission that logic does have some significant uses in the field of law. However, it cannot claim to have made a case for a further submission that expertise in logic is indispensable for lawyers. It may be argued that in their actual work, especially in forensic reasoning, principles and methods of logic as such are rarely invoked, and when they are, this may appear to be only a kind of intellectual luxury which need not be displayed and sometimes even should not be displayed so as to avoid the impression that the reasoner is flaunting his learning.

To dispel scepticism about the role of logic in the service of law, extensive analysis of a variety of actual legal problems requiring logical attention is needed. This task is to be performed, but to lay a foundation for it, an aerial survey of logic in its application to legal thought was first to be provided. All that can be done in the following lines is to say why the belief may be entertained that logic has not only certain uses in legal thought but a significant application in this thought.

In legal reasoning formal consistency of thought is an end constantly pursued, even though it is not always achieved. To assure this consistency, ordinary common sense logic is employed, a logic expressed in an imprecise manner by ordinary language. In the contexts of reasoning which include unstated assumptions and premisses, these imprecisions are ironed out, especially where the partners of reasoning are intellectually disciplined and intellectually honest. However, the art of reasoning in an informal manner about matters in which consistency of thought is significant cannot be easily acquired. The process of becoming conversant and skilful in this art is accelerated by studying the rigorous principles and methods underlying it. Thus the study of logic in the service of law promotes legal education by making explicit what otherwise is merely implicit in the wealth and welter of instances of actual legal reasoning.

There are numerous occasions in legal reasoning in which reasoners try to deceive their opponents or the decision-maker (and sometimes even succeed in deluding themselves) about the logical necessity of their conclusions. To expose the formal defects of their argumentation is often unfeasible by means of informal reasoning. Recourse to principles and methods of logic is frequently the only safe way to identify the sources of fallacies and to ascertain that a fallacy has been committed. There are argumentative situations in which it is impracticable to adduce rigorous proofs about the formal soundness or unsoundness of legal reasoning; however, even if this cannot be done, the reasoner has an advantage over his opponent if he himself knows for what logical reasons a conclusion is sound or unsound. Knowing this, he can find appropriate informal ways of expressing the results of his formal analyses.

Logic is significant for the lawyer in that it helps him to present his reasoning in a well-organised, lucid, and cogent manner. The actual presentation of his reasoning must take into account, of course, the addressees of his train of thought and must be adjusted, by employing appropriate informal ways of expression, to their intellectual background and habits of thought. But this train of thought is more likely to be sound if it is established in awareness of the formal requirements of self-consistent reasoning. The same awareness is important for legal draftsmanship. Antinomies, gaps, ambiguities, and vaguenesses in law can be avoided if the draftsmen are conversant with principles and methods of logic.

There is a wide-spread and tenacious suspicion that preoccupation with formal patterns of legal reasoning cultivates legal formalism and thus affects adversely the effort to achieve justice through law. This suspicion is unwarranted because logical reasoning exposes faults in law which must be remedied in order to achieve justice and it discloses the leeways available to the decision-maker to make just decisions. Rigour of legal reasoning is not the same thing as rigidity of legal reasoning. It may be argued that logical rigour is even conducive to making law and its application flexible, because logical analyses of premisses or data of legal reasoning often show that these are not as rigid as the legal formalist may assume them to be but offer opportunities to the decision-maker to interpret them in accordance with the requirements of expediency and justice. Logical rigour means intellectual integrity and is thus an important ethical requirement in the application of law. To blame logic for shortcomings in the administration of law is very much the same as to blame honesty for evils in the world. It may be that violation of rules of logic is, on some occasions, the only practicable way to bring relief against *dura lex* as it may be that on some occasions deception is the only feasible means for coping with adversities of ordinary life. But both could be justified only as exceptional resorts; their universalisation is out of the question.

Appendix A

A Sign-Constellation Method for Recognising Valid Syllogisms

Valid simple syllogisms can easily be recognised in the procedure described below. In this procedure the following signs are employed:

— (a hyphen) to indicate that a term is distributed.
v (a wedge) to indicate that a term is undistributed.
× (a cross) to indicate that a proposition is positive.
~ (a tilde) to indicate that a proposition is negative.

To establish sign-constellations characterising the four propositional forms, either a cross or a tilde is placed between the other signs. The four propositional forms are characterised by the following sign-constellations:

S *a* P: — × v
S *e* P: — ~ —
S *i* P: v × v
S *o* P: v ~ —

To ascertain whether a syllogism is valid, proceed as follows:

I. Express its schema by writing the major premiss first and the minor premiss under it.

For example:

P *a* M

M *e* S

II. Place the sign-constellation of the major premiss below its form and the sign-constellation of the minor premiss above its form.

For example:

P *a* M
— × v

— ~ —
M *e* S

III. (a) Place a tilde between S and P if a tilde appears in exactly one of the sign-constellations and if a hyphen appears with P and at least with one of the Ms.

For example:

```
P a M
— × v
  ~
— ~ —
M e S
```

(b) Place a cross between S and P if a cross appears in both sign-constellations and if a hyphen appears at least with one of the Ms.

For example:

```
P i M
v × v
  ×
— × v
M a S
```

If the above described steps can be taken, that is, if by following the above instructions either a tilde or a cross can be placed between S and P, the propositional forms in the relevant schema represent premisses of a valid syllogism. The sign-constellation between S and P represents the sign-constellation of the propositional form expressing the conclusion of this syllogism.

In this way the validity of all conventional syllogistic moods can conveniently be recognised. A problem which can be, however, easily solved (as will be shown) arises only in connection with *Modus Bramantip*.

First Figure

Modus Barbara	*Modus Celarent*
M *a* P	M *e* P
— × v	— ~ —
×	~
— × v	— × v
S *a* M	S *a* M

Modus Darii				*Modus Ferio*		
M	*a*	P		M	*e*	P
—	×	∨		—	~	—
	×				~	
∨	×	∨		∨	×	∨
S	*i*	M		S	*i*	M

Second Figure

Modus Cesare				*Modus Camestres*		
P	*e*	M		P	*a*	M
—	~	—		—	×	∨
~				~		
—	×	∨		—	~	—
S	*a*	M		S	*e*	M

Modus Festino				*Modus Baroco*		
P	*e*	M		P	*a*	M
—	~	—		—	×	∨
~				~		
∨	×	∨		∨	~	—
S	*i*	M		S	*o*	M

Third Figure

Modus Darapti				*Modus Disamis*		
M	*a*	P		M	*i*	P
—	×	∨		∨	×	∨
		×				×
—	×	∨		—	×	∨
M	*a*	S		M	*a*	S

Modus Datisi				*Modus Felapton*		
M	*a*	P		M	*e*	P
—	×	∨		—	~	—
		×				~
∨	×	∨		—	×	∨
M	*i*	S		M	*a*	S

Modus Bocardo

M	*o*	P
v	~	—
		~
—	×	v
M	*a*	S

Modus Ferison

M	*e*	P
—	~	—
		~
v	×	v
M	*i*	S

Fourth Figure

Modus Bramantip

P	*a*	M
—	×	v
	×	
—	×	v
M	*a*	S

Modus Dimaris

P	*i*	M
v	×	v
	×	
—	×	v
M	*a*	S

Modus Camenes

P	*a*	M
—	×	v
	~	
—	~	—
M	*e*	S

Modus Fesapo

P	*e*	M
—	~	—
	~	
—	×	v
M	*a*	S

Modus Fresison

P	*e*	M
—	~	—
	~	
v	×	v
M	*i*	S

The above exposition shows that only the conclusion in *Modus Bramantip* produces a sign-constellation (viz. v × —) which is not one characterising any of the four propositional forms of traditional logic. However, this sign-constellation still represents a possible distribution of terms in a particular positive proposition and if it appears in a syllogistic inference, this shows only that this inference actualises that possibility, which *Modus Bramantip* in fact does. It is to be noted that P *a* S (— × v) is a valid conclusion from the premisses of this mood; S *i* P is a weakened conclusion (resulting from conversion by limitation) and therefore does not appear for the same reasons as other weakened conclusions (*Barbari*, *Celaront*, *Cesaro*, *Camestros*, and *Camenos*) do not.

Appendix B

A Short-Cut Tabular Method

In this Appendix a decision-procedure method is described in relation to protological calculus. It is applicable, *mutatis mutandis*, to propositional calculus and to extensional calculus. While the tabular method is adequate for determining whether a compound is firm, loose, or pliant — adequate in the sense that a result will always be obtained no matter how long this may take — it may prove extremely cumbersome and lengthy where a compound contains more than just a few elements. Thus a compound with seven different elements, a not unreasonable number, would require a table with 128 rows. Accordingly, a simpler method has been devised to determine whether a compound is either firm or infirm (if the latter, either loose or pliant). This method may be called "*reductio ad absurdum* method for assigning marks". When applicable, it is swift and satisfactory; however, unfortunately it is not applicable in every case for reasons given later. Its operational rules are:

(1) Assign the mark "—" to the first operator of the compound.
(2) Assign marks to the units of the compound necessary to yield a "—" for the first operator.
(3) Proceed to assign marks to units necessary to yield marks already assigned.
(4) Where at any stage a mark is necessarily assigned to an element, assign the same mark to the element wherever it occurs in the compound.
(5) Derive marks for operators when marks for the units subject to the operator have been assigned by any of the above rules.
(6) Proceed with Rules 3, 4, or 5 in any order until no further step is compelled by these rules.

There are the following possibilities:

(a) The procedure yields marks for the elements so that when these marks are applied consistently, they produce a "—" under the first operator of the compound. In this case the compound is infirm.

(b) The procedure yields some inconsistency (viz. incorrect use of operators; mark required to be assigned to an element is opposite to mark previously assigned to the same element). In this case the compound is firm.

(c) The procedure yields a situation with no compulsion as to how to proceed in assigning marks. In this case the method is not applicable and one cannot tell whether the compound is firm or infirm. It then becomes necessary to resort to some other method, for example, the full tabular method.

In the following some examples are provided to clarify this procedure. Table V in Chapter II, section 2, which is relevant to the procedure, is here reproduced for convenient reference:

x	y	Cxy	Axy	Kxy	Exy	Dxy	Ixy	Jxy	Oxy
+	+	+	+	+	+	+	–	–	–
+	–	–	+	–	–	+	–	+	+
–	+	+	+	–	–	–	–	+	+
–	–	+	–	–	+	+	+	+	–

A. CKCxyCzyCxz is the compound which is required to be identified as either firm or infirm.

Step I: CKCxyCzyCxz

–   Rule (1)

Step II: CKCxyCzyCxz

– +   –   Rule (2) — because C has "—" only when the first unit subject to it is "+" and the second unit subject to it is "–". See the Table.

Step III: CKCxyCzyCxz

− + + + − Rule (3) — because K has "+" only when both units subject to it are "+". See the Table.

Step IV: CKCxyCzyCxz

− + + + − + − Rule (3) — because C has "−" only when the units subject to it have "+" and "−" in that order. See Step II and the Table.

Step V: CKCxyCzyCxz

− + + + + − − + − Rule (4) — in Step IV "+" and "−" were necessarily assigned to x and z respectively; therefore "+" is assigned to x wherever x occurs and "−" is assigned to z wherever z occurs.

Step VI: CKCxyCzyCxz

− + + + + + − − + − Rule (3) — since C in Cxy is "+" and x is "+", y must be "+"; for if y were "−", C would be "−". See the Table.

Step VII: CKCxyCzyCxz

− + + + + + − + − + − Rule (4) — since Step VI established "+" as the mark of y, "+" is assigned to y wherever occurs.

This completes the procedure and it can be seen that by assigning "+" to x, "+" to y, and "−" to z, the mark under the first operator is necessarily "−". Therefore the compound CKCxyCzyCxz is infirm.

B. JKKxyAzxIyz is the compound which is required to be identified as either firm or infirm.

Step I: JKKxyAzxIyz

− Rule (1)

```
Step II:   JKKxyAzxIyz
           −+       +      Rule (2)

Step III:  JKKxyAzxIyz
           −++   +  +      Rule (3)

Step IV:   JKKxyAzxIyz
           −++   +  +−−    Rule (3)

Step V:    JKKxyAzxIyz
           −++ −+− +−−     Rule (4)
```

But now an inconsistency is reached, viz. in the compound Kxy "+" appears under K and "−" appears under y (which is a unit subject to this operator). For K can only have "+" when both units subject to it have "+"; yet here y proves to have "−". Once an inconsistency has been reached, there is no need to go further and the compound is established as a firm compound, because there is no consistent way of assigning marks to the elements which will produce "−" under the first operator of the compound. To remove any trace of doubt, the table for JKKxyAzxIyz is now set out, which confirms that this compound is firm.

*J	K	K	x	y	A	z	x	I	y	z
+	+	+	+	+	+	+	+	−	+	+
+	+	+	+	+	+	−	+	−	+	−
+	−	−	+	−	+	+	+	−	−	+
+	−	−	+	−	+	−	+	+	−	−
+	−	−	−	+	+	+	−	−	+	+
+	−	−	−	+	−	−	−	−	+	−
+	−	−	−	−	+	+	−	−	−	+
+	−	−	−	−	−	−	−	+	−	−

C. $CKCxzAAzyKCx_1y_1z_1Kxy$ is the compound which is required to be identified as either firm or infirm.

```
Step I:    CKCxzAAzyKCx₁y₁z₁Kxy
           −                        Rule (1)
```

Step II: $\underline{CKCxzAAzyKCx_1y_1z_1Kxy}$

− + − Rule (2)

Step III: $\underline{CKCxzAAzyKCx_1y_1z_1Kxy}$

− + + + − Rule (3)

This is as far as it is possible to go. "+" under C in Cxz does not compel the assignment of a particular mark for x or z, and similarly, "+" under A in $AAzyKCx_1y_1z_1$ and "—" under K in Kxy do not compel the assignment of any further particular marks. When this situation arises, the method of assigning marks can proceed by trial-and-error assignments, which diminishes its advantage of speed and simplicity. However, it is not to be thought that merely because marks are not assigned to every part of a compound this method is inapplicable. There are instances where this method succeeds in identifying a compound as either firm or infirm even though marks are not assigned to every part of it, as is shown in the next example which is almost identical with Example C.

D. $CKCxzAAzyKCx_1y_1z_1Cxy$ is the compound which is required to be identified as either firm or infirm.

Step I: $\underline{CKCxzAAzyKCx_1y_1z_1Cxy}$

− Rule (1)

Step II: $\underline{CKCxzAAzyKCx_1y_1z_1Cxy}$

− + − Rule (2)

Step III: $\underline{CKCxzAAzyKCx_1y_1z_1Cxy}$

− + + + − Rule (3)

Step IV: $\underline{CKCxzAAzyKCx_1y_1z_1Cxy}$

− + + + − + − Rule (3)

Step V: $\underline{CKCxzAAzyKCx_1y_1z_1Cxy}$

− + + + + − − + − Rule (4)

Step VI: $\underline{CKCxzAAzyKCx_1y_1z_1Cxy}$

− + + + + + − − + − Rule (3)

Step VII: $\underline{CKCxzAAzyKCx_1y_1z_1Cxy}$

−+++++ +− −+− Rule (4)

Step VIII: $\underline{CKCxzAAzyKCx_1y_1z_1Cxy}$

−+++++++− −+− Rule (5)

This is as far as it is possible to go. However, there is no need to go any further, for the ascertainment of the marks for $KCx_1y_1z_1$ is not necessary. Whether x_1, y_1 or z_1 have "+" or "−" and whether K and C in $KCx_1y_1z_1$ have "+" or "−", the first A in $AAzyKCx_1y_1z_1$ will always have "+", because the second A therein has been shown to have necessarily "+". Hence it has been shown that whenever x has "+", y has "−" and z has "+" (whatever marks x_1, y_1, and z_1 may have), the mark for the first operator in the above compound is "−"; therefore this compound is infirm. It is to be noted that to show this compound to be infirm by the full tabular method would require a table with 64 rows and 17 columns, so that the simplicity and speed of the present method in comparison with the full tabular method is appreciable.

The astute reader may now have concluded that this short-cut method is short only when the first operator of a compound is A, C, D, or J (or N where the second operator is K or I, etc.), since when the first operator is K or I (or N followed by A, C, D, or J) there are three different combinations of marks which would make the first operator "−", while with E and O there are two such different combinations. The present method could still be used in these cases, treating each combination in turn although the advantage of speed is thereby lost.

Appendix C

The Normal Forms Methods as a Decision-Procedure

In this Appendix an ingenious method devised by David Hilbert is described by which it is possible to identify whether a propositional compound is tautologous. It can be applied, *mutatis mutandis*, to extensional calculus and can be generalised to be applicable to protological calculus. Although this method is somewhat cumbersome, its advantage lies in that after a given formula is reduced (according to the rules stated below) to a formula in "normal form", mere inspection of the resultant formula indicates whether the original formula is a tautology (and hence whether the argument, if any, represented by such original formula is valid).

There are two methods of this procedure: either by conjunctive normal form or by alternational normal form (also known as disjunctive normal form).

A. Conjunctive Normal Form

A formula is said to be in conjunctive normal form if it is of the form *KK...pqr...*, where each of the conjuncts (*p*, *q*, *r*, ...) is either

(a) a simple formula or the negation of a simple formula (e. g. *p*, *q*, *Np*, *Nq*), or
(b) an alternation, being an alternation of simple formulae or an alternation of the negations of simple formulae or an alternation consisting of simple formulae and the negations of simple formulae (e. g. *Apq*, *ANpNq*, *ApNq*, *AAApqNprNq*).

Thus, in conjunctive normal form, if *N* appears at all, it may appear only before a simple formula, never before an operator. Hence the following are not in conjunctive normal form: *NApq*, *NKpq*, *KCpqApr*.

To derive conjunctive normal form, proceed as follows:

(1) Eliminate all operators other than *A*, *K*, or *N* by using the following laws of equivalence:

**ECpqANpq*
**EDpqApNq*
**EJpqANpNq*
**EIpqKNpNq*
**EOpqKApqANpNq*
**EEpqKANpqANqp*
**EEpqAKpqKNpNq*

(2) Apply De Morgan laws by replacing *NKpq* or *NApq* wherever they occur by *ANpNq* or *KNpNq* respectively.

(3) Eliminate all pairs of consecutive *N* signs wherever occuring by virtue of the law of double negation.
(4) Apply the equivalences of association, commutation, and distribution if necessary in order that all conjuncts are either simple formulae, the negations of simple formulae, or alternations as permitted above.
(5) Continue to apply any of the above instructions until conjunctive normal form is reached.

It is customary to group all *K* signs together at the beginning of the formula and all *A* signs together at the beginning of their respective alternation and to have all simple formulae or their negations in alphabetical order. Thus, though *KApAqrKAANqAspANprAqs* is already in conjunctive normal form, it is customarily written as *KKAApqrAAAApNpNqrsAqs*. These rearrangements are permissible under the laws of commutation and association.

Once conjunctive normal form has been reached, inspect this form to determine whether every conjunct is a tautology. If so, the original formula is a tautology, and any argument form represented by such a formula is valid. If any conjunct is not a tautology, the original formula is not a tautology and any relevant argument form is invalid.

Since a simple formula or the negation of a simple formula is not a tautology, a formula in conjunctive normal form which has a simple formula or the negation of a simple formula as a conjunct is not a tautology. To be a tautology, a conjunct must be an alternation in the following circumstances:

(a) Any alternation consisting of any formula and the negation of that formula is a tautology.
(b) Any alternation which has a tautology as one of its alternants is itself a tautology.

For example, *ApNp* is a tautology, whether *p* represents either a simple or a complex proposition. *AApNpq* is a tautology whatever propositions (either simple or complex) *p* and *q* may represent. Note also that *AApqNp* could also be written as *ApAqNp*, *ApANpq*, ... under the equivalences by association and commutation.

Example

Determine by the conjunctive normal form method whether the formula *CKCpqCqrCpr* is a tautology.

1.	*CKCpqCqrCpr*	
2.	*ANKCpqCqrCpr*	Rule (1)
3.	*ANKANpqANqrANpr*	Rule (1)
4.	*AANANpqNANqrANpr*	De M.
5.	*AAKNNpNqKNNqNrANpr*	De M.
6.	*AAKpNqKqNrANpr*	D. N.
7.	*AAANprKpNqKqNr*	Comm. and Assoc.
8.	*AKAANprpAANprNqKqNr*	Dist.
9.	*KAKAANprpAANprNqqAKAANprpAANprNqNr*	Dist.
10.	*KAqKAANprpAANprNqANrKAANprpAANprNq*	Comm.
11.	*KKAqAANprpAqAANprNqKANrAANprpANrAANprNq*	Dist.
12.	*KKKAqAANprpAqAANprNqANrAANprpANrAANprNq*	Assoc.
13.	*KKKAAApNpqrAAANpqNqrAAApNprNrAAANpNqrNr*	Assoc. and Comm.

On inspecting this final formula (11, 12, and 13 are all in conjunctive normal form, but 13 is preferred for the reasons given above), each of the four conjuncts is an alternation within which two of the alternants are a formula and its negation (e. g. in *AAApNpqr* they are *p* are *Np*, in *AAANpqNqr* they are *q* and *Nq*, etc.). Hence each conjunct is a tautology; therefore the whole formula is a tautology and the original formula is also a tautology. The argument expressed by the original formula (hypothetic syllogism) is therefore valid.

B. Alternational Normal Form

A formula is in alternational normal form if it is of the form *AA*. . .*pqr*. . ., where each of the alternants (*p*, *q*, *r*, . . .) is either

(a) a simple formula or the negation of a simple formula (e. g. *p*, *q*, *Np*, *Nq*), or

(b) a conjunction, being a conjunction of simple formulae or a conjunction of the negations of simple formulae or a conjunction consisting of simple formulae and the negation of simple formulae (e. g. *Kpq*, *KKNpNqNr*, *KpNr*, *KKKpqNrs*).

Thus in alternational normal form, just as in conjunctive normal form, if *N* appears at all, it may only appear before a simple formula, never before an operator. Hence the following are not in alternational normal form: *NApq*, *NKpq*, *AKpqKrAqNp*.

To derive alternational normal form, proceed as follows:

(1) Eliminate all operators other than *A*, *K*, or *N*, as was done for conjunctive normal form.
(2) Apply De Morgan laws by replacing *NKpq* or *NApq* wherever they occur by *ANpNq* or *KNpNq* respectively.
(3) Eliminate all pairs of consecutive *N* signs wherever occurring by virtue of the law of double negation.
(4) Apply the equivalences of association, commutation, and distribution as necessary in order that all alternants are either simple formulae, the negations of simple formulae, or conjunctions as permitted above.
(5) Apply the law of autology to eliminate repetition.
(6) Eliminate dyslogous conjunctions.

If, after the above instructions have been followed, either everything is eliminated or the alternation has only dyslogous alternants, the original formula is a dyslogy. If the alternation has any two alternants being a simple formula and the negation of that formula, the whole formula is a tautology and the original formula is a tautology; the argument, if any, represented by the original formula is therefore valid.

If, however, the formula cannot be thus identified as being either a dyslogy or a tautology, each alternant which is not a conjunction in which every simple formula or its negation appears is to be expanded so that such alternant does include every simple formula or its negation. For this purpose, the equivalence of p *with* AKpqKpNq *is used so that, for instance, if an alternant consisted only of* Np *where* p *and* q *were both simple formulae within the inspected formula,* Np *would be replaced by* AKNpqKNpNq. *After this has been done throughout the formula, the laws of distribution, association, and commutation are again applied until the formula is again in alternational normal form. It is then inspected and it is a tautology if and only if every amphilogous conjunction of all the simple formulae in the inspected formula is present among the alternants of the formula. If every amphilogous conjunction of all the simple formulae in the inspected formula is not present among the alternants of the formula, the formula is not a tautology and the argument, if any, represented by it is invalid.*

It is to be noted that for *p* and *q*, the complete list of amphilogous conjunctions is *Kpq*, *KpNq*, *KNpq*, *KNpNq* (or the equivalents of these by commutation); for *p*, *q*, and *r*, the complete list is *KKpqr*, *KKpqNr*, *KKpNqr*, *KKpNqNr*, *KKNpqr*, *KKNpqNr*, *KKNpNqr*, *KKNpNqNr*.

Examples

Determine by the alternational normal form method whether *DNpNJNpq* is a tautology.

1.	*DNpNJNpq*	
2.	*DNpNNKNpq*	Rule (1)
3.	*DNpKNpq*	D. N.
4.	*ANpNKNpq*	Rule (1)
5.	*ANpANNpNq*	De M.
6.	*ANpApNq*	D. N.
7.	*AApNpNq*	Assoc. and Comm.

On inspecting this final formula, it proves that both a simple formula and its negation appear as alternants; hence the formula is a tautology.

Determine by the alternational normal form method whether *CKCpqpq* is a tautology.

1.	*CKCpqpq*	
2.	*ANKCpqpq*	Rule (1)
3.	*ANKANpqpq*	Rule (1)
4.	*AANANpqNpq*	De M.
5.	*AAKNNpNqNpq*	De M.
6.	*AAKpNqNpq*	D. N.

This last formula is in alternational normal form, but the three alternants: *KpNq*, *Np*, and *q* are not all dyslogies, nor is any one of them a tautology. Therefore *Np* is to be replaced by *AKNpqKNpNq* and *q* by *AKqpKqNp*.

The decision-procedure continues as follows:

7.	*AAKpNqAKNpqKNpNqAKqpKqNp*	
8.	*AAKpNqAKNpqKNpNqAKpqKNpq*	Comm.
9.	*AAAAKpqKpNqKNpqKNpqKNpNq*	Assoc.
10.	*AAAKpqKpNqKNpqKNpNq*	Rule (5)

On inspecting this final formula, it proves that every amphilogous conjunction of *p* and *q* is present among the alternants of the formula. Hence the formula is a tautology and therefore the original formula is a tautology.

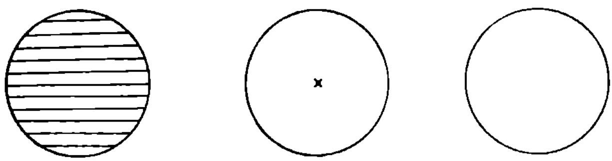
Appendix D

Vennian Diagrams

A diagrammatic method devised by John Venn has proved most helpful for ascertaining the validity of inferences operating with class concepts. It has therefore important application in the term-logical inferences of traditional logic. It is also applicable in extensional calculus of modern logic for ascertaining the validity of extensional derivations. Usually the application of the method proceeds from the assumption that universal propositions do not have existential import. This assumption is not one on which traditional logic as *traditionally conceived* is based. Nevertheless the central idea of Venn diagrams is valuable for providing a technique which would accord with the conception of traditional logic that all categoric propositions do have existential import. This technique may be called "*Vennian diagrams*". The following is a comparative exposition of Venn diagrams and Vennian diagrams.

The basic diagrams in both Venn and Vennian approaches are circles. To show that a class has no members, i. e. is void (empty), the relevant area is shaded out. To show that a class has at least one member, i. e. is filled (not empty), a cross is placed in the relevant area. A blank space (neither shaded out nor with a cross in it) indicates that no claim is made as to whether the class does or does not have members.

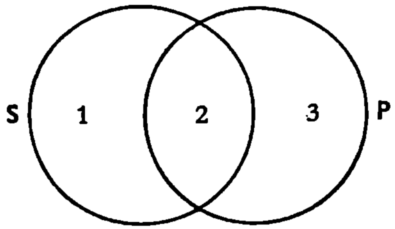
Diagram I



$S = O$ $S \neq O$ S?

To express class relations, both methods employ intersecting circles, in which the compartments arising from intersection are marked by arabic numerals.

Diagram II



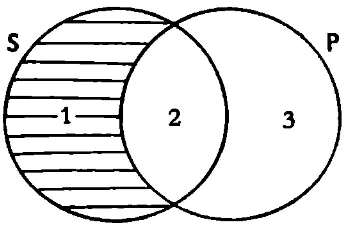
1: Class "S and *non*-P"
2: Class "S and P"
3: Class "*non*-S and P"

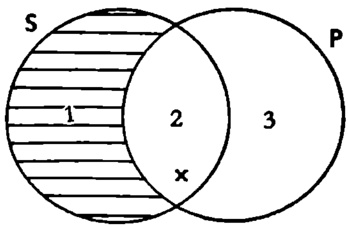
Since Vennian diagrams postulate that each class in a categoric proposition has at least one member, thereby giving existential import not only to particular propositions (which Venn diagrams do) but also to universal propositions (which Venn diagrams do not), they are capable of validating certain immediate as well as mediate (syllogistic) inferences which proved to be "invalid" by the use of Venn diagrams but have been accepted as valid in traditional expositions of traditional logic.

In the following, for each propositional form of traditional logic the Venn diagram is given first and then the Vennian diagram, highlighting their differences.

Diagram III

Venn S *a* P Vennian



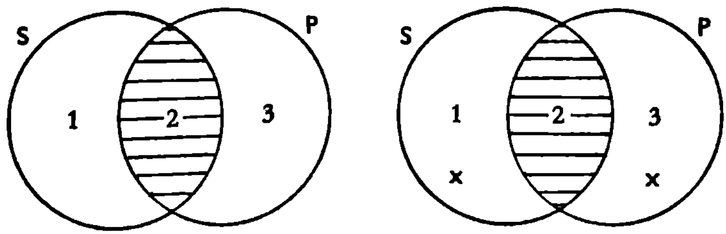


The Vennian diagram differs by virtue of the "x" in compartment 2. The cross is placed in this compartment to show that the class S in S *a* P has at least one member. To show that the class P has also at least one member, no further cross is necessary, for the cross in compartment 2 falls within the class P and is sufficient to show that P, too, has at least one member. Indeed, it would be impermissible to place a cross in compartment 3 to indicate that P has at least one member; for to do so would carry the further implication that there is at least one member of P which is outside the class S and nothing in the premiss S *a* P warrants this. A further proposition (P *o* S) would be required to justify this.

The advantage of the Vennian diagram here is that it permits the immediate inferences "S *a* P, *therefore* P *i* S" (conversion by limitation) and "S *a* P, *therefore* S *i* P" (subalternation). On the Venn diagram above, although the premiss has been diagrammed, neither conclusion appears and hence the inference is "invalid". In contrast, the conclusions do appear on the Vennian diagram by virtue of the cross in compartment 2.

Diagram IV

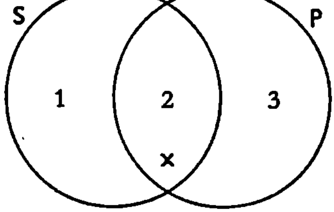
Venn S *e* P Vennian

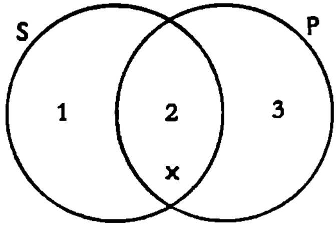


The Vennian diagram differs by virtue of the two crosses, one in the range of S (compartment 1) and one in the range of P (compartment 3), thus showing that S and P both have members. The advantage of this is that it permits the immediate inferences "S *e* P, *therefore* S *o* P" (subalternation) and "S *e* P, *therefore* P *o* S" (conversion and subalternation), whereas these prove "invalid" by reference to the Venn diagram.

Diagram V

Venn S *i* P Vennian

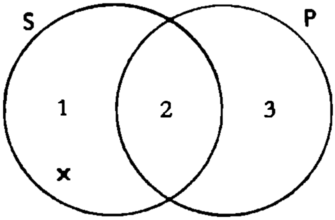


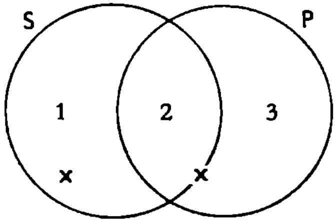


The diagrams here are identical; the cross appears in compartment 2 of both diagrams, indicating that S as well as P have members.

Diagram VI

Venn S *o* P Vennian



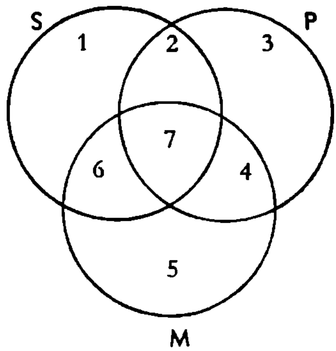


The Vennian diagram differs by virtue of the cross on the border between compartments 2 and 3. It is necessary to show that class P has at least one member and this is indicated by placing a cross inside the range of P. The problem is in which compartment to place the cross (note that S *o* P has no converse). If a cross is placed in compartment 2, the affirmation "S *i* P *or* P *i* S" is made, which is not warranted from S *o* P; in addition, it would make the inference "S *o* P, *therefore* S *i* P" valid, thus affecting the meaning of "*some*" as conceived in traditional logic. If a cross is placed in compartment 3, the inference "S *o* P, *therefore* P *o* S" would appear as valid, thus giving S *o* P a converse, which traditional logic does not admit. If a cross is placed in both compartment 2 and 3, both the foregoing difficulties arise. However, even though

P *i* S cannot be affirmed singly and P *o* S cannot be affirmed singly, the disjunction "P *i* S *or* P *o* S" can be affirmed, since they are subcontraries and cannot both be false. This disjunction is represented by placing the cross on the border between compartments 2 and 3. It is to be noted that this disjunction does not appear on the Venn diagram.

For representing syllogistic inferences, both the Venn and Vennian diagrams employ three intersecting circles.

Diagram VII



1: Class S *non*-P *non*-M
2: Class S P *non*-M
3: Class *non*-S P *non*-M
4: Class *non*-S P M
5: Class *non*-S *non*-P M
6: Class S *non*-P M
7: Class S P M

The following samples of syllogistic inferences will highlight the differences between the Venn and Vennian approaches:

First Figure

Modus Barbara	*Modus Barbari*
M *a* P	M *a* P
S *a* M	S *a* M
S *a* P	S *i* P

Diagram VIII (Venn)

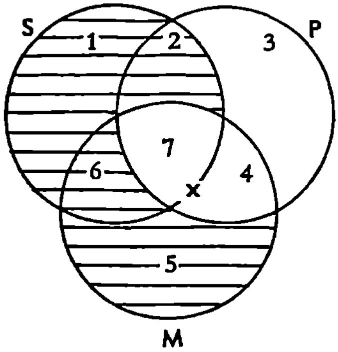
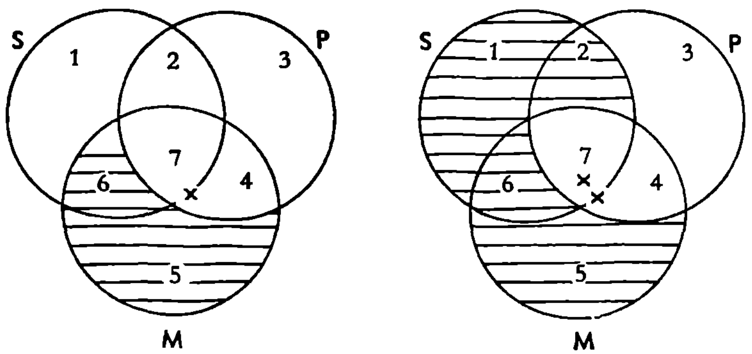


Diagram IX (Vennian)



Stage I: Premiss M *a* P

Stage II: Premiss S *a* M

Note that *Modus Barbara* appears as valid in both diagrams, whereas *Modus Barbari* appears valid only in the Vennian diagram. Note also that nothing hinges on the cross on the border between compartments 4 and 7 in the Vennian diagram; the other cross is sufficient to indicate that all classes involved in the inference have members. In the first stage of the

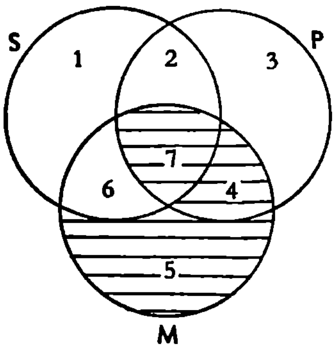
Vennian diagram, the major premiss (M *a* P) is expressed by shading out areas 5 and 6. In the second stage of this diagram, built on the first stage, the minor premiss is expressed by shading out areas 1 and 2.

Third Figure

Modus Felapton

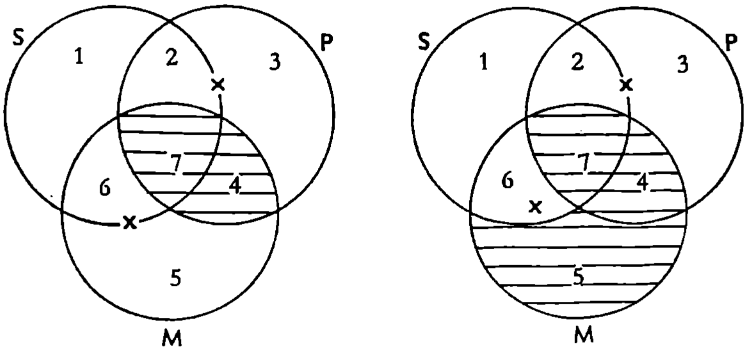
M *e* P
M *a* S
S *o* P

Diagram X (Venn)



This inference appears as invalid in the above Venn diagram. In the following Vennian diagram it appears as valid. Note that in the latter the effect of the minor premiss is to "push" the cross, which was previously placed on the border between areas 5 and 6 (because the major premiss leaves undecided to which compartment it belongs) into area 6 (because according to the minor premiss area 5 represents a void class and therefore the cross cannot be placed there).

Diagram XI (Vennian)



Stage I: Premiss M *e* P

Stage II: Premiss S *a* P

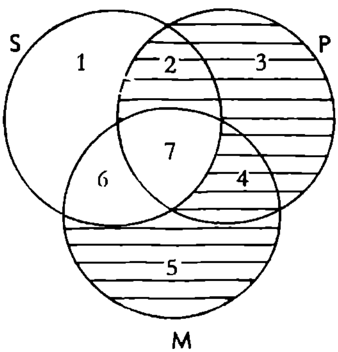
Fourth Figure

Modus Bramantip

P *a* M
M *a* S

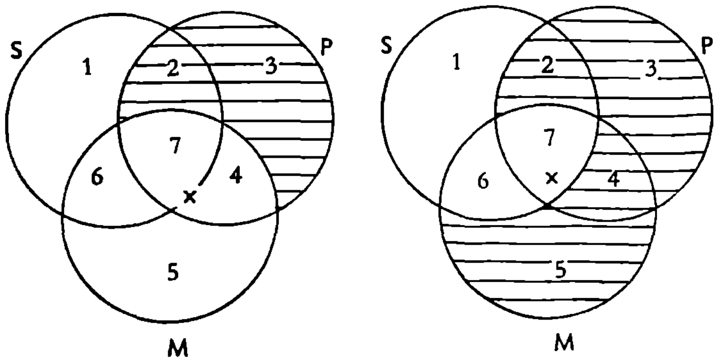
S *i* P

Diagram XII (Venn)



This inference appears as invalid in the above Venn diagram. In the following Vennian diagram it appears as valid. Note that in the latter (second stage) the cross is "pushed" from the border between areas 4 and 7 into compartment 7.

Diagram XIII (Vennian)



Stage I: Premiss P *a* M Stage II: Premiss M *a* S

The position of the crosses in Vennian diagrams is important and a cross may sometimes have to "straddle" three or even four compartments. For example, in diagramming *Modus Bocardo*, the cross to show that P has members would have to straddle compartments 2, 3, and 4, since the members of P could fall within any one of these.

An alternative method is to use dashes instead of crosses. This is quite feasible. One difficulty that can arise here is where four compartments have to be straddled. With a cross this can be accomplished, but it requires more than one dash.

Appendix E

A Method of Eliminating Hypotact-Variables from Predicational Calculus

In chapter II, section 4, where techniques of predicational calculus were explained, $x, y, z, \ldots$ were used to represent hypotact-variables and $F, G, H, \ldots$ to represent predicators. Propositions such as "*All* trespassers *are* tortfeasors", "*No* licencees *are* trespassers" and "*Some* minors *are not* juvenile deliquents" were symbolised (with a caveat irrelevant for the present purpose) as $\Pi xCFxGx$, $\Pi xCFxNGx$, and $\Sigma xKFxNGx$ respectively. In addition, more complex examples were given requiring the use of more than one hypotact-variable sign.

This appendix indicates a method of eliminating hypotact-variable signs from those formulae which do not involve more than one hypotact-variable (such as those examples listed above). Such formulae can be symbolised without any hypotact sign at all. For example, the above formulae would appear as ΠCfg, $\Pi CfNg$, and $\Sigma KfNg$, in which lower case letters are used to represent predicators.

The purpose that these expressions would serve would be identical to that served by formulae containing a hypotact-variable sign. They are merely less cumbersome and can be regarded as elliptical expressions of the latter. In translating such elliptical expressions into ordinary language, Π may be rendered as "for all instances" and Σ as "for some instances". Thus ΠCfg could read "for all instances, *if* a trespasser *then* a tortfeasor", i.e. all trespassers are tortfeasors.

Set out below are some examples showing how ordinary language can be symbolised using this simplified notation and also what the corresponding symbolisation in the ordinary notation of predicational calculus would be.

(1) Some contracts may be void for uncertainty.
f: "contract", *g*: "may be void for uncertainty"

Unabbreviated Formula	Abbreviated Formula
$\Sigma xKFxGx$	ΣKfg

(2) Equity judges may award damages instead of granting an injunction.
f: "equity judge", *g*: "person who may award damages instead of granting an injunction"

Unabbreviated Formula	Abbreviated Formula
$\Pi xCFxGx$	ΠCfg

(3) If a will is valid, it must be signed by the testator and two witnesses.
f: "will", *g*: "valid", *h*: "must be signed by the testator and two witnesses"

Unabbreviated Formula	Abbreviated Formula
$\Pi xCKFxGxHx$	$\Pi CKfgh$

(4) A contract has consideration or it is not binding.
f: "contract", *g*: "has consideration", *h*: "is binding"

Unabbreviated Formula	Abbreviated Formula
ΠxCFxAGxNHx	*ΠCfAgNh*

(5) Some marriages are voidable and some are not.
f: "marriage", *g*: "voidable"

Unabbreviated Formula	Abbreviated Formula
KΣxKFxGxΣxKFxNGx	*KΣKfgΣKfNg*

Appendix F

Notations of Modern Logic

The notation employed in this Compendium to express the formulae of modern logic is based on a system devised by Jan Łukasiewicz for propositional calculus, but differs from it in that the symbols *D*, *I*, *J*, and *O* have a different meaning. The essential differences of notations used by other logicians are that they employ special symbols for the operators rather than capital letters and that the dyadic operators are placed between the elements governed instead of in front of the elements. One of the most commonly used of such systems is that employed by Alfred North Whitehead and Bertrand Russell in their *Principia Mathematica* (vol. I, 1910). A comparison of their notation and this Compendium's notation is set out below so that the reader will have some guidance when faced with works employing other notations.

Compendium	*Principia Mathematica*
Np	$\sim p$
Apq	$p \vee q$
Cpq	$p \supset q$
Dpq	Nil, but $p \vee \sim q$ can be used
Epq	$p \equiv q$
Ipq	Nil, but $\sim p \cdot \sim q$ can be used
Jpq	Nil, but $\sim p \vee \sim q$ can be used
Kpq	$p \cdot q$
Opq	Nil, but $p \equiv \sim q$ can be used
Πx	(x)
Σx	$(\exists x)$

Variations of the *Principia Mathematica* notation are quite common, some of the more usual alternatives being: $p \,\&\, q$ for $p \cdot q$, $p \rightarrow q$ for $p \supset q$, and $p \leftrightarrow q$ for $p \equiv q$. A notable variation often used for negation is the writing of Np as $\overline{p}$ and expressions such as $NKpq$ and $NApCpq$ as $\overline{p \,\&\, q}$ and $\overline{p \vee (p \rightarrow q)}$ respectively. It is to be noted that a bar above a formula indicating that it is negated can also advantageously be employed in the Polish notation adopted in this Compendium. Thus $NApNq$ can be rendered as $\overline{Ap\overline{q}}$.

Acquaintance with such different notations is an asset when reading works on modern logic. The main advantages of the notation of this Compendium are that it dispenses with the need for brackets, which in other notations can become excessive, that it can be expressed by symbols available on ordinary typewriters (certain special symbols can be so expressed by using a little ingenuity), and that concise logical expressions are possible in it without proliferation of specific symbols. Its main disadvantages are that logical structure, especially of complex formulae, is often difficult to discern and that translation of formulae into ordinary language is not as direct as it otherwise could be.

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An excellent expedient for learning the handling of the formulae expressed in Polish notation and for acquiring techniques of formal proof is Layman E. Allen's logic game *WFF'n Proof* (originally issued in 1962).

Exercises in Logic? p.14

Traditional logic

(cf. Strawson)
+ Quine

Theory of logic.

P.37

P.38 cf. 'language-game'

The tautological calculus too abstract for an introduction.
Is there a language more formal than common language?
An uninterpreted sign formally rules formulated in language. Which rules do we formulate?

53-54 – the laws' usage = summary reminders to those with some knowledge of math logic, useless to others.

Then when the lessons learned, even the transformations symbols itself becomes formalism.

64. Deontic-formations? (p. 79, 87 – explanation)

75ff. Propositions formulations communicate the meaning more simply. (contrast the simplicity of symbolism in maths.)
Where the variables can be expressed, no proof needed.
Where not easily recog. – proof construction more difficult – (a) ambiguities etc. are to be resolved before symbolization possible
(ii) No common method of proof construction. eg. in maths.

87 – Imperative & statements

88 – 'may' – signs.

1–92 Closed & open legal systems.

16 = higher & lower norms.

85 – Laws & exhortations

Logic / the Ezra – p. 10.
Law formalizes, but the ... has to be interpreted to ... appear
(1) Meaning
(2) Value
Where semantic considerations predominate logical formalization of limited value.

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